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Block-Wise Equal Sums Magic Squares of Orders $6k$

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Abstract

*In this paper, we shall construct **block-wise** magic squares of orders 12, 18, 24, 30 and 36. The construction is in a such way the each block is a magic square of order 6 with equal magic sums.*

Contents

1 Introduction

In previous works, the author [19, 20, 21, 22] constructed **block-wise** magic squares of orders 8 to 36 in different situations. The idea of magic rectangles is used in two different situations. One in construction [22] of **block-wise** pan diagonal magic squares of orders 15, 21, 27 and 33. Secondly, the new idea of magic crosses [23] are introduced first time in literature. In the above works, we don't have block-wise construction of magic squares as blocks of order 6 with equal magic sums in each case. This is the aim of this work. We shall construct magic squares of order 12, 18, 24, 30 and 36 with the property that each block is a magic square of order 6 with equal magic sums. In order to bring these magic square we shall use the following magic square of order 6 [3]:

Example 1. *Let's consider a magic square of order 6.*

						111
1	*35*	*34*	*33*	*2*	*6*	*111*
30	*8*	*28*	*9*	*11*	*25*	*111*
24	*23*	*15*	*16*	*20*	*13*	*111*
18	*14*	*21*	*22*	*17*	*19*	*111*
7	*26*	*10*	*27*	*29*	*12*	*111*
31	*5*	*3*	*4*	*32*	*36*	*111*
111	*111*	*111*	*111*	*111*	*111*	*111*

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2 Block-Wise Equal Sums Magic Squares

2.1 Magic Square of Order 12

In this subsection, we shall give construction of a magic square of order 12 in four blocks of magic squares of order 6 with equal magic sums. In order to construct it let's divide 144 numbers in four equal parts according to following distribution.

Distribution 1. *Let's consider the following distribution of 144 numbers in 4 blocks of 36 each giving equal sum:*

	1	2	3	4	5	6	...	...	31	32	33	34	35	36	Total
A1	1	8	9	16	17	24	...	...	121	128	129	136	137	144	2610
A2	2	7	10	15	18	23	...	...	122	127	130	135	138	143	2610
A3	3	6	11	14	19	22	...	...	123	126	131	134	139	142	2610
A4	4	5	12	13	20	21	...	...	124	125	132	133	140	141	2610

Let's construct 4 blocks of magic squares of order 6 according to Example 1, and put them according to following structure.

Structure 1. *Let's consider 4 blocks of order 6 as below:*

A1	A2
A3	A4

Example 2. *4 blocks of magic squares constructed according to Example 1 by using data given in Distribution 1, and putting them according to Structure 1, we get a magic square of order 12 given by*

												870
1	137	136	129	8	24	2	138	135	130	7	23	870
120	32	112	33	41	97	119	31	111	34	42	98	870
96	89	57	64	80	49	95	90	58	63	79	50	870
72	56	81	88	65	73	71	55	82	87	66	74	870
25	104	40	105	113	48	26	103	39	106	114	47	870
121	17	9	16	128	144	122	18	10	15	127	143	870
3	139	134	131	6	22	4	140	133	132	5	21	870
118	30	110	35	43	99	117	29	109	36	44	100	870
94	91	59	62	78	51	93	92	60	61	77	52	870
70	54	83	86	67	75	69	53	84	85	68	76	870
27	102	38	107	115	46	28	101	37	108	116	45	870
123	19	11	14	126	142	124	20	12	13	125	141	870
870	870	870	870	870	870	870	870	870	870	870	870	870

In this case, the magic sum is $S_{12\times 12} := 870$, and all four blocks of order 6 are magic squares with equal magic sums $S_{6\times 6} := 435$.

2.2 Magic Square of Order 18

In this subsection, we shall give construction of a magic square of order 18 in 9 blocks of magic squares of order 6 with equal magic sums. In order to construct it let's divide 324 numbers in 9 equal parts according to following distribution.

Distribution 2. *Let's consider the following distribution of 1 to 324 numbers in 9 blocks to construct* ***magic square*** *of order 18:*

	1	2	3	4	5	6	…	…	31	32	33	34	35	36	Total
A1	1	18	19	36	37	54	…	…	271	288	289	306	307	324	5850
A2	2	17	20	35	38	53	…	…	272	287	290	305	308	323	5850
A3	3	16	21	34	39	52	…	…	273	286	291	304	309	322	5850
A4	4	15	22	33	40	51	…	…	274	285	292	303	310	321	5850
A5	5	14	23	32	41	50	…	…	275	284	293	302	311	320	5850
A6	6	13	24	31	42	49	…	…	276	283	294	301	312	319	5850
A7	7	12	25	30	43	48	…	…	277	282	295	300	313	318	5850
A8	8	11	26	29	44	47	…	…	278	281	296	299	314	317	5850
A9	9	10	27	28	45	46	…	…	279	280	297	298	315	316	5850

Let's construct 9 blocks of magic squares of order 6 according to Example 1, and put them according to following structure.

Structure 2. *Let's consider 9 blocks of order 6 as below:*

A1	A2	A3
A4	A5	A6
A7	A8	A9

Example 3. *4 blocks of magic squares constructed according to Example 1 by using data given in Distribution 2, and putting them according to Structure 2, we get a magic square of order 18 given by*

																		2925
1	307	306	289	18	54	2	308	305	290	17	53	3	309	304	291	16	52	2925
270	72	252	73	91	217	269	71	251	74	92	218	268	70	250	75	93	219	2925
216	199	127	144	180	109	215	200	128	143	179	110	214	201	129	142	178	111	2925
162	126	181	198	145	163	161	125	182	197	146	164	160	124	183	196	147	165	2925
55	234	90	235	253	108	56	233	89	236	254	107	57	232	88	237	255	106	2925
271	37	19	36	288	324	272	38	20	35	287	323	273	39	21	34	286	322	2925
4	310	303	292	15	51	5	311	302	293	14	50	6	312	301	294	13	49	2925
267	69	249	76	94	220	266	68	248	77	95	221	265	67	247	78	96	222	2925
213	202	130	141	177	112	212	203	131	140	176	113	211	204	132	139	175	114	2925
159	123	184	195	148	166	158	122	185	194	149	167	157	121	186	193	150	168	2925
58	231	87	238	256	105	59	230	86	239	257	104	60	229	85	240	258	103	2925
274	40	22	33	285	321	275	41	23	32	284	320	276	42	24	31	283	319	2925
7	313	300	295	12	48	8	314	299	296	11	47	9	315	298	297	10	46	2925
264	66	246	79	97	223	263	65	245	80	98	224	262	64	244	81	99	225	2925
210	205	133	138	174	115	209	206	134	137	173	116	208	207	135	136	172	117	2925
156	120	187	192	151	169	155	119	188	191	152	170	154	118	189	190	153	171	2925
61	228	84	241	259	102	62	227	83	242	260	101	63	226	82	243	261	100	2925
277	43	25	30	282	318	278	44	26	29	281	317	279	45	27	28	280	316	2925
2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925

The above magic square is with magic sum $S_{18\times18} = 2925$, and all the nine blocks of order 6 are magic squares with equal magic sums $S_{6\times6} := 975$.

2.3 Magic Square of Order 24

In this subsection, we shall give construction of a magic square of order 24 in 16 blocks of magic squares of order 6 with equal magic sums. In order to construct it let's divide 576 numbers in 16 equal parts according to following distribution.

Distribution 3. *Let's consider the following distribution of 1 to 576 numbers in 16 blocks of equal sums:*

	1	2	3	4	5	6	...	...	31	32	33	34	35	36	Total
A1	1	32	33	64	65	96	...	...	481	512	513	544	545	576	10386
A2	2	31	34	63	66	95	...	...	482	511	514	543	546	575	10386
A3	3	30	35	62	67	94	...	...	483	510	515	542	547	574	10386
A4	4	29	36	61	68	93	...	...	484	509	516	541	548	573	10386
A5	5	28	37	60	69	92	...	...	485	508	517	540	549	572	10386
A6	6	27	38	59	70	91	...	...	486	507	518	539	550	571	10386
A7	7	26	39	58	71	90	...	...	487	506	519	538	551	570	10386
A8	8	25	40	57	72	89	...	...	488	505	520	537	552	569	10386
A9	9	24	41	56	73	88	...	...	489	504	521	536	553	568	10386
A10	10	23	42	55	74	87	...	...	490	503	522	535	554	567	10386
A11	11	22	43	54	75	86	...	...	491	502	523	534	555	566	10386
A12	12	21	44	53	76	85	...	...	492	501	524	533	556	565	10386
A13	13	20	45	52	77	84	...	...	493	500	525	532	557	564	10386
A14	14	19	46	51	78	83	...	...	494	499	526	531	558	563	10386
A15	15	18	47	50	79	82	...	...	495	498	527	530	559	562	10386
A16	16	17	48	49	80	81	...	...	496	497	528	529	560	561	10386

Let's construct 16 blocks of magic squares of order 6 according to Example 1, and put them according to following structure.

Structure 3. *Let's consider 16 blocks of order 6 as below:*

A1	A2	A3	A4
A5	A6	A7	A8
A9	A10	A11	A12
A13	A14	A15	A16

Example 4. *16 blocks of magic squares constructed according to Example 1 by using data given in Distribution 3, and putting them according to Structure 3, we get a magic square of order 24 given by*

1	2	3	4	5	6	7	8	9	10	11	12
I											
1	545	544	513	32	96	2	546	543	514	31	95
480	128	448	129	161	385	479	127	447	130	162	386
384	353	225	256	320	193	383	354	226	255	319	194
288	224	321	352	257	289	287	223	322	351	258	290
97	416	160	417	449	192	98	415	159	418	450	191
481	65	33	64	512	576	482	66	34	63	511	575
5	549	540	517	28	92	6	550	539	518	27	91
476	124	444	133	165	389	475	123	443	134	166	390
380	357	229	252	316	197	379	358	230	251	315	198
284	220	325	348	261	293	283	219	326	347	262	294
101	412	156	421	453	188	102	411	155	422	454	187
485	69	37	60	508	572	486	70	38	59	507	571
9	553	536	521	24	88	10	554	535	522	23	87
472	120	440	137	169	393	471	119	439	138	170	394
376	361	233	248	312	201	375	362	234	247	311	202
280	216	329	344	265	297	279	215	330	343	266	298
105	408	152	425	457	184	106	407	151	426	458	183
489	73	41	56	504	568	490	74	42	55	503	567
13	557	532	525	20	84	14	558	531	526	19	83
468	116	436	141	173	397	467	115	435	142	174	398
372	365	237	244	308	205	371	366	238	243	307	206
276	212	333	340	269	301	275	211	334	339	270	302
109	404	148	429	461	180	110	403	147	430	462	179
493	77	45	52	500	564	494	78	46	51	499	563
6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

13	14	15	16	17	18	19	20	21	22	23	24	
II												6924
3	547	542	515	30	94	4	548	541	516	29	93	6924
478	126	446	131	163	387	477	125	445	132	164	388	6924
382	355	227	254	318	195	381	356	228	253	317	196	6924
286	222	323	350	259	291	285	221	324	349	260	292	6924
99	414	158	419	451	190	100	413	157	420	452	189	6924
483	67	35	62	510	574	484	68	36	61	509	573	6924
7	551	538	519	26	90	8	552	537	520	25	89	6924
474	122	442	135	167	391	473	121	441	136	168	392	6924
378	359	231	250	314	199	377	360	232	249	313	200	6924
282	218	327	346	263	295	281	217	328	345	264	296	6924
103	410	154	423	455	186	104	409	153	424	456	185	6924
487	71	39	58	506	570	488	72	40	57	505	569	6924
11	555	534	523	22	86	12	556	533	524	21	85	6924
470	118	438	139	171	395	469	117	437	140	172	396	6924
374	363	235	246	310	203	373	364	236	245	309	204	6924
278	214	331	342	267	299	277	213	332	341	268	300	6924
107	406	150	427	459	182	108	405	149	428	460	181	6924
491	75	43	54	502	566	492	76	44	53	501	565	6924
15	559	530	527	18	82	16	560	529	528	17	81	6924
466	114	434	143	175	399	465	113	433	144	176	400	6924
370	367	239	242	306	207	369	368	240	241	305	208	6924
274	210	335	338	271	303	273	209	336	337	272	304	6924
111	402	146	431	463	178	112	401	145	432	464	177	6924
495	79	47	50	498	562	496	80	48	49	497	561	6924
6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Combining partes (I) and (II) we get the required result. In this case the magic sum is $S_{24\times 24} = 6924$, and all the 16 blocks of order 6 are magic squares with equal magic sums $S_{6\times 6} := 1731$.

2.4 Magic Square of Order 30

In this subsection, we shall give construction of a magic square of order 30 in 25 blocks of magic squares of order 6 with equal magic sums. In order to construct it let's divide 900 numbers in 25 equal parts according to following distribution.

Distribution 4. *Let's consider the following distribution of 1 to 900 numbers in 25 blocks of equal sums:*

	1	2	3	4	5	6	...	...	31	32	33	34	35	36	Total
A1	1	50	51	100	101	150	...	...	751	800	801	850	851	900	16218
A2	2	49	52	99	102	149	...	...	752	799	802	849	852	899	16218
A3	3	48	53	98	103	148	...	...	753	798	803	848	853	898	16218
A4	4	47	54	97	104	147	...	...	754	797	804	847	854	897	16218
A5	5	46	55	96	105	146	...	...	755	796	805	846	855	896	16218
A6	6	45	56	95	106	145	...	...	756	795	806	845	856	895	16218
A7	7	44	57	94	107	144	...	...	757	794	807	844	857	894	16218
A8	8	43	58	93	108	143	...	...	758	793	808	843	858	893	16218
A9	9	42	59	92	109	142	...	...	759	792	809	842	859	892	16218
A10	10	41	60	91	110	141	...	...	760	791	810	841	860	891	16218
A11	11	40	61	90	111	140	...	...	761	790	811	840	861	890	16218
A12	12	39	62	89	112	139	...	...	762	789	812	839	862	889	16218
A13	13	38	63	88	113	138	...	...	763	788	813	838	863	888	16218
A14	14	37	64	87	114	137	...	...	764	787	814	837	864	887	16218
A15	15	36	65	86	115	136	...	...	765	786	815	836	865	886	16218
A16	16	35	66	85	116	135	...	...	766	785	816	835	866	885	16218
A17	17	34	67	84	117	134	...	...	767	784	817	834	867	884	16218
A18	18	33	68	83	118	133	...	...	768	783	818	833	868	883	16218
A19	19	32	69	82	119	132	...	...	769	782	819	832	869	882	16218
A20	20	31	70	81	120	131	...	...	770	781	820	831	870	881	16218
A21	21	30	71	80	121	130	...	...	771	780	821	830	871	880	16218
A22	22	29	72	79	122	129	...	...	772	779	822	829	872	879	16218
A23	23	28	73	78	123	128	...	...	773	778	823	828	873	878	16218
A24	24	27	74	77	124	127	...	...	774	777	824	827	874	877	16218
A25	25	26	75	76	125	126	...	...	775	776	825	826	875	876	16218

Let's construct 25 magic squares of order 6 according to Example 1, and put them according to following structure.

Structure 4. *Let's consider 25 blocks of magic squares of order 6 as below:*

A1	A2	A3	A4	A5
A6	A7	A8	A9	A10
A11	A12	A13	A14	A15
A16	A17	A18	A19	A20
A21	A22	A23	A24	A25

Example 5. *25 blocks of magic squares constructed according to Example 1 by using data given in Distribution 4, and putting them according to Structure 4, we get a magic square of order 30 given by*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
I																	
1	851	850	801	50	150	2	852	849	802	49	149	3	853	848	803	48	148
750	200	700	201	251	601	749	199	699	202	252	602	748	198	698	203	253	603
600	551	351	400	500	301	599	552	352	399	499	302	598	553	353	398	498	303
450	350	501	550	401	451	449	349	502	549	402	452	448	348	503	548	403	453
151	650	250	651	701	300	152	649	249	652	702	299	153	648	248	653	703	298
751	101	51	100	800	900	752	102	52	99	799	899	753	103	53	98	798	898
6	856	845	806	45	145	7	857	844	807	44	144	8	858	843	808	43	143
745	195	695	206	256	606	744	194	694	207	257	607	743	193	693	208	258	608
595	556	356	395	495	306	594	557	357	394	494	307	593	558	358	393	493	308
445	345	506	545	406	456	444	344	507	544	407	457	443	343	508	543	408	458
156	645	245	656	706	295	157	644	244	657	707	294	158	643	243	658	708	293
756	106	56	95	795	895	757	107	57	94	794	894	758	108	58	93	793	893
11	861	840	811	40	140	12	862	839	812	39	139	13	863	838	813	38	138
740	190	690	211	261	611	739	189	689	212	262	612	738	188	688	213	263	613
590	561	361	390	490	311	589	562	362	389	489	312	588	563	363	388	488	313
440	340	511	540	411	461	439	339	512	539	412	462	438	338	513	538	413	463
161	640	240	661	711	290	162	639	239	662	712	289	163	638	238	663	713	288
761	111	61	90	790	890	762	112	62	89	789	889	763	113	63	88	788	888
16	866	835	816	35	135	17	867	834	817	34	134	18	868	833	818	33	133
735	185	685	216	266	616	734	184	684	217	267	617	733	183	683	218	268	618
585	566	366	385	485	316	584	567	367	384	484	317	583	568	368	383	483	318
435	335	516	535	416	466	434	334	517	534	417	467	433	333	518	533	418	468
166	635	235	666	716	285	167	634	234	667	717	284	168	633	233	668	718	283
766	116	66	85	785	885	767	117	67	84	784	884	768	118	68	83	783	883
21	871	830	821	30	130	22	872	829	822	29	129	23	873	828	823	28	128
730	180	680	221	271	621	729	179	679	222	272	622	728	178	678	223	273	623
580	571	371	380	480	321	579	572	372	379	479	322	578	573	373	378	478	323
430	330	521	530	421	471	429	329	522	529	422	472	428	328	523	528	423	473
171	630	230	671	721	280	172	629	229	672	722	279	173	628	228	673	723	278
771	121	71	80	780	880	772	122	72	79	779	879	773	123	73	78	778	878
13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515

19	20	21	22	23	24	25	26	27	28	29	30	
II												13515
4	854	847	804	47	147	5	855	846	805	46	146	13515
747	197	697	204	254	604	746	196	696	205	255	605	13515
597	554	354	397	497	304	596	555	355	396	496	305	13515
447	347	504	547	404	454	446	346	505	546	405	455	13515
154	647	247	654	704	297	155	646	246	655	705	296	13515
754	104	54	97	797	897	755	105	55	96	796	896	13515
9	859	842	809	42	142	10	860	841	810	41	141	13515
742	192	692	209	259	609	741	191	691	210	260	610	13515
592	559	359	392	492	309	591	560	360	391	491	310	13515
442	342	509	542	409	459	441	341	510	541	410	460	13515
159	642	242	659	709	292	160	641	241	660	710	291	13515
759	109	59	92	792	892	760	110	60	91	791	891	13515
14	864	837	814	37	137	15	865	836	815	36	136	13515
737	187	687	214	264	614	736	186	686	215	265	615	13515
587	564	364	387	487	314	586	565	365	386	486	315	13515
437	337	514	537	414	464	436	336	515	536	415	465	13515
164	637	237	664	714	287	165	636	236	665	715	286	13515
764	114	64	87	787	887	765	115	65	86	786	886	13515
19	869	832	819	32	132	20	870	831	820	31	131	13515
732	182	682	219	269	619	731	181	681	220	270	620	13515
582	569	369	382	482	319	581	570	370	381	481	320	13515
432	332	519	532	419	469	431	331	520	531	420	470	13515
169	632	232	669	719	282	170	631	231	670	720	281	13515
769	119	69	82	782	882	770	120	70	81	781	881	13515
24	874	827	824	27	127	25	875	826	825	26	126	13515
727	177	677	224	274	624	726	176	676	225	275	625	13515
577	574	374	377	477	324	576	575	375	376	476	325	13515
427	327	524	527	424	474	426	326	525	526	425	475	13515
174	627	227	674	724	277	175	626	226	675	725	276	13515
774	124	74	77	777	877	775	125	75	76	776	876	13515
13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515

Combining parts I and II we get the required result. In this case, the magic square sum is $S_{30\times 30} = 13515$, and all the 25 blocks of order 6 are magic squares with equal magic sums $S_{6\times 6} := 2703$.

2.5 Magic Square of Order 36

In this subsection, we shall give construction of a magic square of order 36 in 36 blocks of magic squares of order 6 with equal magic sums. In order to construct it let's divide 1296 numbers in 36 equal parts according to following distribution.

Distribution 5. *Let's consider the following distribution of 1 to 1296 numbers in 36 blocks of equal sums:*

	1	2	3	4	5	6	…	…	31	32	33	34	35	36	Total
A1	1	72	73	144	145	216	…	…	1081	1152	1153	1224	1225	1296	23346
A2	2	71	74	143	146	215	…	…	1082	1151	1154	1223	1226	1295	23346
A3	3	70	75	142	147	214	…	…	1083	1150	1155	1222	1227	1294	23346
A4	4	69	76	141	148	213	…	…	1084	1149	1156	1221	1228	1293	23346
A5	5	68	77	140	149	212	…	…	1085	1148	1157	1220	1229	1292	23346
A6	6	67	78	139	150	211	…	…	1086	1147	1158	1219	1230	1291	23346
A7	7	66	79	138	151	210	…	…	1087	1146	1159	1218	1231	1290	23346
A8	8	65	80	137	152	209	…	…	1088	1145	1160	1217	1232	1289	23346
A9	9	64	81	136	153	208	…	…	1089	1144	1161	1216	1233	1288	23346
A10	10	63	82	135	154	207	…	…	1090	1143	1162	1215	1234	1287	23346
A11	11	62	83	134	155	206	…	…	1091	1142	1163	1214	1235	1286	23346
A12	12	61	84	133	156	205	…	…	1092	1141	1164	1213	1236	1285	23346
A13	13	60	85	132	157	204	…	…	1093	1140	1165	1212	1237	1284	23346
A14	14	59	86	131	158	203	…	…	1094	1139	1166	1211	1238	1283	23346
A15	15	58	87	130	159	202	…	…	1095	1138	1167	1210	1239	1282	23346
A16	16	57	88	129	160	201	…	…	1096	1137	1168	1209	1240	1281	23346
A17	17	56	89	128	161	200	…	…	1097	1136	1169	1208	1241	1280	23346
A18	18	55	90	127	162	199	…	…	1098	1135	1170	1207	1242	1279	23346
A19	19	54	91	126	163	198	…	…	1099	1134	1171	1206	1243	1278	23346
A20	20	53	92	125	164	197	…	…	1100	1133	1172	1205	1244	1277	23346
A21	21	52	93	124	165	196	…	…	1101	1132	1173	1204	1245	1276	23346
A22	22	51	94	123	166	195	…	…	1102	1131	1174	1203	1246	1275	23346
A23	23	50	95	122	167	194	…	…	1103	1130	1175	1202	1247	1274	23346
A24	24	49	96	121	168	193	…	…	1104	1129	1176	1201	1248	1273	23346
A25	25	48	97	120	169	192	…	…	1105	1128	1177	1200	1249	1272	23346
A26	26	47	98	119	170	191	…	…	1106	1127	1178	1199	1250	1271	23346
A27	27	46	99	118	171	190	…	…	1107	1126	1179	1198	1251	1270	23346
A28	28	45	100	117	172	189	…	…	1108	1125	1180	1197	1252	1269	23346
A29	29	44	101	116	173	188	…	…	1109	1124	1181	1196	1253	1268	23346
A30	30	43	102	115	174	187	…	…	1110	1123	1182	1195	1254	1267	23346
A31	31	42	103	114	175	186	…	…	1111	1122	1183	1194	1255	1266	23346
A32	32	41	104	113	176	185	…	…	1112	1121	1184	1193	1256	1265	23346
A33	33	40	105	112	177	184	…	…	1113	1120	1185	1192	1257	1264	23346
A34	34	39	106	111	178	183	…	…	1114	1119	1186	1191	1258	1263	23346
A35	35	38	107	110	179	182	…	…	1115	1118	1187	1190	1259	1262	23346
A36	36	37	108	109	180	181	…	…	1116	1117	1188	1189	1260	1261	23346

Let's construct 36 magic squares of order 6 according to Example 1, and put them according to following structure.

Structure 5. *Let's consider 36 blocks of magic squares of order 6 as below:*

A1	A2	A3	A4	A5	A6
A7	A8	A9	A10	A11	A12
A13	A14	A15	A16	A17	A18
A19	A20	A21	A22	A23	A24
A25	A26	A27	A28	A29	A30
A31	A32	A33	A34	A35	A36

Example 6. *36 blocks of magic squares constructed according to Example 1 by using data given in Distribution 5, and putting them according to Structure 5, we get a magic square of order 36 given by*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
I																	
1	1225	1224	1153	72	216	2	1226	1223	1154	71	215	3	1227	1222	1155	70	214
1080	288	1008	289	361	865	1079	287	1007	290	362	866	1078	286	1006	291	363	867
864	793	505	576	720	433	863	794	506	575	719	434	862	795	507	574	718	435
648	504	721	792	577	649	647	503	722	791	578	650	646	502	723	790	579	651
217	936	360	937	1009	432	218	935	359	938	1010	431	219	934	358	939	1011	430
1081	145	73	144	1152	1296	1082	146	74	143	1151	1295	1083	147	75	142	1150	1294
7	1231	1218	1159	66	210	8	1232	1217	1160	65	209	9	1233	1216	1161	64	208
1074	282	1002	295	367	871	1073	281	1001	296	368	872	1072	280	1000	297	369	873
858	799	511	570	714	439	857	800	512	569	713	440	856	801	513	568	712	441
642	498	727	786	583	655	641	497	728	785	584	656	640	496	729	784	585	657
223	930	354	943	1015	426	224	929	353	944	1016	425	225	928	352	945	1017	424
1087	151	79	138	1146	1290	1088	152	80	137	1145	1289	1089	153	81	136	1144	1288
13	1237	1212	1165	60	204	14	1238	1211	1166	59	203	15	1239	1210	1167	58	202
1068	276	996	301	373	877	1067	275	995	302	374	878	1066	274	994	303	375	879
852	805	517	564	708	445	851	806	518	563	707	446	850	807	519	562	706	447
636	492	733	780	589	661	635	491	734	779	590	662	634	490	735	778	591	663
229	924	348	949	1021	420	230	923	347	950	1022	419	231	922	346	951	1023	418
1093	157	85	132	1140	1284	1094	158	86	131	1139	1283	1095	159	87	130	1138	1282
19	1243	1206	1171	54	198	20	1244	1205	1172	53	197	21	1245	1204	1173	52	196
1062	270	990	307	379	883	1061	269	989	308	380	884	1060	268	988	309	381	885
846	811	523	558	702	451	845	812	524	557	701	452	844	813	525	556	700	453
630	486	739	774	595	667	629	485	740	773	596	668	628	484	741	772	597	669
235	918	342	955	1027	414	236	917	341	956	1028	413	237	916	340	957	1029	412
1099	163	91	126	1134	1278	1100	164	92	125	1133	1277	1101	165	93	124	1132	1276
25	1249	1200	1177	48	192	26	1250	1199	1178	47	191	27	1251	1198	1179	46	190
1056	264	984	313	385	889	1055	263	983	314	386	890	1054	262	982	315	387	891
840	817	529	552	696	457	839	818	530	551	695	458	838	819	531	550	694	459
624	480	745	768	601	673	623	479	746	767	602	674	622	478	747	766	603	675
241	912	336	961	1033	408	242	911	335	962	1034	407	243	910	334	963	1035	406
1105	169	97	120	1128	1272	1106	170	98	119	1127	1271	1107	171	99	118	1126	1270
31	1255	1194	1183	42	186	32	1256	1193	1184	41	185	33	1257	1192	1185	40	184
1050	258	978	319	391	895	1049	257	977	320	392	896	1048	256	976	321	393	897
834	823	535	546	690	463	833	824	536	545	689	464	832	825	537	544	688	465
618	474	751	762	607	679	617	473	752	761	608	680	616	472	753	760	609	681
247	906	330	967	1039	402	248	905	329	968	1040	401	249	904	328	969	1041	400
1111	175	103	114	1122	1266	1112	176	104	113	1121	1265	1113	177	105	112	1120	1264
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	II
II																		23346
4	1228	1221	1156	69	213	5	1229	1220	1157	68	212	6	1230	1219	1158	67	211	23346
1077	285	1005	292	364	868	1076	284	1004	293	365	869	1075	283	1003	294	366	870	23346
861	796	508	573	717	436	860	797	509	572	716	437	859	798	510	571	715	438	23346
645	501	724	789	580	652	644	500	725	788	581	653	643	499	726	787	582	654	23346
220	933	357	940	1012	429	221	932	356	941	1013	428	222	931	355	942	1014	427	23346
1084	148	76	141	1149	1293	1085	149	77	140	1148	1292	1086	150	78	139	1147	1291	23346
10	1234	1215	1162	63	207	11	1235	1214	1163	62	206	12	1236	1213	1164	61	205	23346
1071	279	999	298	370	874	1070	278	998	299	371	875	1069	277	997	300	372	876	23346
855	802	514	567	711	442	854	803	515	566	710	443	853	804	516	565	709	444	23346
639	495	730	783	586	658	638	494	731	782	587	659	637	493	732	781	588	660	23346
226	927	351	946	1018	423	227	926	350	947	1019	422	228	925	349	948	1020	421	23346
1090	154	82	135	1143	1287	1091	155	83	134	1142	1286	1092	156	84	133	1141	1285	23346
16	1240	1209	1168	57	201	17	1241	1208	1169	56	200	18	1242	1207	1170	55	199	23346
1065	273	993	304	376	880	1064	272	992	305	377	881	1063	271	991	306	378	882	23346
849	808	520	561	705	448	848	809	521	560	704	449	847	810	522	559	703	450	23346
633	489	736	777	592	664	632	488	737	776	593	665	631	487	738	775	594	666	23346
232	921	345	952	1024	417	233	920	344	953	1025	416	234	919	343	954	1026	415	23346
1096	160	88	129	1137	1281	1097	161	89	128	1136	1280	1098	162	90	127	1135	1279	23346
22	1246	1203	1174	51	195	23	1247	1202	1175	50	194	24	1248	1201	1176	49	193	23346
1059	267	987	310	382	886	1058	266	986	311	383	887	1057	265	985	312	384	888	23346
843	814	526	555	699	454	842	815	527	554	698	455	841	816	528	553	697	456	23346
627	483	742	771	598	670	626	482	743	770	599	671	625	481	744	769	600	672	23346
238	915	339	958	1030	411	239	914	338	959	1031	410	240	913	337	960	1032	409	23346
1102	166	94	123	1131	1275	1103	167	95	122	1130	1274	1104	168	96	121	1129	1273	23346
28	1252	1197	1180	45	189	29	1253	1196	1181	44	188	30	1254	1195	1182	43	187	23346
1053	261	981	316	388	892	1052	260	980	317	389	893	1051	259	979	318	390	894	23346
837	820	532	549	693	460	836	821	533	548	692	461	835	822	534	547	691	462	23346
621	477	748	765	604	676	620	476	749	764	605	677	619	475	750	763	606	678	23346
244	909	333	964	1036	405	245	908	332	965	1037	404	246	907	331	966	1038	403	23346
1108	172	100	117	1125	1269	1109	173	101	116	1124	1268	1110	174	102	115	1123	1267	23346
34	1258	1191	1186	39	183	35	1259	1190	1187	38	182	36	1260	1189	1188	37	181	23346
1047	255	975	322	394	898	1046	254	974	323	395	899	1045	253	973	324	396	900	23346
831	826	538	543	687	466	830	827	539	542	686	467	829	828	540	541	685	468	23346
615	471	754	759	610	682	614	470	755	758	611	683	613	469	756	757	612	684	23346
250	903	327	970	1042	399	251	902	326	971	1043	398	252	901	325	972	1044	397	23346
1114	178	106	111	1119	1263	1115	179	107	110	1118	1262	1116	180	108	109	1117	1261	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346

Combining parts I and II we get the required result. In this case, the magic sum is $S_{36\times 36} = 23346$, and all the 36 blocks of order 6 are magic squares with equal magic sums $S_{6\times 6} := 3891$.

3 Final Comments

In the previous works [19, 20, 21], the author constructed magic squares of order 12, 18, 24, 30 and 36 as sub-blocks of orders 6, but none of them are of equal magic sums. In this work, we are able construct these magic squares in a systematic way, where sub-blocks of order 6 are magic squares with equal magic sums. This happened specially due magic square of order 6 considered in Example 1. The construction of this magic square is due to [3]. The same process can be extended for further orders, such as, 42, 48, etc.

During past years the author worked with magic squares in different situations. These are given in details below:

- **Author's Contributions to Magic Squares**

The item-wise author's work on magic squares is as follows:

(i) **Digital numbers** magic squares - [4, 5, 6, 7, 8, 9];
(ii) **Block-wise construction** of **bimagic squares** - [10];
(iii) Connections with **genetic tables** and **Shannon's entropy** - [11];
(iv) **Selfie** and **palindromic-type** magic squares - [12];
(v) **Intervally distributed** and **block-wise** magic squares - [13, 14, 15];
(vi) **Multi-digits** magic squares - [16];
(vii) **Perfect square sum** magic squares with **uniformity**, **minimum sum** and **Pythagorean triples**- [17, 18];
(viii) **Block-wise** equal and unequal sums magic squares of orders $4k$ and $3k$ - [19, 20, 21];
(ix) **Magic rectangles** in construction of **block-wise pan magic squares** - [22];
(x) **Magic crosses:** repeated and non repeated entries - [23].

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