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Magic Crosses: Repeated and Non Repeated Entries

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Abstract

The idea of magic rectangles is well known in literature [1, 3, 4]. Using this idea we brought for the first time in history a new concept on magic crosses. The work is divided in two groups. One on ***orders (odd, odd)*** *and another on* ***orders (even, even)****. Within the* ***orders (odd, odd)****, the work is on magic crosses of type* $(3, 2n+3)$, $(5, 2n+5)$,... $n = 1, 2, ...$ *Within orders* ***orders (even, even)*** *the work is on magic crosses of orders* $(4n, 4m)$, $(4n, 2n+2)$, $2 \times (even, odd)$, *etc. In all the case, we used the same number of entries as of magic rectangles to bring magic squares. In case of lower rows and columns of magic crosses the entries are repeated. For non repeated entries we worked with orders (4,12), (5,15), (6,18), (8,24) and (10,30). In this case the, the magic squares are of equal magic sums. The inspiration of this is due to classical magic square of Nārāṇyana [2] done in 14th century (1356AD).*

Contents

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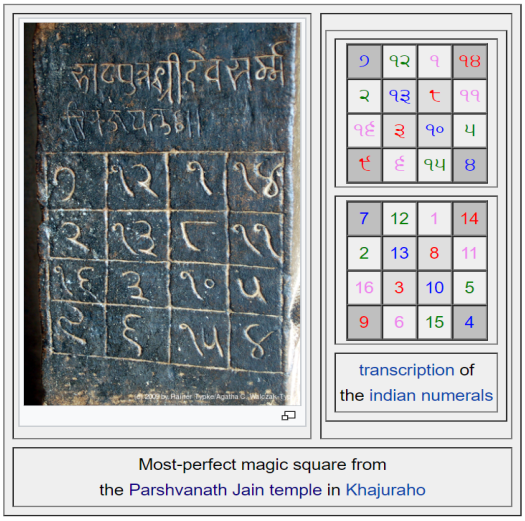
1 Historical Notes

The **Khajuraho magic square** of order 4 is famous in the literature as one of the most **most perfect magic square** of order 4¿ It is studied around 10th century. The original plate of this magic square seen at **Parshvanath Jain temple in Khajuraho** - (*Link: Wikipedia - https://goo.gl/nsYn2j*):

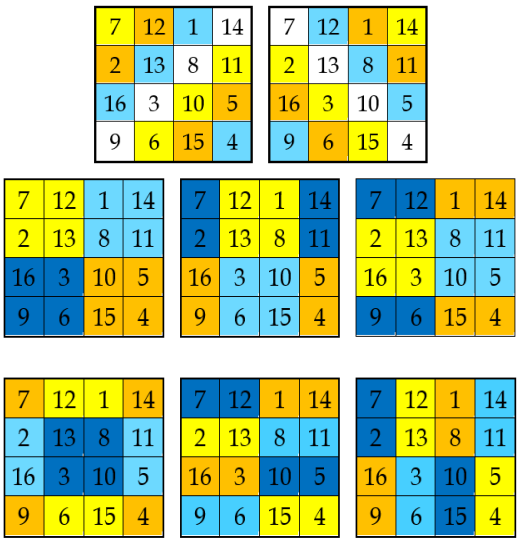


It is also pan diagonal magic square of order 4 given in example below.

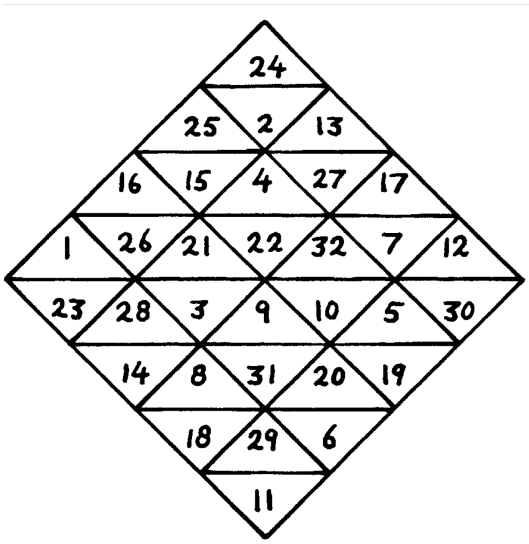
Example 1. *Let's rewrite **Khajuraho magic square** as pan magic square of order 4.*

		34	34	34	34
	7	12	1	14	34
34	2	13	8	11	34
34	16	3	10	5	34
34	9	6	15	4	34
	34	34	34	34	34

Below are some properties in colors resulting magic square sums for each color:



During 14th centaury (1356AD)[2] Nārānyana constructed a magic square of 32 numbers instead of 16. See below:



Vajra or Diamond

Making 45^o rotation on left, the above magic square can be written as

24		13		17		12		66
	2		27		7		30	66
25		4		32		5		66
	15		22		10		19	66
16		21		9		20		66
	26		3		31		6	66
1		28		8		29		66
	23		14		18		11	66
66	66	66	66	66	66	66	66	132

The real construction is based on two magic squares of order 4. Let's see how it constructed.

Example 2. *Let's consider modified version of Khajuraho's magic square of Example 1 given by*

		34	*34*	*34*	*34*
	1	*8*	*13*	*12*	*34*
34	*14*	*11*	*2*	*7*	*34*
34	*4*	*5*	*16*	*9*	*34*
34	*15*	*10*	*3*	*6*	*34*
	34	*34*	*34*	*34*	*34*

Let's divide the numbers 1 to 32 in two equal parts as:

1	4	5	8	9	12	13	16	17	20	21	24	25	28	29	32	264
2	3	6	7	10	11	14	15	18	19	22	23	26	27	30	31	264

For each row let's calculate a magic square of order 4 according to Example 2.

Example 3. *Let's consider modified version of* ***Khajuraho's magic square*** *of Example 1 given by*

		66	66	66	66
	1	16	25	24	66
66	28	21	4	13	66
66	8	9	32	17	66
66	29	20	5	12	66
	66	66	66	66	66

		66	66	66	66
	2	15	26	23	66
66	27	22	3	14	66
66	7	10	31	18	66
66	30	19	6	11	66
	66	66	66	66	66

Combining two magic squares of order 4 given in Example 3, we get a **magic rectangle** of order (4,8).

Example 4. *The **magic rectangle** of order (4,8) based on Example 3 for the numbers 1 to 32 is given by*

1	16	25	24	2	15	26	23	132
28	21	4	13	27	22	3	14	132
8	9	32	17	7	10	31	18	132
29	20	5	12	30	19	6	11	132
66	66	66	66	66	66	66	66	

According to Datta and Shing [2], there are 32 blocks of 8 elements giving the sums 132. See below these 32 blocks.

1	16	25	24	2	15	26	23
28	21	4	13	27	22	3	14
8	9	32	17	7	10	31	18
29	20	5	12	30	19	6	11

1	16	25	24	2	15	26	23
28	21	4	13	27	22	3	14
8	9	32	17	7	10	31	18
29	20	5	12	30	19	6	11

1	16	25	24	2	15	26	23
28	21	4	13	27	22	3	14
8	9	32	17	7	10	31	18
29	20	5	12	30	19	6	11

1	16	25	24	2	15	26	23
28	21	4	13	27	22	3	14
8	9	32	17	7	10	31	18
29	20	5	12	30	19	6	11

1	16	25	24	2	15	26	23
28	21	4	13	27	22	3	14
8	9	32	17	7	10	31	18
29	20	5	12	30	19	6	11

1	16	25	24	30	19	6	11
28	21	4	13	2	15	26	23
8	9	32	17	27	22	3	14
29	20	5	12	7	10	31	18

1	16	25	24	7	10	31	18
28	21	4	13	30	19	6	11
8	9	32	17	2	15	26	23
29	20	5	12	27	22	3	14

1	16	25	24	27	22	3	14
28	21	4	13	7	10	31	18
8	9	32	17	30	19	6	11
29	20	5	12	2	15	26	23

There are much more combinations of 8 numbers giving the sum 132, but we have written only obvious ones.

Thus, we observe that the **magic rectangle** given in Example 3 is fundamental in construction of Nārānyana's magic square with 32 numbers instead of 16. We can write this **magic rectangle** in a symmetric way as **magic cross**. Below are two different ways of writing **magic cross**:

Example 5. *The two magic crosses of order (4,8) are given by*

		1	28	8	29			66
		16	21	9	20			66
1	16	25	24	2	15	26	23	132
28	21	4	13	27	22	3	14	132
8	9	32	17	7	10	31	18	132
29	20	5	12	30	19	6	11	132
		26	3	31	6			66
		23	14	18	11			66
66	66	132	132	132	132	66	66	

		1	16	20	29			66
		28	21	9	8			66
1	28	14	23	2	27	13	24	132
16	21	3	26	15	22	4	25	132
20	9	31	6	19	10	32	5	132
29	8	18	11	30	7	17	12	132
		13	4	32	17			66
		24	25	5	12			66
66	66	132	132	132	132	66	66	

The aim of this paper is to work with **magic crosses** of different types, such as of **orders (odd, odd)** and of **orders (even, even)**. Within the orders (odd, odd), the work is on magic crosses of orders $(3, 2n+3)$, $(5, 2n+5)$,... $n = 1, 2, ...$ Within the orders (even, odd), the work is on magic crosses of orders $(4n, 4m)$, $(4n, 2n+2)$, $2 \times (even, odd)$, etc. In all the case, the same number of entries are the same as of magic rectangles. Moreover, in small rows and columns the entried are repeated. For non repeated entries, we worked with orders (4,12), (5,15), (6,18), (8,24) and (10,30). In this case the, the magic squares are of equal magic sums.

2 Magic Crosses: Repeated Entries

2.1 Magic Crosses of Order $(3, 2n+3)$

The magic crosses constructed in this section are of orders $(3, 2n+3)$, $n = 1, 2, 3, 4, 5, 6, 7$ and 8, i.e, from orders (3,5) to (3,19).

2.1.1 Magic Cross of Order (3,5)

Example 6. *A **magic cross** of order $(3, 5)$ is constructed based on magic rectangle of order $(3, 5)$ for the consecutive numbers 1 to 15. The bigger and smaller rows and columns are of sums 40 and 24 respectively. It is given by*

	14	1	9		24
14	10	4	5	7	40
1	3	8	13	15	40
9	11	12	6	2	40
	2	15	7		24
24	40	40	40	24	

2.1.2 Magic Cross of Order (3,7)

Example 7. *A **magic cross** of order $(3, 7)$ is constructed based on magic rectangle of order $(3, 7)$ for the consecutive numbers 1 to 21. The bigger and smaller rows and columns are of sums 77 and 33 respectively. It is given by*

		14	18	1			33
		2	19	12			33
1	12	13	6	17	20	8	77
18	19	15	11	7	3	4	77
14	2	5	16	9	10	21	77
		20	3	10			33
		8	4	21			33
33	33	77	77	77	33	33	

2.1.3 Magic Cross of Order (3,9)

Example 8. *A **magic cross** of order* $(3,9)$ *is constructed based on magic rectangle of order* $(3,9)$ *for the consecutive numbers 1 to 27. The bigger and smaller rows and columns are of sums 126 and 42 respectively. It is given by*

			22	12	8				42
			9	23	10				42
			11	7	24				42
4	18	20	13	27	2	22	9	11	126
21	5	16	3	14	25	12	23	7	126
17	19	6	26	1	15	8	10	24	126
			4	21	17				42
			18	5	19				42
			20	16	6				42
42	42	42	126	126	126	42	42	42	

2.1.4 Magic Cross of Order (3,11)

Example 9. *A **magic cross** of order* $(3,11)$ *is constructed based on magic rectangle of order* $(3,11)$ *for the consecutive numbers 1 to 33. The bigger and smaller rows and columns are of sums 187 and 51 respectively. It is given by*

				28	1	22					51
				2	20	29					51
				18	30	3					51
				7	23	21					51
22	29	3	7	24	9	26	13	16	32	6	187
1	20	30	23	19	17	15	11	4	14	33	187
28	2	18	21	8	25	10	27	31	5	12	187
				27	11	13					51
				16	4	31					51
				32	14	5					51
				6	33	12					51
51	51	51	51	187	187	187	51	51	51	51	

2.1.5 Magic Cross of Order (3,13)

Example 10. *A **magic cross** of order* $(3,13)$ *is constructed based on magic rectangle of order* $(3,13)$ *for the consecutive numbers 1 to 39. The bigger and smaller rows and columns are of sums 260 and 60 respectively. It is given by*

					16	12	32						60
					33	13	14						60
					19	4	37						60
					38	17	5						60
					15	39	6						60
34	2	21	26	8	22	10	31	16	33	37	5	15	260
1	23	36	27	28	29	20	11	12	13	4	17	39	260
25	35	3	7	24	9	30	18	32	14	19	38	6	260
					25	1	34						60
					2	23	35						60
					21	36	3						60
					7	27	26						60
					24	28	8						60
60	60	60	60	60	260	260	260	60	60	60	60	60	

2.1.6 Magic Cross of Order (3,15)

Example 11. *A **magic cross** of order* $(3,15)$ *is constructed based on magic rectangle of order* $(3,15)$ *for the consecutive numbers 1 to 45. The bigger and smaller rows and columns are of sums 345 and 69 respectively. It is given by*

						36	14	19						69
						17	15	37						69
						22	5	42						69
						43	6	20						69
						44	7	18						69
						16	8	45						69
1	28	26	4	9	27	25	12	35	36	17	22	43	44	16	345
38	39	40	41	31	32	33	23	13	14	15	5	6	7	8	345
30	2	3	24	29	10	11	34	21	19	37	42	20	18	45	345
						30	38	1						69
						2	39	28						69
						26	40	3						69
						4	41	24						69
						9	31	29						69
						27	32	10						69
69	69	69	69	69	69	345	345	345	69	69	69	69	69	69	

2.1.7 Magic Cross of Order (3,17)

Example 12. *A **magic cross** of order* $(3,17)$ *is constructed based on magic rectangle of order* $(3,17)$ *for the consecutive numbers 1 to 51. The bigger and smaller rows and columns are of sums 442 and 78 respectively. It is given by*

							15	22	41								78
							42	16	20								78
							18	17	43								78
							48	5	25								78
							23	6	49								78
							50	7	21								78
							51	8	19								78
1	31	29	4	9	32	11	28	39	40	15	42	18	25	49	50	19	442
44	45	46	47	35	36	30	38	26	14	22	16	17	5	6	7	8	442
33	2	3	27	34	10	37	12	13	24	41	20	43	48	23	21	51	442
							1	44	33								78
							31	45	2								78
							29	46	3								78
							4	47	27								78
							9	35	34								78
							32	36	10								78
							11	30	37								78
78	78	78	78	78	78	78	442	442	442	78	78	78	78	78	78	78	

2.1.8 Magic Cross of Order (3,19)

Example 13. *A **magic cross** of order* $(3,19)$ *is constructed based on magic rectangle of order* $(3,19)$ *for the consecutive numbers 1 to 57. The bigger and smaller rows and columns are of sums 551 and 87 respectively. It is given by*

								25	17	45								87
								46	18	23								87
								21	19	47								87
								28	6	53								87
								54	7	26								87
								55	8	24								87
								9	22	56								87
								20	57	10								87
38	49	3	32	5	11	35	13	42	15	44	25	46	21	28	54	24	56	10	551
1	36	50	51	52	39	40	41	31	29	27	17	18	19	6	7	8	22	57	551
48	2	34	4	30	37	12	33	14	43	16	45	23	47	53	26	55	9	20	551
								38	1	48								87
								49	36	2								87
								3	50	34								87
								32	51	4								87
								5	52	30								87
								11	39	37								87
								35	40	12								87
								33	41	13								87
87	87	87	87	87	87	87	87	551	551	551	87	87	87	87	87	87	87	87	

2.2 Magic Crosses of Order $(5, 2n+5)$

The **magic crosses** constructed in this section are of order $(3, 2n+5)$, and are magic crosses. See below some examples.

2.2.1 Magic Cross of Order (5,7)

Example 14. *A **magic cross** of order $(5,7)$ constructed based on a **magic rectangle** of order $(5,7)$ for the consecutive numbers 1 to 35. The bigger and smaller rows and columns are of sums 126 and 90 respectively. It is given by*

	15	1	28	32	14		90
15	26	13	6	20	24	22	126
1	33	27	11	31	19	4	126
28	2	29	18	7	34	8	126
32	17	5	25	9	3	35	126
14	12	16	30	23	10	21	126
	21	35	8	4	22		90
90	126	126	126	126	126	90	

2.2.2 Magic Cross of Order (5,9)

Example 15. *A **magic cross** of order $(5,9)$ constructed based on a **magic rectangle** of order $(5,9)$ for the consecutive numbers 1 to 45. The bigger and smaller rows and columns are of sums 207 and 115 respectively. It is given by*

		20	17	35	42	1			115
		43	22	33	2	15			115
20	43	19	21	7	12	9	31	45	207
17	22	18	38	14	40	10	44	4	207
35	33	5	16	23	30	41	13	11	207
42	2	36	6	32	8	28	24	29	207
1	15	37	34	39	25	27	3	26	207
		3	24	13	44	31			115
		26	29	11	4	45			115
115	115	207	207	207	207	207	115	115	

2.2.3 Magic Cross of Order (5,11)

Example 16. *A **magic cross** of order $(5,11)$ constructed based on a **magic rectangle** of order $(5,11)$ for the consecutive numbers 1 to 55. The bigger and smaller rows and columns are of sums 308 and 140 respectively. It is given by*

			53	4	16	38	29				140
			36	54	14	31	5				140
			55	34	12	6	33				140
23	51	27	7	19	9	15	13	53	36	55	308
50	25	18	21	26	17	48	11	4	54	34	308
44	42	40	24	46	28	10	32	16	14	12	308
22	2	52	45	8	39	30	35	38	31	6	308
1	20	3	43	41	47	37	49	29	5	33	308
			1	22	44	50	23				140
			20	2	42	25	51				140
			3	52	40	18	27				140
140	140	140	308	308	308	308	308	140	140	140	

2.2.4 Magic Cross of Order (5,13)

Example 17. *A **magic cross** of order* $(5,13)$ *constructed based on a **magic rectangle** of order* $(5,13)$ *for the consecutive numbers 1 to 65. The bigger and smaller rows and columns are of sums 429 and 165 respectively. It is given by*

				1	25	51	60	28				165
				30	61	49	2	23				165
				62	32	47	21	3				165
				52	27	53	7	26				165
1	30	62	52	54	9	10	11	16	40	63	43	38	429
25	61	32	27	8	22	20	18	42	59	45	64	6	429
51	49	47	53	29	31	33	35	37	13	19	17	15	429
60	2	21	7	24	48	46	44	58	39	34	5	41	429
28	23	3	26	50	55	56	57	12	14	4	36	65	429
				14	39	13	59	40				165
				4	34	19	45	63				165
				36	5	17	64	43				165
				65	41	15	6	38				165
165	165	165	165	429	429	429	429	429	165	165	165	165	

2.2.5 Magic Cross of Order (5,15)

Example 18. *A **magic cross** of order* $(5,15)$ *constructed based on a **magic rectangle** of order* $(5,15)$ *for the consecutive numbers 1 to 75. The bigger and smaller rows and columns are of sums 570 and 190 respectively. It is given by*

					31	30	60	68	1						190
					33	28	58	69	2						190
					3	70	56	26	35						190
					71	37	54	4	24						190
					61	32	59	9	29						190
31	33	3	71	61	27	11	12	13	19	47	52	41	74	75	570
30	28	70	37	32	34	25	23	21	66	67	72	50	7	8	570
60	58	56	54	59	62	36	38	40	14	17	22	20	18	16	570
68	69	26	4	9	10	55	53	51	42	44	39	6	48	46	570
1	2	35	24	29	57	63	64	65	49	15	5	73	43	45	570
					15	44	17	67	47						190
					5	39	22	72	52						190
					73	6	20	50	41						190
					43	48	18	7	74						190
					45	46	16	8	75						190
190	190	190	190	190	570	570	570	570	570	190	190	190	190	190	

2.2.6 Magic Cross of Order (5,17)

Example 19. *A **magic cross** of order* $(5,17)$ *constructed based on a **magic rectangle** of order* $(5,17)$ *for the consecutive numbers 1 to 85. The bigger and smaller rows and columns are of sums 731 and 215 respectively. It is given by*

						36	33	67	78	1						215
						79	38	65	2	31						215
						3	29	63	40	80						215
						81	42	61	27	4						215
						69	35	68	34	9						215
						10	32	66	37	70						215
36	79	3	81	69	10	39	41	13	24	15	16	77	82	6	55	85	731
33	38	29	42	35	32	30	72	26	74	22	49	52	59	46	84	8	731
67	65	63	61	68	66	11	28	43	58	75	20	18	25	23	21	19	731
78	2	40	27	34	37	64	12	60	14	56	54	51	44	57	48	53	731
1	31	80	4	9	70	71	62	73	45	47	76	17	5	83	7	50	731
						76	54	20	49	16						215
						17	51	18	52	77						215
						5	44	25	59	82						215
						83	57	23	46	6						215
						7	48	21	84	55						215
						50	53	19	8	85						215
215	215	215	215	215	215	731	731	731	731	731	215	215	215	215	215	215	

2.2.7 Magic Cross of Order (5,19)

Example 20. *A **magic cross** of order* $(5,19)$ *constructed based on a **magic rectangle** of order* $(5,19)$ *for the consecutive numbers 1 to 95. The bigger and smaller rows and columns are of sums 912 and 240 respectively. It is given by*

							18	54	23	61	84							240
							85	59	21	56	19							240
							91	66	28	49	6							240
							7	51	26	64	92							240
							93	8	24	62	53							240
							60	94	22	55	9							240
							95	58	20	10	57							240
39	87	43	4	90	77	12	13	31	15	27	25	18	85	91	7	93	60	95	912
86	41	34	32	47	40	35	33	46	29	82	17	54	59	66	51	8	94	58	912
76	74	72	70	68	75	73	44	80	48	16	52	23	21	28	26	24	22	20	912
38	2	88	45	30	37	42	79	14	67	50	63	61	56	49	64	62	55	10	912
1	36	3	89	5	11	78	71	69	81	65	83	84	19	6	92	53	9	57	912
							1	38	76	86	39							240
							36	2	74	41	87							240
							3	88	72	34	43							240
							89	45	70	32	4							240
							5	30	68	47	90							240
							11	37	75	40	77							240
							78	42	73	35	12							240
240	240	240	240	240	240	240	912	912	912	912	912	240	240	240	240	240	240	240	

2.3 Magic Crosses of Order $(7, 2n+7)$

2.3.1 Magic Cross of Order (7,9)

Example 21. *A magic cross of order* $(7,9)$ *constructed based on a magic rectangle of order* $(7,9)$ *for the consecutive numbers 1 to 63. The bigger and smaller rows and columns are of sums 288 and 224 respectively. It is given by*

		60	13	29	49	45	22	6		224
	58	7	63	2	14	8	55	21	60	288
	42	48	10	53	5	47	18	52	13	288
	19	33	23	34	39	40	27	44	29	288
	15	61	36	38	32	26	28	3	49	288
	35	20	37	24	25	30	41	31	45	288
	51	12	46	17	59	11	54	16	22	288
	4	43	9	56	50	62	1	57	6	288
		4	51	35	15	19	42	58		224
224	288	288	288	288	288	288	288	224		

2.3.2 Magic Cross of Order (7,11)

Example 22. *A magic cross of order* $(7,11)$ *constructed based on a magic rectangle of order* $(7,11)$ *for the consecutive numbers 1 to 77. The bigger and smaller rows and columns are of sums 429 and 273 respectively. It is given by*

		71	18	54	36	27	62	5			273
		69	64	28	55	34	20	3			273
75	73	4	67	10	17	2	33	8	71	69	429
58	16	52	22	57	6	65	12	59	18	64	429
44	51	25	29	46	31	48	35	38	54	28	429
23	42	63	1	41	39	37	77	15	36	55	429
50	24	40	43	30	47	32	49	53	27	34	429
14	60	19	66	13	72	21	56	26	62	20	429
9	7	70	45	76	61	68	11	74	5	3	429
		9	58	50	23	44	14	75			273
		7	16	24	42	51	60	73			273
273	273	429	429	429	429	429	429	429	273	273	

2.3.3 Magic Cross of Order (7,13)

Example 23. *A magic cross of order* $(7,13)$ *constructed based on a magic rectangle of order* $(7,13)$ *for the consecutive numbers 1 to 91. The bigger and smaller rows and columns are of sums 598 and 322 respectively. It is given by*

			88	17	63	69	45	30	10				322
			5	74	31	43	64	22	83				322
			86	19	41	65	32	71	8				322
84	9	82	1	90	3	20	11	80	39	88	5	86	598
21	70	62	78	15	76	7	68	25	66	17	74	19	598
60	28	47	52	34	48	36	57	42	59	63	31	41	598
27	49	23	79	54	55	46	37	38	13	69	43	65	598
51	61	29	33	50	35	56	44	58	40	45	64	32	598
73	18	75	26	67	24	85	16	77	14	30	22	71	598
6	87	4	53	12	81	72	89	2	91	10	83	8	598
			6	73	51	27	60	21	84				322
			87	18	61	49	28	70	9				322
			4	75	29	23	47	62	82				322
322	322	322	598	598	598	598	598	598	598	322	322	322	

2.3.4 Magic Cross of Order (7,15)

Example 24. *A magic cross of order* $(7,15)$ *constructed based on a magic rectangle of order* $(7,15)$ *for the consecutive numbers 1 to 105. The bigger and smaller rows and columns are of sums 795 and 371 respectively. It is given by*

				35	26	52	95	72	86	5					371
				10	81	73	21	50	36	100					371
				97	24	74	37	48	84	7					371
				94	87	46	38	75	27	4					371
102	99	6	101	15	92	13	23	3	104	1	35	10	97	94	795
79	22	70	20	76	29	78	8	88	17	90	26	81	24	87	795
31	58	56	34	39	57	55	42	65	66	47	52	73	74	46	795
68	69	85	11	61	62	63	53	43	44	45	95	21	37	38	795
60	32	33	54	59	40	41	64	51	49	67	72	50	48	75	795
19	82	25	80	16	89	18	98	28	77	30	86	36	84	27	795
12	9	96	71	105	2	103	83	93	14	91	5	100	7	4	795
				12	19	60	68	31	79	102					371
				9	82	32	69	58	22	99					371
				96	25	33	85	56	70	6					371
				71	80	54	11	34	20	101					371
371	371	371	371	795	795	795	795	795	795	795	371	371	371	371	

2.3.5 Magic Cross of Order (7,17)

Example 25. *A magic cross of order* $(7,17)$ *constructed based on a magic rectangle of order* $(7,17)$ *for the consecutive numbers 1 to 119 The bigger and smaller rows and columns are of sums 1020 and 420 respectively. It is given by*

					110	27	35	78	67	95	8						420
					11	92	65	79	36	24	113						420
					108	80	63	29	37	97	6						420
					13	90	38	115	61	22	81						420
					119	18	43	69	68	86	17						420
110	11	108	13	119	2	117	4	26	14	105	16	103	39	114	7	112	1020
27	92	80	90	18	101	20	99	9	89	32	87	34	98	23	96	25	1020
35	65	63	38	43	66	45	62	73	74	49	76	52	59	83	84	53	1020
78	79	29	115	69	70	64	72	60	48	56	50	51	5	91	41	42	1020
67	36	37	61	68	44	71	46	47	58	75	54	77	82	57	55	85	1020
95	24	97	22	86	33	88	31	111	21	100	19	102	30	40	28	93	1020
8	113	6	81	17	104	15	106	94	116	3	118	1	107	12	109	10	1020
					1	102	77	51	52	34	103						420
					107	30	82	5	59	98	39						420
					12	40	57	91	83	23	114						420
					109	28	55	41	84	96	7						420
					10	93	85	42	53	25	112						420
420	420	420	420	420	1020	1020	1020	1020	1020	1020	1020	420	420	420	420	420	

2.3.6 Magic Cross of Order (7,19)

Example 26. *A magic cross of order* $(7,19)$ *constructed based on a magic rectangle of order* $(7,19)$ *for the consecutive numbers 1 to 133. The bigger and smaller rows and columns are of sums 1273 and 469*

respectively. It is given by

						129	100	76	39	86	24	15							469
						125	28	87	74	40	104	11							469
						8	88	41	107	72	31	122							469
						127	26	70	13	42	102	89							469
						6	109	43	90	68	33	120							469
						115	38	49	77	75	114	1							469
129	125	8	127	6	115	18	117	16	29	4	131	2	133	14	45	12	123	119	1273
100	28	88	26	109	38	97	36	99	10	111	22	113	20	101	32	103	30	110	1273
76	87	41	70	43	49	73	51	80	53	82	63	84	59	66	92	62	94	48	1273
39	74	107	13	90	77	78	79	69	67	65	55	56	57	44	121	27	60	95	1273
86	40	72	42	68	75	50	71	52	81	54	83	61	85	91	64	93	47	58	1273
24	104	31	102	33	114	21	112	23	124	35	98	37	96	25	108	46	106	34	1273
15	11	122	89	120	1	132	3	130	105	118	17	116	19	128	7	126	9	5	1273
						19	96	85	57	59	20	133							469
						128	25	91	44	66	101	14							469
						7	108	64	121	92	32	45							469
						126	46	93	27	62	103	12							469
						9	106	47	60	94	30	123							469
						5	34	58	95	48	110	119							469
469	469	469	469	469	469	1273	1273	1273	1273	1273	1273	1273	469	469	469	469	469	469	

2.4 Magic Crosses of Order $(9, 2n+9)$

2.4.1 Magic Cross of Order (9,11)

Example 27. *A magic cross of order* $(9,11)$ *constructed based on a magic rectangle of order* $(9,11)$ *for the consecutive numbers 1 to 99. The bigger and smaller rows and columns are of sums 550 and 450 respectively. It is given by*

	99	78	45	72	66	44	23	12	11		450
99	3	95	2	96	17	92	10	38	9	89	550
78	64	16	87	15	6	19	79	18	80	88	550
45	73	49	29	41	31	37	35	75	58	77	550
72	47	40	43	48	39	70	33	26	76	56	550
66	86	7	46	68	50	32	54	93	14	34	550
44	24	74	67	30	61	52	57	60	53	28	550
23	42	25	65	63	69	59	71	51	27	55	550
12	20	82	21	81	94	85	13	84	36	22	550
11	91	62	90	8	83	4	98	5	97	1	550
	1	22	55	28	34	56	77	88	89		450
450	550	550	550	550	550	550	550	550	550	450	

2.4.2 Magic Cross of Order (9,13)

Example 28. *A magic cross of order* $(9,13)$ *constructed based on a magic rectangle of order* $(9,13)$ *for the consecutive numbers 1 to 117. The bigger and smaller rows and columns are of sums 767 and 531 respectively. It is given by*

		12	93	27	51	77	86	54	15	116			531
		108	75	56	87	23	28	49	101	4			531
12	108	8	105	11	109	20	113	3	117	45	114	2	767
93	75	97	26	94	22	7	18	102	14	99	17	103	767
27	56	88	78	80	35	36	37	42	66	89	69	64	767
51	87	58	53	34	48	46	44	68	85	71	90	32	767
77	23	112	79	55	57	59	61	63	39	6	95	41	767
86	28	47	33	50	74	72	70	84	65	60	31	67	767
54	49	29	52	76	81	82	83	38	40	30	62	91	767
15	101	19	104	16	100	111	96	24	92	21	43	25	767
116	4	73	1	115	5	98	9	107	13	110	10	106	767
		10	43	62	31	95	90	69	17	114			531
		106	25	91	67	41	32	64	103	2			531
531	531	767	767	767	767	767	767	767	767	767	531	531	

2.4.3 Magic Cross of Order (9,15)

Example 29. *A magic cross of order* $(9,15)$ *constructed based on a magic rectangle of order* $(9,15)$ *for the consecutive numbers 1 to 135. The bigger and smaller rows and columns are of sums 1020 and 612 respectively. It is given by*

			135	106	61	60	90	98	31	16	15				612
			3	88	63	58	118	99	32	28	123				612
			131	20	33	100	11	56	65	110	86				612
135	3	131	7	134	4	130	23	126	12	122	9	50	13	121	1020
106	88	20	114	17	117	21	8	25	109	29	112	26	108	120	1020
61	63	33	101	91	57	41	42	43	49	77	82	71	104	105	1020
60	58	100	67	62	64	55	53	51	96	97	102	80	37	38	1020
90	118	11	84	89	92	66	68	70	44	47	52	125	18	46	1020
98	99	56	34	39	40	85	83	81	72	74	69	36	78	76	1020
31	32	65	54	59	87	93	94	95	79	45	35	103	73	75	1020
16	28	110	24	107	27	111	128	115	19	119	22	116	48	30	1020
15	123	86	127	14	124	10	113	6	132	2	129	5	133	1	1020
			5	116	103	36	125	80	71	26	50				612
			133	48	73	78	18	37	104	108	13				612
			1	30	75	76	46	38	105	120	121				612
612	612	612	1020	1020	1020	1020	1020	1020	1020	1020	1020	612	612	612	

2.4.4 Magic Cross of Order (9,17)

Example 30. *A magic cross of order* $(9,17)$ *constructed based on a magic rectangle of order* $(9,17)$ *for the consecutive numbers 1 to 153. The bigger and smaller rows and columns are of sums 1309 and 693 respectively. It is given by*

				16	121	70	67	101	112	35	19	152					693
				140	99	113	72	31	36	65	133	4					693
				12	125	37	63	148	74	114	23	97					693
				144	27	115	76	95	61	38	129	8					693
16	140	12	144	17	139	13	143	26	147	5	151	1	146	57	150	2	1309
121	99	125	27	120	32	124	28	9	24	132	20	136	25	131	21	135	1309
70	113	37	115	103	44	73	75	47	58	49	50	111	116	40	89	119	1309
67	72	63	76	69	66	64	106	60	108	56	83	86	93	80	118	42	1309
101	31	148	95	102	100	45	62	77	92	109	54	52	59	6	123	53	1309
112	36	74	61	68	71	98	46	94	48	90	88	85	78	91	82	87	1309
35	65	114	38	43	104	105	96	107	79	81	110	51	39	117	41	84	1309
19	133	23	129	18	134	22	130	145	126	30	122	34	127	29	55	33	1309
152	4	97	8	153	3	149	7	128	11	141	15	137	10	142	14	138	1309
				10	127	39	78	59	93	116	25	146					693
				142	29	117	91	6	80	40	131	57					693
				14	55	41	82	123	118	89	21	150					693
				138	33	84	87	53	42	119	135	2					693
693	693	693	693	1309	1309	1309	1309	1309	1309	1309	1309	1309	693	693	693	693	

2.4.5 Magic Cross of Order (9,19)

Example 31. *A magic cross of order* $(9,19)$ *constructed based on a magic rectangle of order* $(9,19)$ *for the consecutive numbers 1 to 171. The bigger and smaller rows and columns are of sums 1634 and 774 respectively. It is given by*

					171	134	77	124	114	76	39	20	19					774
					3	112	125	79	150	40	74	36	155					774
					167	24	81	72	15	126	41	138	110					774
					7	146	42	70	108	83	127	32	159					774
					163	28	128	85	106	68	43	142	11					774
171	3	167	7	163	2	168	6	164	29	160	14	156	18	161	13	62	17	153	1634
134	112	24	146	28	151	23	147	27	10	31	139	35	135	30	140	34	136	152	1634
77	125	81	42	128	115	50	51	69	53	65	63	56	123	129	45	131	98	133	1634
124	79	72	70	85	78	73	71	84	67	120	55	92	97	104	89	46	132	96	1634
114	150	15	108	106	113	111	82	118	86	54	90	61	59	66	64	157	22	58	1634
76	40	126	83	68	75	80	117	52	105	88	101	99	94	87	102	100	93	48	1634
39	74	41	127	43	49	116	109	107	119	103	121	122	57	44	130	91	47	95	1634
20	36	138	32	142	37	137	33	141	162	145	25	149	21	144	26	148	60	38	1634
19	155	110	159	11	154	16	158	12	143	8	166	4	170	9	165	5	169	1	1634
					9	144	44	87	66	104	129	30	161					774
					165	26	130	102	64	89	45	140	13					774
					5	148	91	100	157	46	131	34	62					774
					169	60	47	93	22	132	98	136	17					774
					1	38	95	48	58	96	133	152	153					774
774	774	774	774	774	1634	1634	1634	1634	1634	1634	1634	1634	1634	774	774	774	774	774	

2.5 Magic Cross of Order$(11, 2n + 11)$

2.5.1 Magic Cross of Order (11,13)

Example 32. *A magic cross of order* $(11, 13)$ *constructed based on a magic rectangle of order* $(11, 13)$ *for the consecutive numbers 1 to 143. The bigger and smaller rows and columns are of sums 936 and 782 respectively. It is given by*

	138	19	112	45	67	91	58	97	34	123	8		792
136	9	134	1	142	3	20	63	132	13	140	5	138	936
21	122	23	130	80	128	7	120	25	118	17	126	19	936
110	35	108	27	116	29	46	37	106	65	114	31	112	936
47	96	88	104	41	102	33	94	51	92	43	100	45	936
86	54	73	78	60	74	62	83	68	85	89	57	67	936
53	75	49	105	15	133	72	11	129	39	95	69	91	936
77	87	55	59	76	61	82	70	84	66	71	90	58	936
99	44	101	52	93	50	111	42	103	40	56	48	97	936
32	113	30	79	38	107	98	115	28	117	36	109	34	936
125	18	127	26	119	24	137	16	64	14	121	22	123	936
6	139	4	131	12	81	124	141	2	143	10	135	8	936
	6	125	32	99	77	53	86	47	110	21	136		792
792	936	936	936	936	936	936	936	936	936	936	936	792	

2.5.2 Magic Cross of Order (11,15)

Example 33. *A magic cross of order* $(11, 15)$ *constructed based on a magic rectangle of order* $(11, 15)$ *for the consecutive numbers 1 to 165. The bigger and smaller rows and columns are of sums 1245 and 913 respectively. It is given by*

		157	24	127	54	104	67	78	114	37	144	7			913
		154	147	124	117	76	68	105	57	34	27	4			913
162	159	6	161	15	152	13	23	3	74	1	155	10	157	154	1245
139	22	145	20	91	29	138	8	148	17	150	26	141	24	147	1245
132	129	36	131	45	122	43	53	33	134	31	65	40	127	124	1245
109	52	100	50	106	59	108	38	118	47	120	56	111	54	117	1245
61	88	86	64	69	87	85	72	95	96	77	82	103	104	76	1245
98	99	115	41	136	2	93	83	73	164	30	125	51	67	68	1245
90	62	63	84	89	70	71	94	81	79	97	102	80	78	105	1245
49	112	55	110	46	119	48	128	58	107	60	116	66	114	57	1245
42	39	126	101	135	32	133	113	123	44	121	35	130	37	34	1245
19	142	25	140	16	149	18	158	28	137	75	146	21	144	27	1245
12	9	156	11	165	92	163	143	153	14	151	5	160	7	4	1245
		12	19	42	49	90	98	61	109	132	139	162			913
		9	142	39	112	62	99	88	52	129	22	159			913
913	913	1245	1245	1245	1245	1245	1245	1245	1245	1245	1245	1245	913	913	

2.5.3 Magic Cross of Order (11,17)

Example 34. *A magic cross of order* $(11,17)$ *constructed based on a magic rectangle of order* $(11,17)$ *for the consecutive numbers 1 to 187. The bigger and smaller rows and columns are of sums 1598 and 1034 respectively. It is given by*

			182	23	148	57	117	159	91	125	46	74	12				1034
			7	164	75	130	118	41	89	62	143	28	177				1034
			180	25	146	59	87	127	119	76	44	161	10				1034
178	11	176	13	187	2	185	4	26	14	173	16	171	73	182	7	180	1598
27	160	114	158	18	169	20	167	9	157	32	155	34	166	23	164	25	1598
144	45	142	47	153	36	151	38	60	48	139	50	137	39	148	75	146	1598
112	126	63	124	52	135	54	133	43	123	66	121	68	132	57	130	59	1598
69	99	97	72	77	100	79	96	107	108	83	110	86	93	117	118	87	1598
61	147	29	183	103	104	98	106	94	82	90	84	85	5	159	41	127	1598
101	70	71	95	102	78	105	80	81	92	109	88	111	116	91	89	119	1598
129	58	131	56	120	67	122	65	145	55	134	53	136	64	125	62	76	1598
42	113	40	149	51	138	49	140	128	150	37	152	35	141	46	143	44	1598
163	24	165	22	154	33	156	31	179	21	168	19	170	30	74	28	161	1598
8	181	6	115	17	172	15	174	162	184	3	186	1	175	12	177	10	1598
			8	163	42	129	101	61	69	112	144	27	178				1034
			181	24	113	58	70	147	99	126	45	160	11				1034
			6	165	40	131	71	29	97	63	142	114	176				1034
1034	1034	1034	1598	1598	1598	1598	1598	1598	1598	1598	1598	1598	1598	1034	1034	1034	

2.5.4 Magic Cross of Order (11,19)

Example 35. *A magic cross of order* $(11,19)$ *constructed based on a magic rectangle of order* $(11,19)$ *for the consecutive numbers 1 to 209. The bigger and smaller rows and columns are of sums 1995 and 1155 respectively. It is given by*

				197	32	83	70	130	159	102	146	45	184	7					1155
				12	179	50	141	100	65	131	84	164	27	202					1155
				199	30	161	68	132	98	85	144	47	182	9					1155
				195	186	157	148	86	133	96	72	43	34	5					1155
205	201	8	203	6	191	18	193	16	29	4	207	2	95	14	197	12	199	195	1995
176	28	183	26	128	38	173	36	175	10	187	22	189	20	177	32	179	30	186	1995
167	163	46	165	44	153	56	155	54	67	42	169	40	171	52	83	50	161	157	1995
138	66	126	64	147	76	135	74	137	48	149	60	151	58	139	70	141	68	148	1995
114	125	79	108	81	87	111	89	118	91	120	101	122	97	104	130	100	132	86	1995
77	112	145	51	185	1	116	117	107	105	103	93	94	209	25	159	65	98	133	1995
124	78	110	80	106	113	88	109	90	119	92	121	99	123	129	102	131	85	96	1995
62	142	69	140	71	152	59	150	61	162	73	136	75	134	63	146	84	144	72	1995
53	49	160	127	158	39	170	41	168	143	156	55	154	57	166	45	164	47	43	1995
24	180	31	178	33	190	21	188	23	200	35	174	37	172	82	184	27	182	34	1995
15	11	198	13	196	115	208	3	206	181	194	17	192	19	204	7	202	9	5	1995
				15	24	53	62	124	77	114	138	167	176	205					1155
				11	180	49	142	78	112	125	66	163	28	201					1155
				198	31	160	69	110	145	79	126	46	183	8					1155
				13	178	127	140	80	51	108	64	165	26	203					1155
1155	1155	1155	1155	1995	1995	1995	1995	1995	1995	1995	1995	1995	1995	1995	1155	1155	1155	1155	

2.6 Magic Cross of Order $(13, 2n + 13)$

2.6.1 Magic Cross of Order (13,15)

Example 36. *A magic cross of order* $(13, 15)$ *constructed based on a magic rectangle of order* $(13, 15)$ *for the consecutive numbers 1 to 195. The bigger and smaller rows and columns are of sums 1470 and 1274 respectively. It is given by*

	195	166	165	136	91	90	120	128	61	46	45	16	15	1274
195	3	191	7	194	4	190	23	186	12	77	9	185	13	181	1470
166	178	20	114	17	177	21	8	25	169	29	172	26	168	180	1470
165	33	161	37	164	34	160	53	156	42	152	39	80	43	151	1470
136	118	50	144	47	147	51	38	55	139	59	142	56	138	150	1470
91	93	63	131	121	87	71	72	73	79	107	112	101	134	135	1470
90	88	130	97	92	94	85	83	81	126	127	132	110	67	68	1470
120	148	41	174	14	122	96	98	100	74	182	22	155	48	76	1470
128	129	86	64	69	70	115	113	111	102	104	99	66	108	106	1470
61	62	95	84	89	117	123	124	125	109	75	65	133	103	105	1470
46	58	140	54	137	57	141	158	145	49	149	52	146	78	60	1470
45	153	116	157	44	154	40	143	36	162	32	159	35	163	31	1470
16	28	170	24	167	27	171	188	175	19	179	82	176	18	30	1470
15	183	11	187	119	184	10	173	6	192	2	189	5	193	1	1470
	1	30	31	60	105	106	76	68	135	150	151	180	81		1174
1274	1470	1470	1470	1470	1470	1470	1470	1470	1470	1470	1470	1470	1370	1274	

2.6.2 Magic Cross of Order (13,17)

Example 37. *A magic cross of order* $(13,17)$ *constructed based on a magic rectangle of order* $(13,17)$ *for the consecutive numbers 1 to 221. The bigger and smaller rows and columns are of sums 1887 and 1443 respectively. It is given by*

		218	21	184	55	123	152	157	116	75	89	48	191	14		1443
		2	203	36	169	153	76	87	121	118	67	172	33	206		1443
16	208	12	212	17	207	13	211	26	215	5	219	86	214	6	218	2	1887
189	31	193	129	188	32	192	28	9	24	200	20	204	25	199	21	203	1887
50	174	46	178	51	173	47	177	60	181	39	185	35	180	91	184	36	1887
155	133	159	61	154	66	158	62	43	58	166	54	170	59	165	55	169	1887
104	147	71	149	137	78	107	109	81	92	83	84	145	150	74	123	153	1887
101	106	97	110	103	100	98	140	94	142	90	117	120	127	114	152	76	1887
135	65	182	27	221	134	79	96	111	126	143	88	1	195	40	157	87	1887
146	70	108	95	102	105	132	80	128	82	124	122	119	112	125	116	121	1887
69	99	148	72	77	138	139	130	141	113	115	144	85	73	151	75	118	1887
53	167	57	163	52	168	56	164	179	160	64	156	68	161	63	89	67	1887
186	38	131	42	187	37	183	41	162	45	175	49	171	44	176	48	172	1887
19	201	23	197	18	202	22	198	213	194	30	190	34	93	29	191	33	1887
220	4	216	8	136	3	217	7	196	11	209	15	205	10	210	14	206	1887
		220	19	186	53	69	146	135	101	104	155	50	189	16			1443
		4	201	38	167	99	70	65	106	147	133	174	31	208			1443
1443	1443	1887	1887	1887	1887	1887	1887	1887	1887	1887	1887	1887	1887	1887	1443	1443	

2.6.3 Magic Cross of Order (13,19)

Example 38. *A magic cross of order* $(13,19)$ *constructed based on a magic rectangle of order* $(13,19)$ *for the consecutive numbers 1 to 247. The bigger and smaller rows and columns are of sums 2356 and 1612 respectively. It is given by*

			247	210	209	172	115	162	152	114	77	58	57	20	19				1612
			3	226	41	150	163	117	188	78	112	74	193	36	231				1612
			243	24	205	62	119	110	53	164	79	176	148	214	15				1612
247	3	243	7	239	2	244	6	240	29	236	14	232	18	104	13	233	17	229	2356
210	226	24	146	28	227	23	223	27	10	31	215	35	211	30	216	34	212	228	2356
209	41	205	45	201	40	206	44	202	67	198	52	194	56	199	51	100	55	191	2356
172	150	62	184	66	189	61	185	65	48	69	177	73	173	68	178	72	174	190	2356
115	163	119	80	166	153	88	89	107	91	103	101	94	161	167	83	169	136	171	2356
162	117	110	108	123	116	111	109	122	105	158	93	130	135	142	127	84	170	134	2356
152	188	53	222	11	151	149	120	156	124	92	128	99	97	237	26	195	60	96	2356
114	78	164	121	106	113	118	155	90	143	126	139	137	132	125	140	138	131	86	2356
77	112	79	165	81	87	154	147	145	157	141	159	160	95	82	168	129	85	133	2356
58	74	176	70	180	75	175	71	179	200	183	63	187	59	182	64	186	98	76	2356
57	193	148	197	49	192	54	196	50	181	46	204	42	208	47	203	43	207	39	2356
20	36	214	32	218	37	213	33	217	238	221	25	225	21	220	102	224	22	38	2356
19	231	15	235	144	230	16	234	12	219	8	242	4	246	9	241	5	245	1	2356
			5	224	43	186	129	138	195	84	169	72	100	34	233				1612
			245	22	207	98	85	131	60	170	136	174	55	212	17				1612
			1	38	39	76	133	86	96	134	171	190	191	228	229				1612
1612	1612	1612	2356	2356	2356	2356	2356	2356	2356	2356	2356	2356	2356	2356	2356	1612	1612	1612	

2.7 Magic Cross of Order $(15, 2n + 15)$

2.7.1 Magic Cross of Order (15,17)

Example 39. *A magic cross of order* $(15, 17)$ *constructed based on a magic rectangle of order* $(15, 17)$ *for the consecutive numbers 1 to 255. The bigger and smaller rows and columns are of sums 2176 and 1920 respectively. It is given by*

	248	25	214	59	180	93	121	161	153	110	78	195	44	229	10		1920
246	11	244	13	255	2	253	4	26	14	241	118	239	5	250	7	248	2176
27	228	29	226	137	237	20	235	9	225	32	223	34	234	23	232	25	2176
212	45	210	47	221	36	219	38	60	48	207	50	205	107	216	41	214	2176
61	194	148	192	52	203	54	201	43	191	66	189	68	200	57	198	59	2176
178	79	176	81	187	70	185	72	94	82	173	84	171	73	182	109	180	2176
146	160	97	158	86	169	88	167	77	157	100	155	102	166	91	164	93	2176
103	133	131	106	111	134	113	130	141	142	117	144	120	127	151	152	121	2176
95	181	63	217	18	240	132	140	128	116	124	16	238	39	193	75	161	2176
135	104	105	129	136	112	139	114	115	126	143	122	145	150	125	123	153	2176
163	92	165	90	154	101	156	99	179	89	168	87	170	98	159	96	110	2176
76	147	74	183	85	172	83	174	162	184	71	186	69	175	80	177	78	2176
197	58	199	56	188	67	190	65	213	55	202	53	204	64	108	62	195	2176
42	215	40	149	51	206	49	208	196	218	37	220	35	209	46	211	44	2176
231	24	233	22	222	33	224	31	247	21	236	19	119	30	227	28	229	2176
8	249	6	251	17	138	15	242	230	252	3	254	1	243	12	245	10	2176
	8	231	42	197	76	163	135	95	103	146	178	61	212	27	246		1920
1920	2176	2176	2176	2176	2176	2176	2176	2176	2176	2176	2176	2176	2176	2176	2176	1920	

2.7.2 Magic Cross of Order (15,19)

Example 40. *A magic cross of order* $(15,19)$ *constructed based on a magic rectangle of order* $(15,19)$ *for the consecutive numbers 1 to 285. The bigger and smaller rows and columns are of sums 2717 and 2145 respectively. It is given by*

		275	30	237	68	199	106	170	136	123	182	85	220	47	258	9		2145
		271	262	233	224	195	186	124	171	134	110	81	72	43	34	5		2145
281	277	8	279	6	267	18	269	16	29	4	131	2	285	14	273	12	275	271	2717
252	28	259	26	261	38	154	36	251	10	263	22	265	20	253	32	255	30	262	2717
243	239	46	241	44	229	56	231	54	67	42	245	40	133	52	235	50	237	233	2717
214	66	221	64	166	76	211	74	213	48	225	60	227	58	215	70	217	68	224	2717
205	201	84	203	82	191	94	193	92	105	80	207	78	209	90	121	88	199	195	2717
176	104	164	102	185	114	173	112	175	86	187	98	189	96	177	108	179	106	186	2717
152	163	117	146	119	125	149	127	156	129	158	139	160	135	142	168	138	170	124	2717
115	150	183	89	223	39	249	3	145	143	141	283	37	247	63	197	103	136	171	2717
162	116	148	118	144	151	126	147	128	157	130	159	137	161	167	140	169	123	134	2717
100	180	107	178	109	190	97	188	99	200	111	174	113	172	101	184	122	182	110	2717
91	87	198	165	196	77	208	79	206	181	194	93	192	95	204	83	202	85	81	2717
62	218	69	216	71	228	59	226	61	238	73	212	75	210	120	222	65	220	72	2717
53	49	236	51	234	153	246	41	244	219	232	55	230	57	242	45	240	47	43	2717
24	256	31	254	33	266	21	264	23	276	35	250	132	248	25	260	27	258	34	2717
15	11	274	13	272	1	284	155	282	257	270	17	268	19	280	7	278	9	5	2717
		15	24	53	214	91	176	162	115	152	100	205	62	243	252	281			2145
		11	256	49	66	87	104	116	150	163	180	201	218	239	28	277			2145
2145	2145	2717	2717	2717	2717	2717	2717	2717	2717	2717	2717	2717	2717	2717	2717	2717	2145	2145	

2.8 Magic Cross of Order $(17, 2n+17)$

2.8.1 Magic Cross of Order (17,19)

Example 41. *A magic cross of order* $(17,19)$ *constructed based on a magic rectangle of order* $(17,19)$ *for the consecutive numbers 1 to 323. The bigger and smaller rows and columns are of sums 3078 and 2754 respectively. It is given by*

	305	304	267	266	229	228	171	172	134	124	209	114	77	76	39	38	1	
323	3	319	7	315	2	320	6	316	29	312	14	137	18	313	13	309	17	305
286	302	24	298	28	189	23	299	27	10	31	291	35	287	30	292	34	288	304
285	41	281	45	277	40	282	44	278	67	274	52	270	56	142	51	271	55	267
248	264	62	184	66	265	61	261	65	48	69	253	73	249	68	254	72	250	266
247	79	243	83	239	78	244	82	240	105	236	90	232	94	237	89	138	93	229
210	188	100	222	104	227	99	223	103	86	107	215	111	211	106	216	110	212	228
153	201	157	118	204	191	126	127	145	129	141	139	132	199	205	121	207	174	209
200	155	148	146	161	154	149	147	160	143	196	131	168	173	180	165	122	208	172
190	226	91	260	49	303	16	158	194	162	130	166	308	21	275	64	233	98	134
152	116	202	159	144	151	156	193	128	181	164	177	175	170	163	178	176	169	124
115	150	117	203	119	125	192	185	183	195	179	197	198	133	120	206	167	123	171
96	112	214	108	218	113	213	109	217	238	221	101	225	97	220	102	224	136	114
95	231	186	235	87	230	92	234	88	219	84	242	80	246	85	241	81	245	77
58	74	252	70	256	75	251	71	255	276	259	63	263	59	258	140	262	60	76
57	269	53	273	182	268	54	272	50	257	46	280	42	284	47	279	43	283	39
20	36	290	32	294	37	289	33	293	314	297	25	301	135	296	26	300	22	38
19	307	15	311	11	306	187	310	12	295	8	318	4	322	9	317	5	321	1
	19	20	57	58	95	96	153	152	190	200	115	210	247	248	285	286	323	

2754	3078	3078	3078	3078	3078	3078	3078	3078	3078	3078	3078	3078	3078	3078	3078	3078	3078	2754

2.9 Magic Crosses of Order $(4n, 4m)$

This subsection brings **magic crosses** of order $(4n, 4m)$. In this case, all the magic crosses are and are with inner squares as magic squares. See the examples below.

2.9.1 Magic Cross of Order (4,8)

Example 42. *A* ***magic cross*** *of order* $(4,8)$ *constructed based on* ***magic rectangle*** *of order* $(4,8)$ *for the consecutive numbers from 1 to 32. The middle square is magic square of order 4 with magic sum 66. The bigger and smaller rows and columns are of sums 132 and 66 respectively. It is given by*

		1	16	20	29			66
		28	21	9	8			66
1	28	14	23	2	27	13	24	132
16	21	3	26	15	22	4	25	132
20	9	31	6	19	10	32	5	132
29	8	18	11	30	7	17	12	132
		13	4	32	17			66
		24	25	5	12			66
66	66	132	132	132	132	66	66	

We observe that the inner magic square is not of consecutive numbers. We can construct with consecutive numbers. See the example below

Example 43. *A **magic cross** of order* $(4,8)$ *with inner square a magic square of consecutive numbers is given by*

		28	27	6	5			66
		8	7	26	25			66
28	8	9	10	23	24	1	29	132
27	7	21	22	11	12	2	30	132
6	26	20	15	18	13	31	3	132
5	25	16	19	14	17	32	4	132
		1	2	31	32			66
		29	30	3	4			66
66	66	132	132	132	132	66	66	

The inner magic square of order 4 is with consecutive numbers from 9 to 24.

2.9.2 Magic Cross of Order (4,12)

Example 44. *A **magic cross** of order* $(4,12)$ *constructed based on magic rectangle of order* $(4,12)$ *for the consecutive numbers 1 to 48. The middle square is a magic square of order 4 with magic sum 98. The bigger and smaller rows and columns are of sums 294 and 98 respectively. It is given by*

				3	40	10	45					98
				22	33	15	28					98
				39	4	46	9					98
				34	21	27	16					98
1	24	37	36	2	23	38	35	3	22	39	34	294
42	31	6	19	41	32	5	20	40	33	4	21	294
12	13	48	25	11	14	47	26	10	15	46	27	294
43	30	7	18	44	29	8	17	45	28	9	16	294
				1	42	12	43					98
				24	31	13	30					98
				37	6	48	7					98
				36	19	25	18					98
98	98	98	98	294	294	294	294	98	98	98	98	

Above are three magic squares of order 4 of equal magic sums.

2.9.3 Magic Cross of Order (8,12)

Example 45. *A **magic cross** of order* $(8,12)$ *constructed based on magic rectangle of order* $(8,12)$ *for the consecutive numbers from 1 to 96. The bigger and smaller rows and columns are of sums 582 and 388 respectively. It is given by*

		1	2	3	4	93	94	95	96			388
		89	90	91	92	5	6	7	8			388
1	89	88	87	86	85	12	11	10	9	56	48	582
2	90	16	15	14	13	84	83	82	81	55	47	582
3	91	17	18	19	20	77	78	79	80	54	46	582
4	92	73	74	75	76	21	22	23	24	53	45	582
93	5	72	71	70	69	28	27	26	25	44	52	582
94	6	32	31	30	29	68	67	66	65	43	51	582
95	7	33	34	35	36	61	62	63	64	42	50	582
96	8	57	58	59	60	37	38	39	40	41	49	582
		56	55	54	53	44	43	42	41			388
		48	47	46	45	52	51	50	49			388
388	388	582	582	582	582	582	582	582	582	388	388	

The inner square is not a magic square. For inner square as magic square, see the example below

Example 46. *For inner square as magic square, the magic cross of order* $(8,12)$ *is given by*

		88	87	86	85	12	11	10	9			388
		16	15	14	13	84	83	82	81			388
1	89	17	18	19	20	77	78	79	80	88	16	582
2	90	73	74	75	76	21	22	23	24	87	15	582
3	91	72	71	70	69	28	27	26	25	86	14	582
4	92	32	31	30	29	68	67	66	65	85	13	582
93	5	33	34	35	36	61	62	63	64	12	84	582
94	6	57	58	59	60	37	38	39	40	11	83	582
95	7	56	47	54	53	44	43	50	41	10	82	582
96	8	48	55	46	45	52	51	42	49	9	81	582
		1	2	3	4	93	94	95	96			388
		89	90	91	92	5	6	7	8			388
388	388	582	582	582	582	582	582	582	582	388	388	

2.9.4 Magic Cross of Order (12,16)

Example 47. *A* ***magic cross*** *of order* $(12,16)$ *constructed based on magic rectangle of order* $(12,16)$ *for the consecutive numbers from 1 to 192. The bigger and smaller rows and columns are of sums 1544 and 1158 respectively. In this case, the inner square is a magic square of order 12 with magic sum 1158 of numbers from 25 to 168. It is given by*

		1	2	3	4	5	6	187	188	189	190	191	192			1158
		181	182	183	184	185	186	7	8	9	10	11	12			1158
1	181	25	26	27	28	29	30	163	164	165	166	167	168	180	24	1544
2	182	157	158	159	160	161	162	31	32	33	34	35	36	179	23	1544
3	183	156	155	154	153	152	151	42	41	40	39	38	37	178	22	1544
4	184	48	47	46	45	44	43	150	149	148	147	146	145	177	21	1544
5	185	49	50	51	52	53	54	139	140	141	142	143	144	176	20	1544
6	186	133	134	135	136	137	138	55	56	57	58	59	60	175	19	1544
187	7	132	131	130	129	128	127	66	65	64	63	62	61	18	174	1544
188	8	72	71	70	69	68	67	126	125	124	123	122	121	17	173	1544
189	9	73	74	75	76	77	78	115	116	117	118	119	120	16	172	1544
190	10	109	110	111	112	113	114	79	80	81	82	83	84	15	171	1544
191	11	108	95	106	105	104	103	90	89	88	87	98	85	14	170	1544
192	12	96	107	94	93	92	91	102	101	100	99	86	97	13	169	1544
		180	179	178	177	176	175	18	17	16	15	14	13			1158
		24	23	22	21	20	19	174	173	172	171	170	169			1158
1158	1158	1544	1544	1544	1544	1544	1544	1544	1544	1544	1544	1544	1544	1158	1158	

2.10 Magic Crosses of Orders $2 \times (even, odd)$

2.10.1 Magic Cross of Order (4,6)

Example 48. *A magic cross of order* $(4,6)$ *constructed based on magic rectangle of order* $(4,6)$ *for the consecutive numbers from 1 to 24. The bigger and smaller rows and columns are of sums 75 and 50 respectively. It is given by*

	1	19	18	12		50
1	2	3	22	23	24	75
19	20	21	4	5	6	75
18	17	16	9	8	7	75
12	11	10	15	14	13	75
	24	6	7	13		50
50	75	75	75	75	50	

2.10.2 Magic Cross of Order (4,10)

Example 49. *A* ***magic cross*** *of order* $(4,10)$ *constructed based on magic rectangle of order* $(4,10)$ *for the consecutive numbers from 1 to 40. The middle square is a magic square of order 4 with magic sum 82. The bigger and smaller rows and columns are of sums 205 and 82 respectively. It is given by*

			1	31	30	20				82
			2	32	29	19				82
			3	33	28	18				82
1	2	3	4	5	36	37	38	39	40	205
31	32	33	34	35	6	7	8	9	10	205
30	29	28	27	26	15	14	13	12	11	205
20	19	18	17	16	25	24	23	22	21	205
			38	8	13	23				82
			39	9	12	22				82
			40	10	11	21				82
82	82	82	205	205	205	205	82	82	82	

2.10.3 Magic Cross of Order (6,8)

Example 50. *A magic cross of order* $(6,8)$ *constructed based on magic rectangle of order* $(6,8)$ *for the consecutive numbers from 1 to 48. The bigger rows and columns are of sum 196, and lower rows and columns are of same sum for two small rows. It is given by*

	43	44	45	7	8	9		156
1	43	42	12	13	31	30	24	196
2	44	41	11	14	32	29	23	196
3	45	40	10	15	33	28	22	196
46	4	9	39	34	16	21	27	196
47	5	8	38	35	17	20	26	196
48	6	7	37	36	18	19	25	196
	6	5	4	42	41	40		138
147	196	196	196	196	196	196	147	

This is the only example, where we don't have regular magic cross, because two of rows/columns don't have same sums as of other rows/columns. Let's call it **semi-magic cross**.

2.10.4 Magic Cross of Order (6,12)

Example 51. *A* ***magic cross*** *of order* $(6,12)$ *constructed based on magic rectangle of order* $(6,12)$ *for the consecutive numbers from 1 to 72. The bigger and smaller rows and columns are of sums 438 and 219 respectively. It is given by*

			13	14	15	58	59	60				219
			55	56	57	16	17	18				219
			54	53	52	21	20	19				219
13	55	54	24	23	22	51	50	49	28	33	36	438
14	56	53	1	2	3	70	71	72	29	32	35	438
15	57	52	67	68	69	4	5	6	30	31	34	438
58	16	21	66	65	64	9	8	7	43	42	39	438
59	17	20	12	11	10	63	62	61	44	41	38	438
60	18	19	25	26	27	46	47	48	45	40	37	438
			43	44	45	28	29	30				219
			42	41	40	33	32	31				219
			36	35	34	39	38	37				219
219	219	219	438	438	438	438	438	438	219	219	219	

2.11 Magic Crosses of Order $(4n+2, 4m+2)$

The magic crosses given in this subsection are are all regular and the inner square is magic square of order 6.

2.11.1 Magic Cross of Order (6,10)

Example 52. *A **magic cross** of order* $(6,10)$ *constructed based on magic rectangle of order* $(6,10)$ *for the consecutive numbers from 1 to 60. The bigger and smaller rows and columns are of sums 305 and 183 respectively. The inner square is a magic square of order 6 with magic sum 183 for the consecutive numbers from 13 to 48 is given by*

		54	53	52	9	8	7			183
		12	11	10	51	50	49			183
54	12	13	47	46	45	14	18	1	55	305
53	11	42	20	40	21	23	37	2	56	305
52	10	36	35	27	28	32	25	3	57	305
9	51	30	26	33	34	29	31	58	4	305
8	50	19	38	22	39	41	24	59	5	305
7	49	43	17	15	16	44	48	60	6	305
		1	2	3	58	59	60			183
		55	56	57	4	5	6			183
183	183	305	305	305	305	305	305	183	183	

2.11.2 Magic Cross of Order (6,14)

Example 53. *A **magic cross** of order* $(6,14)$ *constructed based on magic rectangle of order* $(6,14)$ *for the consecutive numbers from 1 to 84. The bigger and smaller rows and columns are of sums 595 and 255 respectively. The inner square is a magic square of order 6 with magic sum 255 for the consecutive numbers from 25 to 60 is given by*

				13	14	15	70	71	72					255
				67	68	69	16	17	18					255
				66	65	64	21	20	19					255
				24	23	22	63	62	61					255
13	67	66	24	25	59	58	57	26	30	1	79	78	12	595
14	68	65	23	54	32	52	33	35	49	2	80	77	11	595
15	69	64	22	48	47	39	40	44	37	3	81	76	10	595
70	16	21	63	42	38	45	46	41	43	82	4	9	75	595
71	17	20	62	31	50	34	51	53	36	83	5	8	74	595
72	18	19	61	55	29	27	28	56	60	84	6	7	73	595
				1	2	3	82	83	84					255
				79	80	81	4	5	6					255
				78	77	76	9	8	7					255
				12	11	10	75	74	73					255
255	255	255	255	595	595	595	595	595	595	255	255	255	255	

2.11.3 Magic Cross of Order (10,14)

Example 54. *A **magic cross** of order* $(10,14)$ *constructed based on magic rectangle of order* $(10,14)$ *for the consecutive numbers from 1 to 140. The bigger and smaller rows and columns are of sums 987 and 705 respectively. The inner square is a magic square of order 10 with magic sum 7055 for the consecutive numbers from 21 to 120 is given by*

		130	129	128	127	126	15	14	13	12	11		705
		20	19	18	17	16	125	124	123	122	121		705
130	20	21	119	23	117	116	115	114	28	22	30	1	131	987
129	19	110	32	108	34	106	105	37	33	39	101	2	132	987
128	18	100	99	43	97	45	46	44	48	92	91	3	133	987
127	17	51	89	88	54	86	55	57	83	82	60	4	134	987
126	16	80	62	78	77	65	66	74	73	69	61	5	135	987
15	125	70	72	68	64	75	76	67	63	79	71	136	6	987
14	124	81	59	53	84	56	85	87	58	52	90	137	7	987
13	123	50	42	93	47	95	96	94	98	49	41	138	8	987
12	122	31	102	38	104	36	35	107	103	109	40	139	9	987
11	121	111	29	113	27	25	26	24	118	112	120	140	10	987
		1	2	3	4	5	136	137	138	139	140		705
		131	132	133	134	135	6	7	8	9	10		705
705	705	987	987	987	987	987	987	987	987	987	987	705	705	

3 Magic Crosses: Non Repeated Entries

In the above work, we have seen that there is a repetition of small rows and columns. There are possibilities of constructing magic crosses of different digits except the common part. Below are some examples of magic crosses of different digits, where each part is a magic square of respective order.

3.1 Magic Crosses of Order (4,12)

Example 55. *The magic cross of different values except inner square of order (4,12) for the numbers from 1 to 80 is given by*

			5	36	65	56						162
			66	55	6	35						162
			16	25	76	45						162
			75	46	15	26						162
1	40	61	60	3	38	63	58	2	39	62	59	486
70	51	10	31	68	53	8	33	69	52	9	32	486
20	21	80	41	18	23	78	43	19	22	79	42	486
71	50	11	30	73	48	13	28	72	49	12	29	486
			4	37	64	57						162
			67	54	7	34						162
			17	24	77	44						162
			74	47	14	27						162
162	162	162	162	486	486	486	486	162	162	162	162	

Each block of order 4 is a magic square with magic sum 162.

3.2 Magic Crosses of Order (5,15)

Based on similar lines we can construct magic cross of order (5,15), where each value is different except the inner squares.

Example 56. *The magic cross of different values except inner square of order (5,15) for the numbers from 1 to 125 is given by*

					15	29	68	82	121						315
					93	107	21	40	54						315
					46	65	79	118	7						315
					104	18	32	71	90						315
					57	96	115	4	43						315
17	31	75	89	103	8	47	61	80	119	24	38	52	91	110	945
100	114	3	42	56	86	105	19	33	72	77	116	10	49	63	945
28	67	81	125	14	44	58	97	111	5	35	74	88	102	16	945
106	25	39	53	92	122	11	30	69	83	113	2	41	60	99	945
64	78	117	6	50	55	94	108	22	36	66	85	124	13	27	945
					1	45	59	98	112						315
					84	123	12	26	70						315
					37	51	95	109	23						315
					120	9	48	62	76						315
					73	87	101	20	34						315
315	315	315	315	315	945	945	945	945	945	315	315	315	315	315	

Each block of order 5 is a magic square with magic sum 315.

3.3 Magic Crosses of Order (6,18)

Based on similar lines we can construct magic cross of order (6,18), where each value is different except the inner squares.

Example 57. *The magic cross of different values except inner square of order (6,18) for the numbers from 1 to 180 is given by*

						5	175	166	165	6	26						543
						146	36	136	45	55	125						543
						116	115	75	76	96	65						543
						86	66	105	106	85	95						543
						35	126	46	135	145	56						543
						155	25	15	16	156	176						543
1	171	170	161	10	30	3	173	168	163	8	28	2	172	169	162	9	29	1629
150	40	140	41	51	121	148	38	138	43	53	123	149	39	139	42	52	122	1629
120	111	71	80	100	61	118	113	73	78	98	63	119	112	72	79	99	62	1629
90	70	101	110	81	91	88	68	103	108	83	93	89	69	102	109	82	92	1629
31	130	50	131	141	60	33	128	48	133	143	58	32	129	49	132	142	59	1629
151	21	11	20	160	180	153	23	13	18	158	178	152	22	12	19	159	179	1629
						4	174	167	164	7	27						543
						147	37	137	44	54	124						543
						117	114	74	77	97	64						543
						87	67	104	107	84	94						543
						34	127	47	134	144	57						543
						154	24	14	17	157	177						543
543	543	543	543	543	543	1629	1629	1629	1629	1629	1629	543	543	543	543	543	543	

Each block of order 6 is a magic square with magic sum 543.

3.4 Magic Crosses of Order (8,24)

Based on similar lines we can construct magic cross of order (8,24), where each value is different except the inner squares.

Example 58. *The magic cross of different values except inner square of order (8,24) for the numbers from 1 to 320 is given by*

								76	205	176	25	135	306	275	86								1284
								126	315	266	95	65	216	165	36								1284
								5	196	225	56	106	255	286	155								1284
								115	246	295	146	16	185	236	45								1284
								186	15	46	235	245	116	145	296								1284
								256	105	156	285	195	6	55	226								1284
								215	66	35	166	316	125	96	265								1284
								305	136	85	276	206	75	26	175								1284
80	201	180	21	131	310	271	90	78	203	178	23	133	308	273	88	79	202	179	22	132	309	272	89	3852
130	311	270	91	61	220	161	40	128	313	268	93	63	218	163	38	129	312	269	92	62	219	162	39	3852
1	200	221	60	110	251	290	151	3	198	223	58	108	253	288	153	2	199	222	59	109	252	289	152	3852
111	250	291	150	20	181	240	41	113	248	293	148	18	183	238	43	112	249	292	149	19	182	239	42	3852
190	11	50	231	241	120	141	300	188	13	48	233	243	118	143	298	189	12	49	232	242	119	142	299	3852
260	101	160	281	191	10	51	230	258	103	158	283	193	8	53	228	259	102	159	282	192	9	52	229	3852
211	70	31	170	320	121	100	261	213	68	33	168	318	123	98	263	212	69	32	169	319	122	99	262	3852
301	140	81	280	210	71	30	171	303	138	83	278	208	73	28	173	302	139	82	279	209	72	29	172	3852
								77	204	177	24	134	307	274	87								1284
								127	314	267	94	64	217	164	37								1284
								4	197	224	57	107	254	287	154								1284
								114	247	294	147	17	184	237	44								1284
								187	14	47	234	244	117	144	297								1284
								257	104	157	284	194	7	54	227								1284
								214	67	34	167	317	124	97	264								1284
								304	137	84	277	207	74	27	174								1284
1284	1284	1284	1284	1284	1284	1284	1284	3852	3852	3852	3852	3852	3852	3852	3852	1284	1284	1284	1284	1284	1284	1284	1284	

Each block of order 8 is a magic square with magic sum 1284.

3.5 Magic Crosses of Order (10,30)

Based on similar lines we can construct magic cross of order (10,30), where each value is different except the inner squares.

Example 59. *The magic cross of different values except inner square of order (10,30) for the numbers from 1 to 500 is given by*

										24	391	316	477	188	102	235	449	260	63											2505
										485	52	38	349	441	363	266	160	224	127											2505
										227	424	110	388	99	166	463	291	302	35											2505
										341	277	435	163	2	474	138	66	399	210											2505
										413	488	252	74	216	335	360	27	141	199											2505
										60	185	213	41	377	299	402	124	466	338											2505
										366	249	191	410	135	88	327	452	13	274											2505
										288	113	499	202	324	10	91	385	177	416											2505
										149	16	77	285	460	241	174	313	438	352											2505
										152	310	374	116	263	427	49	238	85	491											2505
1	398	323	484	195	109	237	426	262	70	17	389	314	500	181	125	228	442	253	56	8	380	305	486	197	111	244	433	269	72	7515
487	59	45	326	448	370	273	162	201	134	478	75	31	342	439	356	264	153	217	150	494	61	47	333	430	372	255	169	208	136	7515
234	401	112	395	76	173	470	298	309	37	250	417	103	381	92	164	456	289	325	28	236	408	119	397	83	155	472	280	311	44	7515
348	284	437	170	9	451	145	73	376	212	339	300	428	156	25	467	131	64	392	203	330	286	444	172	11	458	147	55	383	219	7515
420	495	259	51	223	337	362	34	148	176	406	481	275	67	214	328	353	50	139	192	422	497	261	58	205	344	369	36	130	183	7515
62	187	220	48	384	276	409	101	473	345	53	178	206	39	400	292	425	117	464	331	69	194	222	30	386	283	411	108	455	347	7515
373	226	198	412	137	95	334	459	20	251	364	242	189	403	128	81	350	475	6	267	355	233	180	419	144	97	336	461	22	258	7515
295	120	476	209	301	12	98	387	184	423	281	106	492	225	317	3	89	378	200	414	297	122	483	211	308	19	80	394	186	405	7515
126	23	84	287	462	248	151	320	445	359	142	14	100	278	453	239	167	306	431	375	133	5	86	294	469	230	158	322	447	361	7515
159	312	351	123	270	434	26	245	87	498	175	303	367	114	256	450	42	231	78	489	161	319	358	105	272	436	33	247	94	480	7515
										15	382	307	493	179	118	246	440	271	54											2505
										496	68	29	340	432	354	257	171	215	143											2505
										243	415	121	379	90	157	454	282	318	46											2505
										332	293	446	154	18	465	129	57	390	221											2505
										404	479	268	65	207	346	371	43	132	190											2505
										71	196	204	32	393	290	418	115	457	329											2505
										357	240	182	421	146	79	343	468	4	265											2505
										279	104	490	218	315	21	82	396	193	407											2505
										140	7	93	296	471	232	165	304	429	368											2505
										168	321	365	107	254	443	40	229	96	482											2505
2505	2505	2505	2505	2505	2505	2505	2505	2505	2505	7515	7515	7515	7515	7515	7515	7515	7515	7515	7515	2505	2505	2505	2505	2505	2505	2505	2505	2505	2505	

Each block of order 10 is a magic square with magic sum 2505.

4 Final Comments

This paper extends the idea of magic rectangles to magic crosses. The work for the **orders (odd, odd)** and **orders (even, even)**. In the smaller rows and columns there is a repetition of numbers. What we observed that, we have all the possible results, except the case, of order (6,8). In this case, we have magic rectangle, but we have difficulties in making magic cross. In this case we don't have two of small rows are of different sums. This type we called as **semi-magic rectanagle**. For non repeated entries we worked with orders (4,12), (5,15), (6,18), (8,24) and (10,30). In this case the, the magic squares are of equal magic sums. For the non repeated entires, we used blocks of equal sums magic squares. Still, it is an open problem to check the magic crosses with non repeated entries for other situations.

During past years the author worked with magic squares in different situations. These are given in details below:

• Author's Contributions to Magic Squares

The item-wise author's work on magic squares is as follows:

(i) **Digital numbers** magic squares - [5, 6, 7, 8, 9, 10];

(ii) **Block-wise construction** of **bimagic squares** - [11];

(iii) Connections with **genetic tables** and **Shannon's entropy** - [12];

(iv) **Selfie** and **palindromic-type** magic squares - [13];
(v) **Intervally distributed** and **block-wise** magic squares - [14, 15, 16];
(vi) **Multi-digits** magic squares - [17];
(vii) **Perfect square sum** magic squares with **uniformity** and **minimum Sum** - [18, 19];
(viii) **Pythagorean triples** to generate **perfect square sum** magic squares - [19];
(ix) **Block-wise** equal sums **pan magic squares of order 4*k*** - [20];
(x) **Block-wise** equal sums **magic squares of order 3*k*** - [21];
(xi) **Block-wise** unequal sums **magic squares of order 3*k*** - [22];
(xii) **Magic rectangles** in Construction of **block-wise pan magic squares** - [23].

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