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Amicable Numbers With Patterns in Products and Powers

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Abstract

*There are many ways of writing amicable numbers. One with divisions and sums. The other with pair of powers of each other. There is another way to represent is in product. In this paper, we brings **amicable numbers in pairs** in terms of products and powers. The idea of **self-amicable** is also introduced. Few blokcs of **symmetrical amicable numbers** multiples of 10 are also given. Some interesting patterns among amicable numbers are also given.*

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1 Introduction

In the history, there are numbers known by "Amicable numbers" (see Madachy [3], p. 155). There are many different ways of expressing these numbers. Most famous among them is with operation of addition, such as 220 and 284. In this case the summing the divisors of one we get another number. See below:

Divisors of 284 : 1, 2, 4, 71 and 142

Sum : $1+2+4+71+142 := 220$

Divisors of 220 : 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110

Sum : $1+2+4+5+10+11+20+22+44+55+110 := 284$.

More studies on this type of numbers can be seen in [3, 4, 5, 6].

The other type of **amicable numbers in pairs** (ref. Madachy [3], p. 165-167) are in terms of squares of each others, for examples,

$$3869 := 62^2 + 05^2 \Leftrightarrow 6205 := 38^2 + 69^2$$

$$5965 := 77^2 + 06^2 \Leftrightarrow 7706 := 59^2 + 65^2.$$

... (1)

Instead of squares of each others, it may happen with same numbers too, for example,

$$1233 := 12^2 + 33^2$$

$$990100 := 990^2 + 100^2.$$

... (2)

It is not necessary that it happens only with addition, we may have results with subtraction, such as,

$$48 := -4^2 + 8^2$$

$$3468 := -34^2 + 68^2$$

$$416768 := -416^2 + 768^2.$$

... (3)

More on numbers given in (1), (2) and (3) can be seen in Madachy [3], p. 165-167. Also refer Heinz [2].

Let's write the numbers given in (1) as **amicable in pairs**, and given in (2) and (3) as **self-amicable** numbers.

The examples given in (1), (2) and (3) are with addition and potentiation. Let's write below some examples with multiplications:

$$168 := 19 \times 8 + 2 \times 8 \Leftrightarrow 192 := 16 \times 8 + 8 \times 8$$

$$248 := 25 \times 8 + 6 \times 8 \Leftrightarrow 256 := 24 \times 8 + 8 \times 8$$

$$\begin{aligned}
 63 &:= 8 \times 9 - 1 \times 9 \quad \Leftrightarrow \quad 81 := 6 \times 9 + 3 \times 9 \\
 1267 &:= 187 \times 7 - 6 \times 7 \quad \Leftrightarrow \quad 1876 := 1 \times 7 + 267 \times 7 \\
 &\dots \tag{4}
 \end{aligned}$$

$$\begin{aligned}
 144 &:= 14 \times 8 + 4 \times 8 \\
 2664 &:= 2 \times 4 + 664 \times 4 \\
 13332 &:= 1 \times 4 + 3332 \times 4. \\
 &\dots \tag{5}
 \end{aligned}$$

The idea of patterns in amicable numbers is not known in the literature. See below some examples **patterns in pairs of amicable numbers** and **self-amicable numbers**:

$$\begin{aligned}
 165 &:= 33 \times 5 + 0 \times 5 &= 3 \times 5 + 30 \times 5 &\Leftrightarrow & 330 := 1 \times 5 + 65 \times 5 \\
 1665 &:= 333 \times 5 + 0 \times 5 &= 3 \times 5 + 330 \times 5 &\Leftrightarrow & 3330 := 1 \times 5 + 665 \times 5 \\
 16665 &:= 3333 \times 5 + 0 \times 5 &= 3 \times 5 + 3330 \times 5 &\Leftrightarrow & 33330 := 1 \times 5 + 6665 \times 5 \\
 166665 &:= 33333 \times 5 + 0 \times 5 &= 3 \times 5 + 33330 \times 5 &\Leftrightarrow & 333330 := 1 \times 5 + 66665 \times 5 \\
 1666665 &:= 333333 \times 5 + 0 \times 5 &= 3 \times 5 + 333330 \times 5 &\Leftrightarrow & 3333330 := 1 \times 5 + 666665 \times 5
 \end{aligned}$$

$$\begin{aligned}
 48 &:= -4^2 + 8^2 & 140400 &:= -140^2 + 400^2 \\
 484848 &:= -484^2 + 848^2 & 14040000 &:= -1400^2 + 4000^2 \\
 4848484848 &:= -48484^2 + 84848^2 & 1404000000 &:= -14000^2 + 40000^2 \\
 48484848484848 &:= -4848484^2 + 8484848^2 & 140400000000 &:= -140000^2 + 400000^2.
 \end{aligned}$$

The numbers given in (4) and (5) are understood as **amicable in pairs** and **self-amicable numbers** in product. The difference is that the numbers given in (1)-(4) are with **power** while in (5) and (6) are with **product**.

The aim of this work is to bring the numbers of type (1)-(5) with positive and negative signs. Some interesting patterns arising due to these numbers are also given. The **self-amicable numbers** are very much similar to **selfie numbers**, **semi-selfie numbers**, **narcissistic type numbers**, etc. For studies on these numbers refer author’s work [7]-[17].

2 Product-Type Amicable Numbers

As explained in examples (4) and (5) regarding amicable numbers in pairs and self-amicable numbers with the idea of product. This section bring these kind of numbers by using the idea of product. There are two situations, one in pairs and another as self. In the both the cases, there are numbers with only positive coefficients, and with positive and negative coefficients.

2.1 Amicable Numbers in Pairs

Below are examples of **amicable numbers in pairs**. These are divided in two parts. One with positive coefficients and another with positive and negative coefficients.

2.1.1 Positive Coefficients

$$\begin{array}{ll}
 15 := 3 \times 5 + 0 \times 5 & \Leftrightarrow 30 := 1 \times 5 + 5 \times 5 \\
 32 := 4 \times 8 + 0 \times 8 & \Leftrightarrow 40 := 3 \times 8 + 2 \times 8 \\
 96 := 12 \times 8 + 0 \times 8 & \Leftrightarrow 120 := 9 \times 8 + 6 \times 8 \\
 \\
 104 := 11 \times 8 + 2 \times 8 & \Leftrightarrow 112 := 10 \times 8 + 4 \times 8 \\
 104 := 1 \times 8 + 12 \times 8 & \Leftrightarrow 112 := 10 \times 8 + 4 \times 8 \\
 128 := 16 \times 8 + 0 \times 8 & \Leftrightarrow 160 := 12 \times 8 + 8 \times 8 \\
 136 := 15 \times 8 + 2 \times 8 & \Leftrightarrow 152 := 13 \times 8 + 6 \times 8 \\
 150 := 25 \times 5 + 5 \times 5 & \Leftrightarrow 255 := 1 \times 5 + 50 \times 5 \\
 160 := 12 \times 8 + 8 \times 8 & \Leftrightarrow 128 := 16 \times 8 + 0 \times 8 \\
 165 := 33 \times 5 + 0 \times 5 & \Leftrightarrow 330 := 1 \times 5 + 65 \times 5 \\
 165 := 3 \times 5 + 30 \times 5 & \Leftrightarrow 330 := 1 \times 5 + 65 \times 5 \\
 168 := 19 \times 8 + 2 \times 8 & \Leftrightarrow 192 := 16 \times 8 + 8 \times 8 \\
 176 := 18 \times 8 + 4 \times 8 & \Leftrightarrow 184 := 17 \times 8 + 6 \times 8 \\
 176 := 6 \times 8 + 16 \times 8 & \Leftrightarrow 616 := 1 \times 8 + 76 \times 8 \\
 208 := 22 \times 8 + 4 \times 8 & \Leftrightarrow 224 := 20 \times 8 + 8 \times 8 \\
 208 := 2 \times 8 + 24 \times 8 & \Leftrightarrow 224 := 20 \times 8 + 8 \times 8 \\
 232 := 27 \times 8 + 2 \times 8 & \Leftrightarrow 272 := 2 \times 8 + 32 \times 8 \\
 248 := 25 \times 8 + 6 \times 8 & \Leftrightarrow 256 := 24 \times 8 + 8 \times 8 \\
 264 := 5 \times 8 + 28 \times 8 & \Leftrightarrow 528 := 2 \times 8 + 64 \times 8 \\
 270 := 43 \times 6 + 2 \times 6 & \Leftrightarrow 432 := 2 \times 6 + 70 \times 6 \\
 344 := 37 \times 8 + 6 \times 8 & \Leftrightarrow 376 := 3 \times 8 + 44 \times 8 \\
 352 := 44 \times 8 + 0 \times 8 & \Leftrightarrow 440 := 3 \times 8 + 52 \times 8 \\
 352 := 4 \times 8 + 40 \times 8 & \Leftrightarrow 440 := 3 \times 8 + 52 \times 8 \\
 376 := 3 \times 8 + 44 \times 8 & \Leftrightarrow 344 := 37 \times 8 + 6 \times 8 \\
 464 := 54 \times 8 + 4 \times 8 & \Leftrightarrow 544 := 4 \times 8 + 64 \times 8 \\
 483 := 60 \times 7 + 9 \times 7 & \Leftrightarrow 609 := 4 \times 7 + 83 \times 7 \\
 576 := 64 \times 8 + 8 \times 8 & \Leftrightarrow 648 := 5 \times 8 + 76 \times 8 \\
 584 := 71 \times 8 + 2 \times 8 & \Leftrightarrow 712 := 5 \times 8 + 84 \times 8 \\
 696 := 81 \times 8 + 6 \times 8 & \Leftrightarrow 816 := 6 \times 8 + 96 \times 8 \\
 \\
 1134 := 121 \times 9 + 5 \times 9 & \Leftrightarrow 1215 := 1 \times 9 + 134 \times 9 \\
 1176 := 141 \times 8 + 6 \times 8 & \Leftrightarrow 1416 := 1 \times 8 + 176 \times 8 \\
 1184 := 148 \times 8 + 0 \times 8 & \Leftrightarrow 1480 := 1 \times 8 + 184 \times 8 \\
 1650 := 325 \times 5 + 5 \times 5 & \Leftrightarrow 3255 := 1 \times 5 + 650 \times 5 \\
 1665 := 333 \times 5 + 0 \times 5 & \Leftrightarrow 3330 := 1 \times 5 + 665 \times 5 \\
 1665 := 3 \times 5 + 330 \times 5 & \Leftrightarrow 3330 := 1 \times 5 + 665 \times 5 \\
 1776 := 6 \times 8 + 216 \times 8 & \Leftrightarrow 6216 := 1 \times 8 + 776 \times 8 \\
 2360 := 289 \times 8 + 6 \times 8 & \Leftrightarrow 2896 := 2 \times 8 + 360 \times 8
 \end{array}$$

$$\begin{aligned}
2368 &:= 296 \times 8 + 0 \times 8 & \Leftrightarrow & 2960 := 2 \times 8 + 368 \times 8 \\
2499 &:= 350 \times 7 + 7 \times 7 & \Leftrightarrow & 3507 := 2 \times 7 + 499 \times 7 \\
2664 &:= 5 \times 8 + 328 \times 8 & \Leftrightarrow & 5328 := 2 \times 8 + 664 \times 8 \\
2754 &:= 453 \times 6 + 6 \times 6 & \Leftrightarrow & 4536 := 2 \times 6 + 754 \times 6 \\
3544 &:= 437 \times 8 + 6 \times 8 & \Leftrightarrow & 4376 := 3 \times 8 + 544 \times 8 \\
3552 &:= 444 \times 8 + 0 \times 8 & \Leftrightarrow & 4440 := 3 \times 8 + 552 \times 8 \\
3552 &:= 4 \times 8 + 440 \times 8 & \Leftrightarrow & 4440 := 3 \times 8 + 552 \times 8 \\
3759 &:= 533 \times 7 + 4 \times 7 & \Leftrightarrow & 5334 := 3 \times 7 + 759 \times 7 \\
4728 &:= 585 \times 8 + 6 \times 8 & \Leftrightarrow & 5856 := 4 \times 8 + 728 \times 8 \\
4736 &:= 592 \times 8 + 0 \times 8 & \Leftrightarrow & 5920 := 4 \times 8 + 736 \times 8 \\
5912 &:= 733 \times 8 + 6 \times 8 & \Leftrightarrow & 7336 := 5 \times 8 + 912 \times 8 \\
5920 &:= 740 \times 8 + 0 \times 8 & \Leftrightarrow & 7400 := 5 \times 8 + 920 \times 8
\end{aligned}$$

$$\begin{aligned}
11840 &:= 1472 \times 8 + 8 \times 8 & \Leftrightarrow & 14728 := 1 \times 8 + 1840 \times 8 \\
11848 &:= 1479 \times 8 + 2 \times 8 & \Leftrightarrow & 14792 := 1 \times 8 + 1848 \times 8 \\
12558 &:= 1791 \times 7 + 3 \times 7 & \Leftrightarrow & 17913 := 1 \times 7 + 2558 \times 7 \\
16650 &:= 3325 \times 5 + 5 \times 5 & \Leftrightarrow & 33255 := 1 \times 5 + 6650 \times 5 \\
16665 &:= 3333 \times 5 + 0 \times 5 & \Leftrightarrow & 33330 := 1 \times 5 + 6665 \times 5 \\
16665 &:= 3 \times 5 + 3330 \times 5 & \Leftrightarrow & 33330 := 1 \times 5 + 6665 \times 5 \\
17776 &:= 6 \times 8 + 2216 \times 8 & \Leftrightarrow & 62216 := 1 \times 8 + 7776 \times 8 \\
23696 &:= 2958 \times 8 + 4 \times 8 & \Leftrightarrow & 29584 := 2 \times 8 + 3696 \times 8 \\
25116 &:= 3582 \times 7 + 6 \times 7 & \Leftrightarrow & 35826 := 2 \times 7 + 5116 \times 7 \\
26664 &:= 5 \times 8 + 3328 \times 8 & \Leftrightarrow & 53328 := 2 \times 8 + 6664 \times 8 \\
35544 &:= 4437 \times 8 + 6 \times 8 & \Leftrightarrow & 44376 := 3 \times 8 + 5544 \times 8 \\
35552 &:= 4444 \times 8 + 0 \times 8 & \Leftrightarrow & 44440 := 3 \times 8 + 5552 \times 8 \\
35552 &:= 4 \times 8 + 4440 \times 8 & \Leftrightarrow & 44440 := 3 \times 8 + 5552 \times 8 \\
37674 &:= 5373 \times 7 + 9 \times 7 & \Leftrightarrow & 53739 := 3 \times 7 + 7674 \times 7 \\
47392 &:= 5916 \times 8 + 8 \times 8 & \Leftrightarrow & 59168 := 4 \times 8 + 7392 \times 8 \\
47400 &:= 5923 \times 8 + 2 \times 8 & \Leftrightarrow & 59232 := 4 \times 8 + 7400 \times 8 \\
59248 &:= 7402 \times 8 + 4 \times 8 & \Leftrightarrow & 74024 := 5 \times 8 + 9248 \times 8
\end{aligned}$$

$$\begin{aligned}
118512 &:= 14810 \times 8 + 4 \times 8 & \Leftrightarrow & 148104 := 1 \times 8 + 18512 \times 8 \\
138456 &:= 23074 \times 6 + 2 \times 6 & \Leftrightarrow & 230742 := 1 \times 6 + 38456 \times 6 \\
166650 &:= 33325 \times 5 + 5 \times 5 & \Leftrightarrow & 333255 := 1 \times 5 + 66650 \times 5 \\
166665 &:= 33333 \times 5 + 0 \times 5 & \Leftrightarrow & 333330 := 1 \times 5 + 66665 \times 5 \\
166665 &:= 3 \times 5 + 33330 \times 5 & \Leftrightarrow & 333330 := 1 \times 5 + 66665 \times 5 \\
177776 &:= 6 \times 8 + 22216 \times 8 & \Leftrightarrow & 622216 := 1 \times 8 + 77776 \times 8 \\
237024 &:= 29620 \times 8 + 8 \times 8 & \Leftrightarrow & 296208 := 2 \times 8 + 37024 \times 8 \\
237032 &:= 29627 \times 8 + 2 \times 8 & \Leftrightarrow & 296272 := 2 \times 8 + 37032 \times 8 \\
251265 &:= 35886 \times 7 + 9 \times 7 & \Leftrightarrow & 358869 := 2 \times 7 + 51265 \times 7
\end{aligned}$$

$$\begin{aligned}
266664 &:= 5 \times 8 + 33328 \times 8 \Leftrightarrow 533328 := 2 \times 8 + 66664 \times 8 \\
276912 &:= 46148 \times 6 + 4 \times 6 \Leftrightarrow 461484 := 2 \times 6 + 76912 \times 6 \\
355544 &:= 44437 \times 8 + 6 \times 8 \Leftrightarrow 444376 := 3 \times 8 + 55544 \times 8 \\
355552 &:= 44444 \times 8 + 0 \times 8 \Leftrightarrow 444440 := 3 \times 8 + 55552 \times 8 \\
355552 &:= 4 \times 8 + 44440 \times 8 \Leftrightarrow 444440 := 3 \times 8 + 55552 \times 8 \\
376908 &:= 53837 \times 7 + 7 \times 7 \Leftrightarrow 538377 := 3 \times 7 + 76908 \times 7 \\
474064 &:= 59254 \times 8 + 4 \times 8 \Leftrightarrow 592544 := 4 \times 8 + 74064 \times 8 \\
592576 &:= 74064 \times 8 + 8 \times 8 \Leftrightarrow 740648 := 5 \times 8 + 92576 \times 8 \\
592584 &:= 74071 \times 8 + 2 \times 8 \Leftrightarrow 740712 := 5 \times 8 + 92584 \times 8 \\
798579 &:= 88727 \times 9 + 4 \times 9 \Leftrightarrow 887274 := 7 \times 9 + 98579 \times 9
\end{aligned}$$

$$\begin{aligned}
1185176 &:= 148141 \times 8 + 6 \times 8 \Leftrightarrow 1481416 := 1 \times 8 + 185176 \times 8 \\
1185184 &:= 148148 \times 8 + 0 \times 8 \Leftrightarrow 1481480 := 1 \times 8 + 185184 \times 8 \\
1256409 &:= 179487 \times 7 + 0 \times 7 \Leftrightarrow 1794870 := 1 \times 7 + 256409 \times 7 \\
1269840 &:= 2 \times 8 + 158728 \times 8 \Leftrightarrow 2158728 := 1 \times 8 + 269840 \times 8 \\
1333332 &:= 1 \times 4 + 333332 \times 4 \Leftrightarrow 1333332 := 1 \times 4 + 333332 \times 4 \\
1384614 &:= 230769 \times 6 + 0 \times 6 \Leftrightarrow 2307690 := 1 \times 6 + 384614 \times 6 \\
1396824 &:= 3 \times 8 + 174600 \times 8 \Leftrightarrow 3174600 := 1 \times 8 + 396824 \times 8 \\
1523808 &:= 4 \times 8 + 190472 \times 8 \Leftrightarrow 4190472 := 1 \times 8 + 523808 \times 8 \\
1650792 &:= 5 \times 8 + 206344 \times 8 \Leftrightarrow 5206344 := 1 \times 8 + 650792 \times 8 \\
1666650 &:= 333325 \times 5 + 5 \times 5 \Leftrightarrow 3333255 := 1 \times 5 + 666650 \times 5 \\
1666665 &:= 3 \times 5 + 333330 \times 5 \Leftrightarrow 3333330 := 1 \times 5 + 666665 \times 5 \\
1666665 &:= 333333 \times 5 + 0 \times 5 \Leftrightarrow 3333330 := 1 \times 5 + 666665 \times 5 \\
1714284 &:= 4 \times 6 + 285710 \times 6 \Leftrightarrow 4285710 := 1 \times 6 + 714284 \times 6 \\
1777776 &:= 6 \times 8 + 222216 \times 8 \Leftrightarrow 6222216 := 1 \times 8 + 777776 \times 8 \\
1904760 &:= 7 \times 8 + 238088 \times 8 \Leftrightarrow 7238088 := 1 \times 8 + 904760 \times 8 \\
2285712 &:= 2 \times 8 + 285712 \times 8 \Leftrightarrow 2285712 := 2 \times 8 + 285712 \times 8 \\
2370360 &:= 296289 \times 8 + 6 \times 8 \Leftrightarrow 2962896 := 2 \times 8 + 370360 \times 8 \\
2370368 &:= 296296 \times 8 + 0 \times 8 \Leftrightarrow 2962960 := 2 \times 8 + 370368 \times 8 \\
2412696 &:= 3 \times 8 + 301584 \times 8 \Leftrightarrow 3301584 := 2 \times 8 + 412696 \times 8 \\
2512818 &:= 358974 \times 7 + 0 \times 7 \Leftrightarrow 3589740 := 2 \times 7 + 512818 \times 7 \\
2539680 &:= 4 \times 8 + 317456 \times 8 \Leftrightarrow 4317456 := 2 \times 8 + 539680 \times 8 \\
2571426 &:= 3 \times 6 + 428568 \times 6 \Leftrightarrow 3428568 := 2 \times 6 + 571426 \times 6 \\
2666664 &:= 5 \times 8 + 333328 \times 8 \Leftrightarrow 5333328 := 2 \times 8 + 666664 \times 8 \\
2769228 &:= 461538 \times 6 + 0 \times 6 \Leftrightarrow 4615380 := 2 \times 6 + 769228 \times 6 \\
2793648 &:= 6 \times 8 + 349200 \times 8 \Leftrightarrow 6349200 := 2 \times 8 + 793648 \times 8 \\
2920632 &:= 7 \times 8 + 365072 \times 8 \Leftrightarrow 7365072 := 2 \times 8 + 920632 \times 8 \\
3428568 &:= 3 \times 8 + 428568 \times 8 \Leftrightarrow 3428568 := 3 \times 8 + 428568 \times 8 \\
3555544 &:= 444437 \times 8 + 6 \times 8 \Leftrightarrow 4444376 := 3 \times 8 + 555544 \times 8 \\
3555552 &:= 4 \times 8 + 444440 \times 8 \Leftrightarrow 4444440 := 3 \times 8 + 555552 \times 8
\end{aligned}$$

$$\begin{aligned}
3555552 &:= 444444 \times 8 + 0 \times 8 \Leftrightarrow 4444440 := 3 \times 8 + 555552 \times 8 \\
3682536 &:= 5 \times 8 + 460312 \times 8 \Leftrightarrow 5460312 := 3 \times 8 + 682536 \times 8 \\
3769227 &:= 538461 \times 7 + 0 \times 7 \Leftrightarrow 5384610 := 3 \times 7 + 769227 \times 7 \\
3809520 &:= 6 \times 8 + 476184 \times 8 \Leftrightarrow 6476184 := 3 \times 8 + 809520 \times 8 \\
3936504 &:= 7 \times 8 + 492056 \times 8 \Leftrightarrow 7492056 := 3 \times 8 + 936504 \times 8 \\
4698408 &:= 5 \times 8 + 587296 \times 8 \Leftrightarrow 5587296 := 4 \times 8 + 698408 \times 8 \\
4740728 &:= 592585 \times 8 + 6 \times 8 \Leftrightarrow 5925856 := 4 \times 8 + 740728 \times 8 \\
4740736 &:= 592592 \times 8 + 0 \times 8 \Leftrightarrow 5925920 := 4 \times 8 + 740736 \times 8 \\
4825392 &:= 6 \times 8 + 603168 \times 8 \Leftrightarrow 6603168 := 4 \times 8 + 825392 \times 8 \\
4952376 &:= 7 \times 8 + 619040 \times 8 \Leftrightarrow 7619040 := 4 \times 8 + 952376 \times 8 \\
5841264 &:= 6 \times 8 + 730152 \times 8 \Leftrightarrow 6730152 := 5 \times 8 + 841264 \times 8 \\
5925912 &:= 740733 \times 8 + 6 \times 8 \Leftrightarrow 7407336 := 5 \times 8 + 925912 \times 8 \\
5925920 &:= 740740 \times 8 + 0 \times 8 \Leftrightarrow 7407400 := 5 \times 8 + 925920 \times 8 \\
5968248 &:= 7 \times 8 + 746024 \times 8 \Leftrightarrow 7746024 := 5 \times 8 + 968248 \times 8 \\
6730152 &:= 5 \times 8 + 841264 \times 8 \Leftrightarrow 5841264 := 6 \times 8 + 730152 \times 8 \\
6984120 &:= 7 \times 8 + 873008 \times 8 \Leftrightarrow 7873008 := 6 \times 8 + 984120 \times 8.
\end{aligned}$$

2.1.2 Positive and Negative Coefficients

$$\begin{aligned}
63 &:= 8 \times 9 - 1 \times 9 \quad \Leftrightarrow \quad 81 := 6 \times 9 + 3 \times 9 \\
126 &:= 16 \times 9 - 2 \times 9 \quad \Leftrightarrow \quad 162 := 12 \times 9 + 6 \times 9 \\
189 &:= 24 \times 9 - 3 \times 9 \quad \Leftrightarrow \quad 243 := 18 \times 9 + 9 \times 9 \\
385 &:= 61 \times 7 - 6 \times 7 \quad \Leftrightarrow \quad 616 := 3 \times 7 + 85 \times 7 \\
792 &:= 89 \times 9 - 1 \times 9 \quad \Leftrightarrow \quad 891 := 7 \times 9 + 92 \times 9 \\
1267 &:= 187 \times 7 - 6 \times 7 \quad \Leftrightarrow \quad 1876 := 1 \times 7 + 267 \times 7 \\
1716 &:= -4 \times 6 + 290 \times 6 \Leftrightarrow 4290 := -1 \times 6 + 716 \times 6 \\
2574 &:= -3 \times 6 + 432 \times 6 \Leftrightarrow 3432 := -2 \times 6 + 574 \times 6 \\
4563 &:= 510 \times 9 - 3 \times 9 \quad \Leftrightarrow \quad 5103 := 4 \times 9 + 563 \times 9 \\
13860 &:= 2316 \times 6 - 6 \times 6 \Leftrightarrow 23166 := 1 \times 6 + 3860 \times 6 \\
45639 &:= 5078 \times 9 - 7 \times 9 \Leftrightarrow 50787 := 4 \times 9 + 5639 \times 9
\end{aligned}$$

2.1.3 Amicable Numbers Multiples of 10

Below are some pairs of amicable numbers given in terms of blocks. All of them are multiple of 10. Each block is symmetrical in itself.

$$\begin{aligned}
110 &:= 1 \times 10 + 10 \times 10 \Leftrightarrow 110 := 1 \times 10 + 10 \times 10 \\
120 &:= 2 \times 10 + 10 \times 10 \Leftrightarrow 210 := 1 \times 10 + 20 \times 10 \\
130 &:= 3 \times 10 + 10 \times 10 \Leftrightarrow 310 := 1 \times 10 + 30 \times 10 \\
140 &:= 4 \times 10 + 10 \times 10 \Leftrightarrow 410 := 1 \times 10 + 40 \times 10 \\
150 &:= 5 \times 10 + 10 \times 10 \Leftrightarrow 510 := 1 \times 10 + 50 \times 10 \\
160 &:= 6 \times 10 + 10 \times 10 \Leftrightarrow 610 := 1 \times 10 + 60 \times 10 \\
170 &:= 7 \times 10 + 10 \times 10 \Leftrightarrow 710 := 1 \times 10 + 70 \times 10 \\
180 &:= 8 \times 10 + 10 \times 10 \Leftrightarrow 810 := 1 \times 10 + 80 \times 10 \\
190 &:= 9 \times 10 + 10 \times 10 \Leftrightarrow 910 := 1 \times 10 + 90 \times 10
\end{aligned}$$

$$\begin{aligned}
230 &:= 3 \times 10 + 20 \times 10 \Leftrightarrow 320 := 2 \times 10 + 30 \times 10 \\
240 &:= 4 \times 10 + 20 \times 10 \Leftrightarrow 420 := 2 \times 10 + 40 \times 10 \\
250 &:= 5 \times 10 + 20 \times 10 \Leftrightarrow 520 := 2 \times 10 + 50 \times 10 \\
260 &:= 6 \times 10 + 20 \times 10 \Leftrightarrow 620 := 2 \times 10 + 60 \times 10 \\
270 &:= 7 \times 10 + 20 \times 10 \Leftrightarrow 720 := 2 \times 10 + 70 \times 10 \\
280 &:= 8 \times 10 + 20 \times 10 \Leftrightarrow 820 := 2 \times 10 + 80 \times 10 \\
290 &:= 9 \times 10 + 20 \times 10 \Leftrightarrow 920 := 2 \times 10 + 90 \times 10
\end{aligned}$$

$$\begin{aligned}
340 &:= 4 \times 10 + 30 \times 10 \Leftrightarrow 430 := 3 \times 10 + 40 \times 10 \\
350 &:= 5 \times 10 + 30 \times 10 \Leftrightarrow 530 := 3 \times 10 + 50 \times 10 \\
360 &:= 6 \times 10 + 30 \times 10 \Leftrightarrow 630 := 3 \times 10 + 60 \times 10 \\
370 &:= 7 \times 10 + 30 \times 10 \Leftrightarrow 730 := 3 \times 10 + 70 \times 10 \\
380 &:= 8 \times 10 + 30 \times 10 \Leftrightarrow 830 := 3 \times 10 + 80 \times 10 \\
390 &:= 9 \times 10 + 30 \times 10 \Leftrightarrow 930 := 3 \times 10 + 90 \times 10
\end{aligned}$$

$$\begin{aligned}
450 &:= 5 \times 10 + 40 \times 10 \Leftrightarrow 540 := 4 \times 10 + 50 \times 10 \\
460 &:= 6 \times 10 + 40 \times 10 \Leftrightarrow 640 := 4 \times 10 + 60 \times 10 \\
470 &:= 7 \times 10 + 40 \times 10 \Leftrightarrow 740 := 4 \times 10 + 70 \times 10 \\
480 &:= 8 \times 10 + 40 \times 10 \Leftrightarrow 840 := 4 \times 10 + 80 \times 10 \\
490 &:= 9 \times 10 + 40 \times 10 \Leftrightarrow 940 := 4 \times 10 + 90 \times 10
\end{aligned}$$

$$\begin{aligned}
560 &:= 6 \times 10 + 50 \times 10 \Leftrightarrow 650 := 5 \times 10 + 60 \times 10 \\
570 &:= 7 \times 10 + 50 \times 10 \Leftrightarrow 750 := 5 \times 10 + 70 \times 10 \\
580 &:= 8 \times 10 + 50 \times 10 \Leftrightarrow 850 := 5 \times 10 + 80 \times 10 \\
590 &:= 9 \times 10 + 50 \times 10 \Leftrightarrow 950 := 5 \times 10 + 90 \times 10
\end{aligned}$$

$$\begin{aligned}
670 &:= 7 \times 10 + 60 \times 10 \Leftrightarrow 760 := 6 \times 10 + 70 \times 10 \\
680 &:= 8 \times 10 + 60 \times 10 \Leftrightarrow 860 := 6 \times 10 + 80 \times 10 \\
690 &:= 9 \times 10 + 60 \times 10 \Leftrightarrow 960 := 6 \times 10 + 90 \times 10
\end{aligned}$$

$$780 := 8 \times 10 + 70 \times 10 \Leftrightarrow 870 := 7 \times 10 + 80 \times 10$$

$$790 := 9 \times 10 + 70 \times 10 \Leftrightarrow 970 := 7 \times 10 + 90 \times 10$$

$$890 := 9 \times 10 + 80 \times 10 \Leftrightarrow 980 := 8 \times 10 + 90 \times 10$$

$$23230 := 3 \times 10 + 2320 \times 10 \Leftrightarrow 32320 := 2 \times 10 + 3230 \times 10$$

$$24240 := 4 \times 10 + 2420 \times 10 \Leftrightarrow 42420 := 2 \times 10 + 4240 \times 10$$

$$25250 := 5 \times 10 + 2520 \times 10 \Leftrightarrow 52520 := 2 \times 10 + 5250 \times 10$$

$$26260 := 6 \times 10 + 2620 \times 10 \Leftrightarrow 62620 := 2 \times 10 + 6260 \times 10$$

$$27270 := 7 \times 10 + 2720 \times 10 \Leftrightarrow 72720 := 2 \times 10 + 7270 \times 10$$

$$28280 := 8 \times 10 + 2820 \times 10 \Leftrightarrow 82820 := 2 \times 10 + 8280 \times 10$$

$$29290 := 9 \times 10 + 2920 \times 10 \Leftrightarrow 92920 := 2 \times 10 + 9290 \times 10$$

$$34340 := 4 \times 10 + 3430 \times 10 \Leftrightarrow 43430 := 3 \times 10 + 4340 \times 10$$

$$35350 := 5 \times 10 + 3530 \times 10 \Leftrightarrow 53530 := 3 \times 10 + 5350 \times 10$$

$$36360 := 6 \times 10 + 3630 \times 10 \Leftrightarrow 63630 := 3 \times 10 + 6360 \times 10$$

$$37370 := 7 \times 10 + 3730 \times 10 \Leftrightarrow 73730 := 3 \times 10 + 7370 \times 10$$

$$38380 := 8 \times 10 + 3830 \times 10 \Leftrightarrow 83830 := 3 \times 10 + 8380 \times 10$$

$$39390 := 9 \times 10 + 3930 \times 10 \Leftrightarrow 93930 := 3 \times 10 + 9390 \times 10$$

$$45450 := 5 \times 10 + 4540 \times 10 \Leftrightarrow 54540 := 4 \times 10 + 5450 \times 10$$

$$46460 := 6 \times 10 + 4640 \times 10 \Leftrightarrow 64640 := 4 \times 10 + 6460 \times 10$$

$$47470 := 7 \times 10 + 4740 \times 10 \Leftrightarrow 74740 := 4 \times 10 + 7470 \times 10$$

$$48480 := 8 \times 10 + 4840 \times 10 \Leftrightarrow 84840 := 4 \times 10 + 8480 \times 10$$

$$49490 := 9 \times 10 + 4940 \times 10 \Leftrightarrow 94940 := 4 \times 10 + 9490 \times 10$$

$$56560 := 6 \times 10 + 5650 \times 10 \Leftrightarrow 65650 := 5 \times 10 + 6560 \times 10$$

$$57570 := 7 \times 10 + 5750 \times 10 \Leftrightarrow 75750 := 5 \times 10 + 7570 \times 10$$

$$58580 := 8 \times 10 + 5850 \times 10 \Leftrightarrow 85850 := 5 \times 10 + 8580 \times 10$$

$$59590 := 9 \times 10 + 5950 \times 10 \Leftrightarrow 95950 := 5 \times 10 + 9590 \times 10$$

$$67670 := 7 \times 10 + 6760 \times 10 \Leftrightarrow 76760 := 6 \times 10 + 7670 \times 10$$

$$68680 := 8 \times 10 + 6860 \times 10 \Leftrightarrow 86860 := 6 \times 10 + 8680 \times 10$$

$$69690 := 9 \times 10 + 6960 \times 10 \Leftrightarrow 96960 := 6 \times 10 + 9690 \times 10$$

$$78780 := 8 \times 10 + 7870 \times 10 \Leftrightarrow 87870 := 7 \times 10 + 8780 \times 10$$

$$79790 := 9 \times 10 + 7970 \times 10 \Leftrightarrow 97970 := 7 \times 10 + 9790 \times 10$$

$$89890 := 9 \times 10 + 8980 \times 10 \Leftrightarrow 98980 := 8 \times 10 + 9890 \times 10$$

$$\begin{aligned}
1111110 &:= 1 \times 10 + 111110 \times 10 \Leftrightarrow 1111110 := 1 \times 10 + 111110 \times 10 \\
1212120 &:= 2 \times 10 + 121210 \times 10 \Leftrightarrow 2121210 := 1 \times 10 + 212120 \times 10 \\
1313130 &:= 3 \times 10 + 131310 \times 10 \Leftrightarrow 3131310 := 1 \times 10 + 313130 \times 10 \\
1414140 &:= 4 \times 10 + 141410 \times 10 \Leftrightarrow 4141410 := 1 \times 10 + 414140 \times 10 \\
1515150 &:= 5 \times 10 + 151510 \times 10 \Leftrightarrow 5151510 := 1 \times 10 + 515150 \times 10 \\
1616160 &:= 6 \times 10 + 161610 \times 10 \Leftrightarrow 6161610 := 1 \times 10 + 616160 \times 10 \\
1717170 &:= 7 \times 10 + 171710 \times 10 \Leftrightarrow 7171710 := 1 \times 10 + 717170 \times 10 \\
1818180 &:= 8 \times 10 + 181810 \times 10 \Leftrightarrow 8181810 := 1 \times 10 + 818180 \times 10 \\
1919190 &:= 9 \times 10 + 191910 \times 10 \Leftrightarrow 9191910 := 1 \times 10 + 919190 \times 10 \\
\\
2323230 &:= 3 \times 10 + 232320 \times 10 \Leftrightarrow 3232320 := 2 \times 10 + 323230 \times 10 \\
2424240 &:= 4 \times 10 + 242420 \times 10 \Leftrightarrow 4242420 := 2 \times 10 + 424240 \times 10 \\
2525250 &:= 5 \times 10 + 252520 \times 10 \Leftrightarrow 5252520 := 2 \times 10 + 525250 \times 10 \\
2626260 &:= 6 \times 10 + 262620 \times 10 \Leftrightarrow 6262620 := 2 \times 10 + 626260 \times 10 \\
2727270 &:= 7 \times 10 + 272720 \times 10 \Leftrightarrow 7272720 := 2 \times 10 + 727270 \times 10 \\
2828280 &:= 8 \times 10 + 282820 \times 10 \Leftrightarrow 8282820 := 2 \times 10 + 828280 \times 10 \\
2929290 &:= 9 \times 10 + 292920 \times 10 \Leftrightarrow 9292920 := 2 \times 10 + 929290 \times 10 \\
\\
3434340 &:= 4 \times 10 + 343430 \times 10 \Leftrightarrow 4343430 := 3 \times 10 + 434340 \times 10 \\
3535350 &:= 5 \times 10 + 353530 \times 10 \Leftrightarrow 5353530 := 3 \times 10 + 535350 \times 10 \\
3636360 &:= 6 \times 10 + 363630 \times 10 \Leftrightarrow 6363630 := 3 \times 10 + 636360 \times 10 \\
3737370 &:= 7 \times 10 + 373730 \times 10 \Leftrightarrow 7373730 := 3 \times 10 + 737370 \times 10 \\
3838380 &:= 8 \times 10 + 383830 \times 10 \Leftrightarrow 8383830 := 3 \times 10 + 838380 \times 10 \\
3939390 &:= 9 \times 10 + 393930 \times 10 \Leftrightarrow 9393930 := 3 \times 10 + 939390 \times 10 \\
\\
4545450 &:= 5 \times 10 + 454540 \times 10 \Leftrightarrow 5454540 := 4 \times 10 + 545450 \times 10 \\
4646460 &:= 6 \times 10 + 464640 \times 10 \Leftrightarrow 6464640 := 4 \times 10 + 646460 \times 10 \\
4747470 &:= 7 \times 10 + 474740 \times 10 \Leftrightarrow 7474740 := 4 \times 10 + 747470 \times 10 \\
4848480 &:= 8 \times 10 + 484840 \times 10 \Leftrightarrow 8484840 := 4 \times 10 + 848480 \times 10 \\
4949490 &:= 9 \times 10 + 494940 \times 10 \Leftrightarrow 9494940 := 4 \times 10 + 949490 \times 10 \\
\\
5656560 &:= 6 \times 10 + 565650 \times 10 \Leftrightarrow 6565650 := 5 \times 10 + 656560 \times 10 \\
5757570 &:= 7 \times 10 + 575750 \times 10 \Leftrightarrow 7575750 := 5 \times 10 + 757570 \times 10 \\
5858580 &:= 8 \times 10 + 585850 \times 10 \Leftrightarrow 8585850 := 5 \times 10 + 858580 \times 10 \\
5959590 &:= 9 \times 10 + 595950 \times 10 \Leftrightarrow 9595950 := 5 \times 10 + 959590 \times 10 \\
\\
6767670 &:= 7 \times 10 + 676760 \times 10 \Leftrightarrow 7676760 := 6 \times 10 + 767670 \times 10 \\
6868680 &:= 8 \times 10 + 686860 \times 10 \Leftrightarrow 8686860 := 6 \times 10 + 868680 \times 10 \\
6969690 &:= 9 \times 10 + 696960 \times 10 \Leftrightarrow 9696960 := 6 \times 10 + 969690 \times 10
\end{aligned}$$

$$7878780 := 8 \times 10 + 787870 \times 10 \Leftrightarrow 8787870 := 7 \times 10 + 878780 \times 10$$

$$7979790 := 9 \times 10 + 797970 \times 10 \Leftrightarrow 9797970 := 7 \times 10 + 979790 \times 10$$

$$8989890 := 9 \times 10 + 898980 \times 10 \Leftrightarrow 9898980 := 8 \times 10 + 989890 \times 10$$

$$1820 := -8 \times 10 + 190 \times 10 \Leftrightarrow 8190 := -1 \times 10 + 820 \times 10$$

$$2730 := -7 \times 10 + 280 \times 10 \Leftrightarrow 7280 := -2 \times 10 + 730 \times 10$$

$$3640 := -6 \times 10 + 370 \times 10 \Leftrightarrow 6370 := -3 \times 10 + 640 \times 10$$

$$4550 := -5 \times 10 + 460 \times 10 \Leftrightarrow 5460 := -4 \times 10 + 550 \times 10$$

$$181820 := -8 \times 10 + 18190 \times 10 \Leftrightarrow 818190 := -1 \times 10 + 81820 \times 10$$

$$272730 := -7 \times 10 + 27280 \times 10 \Leftrightarrow 727280 := -2 \times 10 + 72730 \times 10$$

$$363640 := -6 \times 10 + 36370 \times 10 \Leftrightarrow 636370 := -3 \times 10 + 63640 \times 10$$

$$454550 := -5 \times 10 + 45460 \times 10 \Leftrightarrow 545460 := -4 \times 10 + 54550 \times 10$$

2.2 Self-Amicable

Self-amicable numbers are understood as representation with its own digits with extra numbers appearing in each expression with multiplication. These numbers are very much similar to semi-selfie numbers [10, 13].

$$12 := 1 \times 4 + 2 \times 4$$

$$18 := 1 \times 2 + 8 \times 2$$

$$21 := 2 \times 7 + 1 \times 7$$

$$24 := 2 \times 4 + 4 \times 4$$

$$27 := 2 \times 3 + 7 \times 3$$

$$36 := 3 \times 4 + 6 \times 4$$

$$42 := 4 \times 7 + 2 \times 7$$

$$45 := 4 \times 5 + 5 \times 5$$

$$48 := 4 \times 4 + 8 \times 4$$

$$54 := 5 \times 6 + 4 \times 6$$

$$63 := 6 \times 7 + 3 \times 7$$

$$64 := 8 \times 8 + 0 \times 8$$

$$72 := 7 \times 8 + 2 \times 8$$

$$81 := 8 \times 9 + 1 \times 9$$

$$84 := 8 \times 7 + 4 \times 7$$

$$105 := 10 \times 7 + 5 \times 7$$

$$108 := 10 \times 6 + 8 \times 6$$

$$126 := 12 \times 7 + 6 \times 7$$

$$132 := 1 \times 4 + 32 \times 4$$

$$144 := 14 \times 8 + 4 \times 8$$

$$147 := 14 \times 7 + 7 \times 7$$

$$162 := 16 \times 9 + 2 \times 9$$

$$168 := 16 \times 7 + 8 \times 7$$

$$189 := 18 \times 7 + 9 \times 7$$

$$198 := 1 \times 2 + 98 \times 2$$

$$216 := 21 \times 8 + 6 \times 8$$

$$231 := 2 \times 7 + 31 \times 7$$

$$243 := 24 \times 9 + 3 \times 9$$

$$264 := 2 \times 4 + 64 \times 4$$

$$288 := 28 \times 8 + 8 \times 8$$

$$297 := 2 \times 3 + 97 \times 3$$

$$324 := 32 \times 9 + 4 \times 9$$

$$396 := 3 \times 4 + 96 \times 4$$

$$405 := 40 \times 9 + 5 \times 9$$

$$462 := 4 \times 7 + 62 \times 7$$

$$486 := 48 \times 9 + 6 \times 9$$

$$495 := 4 \times 5 + 95 \times 5$$

$$567 := 56 \times 9 + 7 \times 9$$

$$594 := 5 \times 6 + 94 \times 6$$

$$648 := 64 \times 9 + 8 \times 9$$

$$693 := 6 \times 7 + 93 \times 7$$

$$729 := 72 \times 9 + 9 \times 9$$

$$792 := 7 \times 8 + 92 \times 8$$

$$891 := 8 \times 9 + 91 \times 9$$

$$1332 := 1 \times 4 + 332 \times 4$$

$$1998 := 1 \times 2 + 998 \times 2$$

$$2331 := 2 \times 7 + 331 \times 7$$

$$2664 := 2 \times 4 + 664 \times 4$$

$$2997 := 2 \times 3 + 997 \times 3$$

$$3996 := 3 \times 4 + 996 \times 4$$

$$4662 := 4 \times 7 + 662 \times 7$$

$$4995 := 4 \times 5 + 995 \times 5$$

$$5994 := 5 \times 6 + 994 \times 6$$

$$6993 := 6 \times 7 + 993 \times 7$$

$$7992 := 7 \times 8 + 992 \times 8$$

$$8991 := 8 \times 9 + 991 \times 9$$

$$13332 := 1 \times 4 + 3332 \times 4$$

$$19998 := 1 \times 2 + 9998 \times 2$$

$$23331 := 2 \times 7 + 3331 \times 7$$

$$26664 := 2 \times 4 + 6664 \times 4$$

$$29997 := 2 \times 3 + 9997 \times 3$$

$$39996 := 3 \times 4 + 9996 \times 4$$

$$46662 := 4 \times 7 + 6662 \times 7$$

$$49995 := 4 \times 5 + 9995 \times 5$$

$$59994 := 5 \times 6 + 9994 \times 6$$

$$69993 := 6 \times 7 + 9993 \times 7$$

$$79992 := 7 \times 8 + 9992 \times 8$$

$$89991 := 8 \times 9 + 9991 \times 9$$

$$133332 := 1 \times 4 + 33332 \times 4$$

$$199998 := 1 \times 2 + 99998 \times 2$$

$$233331 := 2 \times 7 + 33331 \times 7$$

$$266664 := 2 \times 4 + 66664 \times 4$$

$$299997 := 2 \times 3 + 99997 \times 3$$

$$399996 := 3 \times 4 + 99996 \times 4$$

$$466662 := 4 \times 7 + 66662 \times 7$$

$$499995 := 4 \times 5 + 99995 \times 5$$

$$599994 := 5 \times 6 + 99994 \times 6$$

$$699993 := 6 \times 7 + 99993 \times 7$$

$$799992 := 7 \times 8 + 99992 \times 8$$

$$899991 := 8 \times 9 + 99991 \times 9$$

$$1142856 := 1 \times 8 + 142856 \times 8$$

$$1999998 := 1 \times 2 + 999998 \times 2$$

$$2333331 := 2 \times 7 + 333331 \times 7$$

$$2666664 := 2 \times 4 + 666664 \times 4$$

$$2999997 := 2 \times 3 + 999997 \times 3$$

$$3999996 := 3 \times 4 + 999996 \times 4$$

$$4571424 := 4 \times 8 + 571424 \times 8$$

$$4666662 := 4 \times 7 + 666662 \times 7$$

$$4999995 := 4 \times 5 + 999995 \times 5$$

$$5714280 := 5 \times 8 + 714280 \times 8$$

$$5999994 := 5 \times 6 + 999994 \times 6$$

$$6857136 := 6 \times 8 + 857136 \times 8$$

$$6999993 := 6 \times 7 + 999993 \times 7$$

$$7999992 := 7 \times 8 + 999992 \times 8$$

$$8999991 := 8 \times 9 + 999991 \times 9$$

$$1144 := -1 \times 8 + 144 \times 8$$

$$2288 := -2 \times 8 + 288 \times 8$$

$$3432 := -3 \times 8 + 432 \times 8$$

$$4576 := -4 \times 8 + 576 \times 8$$

$$5720 := -5 \times 8 + 720 \times 8$$

$$6864 := -6 \times 8 + 864 \times 8$$

3 Power-Type Amicable Numbers

As explained in examples (1), (2) and (3) regarding amicable numbers in pairs and self-amicable numbers with the idea of power. This section bring these kinds of numbers by using the idea of product. There are two situations, one in pairs and another as self. In the both the cases, there are numbers with only positive coefficients, and with positive and negative coefficients.

3.1 Amicable in Pairs

3.1.1 Positive Coefficients

$$\begin{aligned}
 3869 &:= 62^2 + 5^2 && \Leftrightarrow && 6205 &:= 38^2 + 69^2 \\
 5965 &:= 77^2 + 6^2 && \Leftrightarrow && 7706 &:= 59^2 + 65^2 \\
 43354 &:= 127^2 + 165^2 && \Leftrightarrow && 127165 &:= 43^2 + 354^2 \\
 137461 &:= 231^2 + 290^2 && \Leftrightarrow && 231290 &:= 137^2 + 461^2 \\
 1261485 &:= 222^2 + 1101^2 && \Leftrightarrow && 2221101 &:= 126^2 + 1485^2 \\
 1528804 &:= 298^2 + 1200^2 && \Leftrightarrow && 2981200 &:= 1528^2 + 804^2 \\
 7414650 &:= 2217^2 + 1581^2 && \Leftrightarrow && 22171581 &:= 741^2 + 4650^2
 \end{aligned}$$

3.1.2 Positive and Negative Coefficients

$$\begin{aligned}
 16 &:= -3^2 + 5^2 && \Leftrightarrow && 35 &:= -1^2 + 6^2 \\
 28 &:= -6^2 + 8^2 && \Leftrightarrow && 68 &:= 2^2 + 8^2 \\
 240 &:= 16^2 - 4^2 && \Leftrightarrow && 1604 &:= 2^2 + 40^2 \\
 316 &:= -3^3 + 0007^3 && \Leftrightarrow && 30007 &:= 31^3 + 6^3 \\
 369 &:= 12^2 + 15^2 && \Leftrightarrow && 1215 &:= 36^2 - 9^2 \\
 1155 &:= -31^2 + 46^2 && \Leftrightarrow && 3146 &:= 11^2 + 55^2 \\
 2205 &:= 42^2 + 21^2 && \Leftrightarrow && 42021 &:= -2^2 + 205^2 \\
 2880 &:= 56^2 - 16^2 && \Leftrightarrow && 5616 &:= -28^2 + 80^2 \\
 21384 &:= 147^2 - 15^2 && \Leftrightarrow && 147015 &:= -21^2 + 384^2 \\
 42471 &:= -220^2 + 77^2 && \Leftrightarrow && 220077 &:= -42^2 + 471^2 \\
 60912 &:= 371^2 - 00277^2 && \Leftrightarrow && 37100277 &:= 6091^2 - 2^2 \\
 88836 &:= 706^2 - 640^2 && \Leftrightarrow && 706640 &:= 88^2 + 836^2 \\
 96525 &:= -266^2 + 409^2 && \Leftrightarrow && 266409 &:= -96^2 + 525^2 \\
 134784 &:= -596^2 + 700^2 && \Leftrightarrow && 596700 &:= -134^2 + 784^2 \\
 150975 &:= -95^2 + 0400^2 && \Leftrightarrow && 950400 &:= -15^2 + 0975^2 \\
 152207 &:= -487^2 + 624^2 && \Leftrightarrow && 4870624 &:= -15^2 + 2207^2 \\
 161616 &:= -353^2 + 535^2 && \Leftrightarrow && 353535 &:= -161^2 + 616^2
 \end{aligned}$$

$$\begin{aligned}
 186745 &:= 52^2 + 429^2 && \Leftrightarrow && 520429 &:= -186^2 + 745^2 \\
 275808 &:= 577^2 - 239^2 && \Leftrightarrow && 577239 &:= -275^2 + 808^2 \\
 373599 &:= -1295^2 + 1432^2 && \Leftrightarrow && 12951432 &:= -37^2 + 3599^2 \\
 384912 &:= 684^2 - 288^2 && \Leftrightarrow && 684288 &:= -384^2 + 912^2 \\
 970299 &:= 1030^2 - 301^2 && \Leftrightarrow && 1030301 &:= 970^2 + 299^2 \\
 1072519 &:= -5259^2 + 5360^2 && \Leftrightarrow && 5259005360 &:= -1^2 + 72519^2 \\
 1094016 &:= 1196^2 - 580^2 && \Leftrightarrow && 1196580 &:= 1094^2 - 16^2 \\
 1956636 &:= -342^2 + 1440^2 && \Leftrightarrow && 3421440 &:= 1956^2 - 636^2 \\
 2464407 &:= 1936^2 - 1133^2 && \Leftrightarrow && 19361133 &:= -246^2 + 4407^2 \\
 5437256 &:= 5235^2 - 4687^2 && \Leftrightarrow && 52354687 &:= -543^2 + 7256^2 \\
 5668080 &:= 6496^2 - 6044^2 && \Leftrightarrow && 64966044 &:= -566^2 + 8080^2 \\
 6445779 &:= 3298^2 - 2105^2 && \Leftrightarrow && 32982105 &:= -644^2 + 5779^2 \\
 8556048 &:= 732^2 + 02832^2 && \Leftrightarrow && 73202832 &:= 8556^2 - 48^2.
 \end{aligned}$$

3.1.3 With 0 and 1

Below are some amicable pairs just with two digits 0 and 1. There are much more possibilities, but we have written only few of them.

$$\begin{aligned}
 1001 &:= 10^3 + 00001^3 && \Leftrightarrow && 1000001 && := 100^3 + 1^3 \\
 10001 &:= 10^4 + 0000001^4 && \Leftrightarrow && 100000001 && := 100^4 + 1^4 \\
 10001 &:= 100^2 + 0001^2 && \Leftrightarrow && 1000001 && := 1000^2 + 1^2 \\
 10001 &:= 10^4 + 00000000001^4 && \Leftrightarrow && 1000000000001 && := 1000^4 + 1^4 \\
 100001 &:= 10^5 + 000000001^5 && \Leftrightarrow && 10000000001 && := 100^5 + 1^5 \\
 100001 &:= 10^5 + 000000000000001^5 && \Leftrightarrow && 10000000000000001 && := 1000^5 + 1^5 \\
 100001 &:= 10^5 + 000000000000000000001^5 && \Leftrightarrow && 1000000000000000000001 && := 10000^5 + 1^5 \\
 1000001 &:= 10^6 + 00000000001^6 && \Leftrightarrow && 1000000000001 && := 100^6 + 0001^6 \\
 1000001 &:= 100^3 + 0000001^3 && \Leftrightarrow && 1000000001 && := 1000^3 + 001^3 \\
 1000001 &:= 10^6 + 0000000000000000001^6 && \Leftrightarrow && 100000000000000000001 && := 1000^6 + 001^6 \\
 1000001 &:= 1000^2 + 00001^2 && \Leftrightarrow && 100000001 && := 10000^2 + 01^2 \\
 1000001 &:= 100^3 + 0000000001^3 && \Leftrightarrow && 1000000000001 && := 10000^3 + 01^3 \\
 1000001 &:= 1000^2 + 0000001^2 && \Leftrightarrow && 10000000001 && := 100000^2 + 1^2 \\
 1000001 &:= 100^3 + 000000000000001^3 && \Leftrightarrow && 10000000000000001 && := 100000^3 + 1^3 \\
 1010000 &:= 1000^2 + 00100^2 && \Leftrightarrow && 100000100 && := 10^2 + 10000^2. \\
 \\
 1000001 &:= 10^6 + 0000000000000000000000001^6 && \Leftrightarrow && 10000000000000000000000001 && := 10000^6 + 01^6 \\
 1000001 &:= 10^6 + 0000000000000000000000000000001^6 && \Leftrightarrow && 10000000000000000000000000000001 && := 100000^6 + 1^6.
 \end{aligned}$$

3.2 Self-Amicable

$$\begin{aligned}
 100 &:= 10^2 + 0^2 & 16128 &:= -16^2 + 128^2 \\
 101 &:= 10^2 + 1^2 & 34188 &:= -34^2 + 188^2 \\
 407 &:= 4^3 + 7^3 & 140400 &:= -140^2 + 400^2 \\
 1000 &:= 10^3 + 00^3 & 190476 &:= -190^2 + 476^2 \\
 1001 &:= 10^3 + 01^3 & 216513 &:= -216^2 + 513^2 \\
 1233 &:= 12^2 + 33^2 & 300625 &:= -300^2 + 625^2 \\
 8833 &:= 88^2 + 33^2 & 334668 &:= -334^2 + 668^2 \\
 10000 &:= 100^2 + 00^2 & 416768 &:= -416^2 + 768^2 \\
 10000 &:= 10^4 + 000^4 & 484848 &:= -484^2 + 848^2 \\
 10001 &:= 100^2 + 01^2 & 530901 &:= -530^2 + 901^2 \\
 10001 &:= 10^4 + 001^4 & 1010100 &:= 1010^2 - 100^2 \\
 10100 &:= 10^2 + 100^2 & 1016127 &:= 1016^2 - 127^2 \\
 340067 &:= 34^3 + 0067^3 & 1034187 &:= 1034^2 - 187^2 \\
 990100 &:= 990^2 + 100^2 & 1140399 &:= 1140^2 - 399^2 \\
 1000000 &:= 1000^2 + 000^2 & 1190475 &:= 1190^2 - 475^2 \\
 5882353 &:= 588^2 + 2353^2 & 1216512 &:= 1216^2 - 512^2 \\
 & & 1300624 &:= 1300^2 - 624^2 \\
 48 &:= -4^2 + 8^2 & 1334667 &:= 1334^2 - 667^2 \\
 147 &:= 14^2 - 7^2 & 1416767 &:= 1416^2 - 767^2 \\
 3468 &:= -34^2 + 68^2 & 1484847 &:= 1484^2 - 847^2 \\
 10101 &:= -10^2 + 101^2 & 1530900 &:= 1530^2 - 900^2 \\
 13467 &:= 134^2 - 67^2 & &
 \end{aligned}$$

4 Patterns in Amicable Numbers

This section brings patterns in amicable numbers in two different situations. One with operation of multiplication and another with powers. In case of product we have results in both types, i.e., in pairs as well as self-type. In case of powers, we have patterns only with self-amicable numbers.

4.1 Product-Type Patterns in Amicable Numbers

This section brings patterns in amicable numbers. These patterns are in two different subsections. One with pairs, and another with self-amicable numbers.

4.1.1 Patterns in Pair of Amicable Numbers

Below are some examples of patterns in pair of amicable numbers. We have written only up to 4th step. Further steps follows by extending in an obvious way.

$$\begin{aligned}
 165 &:= 33 \times 5 + 0 \times 5 &= 3 \times 5 + 30 \times 5 &\Leftrightarrow 330 := 1 \times 5 + 65 \times 5 \\
 1665 &:= 333 \times 5 + 0 \times 5 &= 3 \times 5 + 330 \times 5 &\Leftrightarrow 3330 := 1 \times 5 + 665 \times 5 \\
 16665 &:= 3333 \times 5 + 0 \times 5 &= 3 \times 5 + 3330 \times 5 &\Leftrightarrow 33330 := 1 \times 5 + 6665 \times 5 \\
 166665 &:= 33333 \times 5 + 0 \times 5 &= 3 \times 5 + 33330 \times 5 &\Leftrightarrow 333330 := 1 \times 5 + 66665 \times 5 \\
 1666665 &:= 333333 \times 5 + 0 \times 5 &= 3 \times 5 + 333330 \times 5 &\Leftrightarrow 3333330 := 1 \times 5 + 666665 \times 5
 \end{aligned}$$

$$\begin{aligned}
 352 &:= 44 \times 8 + 0 \times 8 &= 4 \times 8 + 40 \times 8 &\Leftrightarrow 440 := 3 \times 8 + 52 \times 8 \\
 3552 &:= 444 \times 8 + 0 \times 8 &= 4 \times 8 + 440 \times 8 &\Leftrightarrow 4440 := 3 \times 8 + 552 \times 8 \\
 35552 &:= 4444 \times 8 + 0 \times 8 &= 4 \times 8 + 4440 \times 8 &\Leftrightarrow 44440 := 3 \times 8 + 5552 \times 8 \\
 355552 &:= 44444 \times 8 + 0 \times 8 &= 4 \times 8 + 44440 \times 8 &\Leftrightarrow 444440 := 3 \times 8 + 55552 \times 8 \\
 3555552 &:= 444444 \times 8 + 0 \times 8 &= 4 \times 8 + 444440 \times 8 &\Leftrightarrow 4444440 := 3 \times 8 + 555552 \times 8
 \end{aligned}$$

$$\begin{aligned}
 176 &:= 6 \times 8 + 16 \times 8 &\Leftrightarrow 616 := 1 \times 8 + 76 \times 8 \\
 1776 &:= 6 \times 8 + 216 \times 8 &\Leftrightarrow 6216 := 1 \times 8 + 776 \times 8 \\
 17776 &:= 6 \times 8 + 2216 \times 8 &\Leftrightarrow 62216 := 1 \times 8 + 7776 \times 8 \\
 177776 &:= 6 \times 8 + 22216 \times 8 &\Leftrightarrow 622216 := 1 \times 8 + 77776 \times 8 \\
 1777776 &:= 6 \times 8 + 222216 \times 8 &\Leftrightarrow 6222216 := 1 \times 8 + 777776 \times 8
 \end{aligned}$$

$$\begin{aligned}
 264 &:= 5 \times 8 + 28 \times 8 &\Leftrightarrow 528 := 2 \times 8 + 64 \times 8 \\
 2664 &:= 5 \times 8 + 328 \times 8 &\Leftrightarrow 5328 := 2 \times 8 + 664 \times 8 \\
 26664 &:= 5 \times 8 + 3328 \times 8 &\Leftrightarrow 53328 := 2 \times 8 + 6664 \times 8 \\
 266664 &:= 5 \times 8 + 33328 \times 8 &\Leftrightarrow 533328 := 2 \times 8 + 66664 \times 8 \\
 2666664 &:= 5 \times 8 + 333328 \times 8 &\Leftrightarrow 5333328 := 2 \times 8 + 666664 \times 8
 \end{aligned}$$

$$\begin{aligned}
 1650 &:= 325 \times 5 + 5 \times 5 &\Leftrightarrow 3255 := 1 \times 5 + 650 \times 5 \\
 16650 &:= 3325 \times 5 + 5 \times 5 &\Leftrightarrow 33255 := 1 \times 5 + 6650 \times 5 \\
 166650 &:= 33325 \times 5 + 5 \times 5 &\Leftrightarrow 333255 := 1 \times 5 + 66650 \times 5 \\
 1666650 &:= 333325 \times 5 + 5 \times 5 &\Leftrightarrow 3333255 := 1 \times 5 + 666650 \times 5
 \end{aligned}$$

$$\begin{aligned}
 3544 &:= 437 \times 8 + 6 \times 8 &\Leftrightarrow 4376 := 3 \times 8 + 544 \times 8 \\
 35544 &:= 4437 \times 8 + 6 \times 8 &\Leftrightarrow 44376 := 3 \times 8 + 5544 \times 8 \\
 355544 &:= 44437 \times 8 + 6 \times 8 &\Leftrightarrow 444376 := 3 \times 8 + 55544 \times 8 \\
 3555544 &:= 444437 \times 8 + 6 \times 8 &\Leftrightarrow 4444376 := 3 \times 8 + 555544 \times 8
 \end{aligned}$$

Below are some patterns obtained from the subsection 2.1.3 on "Amicable Numbers Multiples of 10". These are very obvious, but are good looking.

$$\begin{aligned} 120 &:= 2 \times 10 + 10 \times 10 & \Leftrightarrow & 210 := 1 \times 10 + 20 \times 10 \\ 12120 &:= 2 \times 10 + 1210 \times 10 & \Leftrightarrow & 21210 := 1 \times 10 + 2120 \times 10 \\ 1212120 &:= 2 \times 10 + 121210 \times 10 & \Leftrightarrow & 2121210 := 1 \times 10 + 212120 \times 10 \end{aligned}$$

$$\begin{aligned} 130 &:= 3 \times 10 + 10 \times 10 & \Leftrightarrow & 310 := 1 \times 10 + 30 \times 10 \\ 13130 &:= 3 \times 10 + 1310 \times 10 & \Leftrightarrow & 31310 := 1 \times 10 + 3130 \times 10 \\ 1313130 &:= 3 \times 10 + 131310 \times 10 & \Leftrightarrow & 3131310 := 1 \times 10 + 313130 \times 10 \end{aligned}$$

$$\begin{aligned} 140 &:= 4 \times 10 + 10 \times 10 & \Leftrightarrow & 410 := 1 \times 10 + 40 \times 10 \\ 14140 &:= 4 \times 10 + 1410 \times 10 & \Leftrightarrow & 41410 := 1 \times 10 + 4140 \times 10 \\ 1414140 &:= 4 \times 10 + 141410 \times 10 & \Leftrightarrow & 4141410 := 1 \times 10 + 414140 \times 10 \end{aligned}$$

$$\begin{aligned} 150 &:= 5 \times 10 + 10 \times 10 & \Leftrightarrow & 510 := 1 \times 10 + 50 \times 10 \\ 15150 &:= 5 \times 10 + 1510 \times 10 & \Leftrightarrow & 51510 := 1 \times 10 + 5150 \times 10 \\ 1515150 &:= 5 \times 10 + 151510 \times 10 & \Leftrightarrow & 5151510 := 1 \times 10 + 515150 \times 10 \end{aligned}$$

$$\begin{aligned} 160 &:= 6 \times 10 + 10 \times 10 & \Leftrightarrow & 610 := 1 \times 10 + 60 \times 10 \\ 16160 &:= 6 \times 10 + 1610 \times 10 & \Leftrightarrow & 61610 := 1 \times 10 + 6160 \times 10 \\ 1616160 &:= 6 \times 10 + 161610 \times 10 & \Leftrightarrow & 6161610 := 1 \times 10 + 616160 \times 10 \end{aligned}$$

$$\begin{aligned} 170 &:= 7 \times 10 + 10 \times 10 & \Leftrightarrow & 710 := 1 \times 10 + 70 \times 10 \\ 17170 &:= 7 \times 10 + 1710 \times 10 & \Leftrightarrow & 71710 := 1 \times 10 + 7170 \times 10 \\ 1717170 &:= 7 \times 10 + 171710 \times 10 & \Leftrightarrow & 7171710 := 1 \times 10 + 717170 \times 10 \end{aligned}$$

$$\begin{aligned} 180 &:= 8 \times 10 + 10 \times 10 & \Leftrightarrow & 810 := 1 \times 10 + 80 \times 10 \\ 18180 &:= 8 \times 10 + 1810 \times 10 & \Leftrightarrow & 81810 := 1 \times 10 + 8180 \times 10 \\ 1818180 &:= 8 \times 10 + 181810 \times 10 & \Leftrightarrow & 8181810 := 1 \times 10 + 818180 \times 10 \end{aligned}$$

$$\begin{aligned} 190 &:= 9 \times 10 + 10 \times 10 & \Leftrightarrow & 910 := 1 \times 10 + 90 \times 10 \\ 19190 &:= 9 \times 10 + 1910 \times 10 & \Leftrightarrow & 91910 := 1 \times 10 + 9190 \times 10 \\ 1919190 &:= 9 \times 10 + 191910 \times 10 & \Leftrightarrow & 9191910 := 1 \times 10 + 919190 \times 10 \end{aligned}$$

$$\begin{aligned}
230 &:= 3 \times 10 + 20 \times 10 & \Leftrightarrow & \quad 320 := 2 \times 10 + 30 \times 10 \\
23230 &:= 3 \times 10 + 2320 \times 10 & \Leftrightarrow & \quad 32320 := 2 \times 10 + 3230 \times 10 \\
2323230 &:= 3 \times 10 + 232320 \times 10 & \Leftrightarrow & \quad 3232320 := 2 \times 10 + 323230 \times 10
\end{aligned}$$

$$\begin{aligned}
240 &:= 4 \times 10 + 20 \times 10 & \Leftrightarrow & \quad 420 := 2 \times 10 + 40 \times 10 \\
24240 &:= 4 \times 10 + 2420 \times 10 & \Leftrightarrow & \quad 42420 := 2 \times 10 + 4240 \times 10 \\
2424240 &:= 4 \times 10 + 242420 \times 10 & \Leftrightarrow & \quad 4242420 := 2 \times 10 + 424240 \times 10
\end{aligned}$$

$$\begin{aligned}
250 &:= 5 \times 10 + 20 \times 10 & \Leftrightarrow & \quad 520 := 2 \times 10 + 50 \times 10 \\
25250 &:= 5 \times 10 + 2520 \times 10 & \Leftrightarrow & \quad 52520 := 2 \times 10 + 5250 \times 10 \\
2525250 &:= 5 \times 10 + 252520 \times 10 & \Leftrightarrow & \quad 5252520 := 2 \times 10 + 525250 \times 10
\end{aligned}$$

$$\begin{aligned}
260 &:= 6 \times 10 + 20 \times 10 & \Leftrightarrow & \quad 620 := 2 \times 10 + 60 \times 10 \\
26260 &:= 6 \times 10 + 2620 \times 10 & \Leftrightarrow & \quad 62620 := 2 \times 10 + 6260 \times 10 \\
2626260 &:= 6 \times 10 + 262620 \times 10 & \Leftrightarrow & \quad 6262620 := 2 \times 10 + 626260 \times 10
\end{aligned}$$

$$\begin{aligned}
270 &:= 7 \times 10 + 20 \times 10 & \Leftrightarrow & \quad 720 := 2 \times 10 + 70 \times 10 \\
27270 &:= 7 \times 10 + 2720 \times 10 & \Leftrightarrow & \quad 72720 := 2 \times 10 + 7270 \times 10 \\
2727270 &:= 7 \times 10 + 272720 \times 10 & \Leftrightarrow & \quad 7272720 := 2 \times 10 + 727270 \times 10
\end{aligned}$$

$$\begin{aligned}
280 &:= 8 \times 10 + 20 \times 10 & \Leftrightarrow & \quad 820 := 2 \times 10 + 80 \times 10 \\
28280 &:= 8 \times 10 + 2820 \times 10 & \Leftrightarrow & \quad 82820 := 2 \times 10 + 8280 \times 10 \\
2828280 &:= 8 \times 10 + 282820 \times 10 & \Leftrightarrow & \quad 8282820 := 2 \times 10 + 828280 \times 10
\end{aligned}$$

$$\begin{aligned}
290 &:= 9 \times 10 + 20 \times 10 & \Leftrightarrow & \quad 920 := 2 \times 10 + 90 \times 10 \\
29290 &:= 9 \times 10 + 2920 \times 10 & \Leftrightarrow & \quad 92920 := 2 \times 10 + 9290 \times 10 \\
2929290 &:= 9 \times 10 + 292920 \times 10 & \Leftrightarrow & \quad 9292920 := 2 \times 10 + 929290 \times 10
\end{aligned}$$

$$\begin{aligned}
340 &:= 4 \times 10 + 30 \times 10 & \Leftrightarrow & \quad 430 := 3 \times 10 + 40 \times 10 \\
34340 &:= 4 \times 10 + 3430 \times 10 & \Leftrightarrow & \quad 43430 := 3 \times 10 + 4340 \times 10 \\
3434340 &:= 4 \times 10 + 343430 \times 10 & \Leftrightarrow & \quad 4343430 := 3 \times 10 + 434340 \times 10
\end{aligned}$$

$$\begin{aligned}
350 &:= 5 \times 10 + 30 \times 10 & \Leftrightarrow & \quad 530 := 3 \times 10 + 50 \times 10 \\
35350 &:= 5 \times 10 + 3530 \times 10 & \Leftrightarrow & \quad 53530 := 3 \times 10 + 5350 \times 10 \\
3535350 &:= 5 \times 10 + 353530 \times 10 & \Leftrightarrow & \quad 5353530 := 3 \times 10 + 535350 \times 10
\end{aligned}$$

$$\begin{aligned}
360 &:= 6 \times 10 + 30 \times 10 & \Leftrightarrow & \quad 630 := 3 \times 10 + 60 \times 10 \\
36360 &:= 6 \times 10 + 3630 \times 10 & \Leftrightarrow & \quad 63630 := 3 \times 10 + 6360 \times 10 \\
3636360 &:= 6 \times 10 + 363630 \times 10 & \Leftrightarrow & \quad 6363630 := 3 \times 10 + 636360 \times 10
\end{aligned}$$

$$\begin{aligned}
370 &:= 7 \times 10 + 30 \times 10 & \Leftrightarrow & \quad 730 := 3 \times 10 + 70 \times 10 \\
37370 &:= 7 \times 10 + 3730 \times 10 & \Leftrightarrow & \quad 73730 := 3 \times 10 + 7370 \times 10 \\
3737370 &:= 7 \times 10 + 373730 \times 10 & \Leftrightarrow & \quad 7373730 := 3 \times 10 + 737370 \times 10
\end{aligned}$$

$$\begin{aligned}
380 &:= 8 \times 10 + 30 \times 10 & \Leftrightarrow & \quad 830 := 3 \times 10 + 80 \times 10 \\
38380 &:= 8 \times 10 + 3830 \times 10 & \Leftrightarrow & \quad 83830 := 3 \times 10 + 8380 \times 10 \\
3838380 &:= 8 \times 10 + 383830 \times 10 & \Leftrightarrow & \quad 8383830 := 3 \times 10 + 838380 \times 10
\end{aligned}$$

$$\begin{aligned}
390 &:= 9 \times 10 + 30 \times 10 & \Leftrightarrow & \quad 930 := 3 \times 10 + 90 \times 10 \\
39390 &:= 9 \times 10 + 3930 \times 10 & \Leftrightarrow & \quad 93930 := 3 \times 10 + 9390 \times 10 \\
3939390 &:= 9 \times 10 + 393930 \times 10 & \Leftrightarrow & \quad 9393930 := 3 \times 10 + 939390 \times 10
\end{aligned}$$

$$\begin{aligned}
450 &:= 5 \times 10 + 40 \times 10 & \Leftrightarrow & \quad 540 := 4 \times 10 + 50 \times 10 \\
45450 &:= 5 \times 10 + 4540 \times 10 & \Leftrightarrow & \quad 54540 := 4 \times 10 + 5450 \times 10 \\
4545450 &:= 5 \times 10 + 454540 \times 10 & \Leftrightarrow & \quad 5454540 := 4 \times 10 + 545450 \times 10
\end{aligned}$$

$$\begin{aligned}
460 &:= 6 \times 10 + 40 \times 10 & \Leftrightarrow & \quad 640 := 4 \times 10 + 60 \times 10 \\
46460 &:= 6 \times 10 + 4640 \times 10 & \Leftrightarrow & \quad 64640 := 4 \times 10 + 6460 \times 10 \\
4646460 &:= 6 \times 10 + 464640 \times 10 & \Leftrightarrow & \quad 6464640 := 4 \times 10 + 646460 \times 10
\end{aligned}$$

$$\begin{aligned}
470 &:= 7 \times 10 + 40 \times 10 & \Leftrightarrow & \quad 740 := 4 \times 10 + 70 \times 10 \\
47470 &:= 7 \times 10 + 4740 \times 10 & \Leftrightarrow & \quad 74740 := 4 \times 10 + 7470 \times 10 \\
4747470 &:= 7 \times 10 + 474740 \times 10 & \Leftrightarrow & \quad 7474740 := 4 \times 10 + 747470 \times 10
\end{aligned}$$

$$\begin{aligned}
480 &:= 8 \times 10 + 40 \times 10 & \Leftrightarrow & \quad 840 := 4 \times 10 + 80 \times 10 \\
48480 &:= 8 \times 10 + 4840 \times 10 & \Leftrightarrow & \quad 84840 := 4 \times 10 + 8480 \times 10 \\
4848480 &:= 8 \times 10 + 484840 \times 10 & \Leftrightarrow & \quad 8484840 := 4 \times 10 + 848480 \times 10
\end{aligned}$$

$$\begin{aligned}
490 &:= 9 \times 10 + 40 \times 10 & \Leftrightarrow & \quad 940 := 4 \times 10 + 90 \times 10 \\
49490 &:= 9 \times 10 + 4940 \times 10 & \Leftrightarrow & \quad 94940 := 4 \times 10 + 9490 \times 10 \\
4949490 &:= 9 \times 10 + 494940 \times 10 & \Leftrightarrow & \quad 9494940 := 4 \times 10 + 949490 \times 10
\end{aligned}$$

$$\begin{aligned}
560 &:= 6 \times 10 + 50 \times 10 & \Leftrightarrow & \quad 650 := 5 \times 10 + 60 \times 10 \\
56560 &:= 6 \times 10 + 5650 \times 10 & \Leftrightarrow & \quad 65650 := 5 \times 10 + 6560 \times 10 \\
5656560 &:= 6 \times 10 + 565650 \times 10 & \Leftrightarrow & \quad 6565650 := 5 \times 10 + 656560 \times 10
\end{aligned}$$

$$\begin{aligned}
570 &:= 7 \times 10 + 50 \times 10 & \Leftrightarrow & \quad 750 := 5 \times 10 + 70 \times 10 \\
57570 &:= 7 \times 10 + 5750 \times 10 & \Leftrightarrow & \quad 75750 := 5 \times 10 + 7570 \times 10 \\
5757570 &:= 7 \times 10 + 575750 \times 10 & \Leftrightarrow & \quad 7575750 := 5 \times 10 + 757570 \times 10
\end{aligned}$$

$$\begin{aligned}
580 &:= 8 \times 10 + 50 \times 10 & \Leftrightarrow & \quad 850 := 5 \times 10 + 80 \times 10 \\
58580 &:= 8 \times 10 + 5850 \times 10 & \Leftrightarrow & \quad 85850 := 5 \times 10 + 8580 \times 10 \\
5858580 &:= 8 \times 10 + 585850 \times 10 & \Leftrightarrow & \quad 8585850 := 5 \times 10 + 858580 \times 10
\end{aligned}$$

$$\begin{aligned}
590 &:= 9 \times 10 + 50 \times 10 & \Leftrightarrow & \quad 950 := 5 \times 10 + 90 \times 10 \\
59590 &:= 9 \times 10 + 5950 \times 10 & \Leftrightarrow & \quad 95950 := 5 \times 10 + 9590 \times 10 \\
5959590 &:= 9 \times 10 + 595950 \times 10 & \Leftrightarrow & \quad 9595950 := 5 \times 10 + 959590 \times 10
\end{aligned}$$

$$\begin{aligned}
670 &:= 7 \times 10 + 60 \times 10 & \Leftrightarrow & \quad 760 := 6 \times 10 + 70 \times 10 \\
67670 &:= 7 \times 10 + 6760 \times 10 & \Leftrightarrow & \quad 76760 := 6 \times 10 + 7670 \times 10 \\
6767670 &:= 7 \times 10 + 676760 \times 10 & \Leftrightarrow & \quad 7676760 := 6 \times 10 + 767670 \times 10
\end{aligned}$$

$$\begin{aligned}
680 &:= 8 \times 10 + 60 \times 10 & \Leftrightarrow & \quad 860 := 6 \times 10 + 80 \times 10 \\
68680 &:= 8 \times 10 + 6860 \times 10 & \Leftrightarrow & \quad 86860 := 6 \times 10 + 8680 \times 10 \\
6868680 &:= 8 \times 10 + 686860 \times 10 & \Leftrightarrow & \quad 8686860 := 6 \times 10 + 868680 \times 10
\end{aligned}$$

$$\begin{aligned}
690 &:= 9 \times 10 + 60 \times 10 & \Leftrightarrow & \quad 960 := 6 \times 10 + 90 \times 10 \\
69690 &:= 9 \times 10 + 6960 \times 10 & \Leftrightarrow & \quad 96960 := 6 \times 10 + 9690 \times 10 \\
6969690 &:= 9 \times 10 + 696960 \times 10 & \Leftrightarrow & \quad 9696960 := 6 \times 10 + 969690 \times 10
\end{aligned}$$

$$\begin{aligned}
780 &:= 8 \times 10 + 70 \times 10 & \Leftrightarrow & \quad 870 := 7 \times 10 + 80 \times 10 \\
78780 &:= 8 \times 10 + 7870 \times 10 & \Leftrightarrow & \quad 87870 := 7 \times 10 + 8780 \times 10 \\
7878780 &:= 8 \times 10 + 787870 \times 10 & \Leftrightarrow & \quad 8787870 := 7 \times 10 + 878780 \times 10
\end{aligned}$$

$$\begin{aligned}
790 &:= 9 \times 10 + 70 \times 10 & \Leftrightarrow & \quad 970 := 7 \times 10 + 90 \times 10 \\
79790 &:= 9 \times 10 + 7970 \times 10 & \Leftrightarrow & \quad 97970 := 7 \times 10 + 9790 \times 10 \\
7979790 &:= 9 \times 10 + 797970 \times 10 & \Leftrightarrow & \quad 9797970 := 7 \times 10 + 979790 \times 10
\end{aligned}$$

$$\begin{aligned}
890 &:= 9 \times 10 + 80 \times 10 & \Leftrightarrow & \quad 980 := 8 \times 10 + 90 \times 10 \\
89890 &:= 9 \times 10 + 8980 \times 10 & \Leftrightarrow & \quad 98980 := 8 \times 10 + 9890 \times 10 \\
8989890 &:= 9 \times 10 + 898980 \times 10 & \Leftrightarrow & \quad 9898980 := 8 \times 10 + 989890 \times 10
\end{aligned}$$

4.1.2 Patterns in Self-Amicable Numbers

Below are some examples of patterns in self-amicable numbers. We have written only up to 4th step. Further steps follows by extending in an obvious way.

$$\begin{aligned}
132 &:= 1 \times 4 + 32 \times 4 & 264 &:= 2 \times 4 + 64 \times 4 \\
1332 &:= 1 \times 4 + 332 \times 4 & 2664 &:= 2 \times 4 + 664 \times 4 \\
13332 &:= 1 \times 4 + 3332 \times 4 & 26664 &:= 2 \times 4 + 6664 \times 4 \\
133332 &:= 1 \times 4 + 33332 \times 4 & 266664 &:= 2 \times 4 + 66664 \times 4
\end{aligned}$$

$$\begin{aligned}
198 &:= 1 \times 2 + 98 \times 2 & 297 &:= 2 \times 3 + 97 \times 3 \\
1998 &:= 1 \times 2 + 998 \times 2 & 2997 &:= 2 \times 3 + 997 \times 3 \\
19998 &:= 1 \times 2 + 9998 \times 2 & 29997 &:= 2 \times 3 + 9997 \times 3 \\
199998 &:= 1 \times 2 + 99998 \times 2 & 299997 &:= 2 \times 3 + 99997 \times 3
\end{aligned}$$

$$\begin{aligned}
231 &:= 2 \times 7 + 31 \times 7 & 396 &:= 3 \times 4 + 96 \times 4 \\
2331 &:= 2 \times 7 + 331 \times 7 & 3996 &:= 3 \times 4 + 996 \times 4 \\
23331 &:= 2 \times 7 + 3331 \times 7 & 39996 &:= 3 \times 4 + 9996 \times 4 \\
233331 &:= 2 \times 7 + 33331 \times 7 & 399996 &:= 3 \times 4 + 99996 \times 4
\end{aligned}$$

$$\begin{array}{ll}
462 := 4 \times 7 + 62 \times 7 & 693 := 6 \times 7 + 93 \times 7 \\
4662 := 4 \times 7 + 662 \times 7 & 6993 := 6 \times 7 + 993 \times 7 \\
46662 := 4 \times 7 + 6662 \times 7 & 69993 := 6 \times 7 + 9993 \times 7 \\
466662 := 4 \times 7 + 66662 \times 7 & 699993 := 6 \times 7 + 99993 \times 7 \\
\\
495 := 4 \times 5 + 95 \times 5 & 792 := 7 \times 8 + 92 \times 8 \\
4995 := 4 \times 5 + 995 \times 5 & 7992 := 7 \times 8 + 992 \times 8 \\
49995 := 4 \times 5 + 9995 \times 5 & 79992 := 7 \times 8 + 9992 \times 8 \\
499995 := 4 \times 5 + 99995 \times 5 & 799992 := 7 \times 8 + 99992 \times 8 \\
\\
594 := 5 \times 6 + 94 \times 6 & 891 := 8 \times 9 + 91 \times 9 \\
5994 := 5 \times 6 + 994 \times 6 & 8991 := 8 \times 9 + 991 \times 9 \\
59994 := 5 \times 6 + 9994 \times 6 & 89991 := 8 \times 9 + 9991 \times 9 \\
599994 := 5 \times 6 + 99994 \times 6 & 899991 := 8 \times 9 + 99991 \times 9
\end{array}$$

Below are some patterns obtained from the subsection 2.1.3 on "Amicable Numbers Multiples of 10". These are very obvious, but are good looking.

$$\begin{array}{ll}
110 := 1 \times 10 + 10 \times 10 & 220 := 2 \times 10 + 20 \times 10 \\
1110 := 1 \times 10 + 110 \times 10 & 2220 := 2 \times 10 + 220 \times 10 \\
11110 := 1 \times 10 + 1110 \times 10 & 22220 := 2 \times 10 + 2220 \times 10 \\
111110 := 1 \times 10 + 11110 \times 10 & 222220 := 2 \times 10 + 22220 \times 10 \\
1111110 := 1 \times 10 + 111110 \times 10 & 2222220 := 2 \times 10 + 222220 \times 10 \\
\\
330 := 3 \times 10 + 30 \times 10 & 440 := 4 \times 10 + 40 \times 10 \\
3330 := 3 \times 10 + 330 \times 10 & 4440 := 4 \times 10 + 440 \times 10 \\
33330 := 3 \times 10 + 3330 \times 10 & 44440 := 4 \times 10 + 4440 \times 10 \\
333330 := 3 \times 10 + 33330 \times 10 & 444440 := 4 \times 10 + 44440 \times 10 \\
3333330 := 3 \times 10 + 333330 \times 10 & 4444440 := 4 \times 10 + 444440 \times 10 \\
\\
550 := 5 \times 10 + 50 \times 10 & 660 := 6 \times 10 + 60 \times 10 \\
5550 := 5 \times 10 + 550 \times 10 & 6660 := 6 \times 10 + 660 \times 10 \\
55550 := 5 \times 10 + 5550 \times 10 & 66660 := 6 \times 10 + 6660 \times 10 \\
555550 := 5 \times 10 + 55550 \times 10 & 666660 := 6 \times 10 + 66660 \times 10 \\
5555550 := 5 \times 10 + 555550 \times 10 & 6666660 := 6 \times 10 + 666660 \times 10 \\
\\
770 := 7 \times 10 + 70 \times 10 & 880 := 8 \times 10 + 80 \times 10 \\
7770 := 7 \times 10 + 770 \times 10 & 8880 := 8 \times 10 + 880 \times 10 \\
77770 := 7 \times 10 + 7770 \times 10 & 88880 := 8 \times 10 + 8880 \times 10 \\
777770 := 7 \times 10 + 77770 \times 10 & 888880 := 8 \times 10 + 88880 \times 10 \\
7777770 := 7 \times 10 + 777770 \times 10 & 8888880 := 8 \times 10 + 888880 \times 10
\end{array}$$

$$\begin{aligned}
990 &:= 9 \times 10 + 90 \times 10 \\
9990 &:= 9 \times 10 + 990 \times 10 \\
99990 &:= 9 \times 10 + 9990 \times 10 \\
999990 &:= 9 \times 10 + 99990 \times 10 \\
9999990 &:= 9 \times 10 + 999990 \times 10
\end{aligned}$$

Alternatively, the above patterns can also be written as

$$\begin{array}{ll}
110 := 1 \times 10 + 10 \times 10 & 1110 := 1 \times 10 + 110 \times 10 \\
220 := 2 \times 10 + 20 \times 10 & 2220 := 2 \times 10 + 220 \times 10 \\
330 := 3 \times 10 + 30 \times 10 & 3330 := 3 \times 10 + 330 \times 10 \\
440 := 4 \times 10 + 40 \times 10 & 4440 := 4 \times 10 + 440 \times 10 \\
550 := 5 \times 10 + 50 \times 10 & 5550 := 5 \times 10 + 550 \times 10 \\
660 := 6 \times 10 + 60 \times 10 & 6660 := 6 \times 10 + 660 \times 10 \\
770 := 7 \times 10 + 70 \times 10 & 7770 := 7 \times 10 + 770 \times 10 \\
880 := 8 \times 10 + 80 \times 10 & 8880 := 8 \times 10 + 880 \times 10 \\
990 := 9 \times 10 + 90 \times 10 & 9990 := 9 \times 10 + 990 \times 10 \\
\\
11110 := 1 \times 10 + 1110 \times 10 & 111110 := 1 \times 10 + 11110 \times 10 \\
12120 := 2 \times 10 + 1210 \times 10 & 222220 := 2 \times 10 + 22220 \times 10 \\
13130 := 3 \times 10 + 1310 \times 10 & 333330 := 3 \times 10 + 33330 \times 10 \\
14140 := 4 \times 10 + 1410 \times 10 & 444440 := 4 \times 10 + 44440 \times 10 \\
15150 := 5 \times 10 + 1510 \times 10 & 555550 := 5 \times 10 + 55550 \times 10 \\
16160 := 6 \times 10 + 1610 \times 10 & 666660 := 6 \times 10 + 66660 \times 10 \\
17170 := 7 \times 10 + 1710 \times 10 & 777770 := 7 \times 10 + 77770 \times 10 \\
18180 := 8 \times 10 + 1810 \times 10 & 888880 := 8 \times 10 + 88880 \times 10 \\
19190 := 9 \times 10 + 1910 \times 10 & 999990 := 9 \times 10 + 99990 \times 10
\end{array}$$

$$\begin{aligned}
1111110 &:= 1 \times 10 + 111110 \times 10 \\
2222220 &:= 2 \times 10 + 222220 \times 10 \\
3333330 &:= 3 \times 10 + 333330 \times 10 \\
4444440 &:= 4 \times 10 + 444440 \times 10 \\
5555550 &:= 5 \times 10 + 555550 \times 10 \\
6666660 &:= 6 \times 10 + 666660 \times 10 \\
7777770 &:= 7 \times 10 + 777770 \times 10 \\
8888880 &:= 8 \times 10 + 888880 \times 10 \\
9999990 &:= 9 \times 10 + 999990 \times 10
\end{aligned}$$

4.2 Power-Type Patterns in Self-Amicable Numbers

In this case we have very few examples of patterns with powers.

$$100 := 10^2 + 0^2$$

$$101 := 10^2 + 1^2$$

$$10000 := 100^2 + 00^2$$

$$10001 := 100^2 + 01^2$$

$$1000000 := 1000^2 + 000^2$$

$$1000001 := 1000^2 + 001^2$$

$$100000000 := 10000^2 + 0000^2$$

$$100000001 := 10000^2 + 0001^2$$

$$1000 := 10^3 + 00^3$$

$$1001 := 10^3 + 01^3$$

$$1000000 := 100^3 + 0000^3$$

$$1000001 := 100^3 + 0001^3$$

$$1000000000 := 1000^3 + 000000^3$$

$$1000000001 := 1000^3 + 000001^3$$

$$10000 := 10^4 + 000^4$$

$$10001 := 10^4 + 001^4$$

$$100000000 := 100^4 + 000000^4$$

$$100000001 := 100^4 + 000001^4$$

$$1000000000000 := 1000^4 + 000000000^4$$

$$1000000000001 := 1000^4 + 000000001^4$$

$$48 := -4^2 + 8^2$$

$$140400 := -140^2 + 400^2$$

$$484848 := -484^2 + 848^2$$

$$14040000 := -1400^2 + 4000^2$$

$$4848484848 := -48484^2 + 84848^2$$

$$1404000000 := -14000^2 + 40000^2$$

$$48484848484848 := -4848484^2 + 8484848^2.$$

$$140400000000 := -140000^2 + 400000^2.$$

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