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# Block-Wise Magic and Bimagic Squares

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## Abstract

*This paper summarize some of the results done before by author [31, 32, 33, 34, 36] on different ways are writing* ***block-wise*** *magic squares. In this paper, we shall rewrite some these results without details. The details can be seen in above references. This is done for the magic squares of orders 12 to 36, i.e., orders 12, 15, 16, 18, 20, 21, 24, 25, 27, 28, 30, 32, 33, 35 and 36. In some cases, the magic squares are* ***bimagic*** *or* ***semi-bimagic****. In all the cases, at least one of the block-wise representation is* ***pan diagonal*** *except the orders 18 and 30.*


## Contents



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# 1 Basic Magic Squares

Below are some magic and **bimagic** squares well known in the literature. These are of orders, 3, 4, 5, 6,7, 8, 9, 10, 11 and 14. In some cases, these magic squares are **pan diagonal**, such as of orders 4, 5, 7, 8, 9 and 11. **Bimagic** squares of orders 8 and 9 are also written. The aim writing these magic squares is that they are used to construct block-wise magic or bimagic squares.

## 1.1 Magic Square of Order 3

**Example 1.1.** *Let's consider a magic square of order 3 is given by*

| | | | |
|---|---|---|---|
| | | | 15 |
| 8 | 1 | 6 | 15 |
| 3 | 5 | 7 | 15 |
| 4 | 9 | 2 | 15 |
| 15 | 15 | 15 | 15 |

This magic square is used in the block-wise constructions of magic squares of orders $3k,\ k>2$.

## 1.2 Magic Square of Order 4

**Example 1.2.** *Let's consider a* ***pan magic square*** *of order 4.*

| | | | | | |
|---|---|---|---|---|---|
| | | 34 | 34 | 34 | 34 |
| | 7 | 12 | 1 | 14 | 34 |
| 34 | 2 | 13 | 8 | 11 | 34 |
| 34 | 16 | 3 | 10 | 5 | 34 |
| 34 | 9 | 6 | 15 | 4 | 34 |
| | 34 | 34 | 34 | 34 | 34 |

This magic square is used in the block-wise constructions of magic squares of orders $4k,\ k>1$

## 1.3 Magic Square of Order 5

**Example 1.3.** *Let's consider a* ***pan magic square*** *of order 5 is given by*

| | | | | | | |
|---|---|---|---|---|---|---|
| | | 65 | 65 | 65 | 65 | 65 |
| | 1 | 9 | 12 | 20 | 23 | 65 |
| 65 | 17 | 25 | 3 | 6 | 14 | 65 |
| 65 | 8 | 11 | 19 | 22 | 5 | 65 |
| 65 | 24 | 2 | 10 | 13 | 16 | 65 |
| 65 | 15 | 18 | 21 | 4 | 7 | 65 |
| | 65 | 65 | 65 | 65 | 65 | 65 |

This magic square is used in the block-wise constructions of magic squares of orders $5k,\ k>2$

### 1.4 Magic Square of Order 6

**Example 1.4.** *Let's consider a magic square of order 6.*

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | 111 |
| 1 | 35 | 34 | 33 | 2 | 6 | 111 |
| 30 | 8 | 28 | 9 | 11 | 25 | 111 |
| 24 | 23 | 15 | 16 | 20 | 13 | 111 |
| 18 | 14 | 21 | 22 | 17 | 19 | 111 |
| 7 | 26 | 10 | 27 | 29 | 12 | 111 |
| 31 | 5 | 3 | 4 | 32 | 36 | 111 |
| 111 | 111 | 111 | 111 | 111 | 111 | 111 |

This magic square is used in the block-wise constructions of magic squares of orders $6k,\ k > 1$

### 1.5 Magic Square of Order 7

**Example 1.5.** *Let's consider a **pan magic square** of order 7 is given by*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 175 | 175 | 175 | 175 | 175 | 175 | 175 |
| | 1 | 9 | 17 | 25 | 33 | 41 | 49 | 175 |
| 175 | 40 | 48 | 7 | 8 | 16 | 24 | 32 | 175 |
| 175 | 23 | 31 | 39 | 47 | 6 | 14 | 15 | 175 |
| 175 | 13 | 21 | 22 | 30 | 38 | 46 | 5 | 175 |
| 175 | 45 | 4 | 12 | 20 | 28 | 29 | 37 | 175 |
| 175 | 35 | 36 | 44 | 3 | 11 | 19 | 27 | 175 |
| 175 | 18 | 26 | 34 | 42 | 43 | 2 | 10 | 175 |
| | 175 | 175 | 175 | 175 | 175 | 175 | 175 | 175 |

This magic square is used in the block-wise constructions of magic squares of orders $7k,\ k > 2$

### 1.6 Magic and Bimagic Squares of Order 8

**Example 1.6.** *Let's consider a **pan magic square** of order 8 is given by*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 260 | 260 | 260 | 260 | 260 | 260 | 260 | 260 |
| | 25 | 48 | 1 | 56 | 26 | 47 | 2 | 55 | 260 |
| 260 | 8 | 49 | 32 | 41 | 7 | 50 | 31 | 42 | 260 |
| 260 | 64 | 9 | 40 | 17 | 63 | 10 | 39 | 18 | 260 |
| 260 | 33 | 24 | 57 | 16 | 34 | 23 | 58 | 15 | 260 |
| 260 | 27 | 46 | 3 | 54 | 28 | 45 | 4 | 53 | 260 |
| 260 | 6 | 51 | 30 | 43 | 5 | 52 | 29 | 44 | 260 |
| 260 | 62 | 11 | 38 | 19 | 61 | 12 | 37 | 20 | 260 |
| 260 | 35 | 22 | 59 | 14 | 36 | 21 | 60 | 13 | 260 |
| | 260 | 260 | 260 | 260 | 260 | 260 | 260 | 260 | 260 |

This magic square is used in the block-wise constructions of magic squares of orders $8k,\ k > 1$

**Example 1.7.** *Let's consider a **pan diagonal bimagic** square of order 8 given by*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 260 | 260 | 260 | 260 | 260 | 260 | 260 | 260 |
| | 16 | 41 | 36 | 5 | 27 | 62 | 55 | 18 | 260 |
| 260 | 26 | 63 | 54 | 19 | 13 | 44 | 33 | 8 | 260 |
| 260 | 1 | 40 | 45 | 12 | 22 | 51 | 58 | 31 | 260 |
| 260 | 23 | 50 | 59 | 30 | 4 | 37 | 48 | 9 | 260 |
| 260 | 38 | 3 | 10 | 47 | 49 | 24 | 29 | 60 | 260 |
| 260 | 52 | 21 | 32 | 57 | 39 | 2 | 11 | 46 | 260 |
| 260 | 43 | 14 | 7 | 34 | 64 | 25 | 20 | 53 | 260 |
| 260 | 61 | 28 | 17 | 56 | 42 | 15 | 6 | 35 | 260 |
| | 260 | 260 | 260 | 260 | 260 | 260 | 260 | 260 | 260 |

In this case, each $2 \times 4$ blocks entries sum is same as of magic square, i.e., 260.

This magic square is used in the block-wise constructions of bimagic squares of orders 24, 32, etc. In case of order 24 the magic square is semi-bimagic.

### 1.7 Magic and Bimagic Squares of Order 9

**Example 1.8.** *Let's consider a* ***pan diagonal*** *magic square of order 9 is given by*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 369 | 369 | 369 | 369 | 369 | 369 | 369 | 369 | 369 |
| | 8 | 49 | 66 | 61 | 24 | 38 | 36 | 77 | 10 | 369 |
| 369 | 37 | 63 | 23 | 12 | 35 | 76 | 65 | 7 | 51 | 369 |
| 369 | 78 | 11 | 34 | 50 | 64 | 9 | 22 | 39 | 62 | 369 |
| 369 | 14 | 28 | 81 | 67 | 3 | 53 | 42 | 56 | 25 | 369 |
| 369 | 52 | 69 | 2 | 27 | 41 | 55 | 80 | 13 | 30 | 369 |
| 369 | 57 | 26 | 40 | 29 | 79 | 15 | 1 | 54 | 68 | 369 |
| 369 | 20 | 43 | 60 | 73 | 18 | 32 | 48 | 71 | 4 | 369 |
| 369 | 31 | 75 | 17 | 6 | 47 | 70 | 59 | 19 | 45 | 369 |
| 369 | 72 | 5 | 46 | 44 | 58 | 21 | 16 | 33 | 74 | 369 |
| | 369 | 369 | 369 | 369 | 369 | 369 | 369 | 369 | 369 | 369 |

Additionally it has property that each $3 \times 3$ block is of same sum as of magic square, i.e., $S_9 = 369$. Also each $3 \times 3$ block is a semi-magic square of order 3 (only in rows and columns).

**Example 1.9.** *Let's consider a* ***bimagic*** *square of order 9 given by*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | 369 |
| 1 | 18 | 23 | 35 | 40 | 48 | 60 | 65 | 79 | 369 |
| 33 | 38 | 52 | 55 | 72 | 77 | 8 | 13 | 21 | 369 |
| 62 | 67 | 75 | 6 | 11 | 25 | 28 | 45 | 50 | 369 |
| 27 | 5 | 10 | 49 | 30 | 44 | 74 | 61 | 69 | 369 |
| 47 | 34 | 42 | 81 | 59 | 64 | 22 | 3 | 17 | 369 |
| 76 | 57 | 71 | 20 | 7 | 15 | 54 | 32 | 37 | 369 |
| 14 | 19 | 9 | 39 | 53 | 31 | 70 | 78 | 56 | 369 |
| 43 | 51 | 29 | 68 | 73 | 63 | 12 | 26 | 4 | 369 |
| 66 | 80 | 58 | 16 | 24 | 2 | 41 | 46 | 36 | 369 |
| 369 | 369 | 369 | 369 | 369 | 369 | 369 | 369 | 369 | 369 |

Each $3 \times 3$ block is of equal sum entries as of magic square, i.e., 369. The **bimagic** sum is $Sb_{9\times 9} = 20049$.

The bimagic squares of orders 8 and 9 given in examples 2.20 and 1.9 are the classical onea constructed in 1891 by G. Pfeffermann [13].

## 1.8 Magic Square of Order 10

**Example 1.10.** *Let's consider a magic square of order 10 is given by*

| | | | | | | | | | | 505 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 80 | 65 | 97 | 39 | 22 | 48 | 86 | 53 | 14 | 505 |
| 98 | 12 | 9 | 66 | 90 | 74 | 55 | 33 | 41 | 27 | 505 |
| 47 | 81 | 23 | 79 | 16 | 35 | 94 | 60 | 62 | 8 | 505 |
| 70 | 57 | 88 | 34 | 2 | 91 | 29 | 15 | 76 | 43 | 505 |
| 84 | 99 | 52 | 11 | 45 | 68 | 73 | 7 | 30 | 36 | 505 |
| 13 | 38 | 44 | 10 | 77 | 56 | 82 | 21 | 95 | 69 | 505 |
| 75 | 46 | 40 | 83 | 28 | 19 | 67 | 92 | 4 | 51 | 505 |
| 59 | 24 | 96 | 42 | 61 | 3 | 20 | 78 | 37 | 85 | 505 |
| 26 | 5 | 17 | 58 | 93 | 50 | 31 | 64 | 89 | 72 | 505 |
| 32 | 63 | 71 | 25 | 54 | 87 | 6 | 49 | 18 | 100 | 505 |
| 505 | 505 | 505 | 505 | 505 | 505 | 505 | 505 | 505 | 505 | 505 |

This magic square shall be used to bring 4 blocks of order 10 in magic square of order 20. Also to bring 9 blocks of order 10 in magic square of order 30.

## 1.9 Magic Squares of Order 11

**Example 1.11.** *Let's consider a* ***pan diagonal*** *magic square of order 11 is given by*

| | | 671 | 671 | 671 | 671 | 671 | 671 | 671 | 671 | 671 | 671 | 671 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 13 | 25 | 37 | 49 | 61 | 73 | 85 | 97 | 109 | 121 | 671 |
| 671 | 108 | 120 | 11 | 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 671 |
| 671 | 83 | 95 | 107 | 119 | 10 | 22 | 23 | 35 | 47 | 59 | 71 | 671 |
| 671 | 58 | 70 | 82 | 94 | 106 | 118 | 9 | 21 | 33 | 34 | 46 | 671 |
| 671 | 44 | 45 | 57 | 69 | 81 | 93 | 105 | 117 | 8 | 20 | 32 | 671 |
| 671 | 19 | 31 | 43 | 55 | 56 | 68 | 80 | 92 | 104 | 116 | 7 | 671 |
| 671 | 115 | 6 | 18 | 30 | 42 | 54 | 66 | 67 | 79 | 91 | 103 | 671 |
| 671 | 90 | 102 | 114 | 5 | 17 | 29 | 41 | 53 | 65 | 77 | 78 | 671 |
| 671 | 76 | 88 | 89 | 101 | 113 | 4 | 16 | 28 | 40 | 52 | 64 | 671 |
| 671 | 51 | 63 | 75 | 87 | 99 | 100 | 112 | 3 | 15 | 27 | 39 | 671 |
| 671 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 111 | 2 | 14 | 671 |
| | 671 | 671 | 671 | 671 | 671 | 671 | 671 | 671 | 671 | 671 | 671 | 671 |

This magic square is used in the block-wise construction of magic square of order 33.

## 1.10 Magic Square of Order 14

**Example 1.12.** *Let's consider a magic square of order 14 is given by*

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | 1379 |
| 1 | 95 | 191 | 112 | 138 | 78 | 58 | 32 | 17 | 159 | 146 | 178 | 125 | 49 | 1379 |
| 147 | 16 | 110 | 176 | 188 | 127 | 87 | 117 | 4 | 53 | 70 | 83 | 38 | 163 | 1379 |
| 126 | 162 | 31 | 137 | 172 | 63 | 80 | 6 | 47 | 26 | 97 | 99 | 149 | 184 | 1379 |
| 190 | 111 | 150 | 46 | 155 | 115 | 5 | 77 | 34 | 170 | 23 | 96 | 140 | 67 | 1379 |
| 164 | 84 | 55 | 12 | 121 | 86 | 29 | 101 | 133 | 62 | 173 | 25 | 186 | 148 | 1379 |
| 104 | 10 | 89 | 37 | 21 | 166 | 179 | 154 | 195 | 74 | 44 | 134 | 59 | 113 | 1379 |
| 19 | 35 | 132 | 57 | 3 | 153 | 196 | 165 | 180 | 92 | 122 | 51 | 72 | 102 | 1379 |
| 45 | 65 | 71 | 90 | 36 | 193 | 152 | 181 | 168 | 119 | 130 | 2 | 103 | 24 | 1379 |
| 30 | 43 | 60 | 73 | 94 | 182 | 167 | 194 | 151 | 107 | 7 | 118 | 22 | 131 | 1379 |
| 68 | 144 | 156 | 27 | 75 | 48 | 105 | 93 | 120 | 136 | 183 | 42 | 11 | 171 | 1379 |
| 81 | 187 | 175 | 114 | 69 | 18 | 135 | 52 | 85 | 14 | 106 | 143 | 160 | 40 | 1379 |
| 139 | 174 | 8 | 161 | 142 | 108 | 116 | 15 | 79 | 185 | 39 | 61 | 54 | 98 | 1379 |
| 177 | 129 | 123 | 192 | 56 | 33 | 20 | 64 | 100 | 141 | 82 | 158 | 91 | 13 | 1379 |
| 88 | 124 | 28 | 145 | 109 | 9 | 50 | 128 | 66 | 41 | 157 | 189 | 169 | 76 | 1379 |
| 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 | 1379 |

The inner $4 \times 4$ block is a **pan diagonal magic square** of order 4 with magic sum $S_{4\times4} := 694$. This magic square shall be used to bring 4 blocks of order 14 in magic square of order 28.

# 2 Block-Wise Magic and Bimagic Squares

In this section, we shall give different ways of writing block-wise magic squares of orders 12 to 36. In some cases, the magic squares are **bimagic**. Prime orders magic squares, such as, of orders 11, 13, 17, 19, 23, 29 and 31 are excluded as they are not possible to construct in blocks. Also, the magic squares of type $2\times$ primes, such as 14, 22, 26 and 34 are also excluded as in these cases also, it is not possible to construct them as block-wise. Specifically, the magic squares considered are of orders 12, 15, 16, 18, 20, 21, 24, 25, 27, 28, 30, 32, 33, 35 and 36. These magic squares are already been constructed by author in different works [31, 32, 33, 34]. Here, the aim is to write different possibilities together. The detailed construction can be seen in [31, 32, 33, 34].

## 2.1 Magic Squares of Order 12

In this subsection, we shall present three different ways of writing block-wise magic square of order 12. One as $4 \times 3$, i.e., magic square formed by 9 blocks of **pan diagonal** magic squares of order 4. Second as $3 \times 4$, i.e., magic square formed by 16 blocks of magic squares of order 3. Third as 4 blocks of order 6 with equal magic sums. In the first two cases, the magic square of order 12 is **pan diagonal**, while third case, it is just a magic square.

### 2.1.1 First Approach: 9 Blocks of Order 4

**Example 2.1.** *9 blocks of* ***pan diagonal*** *magic squares of order 4 constructed using Example 1.2 lead us to a* ***pan diagonal*** *magic square of order 12 given by*

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 |
| | 55 | 108 | 1 | 126 | 56 | 107 | 2 | 125 | 57 | 106 | 3 | 124 | 870 |
| 870 | 18 | 109 | 72 | 91 | 17 | 110 | 71 | 92 | 16 | 111 | 70 | 93 | 870 |
| 870 | 144 | 19 | 90 | 37 | 143 | 20 | 89 | 38 | 142 | 21 | 88 | 39 | 870 |
| 870 | 73 | 54 | 127 | 36 | 74 | 53 | 128 | 35 | 75 | 52 | 129 | 34 | 870 |
| 870 | 58 | 105 | 4 | 123 | 59 | 104 | 5 | 122 | 60 | 103 | 6 | 121 | 870 |
| 870 | 15 | 112 | 69 | 94 | 14 | 113 | 68 | 95 | 13 | 114 | 67 | 96 | 870 |
| 870 | 141 | 22 | 87 | 40 | 140 | 23 | 86 | 41 | 139 | 24 | 85 | 42 | 870 |
| 870 | 76 | 51 | 130 | 33 | 77 | 50 | 131 | 32 | 78 | 49 | 132 | 31 | 870 |
| 870 | 61 | 102 | 7 | 120 | 62 | 101 | 8 | 119 | 63 | 100 | 9 | 118 | 870 |
| 870 | 12 | 115 | 66 | 97 | 11 | 116 | 65 | 98 | 10 | 117 | 64 | 99 | 870 |
| 870 | 138 | 25 | 84 | 43 | 137 | 26 | 83 | 44 | 136 | 27 | 82 | 45 | 870 |
| 870 | 79 | 48 | 133 | 30 | 80 | 47 | 134 | 29 | 81 | 46 | 135 | 28 | 870 |
| | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 |

In this case, the magic sum is $S_{12\times12} = 870$. All the $4\times4$ blocks are **pan diagonal** magic square of order 4 with equal magic sums, $S_{4\times4} = 290$.

#### 2.1.2 Second Approach: 12 Blocks of Order 3

**Example 2.2.** *16 blocks of magic squares of order 3 constructed using Example 1.1 and arranging according to Example 1.2 lead us to a* ***pan diagonal*** *magic square of order 12 given by*

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 |
| | 55 | 45 | 68 | 96 | 106 | 83 | 13 | 27 | 2 | 126 | 112 | 137 | 870 |
| 870 | 69 | 56 | 43 | 82 | 95 | 108 | 3 | 14 | 25 | 136 | 125 | 114 | 870 |
| 870 | 44 | 67 | 57 | 107 | 84 | 94 | 26 | 1 | 15 | 113 | 138 | 124 | 870 |
| 870 | 18 | 28 | 5 | 121 | 111 | 134 | 60 | 46 | 71 | 91 | 105 | 80 | 870 |
| 870 | 4 | 17 | 30 | 135 | 122 | 109 | 70 | 59 | 48 | 81 | 92 | 103 | 870 |
| 870 | 29 | 6 | 16 | 110 | 133 | 123 | 47 | 72 | 58 | 104 | 79 | 93 | 870 |
| 870 | 132 | 118 | 143 | 19 | 33 | 8 | 90 | 100 | 77 | 49 | 39 | 62 | 870 |
| 870 | 142 | 131 | 120 | 9 | 20 | 31 | 76 | 89 | 102 | 63 | 50 | 37 | 870 |
| 870 | 119 | 144 | 130 | 32 | 7 | 21 | 101 | 78 | 88 | 38 | 61 | 51 | 870 |
| 870 | 85 | 99 | 74 | 54 | 40 | 65 | 127 | 117 | 140 | 24 | 34 | 11 | 870 |
| 870 | 75 | 86 | 97 | 64 | 53 | 42 | 141 | 128 | 115 | 10 | 23 | 36 | 870 |
| 870 | 98 | 73 | 87 | 41 | 66 | 52 | 116 | 139 | 129 | 35 | 12 | 22 | 870 |
| | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 | 870 |

We observe that it is not possible to have equal magic sums of order 3 as it is impossible to divide magic sum of order 12 by 4, i.e., 870/4:=217.5. In this case, all the $3\times3$ blocks are magic squares of order 3 with different magic sums giving again a **pan diagonal** magic square of order 4 given in example below.

**Example 2.3.** *The magic sums of 16 blocks of magic squares of order 3 appearing in Example 2.2 given above lead us to a* ***pan diagonal*** *magic square of order 4 given by*

| | | | | | |
|---|---|---|---|---|---|
| | | *870* | *870* | *870* | *870* |
| | *168* | *285* | *42* | *375* | *870* |
| *870* | *51* | *366* | *177* | *276* | *870* |
| *870* | *393* | *60* | *267* | *150* | *870* |
| *870* | *258* | *159* | *384* | *69* | *870* |
| | *870* | *870* | *870* | *870* | *870* |

### 2.1.3 Third Approach: 4 Blocks of Order 6

**Example 2.4.** *4 blocks of magic squares of order 6 constructed using Example 1.4 lead us to a magic square of order 12 given by*

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | *870* |
| *1* | *137* | *136* | *129* | *8* | *24* | *2* | *138* | *135* | *130* | *7* | *23* | *870* |
| *120* | *32* | *112* | *33* | *41* | *97* | *119* | *31* | *111* | *34* | *42* | *98* | *870* |
| *96* | *89* | *57* | *64* | *80* | *49* | *95* | *90* | *58* | *63* | *79* | *50* | *870* |
| *72* | *56* | *81* | *88* | *65* | *73* | *71* | *55* | *82* | *87* | *66* | *74* | *870* |
| *25* | *104* | *40* | *105* | *113* | *48* | *26* | *103* | *39* | *106* | *114* | *47* | *870* |
| *121* | *17* | *9* | *16* | *128* | *144* | *122* | *18* | *10* | *15* | *127* | *143* | *870* |
| *3* | *139* | *134* | *131* | *6* | *22* | *4* | *140* | *133* | *132* | *5* | *21* | *870* |
| *118* | *30* | *110* | *35* | *43* | *99* | *117* | *29* | *109* | *36* | *44* | *100* | *870* |
| *94* | *91* | *59* | *62* | *78* | *51* | *93* | *92* | *60* | *61* | *77* | *52* | *870* |
| *70* | *54* | *83* | *86* | *67* | *75* | *69* | *53* | *84* | *85* | *68* | *76* | *870* |
| *27* | *102* | *38* | *107* | *115* | *46* | *28* | *101* | *37* | *108* | *116* | *45* | *870* |
| *123* | *19* | *11* | *14* | *126* | *142* | *124* | *20* | *12* | *13* | *125* | *141* | *870* |
| *870* | *870* | *870* | *870* | *870* | *870* | *870* | *870* | *870* | *870* | *870* | *870* | *870* |

In this case, the magic sum is $S_{12\times 12} := 870$, and all the four blocks of order 6 are magic squares with equal magic sums $S_{6\times 6} := 435$.

## 2.2 Magic Squares of Order 15

In this subsection, we shall present two different ways of writing block-wise magic square of order 15. One as $5\times 3$, i.e., magic square formed by 9 blocks of **pan diagonal** magic squares of order 5 with equal magic sums. The second as $3\times 5$, i.e., magic square formed by 25 blocks semi-magic squares of order 3 (only in rows and columns) with equal semi-magic sums.

### 2.2.1 First Approach: 9 Blocks of Order 5

**Example 2.5.** *9* ***pan diagonal*** *magic squares of order 5 constructed according to Example 1.3 lead us to a* ***pan diagonal*** *magic square of order 15 given by*

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 |
| | 1 | 87 | 111 | 178 | 188 | 3 | 86 | 110 | 179 | 187 | 2 | 85 | 109 | 180 | 189 | 1695 |
| 1695 | 171 | 193 | 8 | 76 | 117 | 170 | 194 | 7 | 78 | 116 | 169 | 195 | 9 | 77 | 115 | 1695 |
| 1695 | 83 | 106 | 177 | 186 | 13 | 82 | 108 | 176 | 185 | 14 | 84 | 107 | 175 | 184 | 15 | 1695 |
| 1695 | 192 | 6 | 88 | 113 | 166 | 191 | 5 | 89 | 112 | 168 | 190 | 4 | 90 | 114 | 167 | 1695 |
| 1695 | 118 | 173 | 181 | 12 | 81 | 119 | 172 | 183 | 11 | 80 | 120 | 174 | 182 | 10 | 79 | 1695 |
| 1695 | 31 | 72 | 96 | 163 | 203 | 33 | 71 | 95 | 164 | 202 | 32 | 70 | 94 | 165 | 204 | 1695 |
| 1695 | 156 | 208 | 38 | 61 | 102 | 155 | 209 | 37 | 63 | 101 | 154 | 210 | 39 | 62 | 100 | 1695 |
| 1695 | 68 | 91 | 162 | 201 | 43 | 67 | 93 | 161 | 200 | 44 | 69 | 92 | 160 | 199 | 45 | 1695 |
| 1695 | 207 | 36 | 73 | 98 | 151 | 206 | 35 | 74 | 97 | 153 | 205 | 34 | 75 | 99 | 152 | 1695 |
| 1695 | 103 | 158 | 196 | 42 | 66 | 104 | 157 | 198 | 41 | 65 | 105 | 159 | 197 | 40 | 64 | 1695 |
| 1695 | 16 | 57 | 126 | 148 | 218 | 18 | 56 | 125 | 149 | 217 | 17 | 55 | 124 | 150 | 219 | 1695 |
| 1695 | 141 | 223 | 23 | 46 | 132 | 140 | 224 | 22 | 48 | 131 | 139 | 225 | 24 | 47 | 130 | 1695 |
| 1695 | 53 | 121 | 147 | 216 | 28 | 52 | 123 | 146 | 215 | 29 | 54 | 122 | 145 | 214 | 30 | 1695 |
| 1695 | 222 | 21 | 58 | 128 | 136 | 221 | 20 | 59 | 127 | 138 | 220 | 19 | 60 | 129 | 137 | 1695 |
| 1695 | 133 | 143 | 211 | 27 | 51 | 134 | 142 | 213 | 26 | 50 | 135 | 144 | 212 | 25 | 49 | 1695 |
| | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 |

In this case, the magic sum is $S_{15\times 15} = 1695$. All the $5\times 5$ blocks are **pan diagonal** magic squares of order 5 with equal magic sums $S_{5\times 5} = 565$.

#### 2.2.2 Second Approach: 25 Blocks of Order 3

**Example 2.6.** *25 blocks of order 3 constructed according to Example 1.1 lead us to a* ***pan diagonal*** *magic square of order 15 given by*

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 |
| | 186 | 65 | 88 | 187 | 74 | 78 | 188 | 75 | 76 | 189 | 64 | 86 | 190 | 62 | 87 | 1695 |
| 1695 | 80 | 193 | 66 | 89 | 183 | 67 | 90 | 181 | 68 | 79 | 191 | 69 | 77 | 192 | 70 | 1695 |
| 1695 | 73 | 81 | 185 | 63 | 82 | 194 | 61 | 83 | 195 | 71 | 84 | 184 | 72 | 85 | 182 | 1695 |
| 1695 | 36 | 200 | 103 | 37 | 209 | 93 | 38 | 210 | 91 | 39 | 199 | 101 | 40 | 197 | 102 | 1695 |
| 1695 | 95 | 43 | 201 | 104 | 33 | 202 | 105 | 31 | 203 | 94 | 41 | 204 | 92 | 42 | 205 | 1695 |
| 1695 | 208 | 96 | 35 | 198 | 97 | 44 | 196 | 98 | 45 | 206 | 99 | 34 | 207 | 100 | 32 | 1695 |
| 1695 | 6 | 215 | 118 | 7 | 224 | 108 | 8 | 225 | 106 | 9 | 214 | 116 | 10 | 212 | 117 | 1695 |
| 1695 | 110 | 13 | 216 | 119 | 3 | 217 | 120 | 1 | 218 | 109 | 11 | 219 | 107 | 12 | 220 | 1695 |
| 1695 | 223 | 111 | 5 | 213 | 112 | 14 | 211 | 113 | 15 | 221 | 114 | 4 | 222 | 115 | 2 | 1695 |
| 1695 | 156 | 50 | 133 | 157 | 59 | 123 | 158 | 60 | 121 | 159 | 49 | 131 | 160 | 47 | 132 | 1695 |
| 1695 | 125 | 163 | 51 | 134 | 153 | 52 | 135 | 151 | 53 | 124 | 161 | 54 | 122 | 162 | 55 | 1695 |
| 1695 | 58 | 126 | 155 | 48 | 127 | 164 | 46 | 128 | 165 | 56 | 129 | 154 | 57 | 130 | 152 | 1695 |
| 1695 | 171 | 20 | 148 | 172 | 29 | 138 | 173 | 30 | 136 | 174 | 19 | 146 | 175 | 17 | 147 | 1695 |
| 1695 | 140 | 178 | 21 | 149 | 168 | 22 | 150 | 166 | 23 | 139 | 176 | 24 | 137 | 177 | 25 | 1695 |
| 1695 | 28 | 141 | 170 | 18 | 142 | 179 | 16 | 143 | 180 | 26 | 144 | 169 | 27 | 145 | 167 | 1695 |
| | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 | 1695 |

In this case, the magic sum is $S_{15\times 15} = 1695$. All the $3\times 3$ blocks are **semi-magic** squares of order 3 with equal **semi-magic** sums, i.e., $S_{3\times 3} = 339$ (in rows and columns).

### 2.3 Magic and Bimagic Squares of Order 16

In this subsection, we shall present two different ways of writing magic squares of order 16. One is **pan diagonal** magic square formed by 16 **pan diagonal** magic squares of order 4 of equal magic sums. The second is **bimagic** square of order 16.

#### 2.3.1 First Approach: Pan Magic Square of Order 16

**Example 2.7.** *A **pan diagonal** magic square of order 16 constructed according to Example 1.2 is given by*

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 |
| | 97 | 192 | 1 | 224 | 98 | 191 | 2 | 223 | 99 | 190 | 3 | 222 | 100 | 189 | 4 | 221 | 2056 |
| 2056 | 32 | 193 | 128 | 161 | 31 | 194 | 127 | 162 | 30 | 195 | 126 | 163 | 29 | 196 | 125 | 164 | 2056 |
| 2056 | 256 | 33 | 160 | 65 | 255 | 34 | 159 | 66 | 254 | 35 | 158 | 67 | 253 | 36 | 157 | 68 | 2056 |
| 2056 | 129 | 96 | 225 | 64 | 130 | 95 | 226 | 63 | 131 | 94 | 227 | 62 | 132 | 93 | 228 | 61 | 2056 |
| 2056 | 101 | 188 | 5 | 220 | 102 | 187 | 6 | 219 | 103 | 186 | 7 | 218 | 104 | 185 | 8 | 217 | 2056 |
| 2056 | 28 | 197 | 124 | 165 | 27 | 198 | 123 | 166 | 26 | 199 | 122 | 167 | 25 | 200 | 121 | 168 | 2056 |
| 2056 | 252 | 37 | 156 | 69 | 251 | 38 | 155 | 70 | 250 | 39 | 154 | 71 | 249 | 40 | 153 | 72 | 2056 |
| 2056 | 133 | 92 | 229 | 60 | 134 | 91 | 230 | 59 | 135 | 90 | 231 | 58 | 136 | 89 | 232 | 57 | 2056 |
| 2056 | 105 | 184 | 9 | 216 | 106 | 183 | 10 | 215 | 107 | 182 | 11 | 214 | 108 | 181 | 12 | 213 | 2056 |
| 2056 | 24 | 201 | 120 | 169 | 23 | 202 | 119 | 170 | 22 | 203 | 118 | 171 | 21 | 204 | 117 | 172 | 2056 |
| 2056 | 248 | 41 | 152 | 73 | 247 | 42 | 151 | 74 | 246 | 43 | 150 | 75 | 245 | 44 | 149 | 76 | 2056 |
| 2056 | 137 | 88 | 233 | 56 | 138 | 87 | 234 | 55 | 139 | 86 | 235 | 54 | 140 | 85 | 236 | 53 | 2056 |
| 2056 | 109 | 180 | 13 | 212 | 110 | 179 | 14 | 211 | 111 | 178 | 15 | 210 | 112 | 177 | 16 | 209 | 2056 |
| 2056 | 20 | 205 | 116 | 173 | 19 | 206 | 115 | 174 | 18 | 207 | 114 | 175 | 17 | 208 | 113 | 176 | 2056 |
| 2056 | 244 | 45 | 148 | 77 | 243 | 46 | 147 | 78 | 242 | 47 | 146 | 79 | 241 | 48 | 145 | 80 | 2056 |
| 2056 | 141 | 84 | 237 | 52 | 142 | 83 | 238 | 51 | 143 | 82 | 239 | 50 | 144 | 81 | 240 | 49 | 2056 |
| | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 |

In this case, the magic sum is $S_{16\times 16} = 2056$. All the $4\times 4$ blocks are **pan diagonal** magic square of order 4 with equal magic sums, $S_{4\times 4} = 514$.

#### 2.3.2 Second Approach: Bimagic Square of Order 16

**Example 2.8.** *A **bimagic** square of order 16 is given by*

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | 2056 |
| 1 | 154 | 239 | 120 | 23 | 144 | 249 | 98 | 44 | 179 | 198 | 93 | 62 | 165 | 212 | 75 | 2056 |
| 232 | 127 | 10 | 145 | 242 | 105 | 32 | 135 | 205 | 86 | 35 | 188 | 219 | 68 | 53 | 174 | 2056 |
| 122 | 225 | 152 | 15 | 112 | 247 | 130 | 25 | 83 | 204 | 189 | 38 | 69 | 222 | 171 | 52 | 2056 |
| 159 | 8 | 113 | 234 | 137 | 18 | 103 | 256 | 182 | 45 | 92 | 195 | 164 | 59 | 78 | 213 | 2056 |
| 46 | 181 | 196 | 91 | 60 | 163 | 214 | 77 | 7 | 160 | 233 | 114 | 17 | 138 | 255 | 104 | 2056 |
| 203 | 84 | 37 | 190 | 221 | 70 | 51 | 172 | 226 | 121 | 16 | 151 | 248 | 111 | 26 | 129 | 2056 |
| 85 | 206 | 187 | 36 | 67 | 220 | 173 | 54 | 128 | 231 | 146 | 9 | 106 | 241 | 136 | 31 | 2056 |
| 180 | 43 | 94 | 197 | 166 | 61 | 76 | 211 | 153 | 2 | 119 | 240 | 143 | 24 | 97 | 250 | 2056 |
| 55 | 176 | 217 | 66 | 33 | 186 | 207 | 88 | 30 | 133 | 244 | 107 | 12 | 147 | 230 | 125 | 2056 |
| 210 | 73 | 64 | 167 | 200 | 95 | 42 | 177 | 251 | 100 | 21 | 142 | 237 | 118 | 3 | 156 | 2056 |
| 80 | 215 | 162 | 57 | 90 | 193 | 184 | 47 | 101 | 254 | 139 | 20 | 115 | 236 | 157 | 6 | 2056 |
| 169 | 50 | 71 | 224 | 191 | 40 | 81 | 202 | 132 | 27 | 110 | 245 | 150 | 13 | 124 | 227 | 2056 |
| 28 | 131 | 246 | 109 | 14 | 149 | 228 | 123 | 49 | 170 | 223 | 72 | 39 | 192 | 201 | 82 | 2056 |
| 253 | 102 | 19 | 140 | 235 | 116 | 5 | 158 | 216 | 79 | 58 | 161 | 194 | 89 | 48 | 183 | 2056 |
| 99 | 252 | 141 | 22 | 117 | 238 | 155 | 4 | 74 | 209 | 168 | 63 | 96 | 199 | 178 | 41 | 2056 |
| 134 | 29 | 108 | 243 | 148 | 11 | 126 | 229 | 175 | 56 | 65 | 218 | 185 | 34 | 87 | 208 | 2056 |
| 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 | 2056 |

In this case, the magic and **bimagic** sums are $S_{4x4} := 2056$ and $Sb_{16x16} := 351576$ respectively. Details of this **bimagic** square can be seen in author's work [26].

### 2.4 Magic Square of Order 18

In this subsection, we shall present two different ways of writing magic square of order 18. One is with 36 blocks of magic squares of order 3 with different magic sums. The second is with 9 blocks of magic squares of order 6 of equal magic sums.

#### 2.4.1 First Approach: 36 Blocks of Order 3

**Example 2.9.** *By the applications of Examples 1.1 and 1.4, we get a magic square of order 18 given by*

| | | | | | | | | | | | | | | | | | | 2925 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 19 | 39 | 2 | 193 | 177 | 212 | 246 | 262 | 227 | 300 | 280 | 317 | 139 | 159 | 122 | 78 | 58 | 95 | 2925 |
| 3 | 20 | 37 | 213 | 194 | 175 | 226 | 245 | 264 | 316 | 299 | 282 | 123 | 140 | 157 | 94 | 77 | 60 | 2925 |
| 38 | 1 | 21 | 176 | 211 | 195 | 263 | 228 | 244 | 281 | 318 | 298 | 158 | 121 | 141 | 59 | 96 | 76 | 2925 |
| 247 | 267 | 230 | 73 | 57 | 92 | 301 | 285 | 320 | 132 | 148 | 113 | 187 | 171 | 206 | 31 | 51 | 14 | 2925 |
| 231 | 248 | 265 | 93 | 74 | 55 | 321 | 302 | 283 | 112 | 131 | 150 | 207 | 188 | 169 | 15 | 32 | 49 | 2925 |
| 266 | 229 | 249 | 56 | 91 | 75 | 284 | 319 | 303 | 149 | 114 | 130 | 170 | 205 | 189 | 50 | 13 | 33 | 2925 |
| 90 | 70 | 107 | 36 | 52 | 17 | 127 | 147 | 110 | 241 | 261 | 224 | 289 | 273 | 308 | 192 | 172 | 209 | 2925 |
| 106 | 89 | 72 | 16 | 35 | 54 | 111 | 128 | 145 | 225 | 242 | 259 | 309 | 290 | 271 | 208 | 191 | 174 | 2925 |
| 71 | 108 | 88 | 53 | 18 | 34 | 146 | 109 | 129 | 260 | 223 | 243 | 272 | 307 | 291 | 173 | 210 | 190 | 2925 |
| 294 | 274 | 311 | 138 | 154 | 119 | 30 | 46 | 11 | 198 | 178 | 215 | 84 | 64 | 101 | 235 | 255 | 218 | 2925 |
| 310 | 293 | 276 | 118 | 137 | 156 | 10 | 29 | 48 | 214 | 197 | 180 | 100 | 83 | 66 | 219 | 236 | 253 | 2925 |
| 275 | 312 | 292 | 155 | 120 | 136 | 47 | 12 | 28 | 179 | 216 | 196 | 65 | 102 | 82 | 254 | 217 | 237 | 2925 |
| 181 | 165 | 200 | 295 | 279 | 314 | 85 | 69 | 104 | 25 | 45 | 8 | 252 | 268 | 233 | 133 | 153 | 116 | 2925 |
| 201 | 182 | 163 | 315 | 296 | 277 | 105 | 86 | 67 | 9 | 26 | 43 | 232 | 251 | 270 | 117 | 134 | 151 | 2925 |
| 164 | 199 | 183 | 278 | 313 | 297 | 68 | 103 | 87 | 44 | 7 | 27 | 269 | 234 | 250 | 152 | 115 | 135 | 2925 |
| 144 | 160 | 125 | 240 | 256 | 221 | 186 | 166 | 203 | 79 | 63 | 98 | 24 | 40 | 5 | 306 | 286 | 323 | 2925 |
| 124 | 143 | 162 | 220 | 239 | 258 | 202 | 185 | 168 | 99 | 80 | 61 | 4 | 23 | 42 | 322 | 305 | 288 | 2925 |
| 161 | 126 | 142 | 257 | 222 | 238 | 167 | 204 | 184 | 62 | 97 | 81 | 41 | 6 | 22 | 287 | 324 | 304 | 2925 |
| 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 |

We observe that it is not possible to have equal magic sums of order 3 as it is impossible to divide magic sum of order 18 by 6, i.e., 2925/6:=487.5. All the 36 blocks are magic squares of order 3 with different magic sums resulting again a magic square of order 6 given in example below.

**Example 2.10.** *The magic sums of magic squares of order 3 of Example 2.9 lead us to magic square of order 6 given by*

| | | | | | | 2925 |
|---|---|---|---|---|---|---|
| 60 | 582 | 735 | 897 | 420 | 231 | 2925 |
| 744 | 222 | 906 | 393 | 564 | 96 | 2925 |
| 267 | 105 | 384 | 726 | 870 | 573 | 2925 |
| 879 | 411 | 87 | 591 | 249 | 708 | 2925 |
| 546 | 888 | 258 | 78 | 753 | 402 | 2925 |
| 429 | 717 | 555 | 240 | 69 | 915 | 2925 |
| 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 |

#### 2.4.2 Second Approach: 9 Blocks of Order 6

**Example 2.11.** *9 blocks of magic squares of order 6 constructed according to Example 1.4 lead us to a magic square of order 18 given by*

| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | 2925 |
| 1 | 307 | 306 | 289 | 18 | 54 | 2 | 308 | 305 | 290 | 17 | 53 | 3 | 309 | 304 | 291 | 16 | 52 | 2925 |
| 270 | 72 | 252 | 73 | 91 | 217 | 269 | 71 | 251 | 74 | 92 | 218 | 268 | 70 | 250 | 75 | 93 | 219 | 2925 |
| 216 | 199 | 127 | 144 | 180 | 109 | 215 | 200 | 128 | 143 | 179 | 110 | 214 | 201 | 129 | 142 | 178 | 111 | 2925 |
| 162 | 126 | 181 | 198 | 145 | 163 | 161 | 125 | 182 | 197 | 146 | 164 | 160 | 124 | 183 | 196 | 147 | 165 | 2925 |
| 55 | 234 | 90 | 235 | 253 | 108 | 56 | 233 | 89 | 236 | 254 | 107 | 57 | 232 | 88 | 237 | 255 | 106 | 2925 |
| 271 | 37 | 19 | 36 | 288 | 324 | 272 | 38 | 20 | 35 | 287 | 323 | 273 | 39 | 21 | 34 | 286 | 322 | 2925 |
| 4 | 310 | 303 | 292 | 15 | 51 | 5 | 311 | 302 | 293 | 14 | 50 | 6 | 312 | 301 | 294 | 13 | 49 | 2925 |
| 267 | 69 | 249 | 76 | 94 | 220 | 266 | 68 | 248 | 77 | 95 | 221 | 265 | 67 | 247 | 78 | 96 | 222 | 2925 |
| 213 | 202 | 130 | 141 | 177 | 112 | 212 | 203 | 131 | 140 | 176 | 113 | 211 | 204 | 132 | 139 | 175 | 114 | 2925 |
| 159 | 123 | 184 | 195 | 148 | 166 | 158 | 122 | 185 | 194 | 149 | 167 | 157 | 121 | 186 | 193 | 150 | 168 | 2925 |
| 58 | 231 | 87 | 238 | 256 | 105 | 59 | 230 | 86 | 239 | 257 | 104 | 60 | 229 | 85 | 240 | 258 | 103 | 2925 |
| 274 | 40 | 22 | 33 | 285 | 321 | 275 | 41 | 23 | 32 | 284 | 320 | 276 | 42 | 24 | 31 | 283 | 319 | 2925 |
| 7 | 313 | 300 | 295 | 12 | 48 | 8 | 314 | 299 | 296 | 11 | 47 | 9 | 315 | 298 | 297 | 10 | 46 | 2925 |
| 264 | 66 | 246 | 79 | 97 | 223 | 263 | 65 | 245 | 80 | 98 | 224 | 262 | 64 | 244 | 81 | 99 | 225 | 2925 |
| 210 | 205 | 133 | 138 | 174 | 115 | 209 | 206 | 134 | 137 | 173 | 116 | 208 | 207 | 135 | 136 | 172 | 117 | 2925 |
| 156 | 120 | 187 | 192 | 151 | 169 | 155 | 119 | 188 | 191 | 152 | 170 | 154 | 118 | 189 | 190 | 153 | 171 | 2925 |
| 61 | 228 | 84 | 241 | 259 | 102 | 62 | 227 | 83 | 242 | 260 | 101 | 63 | 226 | 82 | 243 | 261 | 100 | 2925 |
| 277 | 43 | 25 | 30 | 282 | 318 | 278 | 44 | 26 | 29 | 281 | 317 | 279 | 45 | 27 | 28 | 280 | 316 | 2925 |
| 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 | 2925 |

The above magic square is with magic sum $S_{18\times 18} = 2925$, and all the 9 blocks of order 6 are magic squares with equal magic sums, $S_{6\times 6} := 975$.

### 2.5 Magic Square of Order 20

In this subsection, we shall present three different ways of writing block-wise magic square of order 20. One as $4\times 5$, i.e., magic square formed by 25 blocks of equal magic sums of **pan diagonal** magic squares of order 4. The second as $5\times 4$, i.e., magic square formed by 16 blocks of **pan diagonal** magic squares of order 5 with different magic sums. The third as 4 blocks of equal magic sums of magic squares of order 10. In the first two cases, the magic squares are **pan diagonals**, while in the third case is just a magic square of order 20.

#### 2.5.1 First Approach: 25 Blocks of Order 4

**Example 2.12.** *25 blocks of magic squares of order 4 constructed according to Example 1.2 lead us to a* ***pan diagonal*** *magic square of order 20 is given by*

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 |
| | 151 | 300 | 1 | 350 | 152 | 299 | 2 | 349 | 153 | 298 | 3 | 348 | 154 | 297 | 4 | 347 | 155 | 296 | 5 | 346 | 4010 |
| 4010 | 50 | 301 | 200 | 251 | 49 | 302 | 199 | 252 | 48 | 303 | 198 | 253 | 47 | 304 | 197 | 254 | 46 | 305 | 196 | 255 | 4010 |
| 4010 | 400 | 51 | 250 | 101 | 399 | 52 | 249 | 102 | 398 | 53 | 248 | 103 | 397 | 54 | 247 | 104 | 396 | 55 | 246 | 105 | 4010 |
| 4010 | 201 | 150 | 351 | 100 | 202 | 149 | 352 | 99 | 203 | 148 | 353 | 98 | 204 | 147 | 354 | 97 | 205 | 146 | 355 | 96 | 4010 |
| 4010 | 156 | 295 | 6 | 345 | 157 | 294 | 7 | 344 | 158 | 293 | 8 | 343 | 159 | 292 | 9 | 342 | 160 | 291 | 10 | 341 | 4010 |
| 4010 | 45 | 306 | 195 | 256 | 44 | 307 | 194 | 257 | 43 | 308 | 193 | 258 | 42 | 309 | 192 | 259 | 41 | 310 | 191 | 260 | 4010 |
| 4010 | 395 | 56 | 245 | 106 | 394 | 57 | 244 | 107 | 393 | 58 | 243 | 108 | 392 | 59 | 242 | 109 | 391 | 60 | 241 | 110 | 4010 |
| 4010 | 206 | 145 | 356 | 95 | 207 | 144 | 357 | 94 | 208 | 143 | 358 | 93 | 209 | 142 | 359 | 92 | 210 | 141 | 360 | 91 | 4010 |
| 4010 | 161 | 290 | 11 | 340 | 162 | 289 | 12 | 339 | 163 | 288 | 13 | 338 | 164 | 287 | 14 | 337 | 165 | 286 | 15 | 336 | 4010 |
| 4010 | 40 | 311 | 190 | 261 | 39 | 312 | 189 | 262 | 38 | 313 | 188 | 263 | 37 | 314 | 187 | 264 | 36 | 315 | 186 | 265 | 4010 |
| 4010 | 390 | 61 | 240 | 111 | 389 | 62 | 239 | 112 | 388 | 63 | 238 | 113 | 387 | 64 | 237 | 114 | 386 | 65 | 236 | 115 | 4010 |
| 4010 | 211 | 140 | 361 | 90 | 212 | 139 | 362 | 89 | 213 | 138 | 363 | 88 | 214 | 137 | 364 | 87 | 215 | 136 | 365 | 86 | 4010 |
| 4010 | 166 | 285 | 16 | 335 | 167 | 284 | 17 | 334 | 168 | 283 | 18 | 333 | 169 | 282 | 19 | 332 | 170 | 281 | 20 | 331 | 4010 |
| 4010 | 35 | 316 | 185 | 266 | 34 | 317 | 184 | 267 | 33 | 318 | 183 | 268 | 32 | 319 | 182 | 269 | 31 | 320 | 181 | 270 | 4010 |
| 4010 | 385 | 66 | 235 | 116 | 384 | 67 | 234 | 117 | 383 | 68 | 233 | 118 | 382 | 69 | 232 | 119 | 381 | 70 | 231 | 120 | 4010 |
| 4010 | 216 | 135 | 366 | 85 | 217 | 134 | 367 | 84 | 218 | 133 | 368 | 83 | 219 | 132 | 369 | 82 | 220 | 131 | 370 | 81 | 4010 |
| 4010 | 171 | 280 | 21 | 330 | 172 | 279 | 22 | 329 | 173 | 278 | 23 | 328 | 174 | 277 | 24 | 327 | 175 | 276 | 25 | 326 | 4010 |
| 4010 | 30 | 321 | 180 | 271 | 29 | 322 | 179 | 272 | 28 | 323 | 178 | 273 | 27 | 324 | 177 | 274 | 26 | 325 | 176 | 275 | 4010 |
| 4010 | 380 | 71 | 230 | 121 | 379 | 72 | 229 | 122 | 378 | 73 | 228 | 123 | 377 | 74 | 227 | 124 | 376 | 75 | 226 | 125 | 4010 |
| 4010 | 221 | 130 | 371 | 80 | 222 | 129 | 372 | 79 | 223 | 128 | 373 | 78 | 224 | 127 | 374 | 77 | 225 | 126 | 375 | 76 | 4010 |
| | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 |

In this case, the magic sum is $S_{20\times 20} = 4010$. All the $4\times 4$ blocks are **pan diagonal** magic square of order 4 with the equal magic sums, $S_{4\times 4} = 802$.

### 2.5.2 Second Approach: 16 Blocks of Order 5

**Example 2.13.** *16 blocks of magic squares of order 5 constructed according to Example 1.3 lead us to a* ***pan diagonal*** *magic square of order 20 given by*

| | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 |
| | 7 | 135 | 183 | 311 | 359 | 12 | 140 | 188 | 316 | 364 | 1 | 129 | 177 | 305 | 353 | 14 | 142 | 190 | 318 | 366 | 4010 |
| 4010 | 263 | 391 | 39 | 87 | 215 | 268 | 396 | 44 | 92 | 220 | 257 | 385 | 33 | 81 | 209 | 270 | 398 | 46 | 94 | 222 | 4010 |
| 4010 | 119 | 167 | 295 | 343 | 71 | 124 | 172 | 300 | 348 | 76 | 113 | 161 | 289 | 337 | 65 | 126 | 174 | 302 | 350 | 78 | 4010 |
| 4010 | 375 | 23 | 151 | 199 | 247 | 380 | 28 | 156 | 204 | 252 | 369 | 17 | 145 | 193 | 241 | 382 | 30 | 158 | 206 | 254 | 4010 |
| 4010 | 231 | 279 | 327 | 55 | 103 | 236 | 284 | 332 | 60 | 108 | 225 | 273 | 321 | 49 | 97 | 238 | 286 | 334 | 62 | 110 | 4010 |
| 4010 | 2 | 130 | 178 | 306 | 354 | 13 | 141 | 189 | 317 | 365 | 8 | 136 | 184 | 312 | 360 | 11 | 139 | 187 | 315 | 363 | 4010 |
| 4010 | 258 | 386 | 34 | 82 | 210 | 269 | 397 | 45 | 93 | 221 | 264 | 392 | 40 | 88 | 216 | 267 | 395 | 43 | 91 | 219 | 4010 |
| 4010 | 114 | 162 | 290 | 338 | 66 | 125 | 173 | 301 | 349 | 77 | 120 | 168 | 296 | 344 | 72 | 123 | 171 | 299 | 347 | 75 | 4010 |
| 4010 | 370 | 18 | 146 | 194 | 242 | 381 | 29 | 157 | 205 | 253 | 376 | 24 | 152 | 200 | 248 | 379 | 27 | 155 | 203 | 251 | 4010 |
| 4010 | 226 | 274 | 322 | 50 | 98 | 237 | 285 | 333 | 61 | 109 | 232 | 280 | 328 | 56 | 104 | 235 | 283 | 331 | 59 | 107 | 4010 |
| 4010 | 16 | 144 | 192 | 320 | 368 | 3 | 131 | 179 | 307 | 355 | 10 | 138 | 186 | 314 | 362 | 5 | 133 | 181 | 309 | 357 | 4010 |
| 4010 | 272 | 400 | 48 | 96 | 224 | 259 | 387 | 35 | 83 | 211 | 266 | 394 | 42 | 90 | 218 | 261 | 389 | 37 | 85 | 213 | 4010 |
| 4010 | 128 | 176 | 304 | 352 | 80 | 115 | 163 | 291 | 339 | 67 | 122 | 170 | 298 | 346 | 74 | 117 | 165 | 293 | 341 | 69 | 4010 |
| 4010 | 384 | 32 | 160 | 208 | 256 | 371 | 19 | 147 | 195 | 243 | 378 | 26 | 154 | 202 | 250 | 373 | 21 | 149 | 197 | 245 | 4010 |
| 4010 | 240 | 288 | 336 | 64 | 112 | 227 | 275 | 323 | 51 | 99 | 234 | 282 | 330 | 58 | 106 | 229 | 277 | 325 | 53 | 101 | 4010 |
| 4010 | 9 | 137 | 185 | 313 | 361 | 6 | 134 | 182 | 310 | 358 | 15 | 143 | 191 | 319 | 367 | 4 | 132 | 180 | 308 | 356 | 4010 |
| 4010 | 265 | 393 | 41 | 89 | 217 | 262 | 390 | 38 | 86 | 214 | 271 | 399 | 47 | 95 | 223 | 260 | 388 | 36 | 84 | 212 | 4010 |
| 4010 | 121 | 169 | 297 | 345 | 73 | 118 | 166 | 294 | 342 | 70 | 127 | 175 | 303 | 351 | 79 | 116 | 164 | 292 | 340 | 68 | 4010 |
| 4010 | 377 | 25 | 153 | 201 | 249 | 374 | 22 | 150 | 198 | 246 | 383 | 31 | 159 | 207 | 255 | 372 | 20 | 148 | 196 | 244 | 4010 |
| 4010 | 233 | 281 | 329 | 57 | 105 | 230 | 278 | 326 | 54 | 102 | 239 | 287 | 335 | 63 | 111 | 228 | 276 | 324 | 52 | 100 | 4010 |
| | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 | 4010 |

We observe that it is not possible to have equal magic sums of order 5 as it is impossible to divide magic sum 4010 by 4, i.e., 4010/4:=1002.5. In this case, all the $5\times 5$ blocks are magic squares of order 5 with different magic sums giving a **pan diagonal** magic square of order 4 given in example below.

**Example 2.14.** *A* ***pan diagonal*** *magic square of order 4, formed by magic sums of 16 magic squares of order 5 is given by*

| | | | | | |
|---|---|---|---|---|---|
| | | 4010 | 4010 | 4010 | 4010 |
| | 995 | 1020 | 965 | 1030 | 4010 |
| 4010 | 970 | 1025 | 1000 | 1015 | 4010 |
| 4010 | 1040 | 975 | 1010 | 985 | 4010 |
| 4010 | 1005 | 990 | 1035 | 980 | 4010 |
| | 4010 | 4010 | 4010 | 4010 | 4010 |

### 2.5.3 Third Approach: 4 Blocks of Order 10

**Example 2.15.** *4 blocks of equal sums magic squares of order 10 constructed according to Example 1.10 lead us to a magic square of order 20 given by*

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | **4010** |
| 1 | 177 | 72 | 384 | 336 | 88 | 309 | 153 | 240 | 245 | 2 | 178 | 71 | 383 | 335 | 87 | 310 | 154 | 239 | 246 | **4010** |
| 152 | 64 | 256 | 325 | 100 | 397 | 161 | 228 | 9 | 313 | 151 | 63 | 255 | 326 | 99 | 398 | 162 | 227 | 10 | 314 | **4010** |
| 244 | 236 | 85 | 180 | 392 | 13 | 157 | 321 | 308 | 69 | 243 | 235 | 86 | 179 | 391 | 14 | 158 | 322 | 307 | 70 | **4010** |
| 80 | 333 | 229 | 148 | 301 | 165 | 396 | 4 | 252 | 97 | 79 | 334 | 230 | 147 | 302 | 166 | 395 | 3 | 251 | 98 | **4010** |
| 388 | 81 | 17 | 233 | 169 | 304 | 332 | 260 | 65 | 156 | 387 | 82 | 18 | 234 | 170 | 303 | 331 | 259 | 66 | 155 | **4010** |
| 329 | 248 | 141 | 317 | 73 | 232 | 20 | 385 | 96 | 164 | 330 | 247 | 142 | 318 | 74 | 231 | 19 | 386 | 95 | 163 | **4010** |
| 176 | 5 | 320 | 92 | 224 | 149 | 253 | 77 | 381 | 328 | 175 | 6 | 319 | 91 | 223 | 150 | 254 | 78 | 382 | 327 | **4010** |
| 93 | 389 | 168 | 61 | 257 | 340 | 225 | 316 | 144 | 12 | 94 | 390 | 167 | 62 | 258 | 339 | 226 | 315 | 143 | 11 | **4010** |
| 305 | 160 | 393 | 249 | 8 | 76 | 84 | 172 | 337 | 221 | 306 | 159 | 394 | 250 | 7 | 75 | 83 | 171 | 338 | 222 | **4010** |
| 237 | 312 | 324 | 16 | 145 | 241 | 68 | 89 | 173 | 400 | 238 | 311 | 323 | 15 | 146 | 242 | 67 | 90 | 174 | 399 | **4010** |
| 21 | 197 | 52 | 364 | 356 | 108 | 289 | 133 | 220 | 265 | 22 | 198 | 51 | 363 | 355 | 107 | 290 | 134 | 219 | 266 | **4010** |
| 132 | 44 | 276 | 345 | 120 | 377 | 181 | 208 | 29 | 293 | 131 | 43 | 275 | 346 | 119 | 378 | 182 | 207 | 30 | 294 | **4010** |
| 264 | 216 | 105 | 200 | 372 | 33 | 137 | 341 | 288 | 49 | 263 | 215 | 106 | 199 | 371 | 34 | 138 | 342 | 287 | 50 | **4010** |
| 60 | 353 | 209 | 128 | 281 | 185 | 376 | 24 | 272 | 117 | 59 | 354 | 210 | 127 | 282 | 186 | 375 | 23 | 271 | 118 | **4010** |
| 368 | 101 | 37 | 213 | 189 | 284 | 352 | 280 | 45 | 136 | 367 | 102 | 38 | 214 | 190 | 283 | 351 | 279 | 46 | 135 | **4010** |
| 349 | 268 | 121 | 297 | 53 | 212 | 40 | 365 | 116 | 184 | 350 | 267 | 122 | 298 | 54 | 211 | 39 | 366 | 115 | 183 | **4010** |
| 196 | 25 | 300 | 112 | 204 | 129 | 273 | 57 | 361 | 348 | 195 | 26 | 299 | 111 | 203 | 130 | 274 | 58 | 362 | 347 | **4010** |
| 113 | 369 | 188 | 41 | 277 | 360 | 205 | 296 | 124 | 32 | 114 | 370 | 187 | 42 | 278 | 359 | 206 | 295 | 123 | 31 | **4010** |
| 285 | 140 | 373 | 269 | 28 | 56 | 104 | 192 | 357 | 201 | 286 | 139 | 374 | 270 | 27 | 55 | 103 | 191 | 358 | 202 | **4010** |
| 217 | 292 | 344 | 36 | 125 | 261 | 48 | 109 | 193 | 380 | 218 | 291 | 343 | 35 | 126 | 262 | 47 | 110 | 194 | 379 | **4010** |
| **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** | **4010** |

The above magic square is with magic sum $S_{20\times 20} = 4010$, and all the four blocks of order 10 are magic squares with equal magic sums $S_{10\times 10} := 2005$.

## 2.6 Magic Square of Order 21

In this subsection, we shall present two different ways of writing block-wise **pan diagonal** magic square of order 21. One as $7\times 3$, i.e., magic square formed by 9 blocks of **pan diagonal** magic squares of order 7. Second as $3\times 7$, i.e., magic square formed by 49 blocks semi-magic squares of order 3 with equal semi-magic sums (only in rows and columns). In both the cases the magic squares are **pan diagonal**

### 2.6.1 First Approach: 9 Blocks of Order 7

**Example 2.16.** *A **pan diagonal** magic square of order 21 formed by 9 blocks of equal sums magic squares of order 7 constructed according to Example 1.5 is given by*

| | | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 111 | 155 | 243 | 265 | 375 | 397 | 3 | 110 | 154 | 242 | 266 | 374 | 398 | 2 | 109 | 156 | 241 | 267 | 373 | 399 | 4641 |
| 4641 | 370 | 396 | 19 | 106 | 153 | 239 | 264 | 371 | 395 | 20 | 108 | 152 | 238 | 263 | 372 | 394 | 21 | 107 | 151 | 240 | 262 | 4641 |
| 4641 | 237 | 260 | 369 | 391 | 18 | 124 | 148 | 236 | 259 | 368 | 392 | 17 | 125 | 150 | 235 | 261 | 367 | 393 | 16 | 126 | 149 | 4641 |
| 4641 | 123 | 166 | 232 | 258 | 365 | 390 | 13 | 122 | 167 | 234 | 257 | 364 | 389 | 14 | 121 | 168 | 233 | 256 | 366 | 388 | 15 | 4641 |
| 4641 | 386 | 12 | 118 | 165 | 250 | 253 | 363 | 385 | 11 | 119 | 164 | 251 | 255 | 362 | 387 | 10 | 120 | 163 | 252 | 254 | 361 | 4641 |
| 4641 | 271 | 358 | 384 | 8 | 117 | 160 | 249 | 272 | 360 | 383 | 7 | 116 | 161 | 248 | 273 | 359 | 382 | 9 | 115 | 162 | 247 | 4641 |
| 4641 | 159 | 244 | 270 | 376 | 379 | 6 | 113 | 158 | 245 | 269 | 377 | 381 | 5 | 112 | 157 | 246 | 268 | 378 | 380 | 4 | 114 | 4641 |
| 4641 | 43 | 90 | 134 | 222 | 286 | 354 | 418 | 45 | 89 | 133 | 221 | 287 | 353 | 419 | 44 | 88 | 135 | 220 | 288 | 352 | 420 | 4641 |
| 4641 | 349 | 417 | 61 | 85 | 132 | 218 | 285 | 350 | 416 | 62 | 87 | 131 | 217 | 284 | 351 | 415 | 63 | 86 | 130 | 219 | 283 | 4641 |
| 4641 | 216 | 281 | 348 | 412 | 60 | 103 | 127 | 215 | 280 | 347 | 413 | 59 | 104 | 129 | 214 | 282 | 346 | 414 | 58 | 105 | 128 | 4641 |
| 4641 | 102 | 145 | 211 | 279 | 344 | 411 | 55 | 101 | 146 | 213 | 278 | 343 | 410 | 56 | 100 | 147 | 212 | 277 | 345 | 409 | 57 | 4641 |
| 4641 | 407 | 54 | 97 | 144 | 229 | 274 | 342 | 406 | 53 | 98 | 143 | 230 | 276 | 341 | 408 | 52 | 99 | 142 | 231 | 275 | 340 | 4641 |
| 4641 | 292 | 337 | 405 | 50 | 96 | 139 | 228 | 293 | 339 | 404 | 49 | 95 | 140 | 227 | 294 | 338 | 403 | 51 | 94 | 141 | 226 | 4641 |
| 4641 | 138 | 223 | 291 | 355 | 400 | 48 | 92 | 137 | 224 | 290 | 356 | 402 | 47 | 91 | 136 | 225 | 289 | 357 | 401 | 46 | 93 | 4641 |
| 4641 | 22 | 69 | 176 | 201 | 307 | 333 | 439 | 24 | 68 | 175 | 200 | 308 | 332 | 440 | 23 | 67 | 177 | 199 | 309 | 331 | 441 | 4641 |
| 4641 | 328 | 438 | 40 | 64 | 174 | 197 | 306 | 329 | 437 | 41 | 66 | 173 | 196 | 305 | 330 | 436 | 42 | 65 | 172 | 198 | 304 | 4641 |
| 4641 | 195 | 302 | 327 | 433 | 39 | 82 | 169 | 194 | 301 | 326 | 434 | 38 | 83 | 171 | 193 | 303 | 325 | 435 | 37 | 84 | 170 | 4641 |
| 4641 | 81 | 187 | 190 | 300 | 323 | 432 | 34 | 80 | 188 | 192 | 299 | 322 | 431 | 35 | 79 | 189 | 191 | 298 | 324 | 430 | 36 | 4641 |
| 4641 | 428 | 33 | 76 | 186 | 208 | 295 | 321 | 427 | 32 | 77 | 185 | 209 | 297 | 320 | 429 | 31 | 78 | 184 | 210 | 296 | 319 | 4641 |
| 4641 | 313 | 316 | 426 | 29 | 75 | 181 | 207 | 314 | 318 | 425 | 28 | 74 | 182 | 206 | 315 | 317 | 424 | 30 | 73 | 183 | 205 | 4641 |
| 4641 | 180 | 202 | 312 | 334 | 421 | 27 | 71 | 179 | 203 | 311 | 335 | 423 | 26 | 70 | 178 | 204 | 310 | 336 | 422 | 25 | 72 | 4641 |
| | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 |

The above magic square is with magic sum $S_{21\times21} = 4641$, and all the nine blocks of order 7 are **pan diagonal** magic squares with equal magic sums $S_{7\times7} := 1547$.

#### 2.6.2 Second Approach: 49 Blocks of Order 3

**Example 2.17.** *A **pan diagonal** magic square of order 21 formed by 49 blocks of semi-magic squares of order 3 constructed according to Examples 1.1 and 1.5 is given by*

| | | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 246 | 111 | 306 | 232 | 123 | 308 | 250 | 110 | 303 | 233 | 125 | 305 | 252 | 109 | 302 | 234 | 122 | 307 | 247 | 112 | 304 | 4641 |
| 4641 | 300 | 243 | 120 | 312 | 245 | 106 | 299 | 240 | 124 | 314 | 242 | 107 | 298 | 239 | 126 | 311 | 244 | 108 | 301 | 241 | 121 | 4641 |
| 4641 | 117 | 309 | 237 | 119 | 295 | 249 | 114 | 313 | 236 | 116 | 296 | 251 | 113 | 315 | 235 | 118 | 297 | 248 | 115 | 310 | 238 | 4641 |
| 4641 | 288 | 363 | 12 | 274 | 375 | 14 | 292 | 362 | 9 | 275 | 377 | 11 | 294 | 361 | 8 | 276 | 374 | 13 | 289 | 364 | 10 | 4641 |
| 4641 | 6 | 285 | 372 | 18 | 287 | 358 | 5 | 282 | 376 | 20 | 284 | 359 | 4 | 281 | 378 | 17 | 286 | 360 | 7 | 283 | 373 | 4641 |
| 4641 | 369 | 15 | 279 | 371 | 1 | 291 | 366 | 19 | 278 | 368 | 2 | 293 | 365 | 21 | 277 | 370 | 3 | 290 | 367 | 16 | 280 | 4641 |
| 4641 | 183 | 90 | 390 | 169 | 102 | 392 | 187 | 89 | 387 | 170 | 104 | 389 | 189 | 88 | 386 | 171 | 101 | 391 | 184 | 91 | 388 | 4641 |
| 4641 | 384 | 180 | 99 | 396 | 182 | 85 | 383 | 177 | 103 | 398 | 179 | 86 | 382 | 176 | 105 | 395 | 181 | 87 | 385 | 178 | 100 | 4641 |
| 4641 | 96 | 393 | 174 | 98 | 379 | 186 | 93 | 397 | 173 | 95 | 380 | 188 | 92 | 399 | 172 | 97 | 381 | 185 | 94 | 394 | 175 | 4641 |
| 4641 | 225 | 405 | 33 | 211 | 417 | 35 | 229 | 404 | 30 | 212 | 419 | 32 | 231 | 403 | 29 | 213 | 416 | 34 | 226 | 406 | 31 | 4641 |
| 4641 | 27 | 222 | 414 | 39 | 224 | 400 | 26 | 219 | 418 | 41 | 221 | 401 | 25 | 218 | 420 | 38 | 223 | 402 | 28 | 220 | 415 | 4641 |
| 4641 | 411 | 36 | 216 | 413 | 22 | 228 | 408 | 40 | 215 | 410 | 23 | 230 | 407 | 42 | 214 | 412 | 24 | 227 | 409 | 37 | 217 | 4641 |
| 4641 | 162 | 69 | 432 | 148 | 81 | 434 | 166 | 68 | 429 | 149 | 83 | 431 | 168 | 67 | 428 | 150 | 80 | 433 | 163 | 70 | 430 | 4641 |
| 4641 | 426 | 159 | 78 | 438 | 161 | 64 | 425 | 156 | 82 | 440 | 158 | 65 | 424 | 155 | 84 | 437 | 160 | 66 | 427 | 157 | 79 | 4641 |
| 4641 | 75 | 435 | 153 | 77 | 421 | 165 | 72 | 439 | 152 | 74 | 422 | 167 | 71 | 441 | 151 | 76 | 423 | 164 | 73 | 436 | 154 | 4641 |
| 4641 | 267 | 342 | 54 | 253 | 354 | 56 | 271 | 341 | 51 | 254 | 356 | 53 | 273 | 340 | 50 | 255 | 353 | 55 | 268 | 343 | 52 | 4641 |
| 4641 | 48 | 264 | 351 | 60 | 266 | 337 | 47 | 261 | 355 | 62 | 263 | 338 | 46 | 260 | 357 | 59 | 265 | 339 | 49 | 262 | 352 | 4641 |
| 4641 | 348 | 57 | 258 | 350 | 43 | 270 | 345 | 61 | 257 | 347 | 44 | 272 | 344 | 63 | 256 | 349 | 45 | 269 | 346 | 58 | 259 | 4641 |
| 4641 | 204 | 132 | 327 | 190 | 144 | 329 | 208 | 131 | 324 | 191 | 146 | 326 | 210 | 130 | 323 | 192 | 143 | 328 | 205 | 133 | 325 | 4641 |
| 4641 | 321 | 201 | 141 | 333 | 203 | 127 | 320 | 198 | 145 | 335 | 200 | 128 | 319 | 197 | 147 | 332 | 202 | 129 | 322 | 199 | 142 | 4641 |
| 4641 | 138 | 330 | 195 | 140 | 316 | 207 | 135 | 334 | 194 | 137 | 317 | 209 | 134 | 336 | 193 | 139 | 318 | 206 | 136 | 331 | 196 | 4641 |
| | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 | 4641 |

In this case, the magic sum is $S_{21\times21} = 4641$. All the $3\times3$ blocks are **semi-magic** squares with equal **semi-magic** sums, $S_{3\times3} = 663$ (in rows and columns).

### 2.7 Magic and Semi-Bimagic Square of Order 24

In this subsection, we shall present three different ways of writing block-wise magic square of order 24. One as $4 \times 6$, i.e., magic square formed by 36 blocks of **pan diagonal** magic squares of order 4 with equal magic sums. Second as $3 \times 8$, i.e., magic square formed by 64 blocks of magic squares of order 3 with different magic sums. The third as $6 \times 4$, i.e., 16 blocks of magic squares of order 6 with equal magic sums. In the first two cases, the magic squares of order 24 are **pan diagonal** magic squares, while in the third case it is just a magic square.

#### 2.7.1 First Approach: 36 Blocks of Order 4

**Example 2.18.** *A **pan diagonal** magic square of order 24 formed by 36 blocks of **pan diagonal** magic squares of order 4 constructed according to Example 1.2 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 |
| | 217 | 432 | 1 | 504 | 218 | 431 | 2 | 503 | 219 | 430 | 3 | 502 | 220 | 429 | 4 | 501 | 221 | 428 | 5 | 500 | 222 | 427 | 6 | 499 | 6924 |
| 6924 | 72 | 433 | 288 | 361 | 71 | 434 | 287 | 362 | 70 | 435 | 286 | 363 | 69 | 436 | 285 | 364 | 68 | 437 | 284 | 365 | 67 | 438 | 283 | 366 | 6924 |
| 6924 | 576 | 73 | 360 | 145 | 575 | 74 | 359 | 146 | 574 | 75 | 358 | 147 | 573 | 76 | 357 | 148 | 572 | 77 | 356 | 149 | 571 | 78 | 355 | 150 | 6924 |
| 6924 | 289 | 216 | 505 | 144 | 290 | 215 | 506 | 143 | 291 | 214 | 507 | 142 | 292 | 213 | 508 | 141 | 293 | 212 | 509 | 140 | 294 | 211 | 510 | 139 | 6924 |
| 6924 | 223 | 426 | 7 | 498 | 224 | 425 | 8 | 497 | 225 | 424 | 9 | 496 | 226 | 423 | 10 | 495 | 227 | 422 | 11 | 494 | 228 | 421 | 12 | 493 | 6924 |
| 6924 | 66 | 439 | 282 | 367 | 65 | 440 | 281 | 368 | 64 | 441 | 280 | 369 | 63 | 442 | 279 | 370 | 62 | 443 | 278 | 371 | 61 | 444 | 277 | 372 | 6924 |
| 6924 | 570 | 79 | 354 | 151 | 569 | 80 | 353 | 152 | 568 | 81 | 352 | 153 | 567 | 82 | 351 | 154 | 566 | 83 | 350 | 155 | 565 | 84 | 349 | 156 | 6924 |
| 6924 | 295 | 210 | 511 | 138 | 296 | 209 | 512 | 137 | 297 | 208 | 513 | 136 | 298 | 207 | 514 | 135 | 299 | 206 | 515 | 134 | 300 | 205 | 516 | 133 | 6924 |
| 6924 | 229 | 420 | 13 | 492 | 230 | 419 | 14 | 491 | 231 | 418 | 15 | 490 | 232 | 417 | 16 | 489 | 233 | 416 | 17 | 488 | 234 | 415 | 18 | 487 | 6924 |
| 6924 | 60 | 445 | 276 | 373 | 59 | 446 | 275 | 374 | 58 | 447 | 274 | 375 | 57 | 448 | 273 | 376 | 56 | 449 | 272 | 377 | 55 | 450 | 271 | 378 | 6924 |
| 6924 | 564 | 85 | 348 | 157 | 563 | 86 | 347 | 158 | 562 | 87 | 346 | 159 | 561 | 88 | 345 | 160 | 560 | 89 | 344 | 161 | 559 | 90 | 343 | 162 | 6924 |
| 6924 | 301 | 204 | 517 | 132 | 302 | 203 | 518 | 131 | 303 | 202 | 519 | 130 | 304 | 201 | 520 | 129 | 305 | 200 | 521 | 128 | 306 | 199 | 522 | 127 | 6924 |
| 6924 | 235 | 414 | 19 | 486 | 236 | 413 | 20 | 485 | 237 | 412 | 21 | 484 | 238 | 411 | 22 | 483 | 239 | 410 | 23 | 482 | 240 | 409 | 24 | 481 | 6924 |
| 6924 | 54 | 451 | 270 | 379 | 53 | 452 | 269 | 380 | 52 | 453 | 268 | 381 | 51 | 454 | 267 | 382 | 50 | 455 | 266 | 383 | 49 | 456 | 265 | 384 | 6924 |
| 6924 | 558 | 91 | 342 | 163 | 557 | 92 | 341 | 164 | 556 | 93 | 340 | 165 | 555 | 94 | 339 | 166 | 554 | 95 | 338 | 167 | 553 | 96 | 337 | 168 | 6924 |
| 6924 | 307 | 198 | 523 | 126 | 308 | 197 | 524 | 125 | 309 | 196 | 525 | 124 | 310 | 195 | 526 | 123 | 311 | 194 | 527 | 122 | 312 | 193 | 528 | 121 | 6924 |
| 6924 | 241 | 408 | 25 | 480 | 242 | 407 | 26 | 479 | 243 | 406 | 27 | 478 | 244 | 405 | 28 | 477 | 245 | 404 | 29 | 476 | 246 | 403 | 30 | 475 | 6924 |
| 6924 | 48 | 457 | 264 | 385 | 47 | 458 | 263 | 386 | 46 | 459 | 262 | 387 | 45 | 460 | 261 | 388 | 44 | 461 | 260 | 389 | 43 | 462 | 259 | 390 | 6924 |
| 6924 | 552 | 97 | 336 | 169 | 551 | 98 | 335 | 170 | 550 | 99 | 334 | 171 | 549 | 100 | 333 | 172 | 548 | 101 | 332 | 173 | 547 | 102 | 331 | 174 | 6924 |
| 6924 | 313 | 192 | 529 | 120 | 314 | 191 | 530 | 119 | 315 | 190 | 531 | 118 | 316 | 189 | 532 | 117 | 317 | 188 | 533 | 116 | 318 | 187 | 534 | 115 | 6924 |
| 6924 | 247 | 402 | 31 | 474 | 248 | 401 | 32 | 473 | 249 | 400 | 33 | 472 | 250 | 399 | 34 | 471 | 251 | 398 | 35 | 470 | 252 | 397 | 36 | 469 | 6924 |
| 6924 | 42 | 463 | 258 | 391 | 41 | 464 | 257 | 392 | 40 | 465 | 256 | 393 | 39 | 466 | 255 | 394 | 38 | 467 | 254 | 395 | 37 | 468 | 253 | 396 | 6924 |
| 6924 | 546 | 103 | 330 | 175 | 545 | 104 | 329 | 176 | 544 | 105 | 328 | 177 | 543 | 106 | 327 | 178 | 542 | 107 | 326 | 179 | 541 | 108 | 325 | 180 | 6924 |
| 6924 | 319 | 186 | 535 | 114 | 320 | 185 | 536 | 113 | 321 | 184 | 537 | 112 | 322 | 183 | 538 | 111 | 323 | 182 | 539 | 110 | 324 | 181 | 540 | 109 | 6924 |
| | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 |

In this case, the magic sum is $S_{24\times 24} := 6924$. All the 36 blocks of order 4 are **pan diagonal** magic squares with equal magic sums, $S_{4\times 4} := 1154$.

#### 2.7.2 Second Approach: 64 Blocks of Order 3

**Example 2.19.** *A **pan diagonal** magic square of order 24 formed by 64 blocks of **pan diagonal** magic squares of order 3 constructed according to Examples 1.1 and 1.6 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 |
| | 253 | 231 | 278 | 336 | 358 | 311 | 25 | 51 | 2 | 540 | 514 | 563 | 258 | 232 | 281 | 331 | 357 | 308 | 30 | 52 | 5 | 535 | 513 | 560 | 6924 |
| 6924 | 279 | 254 | 229 | 310 | 335 | 360 | 3 | 26 | 49 | 562 | 539 | 516 | 280 | 257 | 234 | 309 | 332 | 355 | 4 | 29 | 54 | 561 | 536 | 511 | 6924 |
| 6924 | 230 | 277 | 255 | 359 | 312 | 334 | 50 | 1 | 27 | 515 | 564 | 538 | 233 | 282 | 256 | 356 | 307 | 333 | 53 | 6 | 28 | 512 | 559 | 537 | 6924 |
| 6924 | 36 | 58 | 11 | 529 | 507 | 554 | 264 | 238 | 287 | 325 | 351 | 302 | 31 | 57 | 8 | 534 | 508 | 557 | 259 | 237 | 284 | 330 | 352 | 305 | 6924 |
| 6924 | 10 | 35 | 60 | 555 | 530 | 505 | 286 | 263 | 240 | 303 | 326 | 349 | 9 | 32 | 55 | 556 | 533 | 510 | 285 | 260 | 235 | 304 | 329 | 354 | 6924 |
| 6924 | 59 | 12 | 34 | 506 | 553 | 531 | 239 | 288 | 262 | 350 | 301 | 327 | 56 | 7 | 33 | 509 | 558 | 532 | 236 | 283 | 261 | 353 | 306 | 328 | 6924 |
| 6924 | 552 | 526 | 575 | 37 | 63 | 14 | 324 | 346 | 299 | 241 | 219 | 266 | 547 | 525 | 572 | 42 | 64 | 17 | 319 | 345 | 296 | 246 | 220 | 269 | 6924 |
| 6924 | 574 | 551 | 528 | 15 | 38 | 61 | 298 | 323 | 348 | 267 | 242 | 217 | 573 | 548 | 523 | 16 | 41 | 66 | 297 | 320 | 343 | 268 | 245 | 222 | 6924 |
| 6924 | 527 | 576 | 550 | 62 | 13 | 39 | 347 | 300 | 322 | 218 | 265 | 243 | 524 | 571 | 549 | 65 | 18 | 40 | 344 | 295 | 321 | 221 | 270 | 244 | 6924 |
| 6924 | 313 | 339 | 290 | 252 | 226 | 275 | 541 | 519 | 566 | 48 | 70 | 23 | 318 | 340 | 293 | 247 | 225 | 272 | 546 | 520 | 569 | 43 | 69 | 20 | 6924 |
| 6924 | 291 | 314 | 337 | 274 | 251 | 228 | 567 | 542 | 517 | 22 | 47 | 72 | 292 | 317 | 342 | 273 | 248 | 223 | 568 | 545 | 522 | 21 | 44 | 67 | 6924 |
| 6924 | 338 | 289 | 315 | 227 | 276 | 250 | 518 | 565 | 543 | 71 | 24 | 46 | 341 | 294 | 316 | 224 | 271 | 249 | 521 | 570 | 544 | 68 | 19 | 45 | 6924 |
| 6924 | 181 | 207 | 158 | 408 | 382 | 431 | 97 | 75 | 122 | 468 | 490 | 443 | 186 | 208 | 161 | 403 | 381 | 428 | 102 | 76 | 125 | 463 | 489 | 440 | 6924 |
| 6924 | 159 | 182 | 205 | 430 | 407 | 384 | 123 | 98 | 73 | 442 | 467 | 492 | 160 | 185 | 210 | 429 | 404 | 379 | 124 | 101 | 78 | 441 | 464 | 487 | 6924 |
| 6924 | 206 | 157 | 183 | 383 | 432 | 406 | 74 | 121 | 99 | 491 | 444 | 466 | 209 | 162 | 184 | 380 | 427 | 405 | 77 | 126 | 100 | 488 | 439 | 465 | 6924 |
| 6924 | 108 | 82 | 131 | 457 | 483 | 434 | 192 | 214 | 167 | 397 | 375 | 422 | 103 | 81 | 128 | 462 | 484 | 437 | 187 | 213 | 164 | 402 | 376 | 425 | 6924 |
| 6924 | 130 | 107 | 84 | 435 | 458 | 481 | 166 | 191 | 216 | 423 | 398 | 373 | 129 | 104 | 79 | 436 | 461 | 486 | 165 | 188 | 211 | 424 | 401 | 378 | 6924 |
| 6924 | 83 | 132 | 106 | 482 | 433 | 459 | 215 | 168 | 190 | 374 | 421 | 399 | 80 | 127 | 105 | 485 | 438 | 460 | 212 | 163 | 189 | 377 | 426 | 400 | 6924 |
| 6924 | 480 | 502 | 455 | 109 | 87 | 134 | 396 | 370 | 419 | 169 | 195 | 146 | 475 | 501 | 452 | 114 | 88 | 137 | 391 | 369 | 416 | 174 | 196 | 149 | 6924 |
| 6924 | 454 | 479 | 504 | 135 | 110 | 85 | 418 | 395 | 372 | 147 | 170 | 193 | 453 | 476 | 499 | 136 | 113 | 90 | 417 | 392 | 367 | 148 | 173 | 198 | 6924 |
| 6924 | 503 | 456 | 478 | 86 | 133 | 111 | 371 | 420 | 394 | 194 | 145 | 171 | 500 | 451 | 477 | 89 | 138 | 112 | 368 | 415 | 393 | 197 | 150 | 172 | 6924 |
| 6924 | 385 | 363 | 410 | 180 | 202 | 155 | 469 | 495 | 446 | 120 | 94 | 143 | 390 | 364 | 413 | 175 | 201 | 152 | 474 | 496 | 449 | 115 | 93 | 140 | 6924 |
| 6924 | 411 | 386 | 361 | 154 | 179 | 204 | 447 | 470 | 493 | 142 | 119 | 96 | 412 | 389 | 366 | 153 | 176 | 199 | 448 | 473 | 498 | 141 | 116 | 91 | 6924 |
| 6924 | 362 | 409 | 387 | 203 | 156 | 178 | 494 | 445 | 471 | 95 | 144 | 118 | 365 | 414 | 388 | 200 | 151 | 177 | 497 | 450 | 472 | 92 | 139 | 117 | 6924 |
| | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 |

We observe that it is not possible to have equal magic sums of order 3 as it is impossible to divide magic sum of order 24 by 8, i.e., 6924/8:=865.5. In this case, each $3 \times 3$ block is a magic square of order 3 with different magic sums resulting in a **pan diagonal** magic square of order 8 given in example below.

**Example 2.20.** *A **pan diagonal** magic square of order 8 arising due to magic sums of order 3 of Exemple 2.19 is given by*

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 |
| | 762 | 1005 | 78 | 1617 | 771 | 996 | 87 | 1608 | 6924 |
| 6924 | 105 | 1590 | 789 | 978 | 96 | 1599 | 780 | 987 | 6924 |
| 6924 | 1653 | 114 | 969 | 726 | 1644 | 123 | 960 | 735 | 6924 |
| 6924 | 942 | 753 | 1626 | 141 | 951 | 744 | 1635 | 132 | 6924 |
| 6924 | 546 | 1221 | 294 | 1401 | 555 | 1212 | 303 | 1392 | 6924 |
| 6924 | 321 | 1374 | 573 | 1194 | 312 | 1383 | 564 | 1203 | 6924 |
| 6924 | 1437 | 330 | 1185 | 510 | 1428 | 339 | 1176 | 519 | 6924 |
| 6924 | 1158 | 537 | 1410 | 357 | 1167 | 528 | 1419 | 348 | 6924 |
| | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 |

### 2.7.3 Third Approach: 16 Blocks of Order 6

**Example 2.21.** *A magic square of order 24 formed by 16 blocks of equal sums magic squares of order 6 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | 6924 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 545 | 544 | 513 | 32 | 96 | 2 | 546 | 543 | 514 | 31 | 95 | 3 | 547 | 542 | 515 | 30 | 94 | 4 | 548 | 541 | 516 | 29 | 93 | 6924 |
| 480 | 128 | 448 | 129 | 161 | 385 | 479 | 127 | 447 | 130 | 162 | 386 | 478 | 126 | 446 | 131 | 163 | 387 | 477 | 125 | 445 | 132 | 164 | 388 | 6924 |
| 384 | 353 | 225 | 256 | 320 | 193 | 383 | 354 | 226 | 255 | 319 | 194 | 382 | 355 | 227 | 254 | 318 | 195 | 381 | 356 | 228 | 253 | 317 | 196 | 6924 |
| 288 | 224 | 321 | 352 | 257 | 289 | 287 | 223 | 322 | 351 | 258 | 290 | 286 | 222 | 323 | 350 | 259 | 291 | 285 | 221 | 324 | 349 | 260 | 292 | 6924 |
| 97 | 416 | 160 | 417 | 449 | 192 | 98 | 415 | 159 | 418 | 450 | 191 | 99 | 414 | 158 | 419 | 451 | 190 | 100 | 413 | 157 | 420 | 452 | 189 | 6924 |
| 481 | 65 | 33 | 64 | 512 | 576 | 482 | 66 | 34 | 63 | 511 | 575 | 483 | 67 | 35 | 62 | 510 | 574 | 484 | 68 | 36 | 61 | 509 | 573 | 6924 |
| 5 | 549 | 540 | 517 | 28 | 92 | 6 | 550 | 539 | 518 | 27 | 91 | 7 | 551 | 538 | 519 | 26 | 90 | 8 | 552 | 537 | 520 | 25 | 89 | 6924 |
| 476 | 124 | 444 | 133 | 165 | 389 | 475 | 123 | 443 | 134 | 166 | 390 | 474 | 122 | 442 | 135 | 167 | 391 | 473 | 121 | 441 | 136 | 168 | 392 | 6924 |
| 380 | 357 | 229 | 252 | 316 | 197 | 379 | 358 | 230 | 251 | 315 | 198 | 378 | 359 | 231 | 250 | 314 | 199 | 377 | 360 | 232 | 249 | 313 | 200 | 6924 |
| 284 | 220 | 325 | 348 | 261 | 293 | 283 | 219 | 326 | 347 | 262 | 294 | 282 | 218 | 327 | 346 | 263 | 295 | 281 | 217 | 328 | 345 | 264 | 296 | 6924 |
| 101 | 412 | 156 | 421 | 453 | 188 | 102 | 411 | 155 | 422 | 454 | 187 | 103 | 410 | 154 | 423 | 455 | 186 | 104 | 409 | 153 | 424 | 456 | 185 | 6924 |
| 485 | 69 | 37 | 60 | 508 | 572 | 486 | 70 | 38 | 59 | 507 | 571 | 487 | 71 | 39 | 58 | 506 | 570 | 488 | 72 | 40 | 57 | 505 | 569 | 6924 |
| 9 | 553 | 536 | 521 | 24 | 88 | 10 | 554 | 535 | 522 | 23 | 87 | 11 | 555 | 534 | 523 | 22 | 86 | 12 | 556 | 533 | 524 | 21 | 85 | 6924 |
| 472 | 120 | 440 | 137 | 169 | 393 | 471 | 119 | 439 | 138 | 170 | 394 | 470 | 118 | 438 | 139 | 171 | 395 | 469 | 117 | 437 | 140 | 172 | 396 | 6924 |
| 376 | 361 | 233 | 248 | 312 | 201 | 375 | 362 | 234 | 247 | 311 | 202 | 374 | 363 | 235 | 246 | 310 | 203 | 373 | 364 | 236 | 245 | 309 | 204 | 6924 |
| 280 | 216 | 329 | 344 | 265 | 297 | 279 | 215 | 330 | 343 | 266 | 298 | 278 | 214 | 331 | 342 | 267 | 299 | 277 | 213 | 332 | 341 | 268 | 300 | 6924 |
| 105 | 408 | 152 | 425 | 457 | 184 | 106 | 407 | 151 | 426 | 458 | 183 | 107 | 406 | 150 | 427 | 459 | 182 | 108 | 405 | 149 | 428 | 460 | 181 | 6924 |
| 489 | 73 | 41 | 56 | 504 | 568 | 490 | 74 | 42 | 55 | 503 | 567 | 491 | 75 | 43 | 54 | 502 | 566 | 492 | 76 | 44 | 53 | 501 | 565 | 6924 |
| 13 | 557 | 532 | 525 | 20 | 84 | 14 | 558 | 531 | 526 | 19 | 83 | 15 | 559 | 530 | 527 | 18 | 82 | 16 | 560 | 529 | 528 | 17 | 81 | 6924 |
| 468 | 116 | 436 | 141 | 173 | 397 | 467 | 115 | 435 | 142 | 174 | 398 | 466 | 114 | 434 | 143 | 175 | 399 | 465 | 113 | 433 | 144 | 176 | 400 | 6924 |
| 372 | 365 | 237 | 244 | 308 | 205 | 371 | 366 | 238 | 243 | 307 | 206 | 370 | 367 | 239 | 242 | 306 | 207 | 369 | 368 | 240 | 241 | 305 | 208 | 6924 |
| 276 | 212 | 333 | 340 | 269 | 301 | 275 | 211 | 334 | 339 | 270 | 302 | 274 | 210 | 335 | 338 | 271 | 303 | 273 | 209 | 336 | 337 | 272 | 304 | 6924 |
| 109 | 404 | 148 | 429 | 461 | 180 | 110 | 403 | 147 | 430 | 462 | 179 | 111 | 402 | 146 | 431 | 463 | 178 | 112 | 401 | 145 | 432 | 464 | 177 | 6924 |
| 493 | 77 | 45 | 52 | 500 | 564 | 494 | 78 | 46 | 51 | 499 | 563 | 495 | 79 | 47 | 50 | 498 | 562 | 496 | 80 | 48 | 49 | 497 | 561 | 6924 |
| 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 |

The above magic square is with magic sum $S_{24\times 24} = 6924$, and all the 16 blocks of order 6 are magic squares with equal magic sums $S_{6\times 6} := 1731$.

#### 2.7.4 Forth Approach: Semi-Bimagic Square of Order 24

Below is a magic square of order 24 with 9 blocks of equal sums magic squares of order 8. The magic square is **semi-bimagic** square of order 24.

**Example 2.22.** *A **semi-bimagic** square of order 24 formed by 9 blocks of equal sums magic squares of order 8 constructed according to Examples 2.20 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | 6924 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 142 | 411 | 298 | 15 | 273 | 568 | 453 | 148 | 119 | 386 | 323 | 38 | 248 | 545 | 476 | 173 | 96 | 361 | 348 | 61 | 223 | 522 | 499 | 198 | 6924 |
| 268 | 573 | 448 | 153 | 135 | 418 | 291 | 22 | 245 | 548 | 473 | 176 | 110 | 395 | 314 | 47 | 222 | 523 | 498 | 199 | 85 | 372 | 337 | 72 | 6924 |
| 3 | 310 | 423 | 130 | 160 | 441 | 556 | 285 | 26 | 335 | 398 | 107 | 185 | 464 | 533 | 260 | 49 | 360 | 373 | 84 | 210 | 487 | 510 | 235 | 6924 |
| 165 | 436 | 561 | 280 | 10 | 303 | 430 | 123 | 188 | 461 | 536 | 257 | 35 | 326 | 407 | 98 | 211 | 486 | 511 | 234 | 60 | 349 | 384 | 73 | 6924 |
| 304 | 9 | 124 | 429 | 435 | 166 | 279 | 562 | 329 | 32 | 101 | 404 | 458 | 191 | 254 | 539 | 354 | 55 | 78 | 379 | 481 | 216 | 229 | 516 | 6924 |
| 442 | 159 | 286 | 555 | 309 | 4 | 129 | 424 | 467 | 182 | 263 | 530 | 332 | 29 | 104 | 401 | 492 | 205 | 240 | 505 | 355 | 54 | 79 | 378 | 6924 |
| 417 | 136 | 21 | 292 | 574 | 267 | 154 | 447 | 392 | 113 | 44 | 317 | 551 | 242 | 179 | 470 | 367 | 90 | 67 | 342 | 528 | 217 | 204 | 493 | 6924 |
| 567 | 274 | 147 | 454 | 412 | 141 | 16 | 297 | 542 | 251 | 170 | 479 | 389 | 116 | 41 | 320 | 517 | 228 | 193 | 504 | 366 | 91 | 66 | 343 | 6924 |
| 95 | 362 | 347 | 62 | 224 | 521 | 500 | 197 | 144 | 409 | 300 | 13 | 271 | 570 | 451 | 150 | 118 | 387 | 322 | 39 | 249 | 544 | 477 | 172 | 6924 |
| 221 | 524 | 497 | 200 | 86 | 371 | 338 | 71 | 270 | 571 | 450 | 151 | 133 | 420 | 289 | 24 | 244 | 549 | 472 | 177 | 111 | 394 | 315 | 46 | 6924 |
| 50 | 359 | 374 | 83 | 209 | 488 | 509 | 236 | 1 | 312 | 421 | 132 | 162 | 439 | 558 | 283 | 27 | 334 | 399 | 106 | 184 | 465 | 532 | 261 | 6924 |
| 212 | 485 | 512 | 233 | 59 | 350 | 383 | 74 | 163 | 438 | 559 | 282 | 12 | 301 | 432 | 121 | 189 | 460 | 537 | 256 | 34 | 327 | 406 | 99 | 6924 |
| 353 | 56 | 77 | 380 | 482 | 215 | 230 | 515 | 306 | 7 | 126 | 427 | 433 | 168 | 277 | 564 | 328 | 33 | 100 | 405 | 459 | 190 | 255 | 538 | 6924 |
| 491 | 206 | 239 | 506 | 356 | 53 | 80 | 377 | 444 | 157 | 288 | 553 | 307 | 6 | 127 | 426 | 466 | 183 | 262 | 531 | 333 | 28 | 105 | 400 | 6924 |
| 368 | 89 | 68 | 341 | 527 | 218 | 203 | 494 | 415 | 138 | 19 | 294 | 576 | 265 | 156 | 445 | 393 | 112 | 45 | 316 | 550 | 243 | 178 | 471 | 6924 |
| 518 | 227 | 194 | 503 | 365 | 92 | 65 | 344 | 565 | 276 | 145 | 456 | 414 | 139 | 18 | 295 | 543 | 250 | 171 | 478 | 388 | 117 | 40 | 321 | 6924 |
| 120 | 385 | 324 | 37 | 247 | 546 | 475 | 174 | 94 | 363 | 346 | 63 | 225 | 520 | 501 | 196 | 143 | 410 | 299 | 14 | 272 | 569 | 452 | 149 | 6924 |
| 246 | 547 | 474 | 175 | 109 | 396 | 313 | 48 | 220 | 525 | 496 | 201 | 87 | 370 | 339 | 70 | 269 | 572 | 449 | 152 | 134 | 419 | 290 | 23 | 6924 |
| 25 | 336 | 397 | 108 | 186 | 463 | 534 | 259 | 51 | 358 | 375 | 82 | 208 | 489 | 508 | 237 | 2 | 311 | 422 | 131 | 161 | 440 | 557 | 284 | 6924 |
| 187 | 462 | 535 | 258 | 36 | 325 | 408 | 97 | 213 | 484 | 513 | 232 | 58 | 351 | 382 | 75 | 164 | 437 | 560 | 281 | 11 | 302 | 431 | 122 | 6924 |
| 330 | 31 | 102 | 403 | 457 | 192 | 253 | 540 | 352 | 57 | 76 | 381 | 483 | 214 | 231 | 514 | 305 | 8 | 125 | 428 | 434 | 167 | 278 | 563 | 6924 |
| 468 | 181 | 264 | 529 | 331 | 30 | 103 | 402 | 490 | 207 | 238 | 507 | 357 | 52 | 81 | 376 | 443 | 158 | 287 | 554 | 308 | 5 | 128 | 425 | 6924 |
| 391 | 114 | 43 | 318 | 552 | 241 | 180 | 469 | 369 | 88 | 69 | 340 | 526 | 219 | 202 | 495 | 416 | 137 | 20 | 293 | 575 | 266 | 155 | 446 | 6924 |
| 541 | 252 | 169 | 480 | 390 | 115 | 42 | 319 | 519 | 226 | 195 | 502 | 364 | 93 | 64 | 345 | 566 | 275 | 146 | 455 | 413 | 140 | 17 | 296 | 6924 |
| 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 | 6924 |

In this case, the magic square is formed by 9 blocks of magic squares of order 8 with equal magic sums $S_{8\times 8} := 2308$. Out of 9 blocks of order 8, three of them are **bimagic** squares with different **bimagic** sums, and the other 6 are **semi-bimagic** (**bimagic** only in rows and columns) resulting in a **semi-bimagic** square of order 24 (**bimagic** only in rows and columns), with **semi-bimagic** sums are $Sb1_{24\times 24} := 2661124$ (rows and columns), $Sb2_{24\times 24} := 2654292$ (upper diagonal) and $Sb3_{24\times 24} := 2714116$ (lower diagonal).

### 2.8 Bimagic Square of Order 25

In this subsection, we shall present a block-wise **pan diagonal** magic square of order 25 with 25 blocks of **pan diagonal** magic square of order 5. The magic square is also **bimagic**.

**Example 2.23.** *A **pan diagonal** magic square of order 25 formed by 25 blocks of **pan diagonal** magic squares of order 5 constructed according to Example 1.3 is given by*

| | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 220 | 284 | 498 | 562 | 417 | 606 | 75 | 139 | 328 | 183 | 272 | 461 | 530 | 119 | 599 | 38 | 227 | 316 | 385 | 365 | 429 | 518 | 82 | 171 | 7825 |
| 7825 | 484 | 573 | 12 | 201 | 295 | 150 | 339 | 403 | 617 | 56 | 536 | 105 | 194 | 258 | 472 | 302 | 391 | 585 | 49 | 238 | 93 | 157 | 371 | 440 | 504 | 7825 |
| 7825 | 212 | 276 | 495 | 559 | 23 | 603 | 67 | 131 | 350 | 414 | 269 | 458 | 547 | 111 | 180 | 35 | 249 | 313 | 377 | 591 | 446 | 515 | 79 | 168 | 357 | 7825 |
| 7825 | 570 | 9 | 223 | 287 | 476 | 331 | 425 | 614 | 53 | 142 | 122 | 186 | 255 | 469 | 533 | 388 | 577 | 41 | 235 | 324 | 154 | 368 | 432 | 521 | 90 | 7825 |
| 7825 | 298 | 487 | 551 | 20 | 209 | 64 | 128 | 342 | 406 | 625 | 455 | 544 | 108 | 197 | 261 | 241 | 310 | 399 | 588 | 27 | 507 | 96 | 165 | 354 | 443 | 7825 |
| 7825 | 583 | 47 | 236 | 305 | 394 | 374 | 438 | 502 | 91 | 160 | 15 | 204 | 293 | 482 | 571 | 401 | 620 | 59 | 148 | 337 | 192 | 256 | 475 | 539 | 103 | 7825 |
| 7825 | 311 | 380 | 594 | 33 | 247 | 77 | 166 | 360 | 449 | 513 | 493 | 557 | 21 | 215 | 279 | 134 | 348 | 412 | 601 | 70 | 550 | 114 | 178 | 267 | 456 | 7825 |
| 7825 | 44 | 233 | 322 | 386 | 580 | 435 | 524 | 88 | 152 | 366 | 221 | 290 | 479 | 568 | 7 | 612 | 51 | 145 | 334 | 423 | 253 | 467 | 531 | 125 | 189 | 7825 |
| 7825 | 397 | 586 | 30 | 244 | 308 | 163 | 352 | 441 | 510 | 99 | 554 | 18 | 207 | 296 | 490 | 345 | 409 | 623 | 62 | 126 | 106 | 200 | 264 | 453 | 542 | 7825 |
| 7825 | 230 | 319 | 383 | 597 | 36 | 516 | 85 | 174 | 363 | 427 | 282 | 496 | 565 | 4 | 218 | 73 | 137 | 326 | 420 | 609 | 464 | 528 | 117 | 181 | 275 | 7825 |
| 7825 | 415 | 604 | 68 | 132 | 346 | 176 | 270 | 459 | 548 | 112 | 592 | 31 | 250 | 314 | 378 | 358 | 447 | 511 | 80 | 169 | 24 | 213 | 277 | 491 | 560 | 7825 |
| 7825 | 143 | 332 | 421 | 615 | 54 | 534 | 123 | 187 | 251 | 470 | 325 | 389 | 578 | 42 | 231 | 86 | 155 | 369 | 433 | 522 | 477 | 566 | 10 | 224 | 288 | 7825 |
| 7825 | 621 | 65 | 129 | 343 | 407 | 262 | 451 | 545 | 109 | 198 | 28 | 242 | 306 | 400 | 589 | 444 | 508 | 97 | 161 | 355 | 210 | 299 | 488 | 552 | 16 | 7825 |
| 7825 | 329 | 418 | 607 | 71 | 140 | 120 | 184 | 273 | 462 | 526 | 381 | 600 | 39 | 228 | 317 | 172 | 361 | 430 | 519 | 83 | 563 | 2 | 216 | 285 | 499 | 7825 |
| 7825 | 57 | 146 | 340 | 404 | 618 | 473 | 537 | 101 | 195 | 259 | 239 | 303 | 392 | 581 | 50 | 505 | 94 | 158 | 372 | 436 | 291 | 485 | 574 | 13 | 202 | 7825 |
| 7825 | 367 | 431 | 525 | 89 | 153 | 8 | 222 | 286 | 480 | 569 | 424 | 613 | 52 | 141 | 335 | 190 | 254 | 468 | 532 | 121 | 576 | 45 | 234 | 323 | 387 | 7825 |
| 7825 | 100 | 164 | 353 | 442 | 506 | 486 | 555 | 19 | 208 | 297 | 127 | 341 | 410 | 624 | 63 | 543 | 107 | 196 | 265 | 454 | 309 | 398 | 587 | 26 | 245 | 7825 |
| 7825 | 428 | 517 | 81 | 175 | 364 | 219 | 283 | 497 | 561 | 5 | 610 | 74 | 138 | 327 | 416 | 271 | 465 | 529 | 118 | 182 | 37 | 226 | 320 | 384 | 598 | 7825 |
| 7825 | 156 | 375 | 439 | 503 | 92 | 572 | 11 | 205 | 294 | 483 | 338 | 402 | 616 | 60 | 149 | 104 | 193 | 257 | 471 | 540 | 395 | 584 | 48 | 237 | 301 | 7825 |
| 7825 | 514 | 78 | 167 | 356 | 450 | 280 | 494 | 558 | 22 | 211 | 66 | 135 | 349 | 413 | 602 | 457 | 546 | 115 | 179 | 268 | 248 | 312 | 376 | 595 | 34 | 7825 |
| 7825 | 199 | 263 | 452 | 541 | 110 | 590 | 29 | 243 | 307 | 396 | 351 | 445 | 509 | 98 | 162 | 17 | 206 | 300 | 489 | 553 | 408 | 622 | 61 | 130 | 344 | 7825 |
| 7825 | 527 | 116 | 185 | 274 | 463 | 318 | 382 | 596 | 40 | 229 | 84 | 173 | 362 | 426 | 520 | 500 | 564 | 3 | 217 | 281 | 136 | 330 | 419 | 608 | 72 | 7825 |
| 7825 | 260 | 474 | 538 | 102 | 191 | 46 | 240 | 304 | 393 | 582 | 437 | 501 | 95 | 159 | 373 | 203 | 292 | 481 | 575 | 14 | 619 | 58 | 147 | 336 | 405 | 7825 |
| 7825 | 113 | 177 | 266 | 460 | 549 | 379 | 593 | 32 | 246 | 315 | 170 | 359 | 448 | 512 | 76 | 556 | 25 | 214 | 278 | 492 | 347 | 411 | 605 | 69 | 133 | 7825 |
| 7825 | 466 | 535 | 124 | 188 | 252 | 232 | 321 | 390 | 579 | 43 | 523 | 87 | 151 | 370 | 434 | 289 | 478 | 567 | 6 | 225 | 55 | 144 | 333 | 422 | 611 | 7825 |
| | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 | 7825 |

In this case the **pan diagonal** magic square is **bimagic** with magic and **bimagic** sums, $S_{25\times25} := 7825$ and $Sb_{25\times25} := 3263025$ respectively. Each block of order 5 is a **pan diagonal** magic square with equal magic sums, $S_{5\times5} := 1565$. Details of this **bimagic** square can be seen in author's work [27].

### 2.9 Magic Squares of Order 27

In this subsection, we shall present three different ways of writing block-wise **pan diagonal** magic square of order 27. One as 3 × 9, i.e., magic square formed by 81 blocks of equal sums semi-magic squares of order 3. Second as 9 × 3, i.e., magic square formed by 9 blocks of **pan diagonal** magic squares of order 9. Third as, 9 blocks of bimagic squares of order 9 resulting a magic square of order 27.

#### 2.9.1 First Approach: 81 Blocks of Order 3

**Example 2.24.** *A **pan diagonal** magic square of order 27 formed by 81 blocks of **semi-magic** squares of order 3 with equal **semi-magic** sums is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 |
| | 622 | 449 | 24 | 636 | 434 | 25 | 626 | 451 | 18 | 637 | 438 | 20 | 642 | 439 | 14 | 643 | 444 | 8 | 630 | 455 | 10 | 647 | 445 | 3 | 632 | 459 | 4 | 9855 |
| 9855 | 17 | 645 | 433 | 2 | 646 | 447 | 19 | 639 | 437 | 6 | 641 | 448 | 7 | 635 | 453 | 12 | 629 | 454 | 23 | 631 | 441 | 13 | 624 | 458 | 27 | 625 | 443 | 9855 |
| 9855 | 456 | 1 | 638 | 457 | 15 | 623 | 450 | 5 | 640 | 452 | 16 | 627 | 446 | 21 | 628 | 440 | 22 | 633 | 442 | 9 | 644 | 435 | 26 | 634 | 436 | 11 | 648 | 9855 |
| 9855 | 649 | 44 | 402 | 663 | 29 | 403 | 653 | 46 | 396 | 664 | 33 | 398 | 669 | 34 | 392 | 670 | 39 | 386 | 657 | 50 | 388 | 674 | 40 | 381 | 659 | 54 | 382 | 9855 |
| 9855 | 395 | 672 | 28 | 380 | 673 | 42 | 397 | 666 | 32 | 384 | 668 | 43 | 385 | 662 | 48 | 390 | 656 | 49 | 401 | 658 | 36 | 391 | 651 | 53 | 405 | 652 | 38 | 9855 |
| 9855 | 51 | 379 | 665 | 52 | 393 | 650 | 45 | 383 | 667 | 47 | 394 | 654 | 41 | 399 | 655 | 35 | 400 | 660 | 37 | 387 | 671 | 30 | 404 | 661 | 31 | 389 | 675 | 9855 |
| 9855 | 460 | 503 | 132 | 474 | 488 | 133 | 464 | 505 | 126 | 475 | 492 | 128 | 480 | 493 | 122 | 481 | 498 | 116 | 468 | 509 | 118 | 485 | 499 | 111 | 470 | 513 | 112 | 9855 |
| 9855 | 125 | 483 | 487 | 110 | 484 | 501 | 127 | 477 | 491 | 114 | 479 | 502 | 115 | 473 | 507 | 120 | 467 | 508 | 131 | 469 | 495 | 121 | 462 | 512 | 135 | 463 | 497 | 9855 |
| 9855 | 510 | 109 | 476 | 511 | 123 | 461 | 504 | 113 | 478 | 506 | 124 | 465 | 500 | 129 | 466 | 494 | 130 | 471 | 496 | 117 | 482 | 489 | 134 | 472 | 490 | 119 | 486 | 9855 |
| 9855 | 514 | 152 | 429 | 528 | 137 | 430 | 518 | 154 | 423 | 529 | 141 | 425 | 534 | 142 | 419 | 535 | 147 | 413 | 522 | 158 | 415 | 539 | 148 | 408 | 524 | 162 | 409 | 9855 |
| 9855 | 422 | 537 | 136 | 407 | 538 | 150 | 424 | 531 | 140 | 411 | 533 | 151 | 412 | 527 | 156 | 417 | 521 | 157 | 428 | 523 | 144 | 418 | 516 | 161 | 432 | 517 | 146 | 9855 |
| 9855 | 159 | 406 | 530 | 160 | 420 | 515 | 153 | 410 | 532 | 155 | 421 | 519 | 149 | 426 | 520 | 143 | 427 | 525 | 145 | 414 | 536 | 138 | 431 | 526 | 139 | 416 | 540 | 9855 |
| 9855 | 352 | 179 | 564 | 366 | 164 | 565 | 356 | 181 | 558 | 367 | 168 | 560 | 372 | 169 | 554 | 373 | 174 | 548 | 360 | 185 | 550 | 377 | 175 | 543 | 362 | 189 | 544 | 9855 |
| 9855 | 557 | 375 | 163 | 542 | 376 | 177 | 559 | 369 | 167 | 546 | 371 | 178 | 547 | 365 | 183 | 552 | 359 | 184 | 563 | 361 | 171 | 553 | 354 | 188 | 567 | 355 | 173 | 9855 |
| 9855 | 186 | 541 | 368 | 187 | 555 | 353 | 180 | 545 | 370 | 182 | 556 | 357 | 176 | 561 | 358 | 170 | 562 | 363 | 172 | 549 | 374 | 165 | 566 | 364 | 166 | 551 | 378 | 9855 |
| 9855 | 190 | 314 | 591 | 204 | 299 | 592 | 194 | 316 | 585 | 205 | 303 | 587 | 210 | 304 | 581 | 211 | 309 | 575 | 198 | 320 | 577 | 215 | 310 | 570 | 200 | 324 | 571 | 9855 |
| 9855 | 584 | 213 | 298 | 569 | 214 | 312 | 586 | 207 | 302 | 573 | 209 | 313 | 574 | 203 | 318 | 579 | 197 | 319 | 590 | 199 | 306 | 580 | 192 | 323 | 594 | 193 | 308 | 9855 |
| 9855 | 321 | 568 | 206 | 322 | 582 | 191 | 315 | 572 | 208 | 317 | 583 | 195 | 311 | 588 | 196 | 305 | 589 | 201 | 307 | 576 | 212 | 300 | 593 | 202 | 301 | 578 | 216 | 9855 |
| 9855 | 244 | 611 | 240 | 258 | 596 | 241 | 248 | 613 | 234 | 259 | 600 | 236 | 264 | 601 | 230 | 265 | 606 | 224 | 252 | 617 | 226 | 269 | 607 | 219 | 254 | 621 | 220 | 9855 |
| 9855 | 233 | 267 | 595 | 218 | 268 | 609 | 235 | 261 | 599 | 222 | 263 | 610 | 223 | 257 | 615 | 228 | 251 | 616 | 239 | 253 | 603 | 229 | 246 | 620 | 243 | 247 | 605 | 9855 |
| 9855 | 618 | 217 | 260 | 619 | 231 | 245 | 612 | 221 | 262 | 614 | 232 | 249 | 608 | 237 | 250 | 602 | 238 | 255 | 604 | 225 | 266 | 597 | 242 | 256 | 598 | 227 | 270 | 9855 |
| 9855 | 55 | 341 | 699 | 69 | 326 | 700 | 59 | 343 | 693 | 70 | 330 | 695 | 75 | 331 | 689 | 76 | 336 | 683 | 63 | 347 | 685 | 80 | 337 | 678 | 65 | 351 | 679 | 9855 |
| 9855 | 692 | 78 | 325 | 677 | 79 | 339 | 694 | 72 | 329 | 681 | 74 | 340 | 682 | 68 | 345 | 687 | 62 | 346 | 698 | 64 | 333 | 688 | 57 | 350 | 702 | 58 | 335 | 9855 |
| 9855 | 348 | 676 | 71 | 349 | 690 | 56 | 342 | 680 | 73 | 344 | 691 | 60 | 338 | 696 | 61 | 332 | 697 | 66 | 334 | 684 | 77 | 327 | 701 | 67 | 328 | 686 | 81 | 9855 |
| 9855 | 82 | 719 | 294 | 96 | 704 | 295 | 86 | 721 | 288 | 97 | 708 | 290 | 102 | 709 | 284 | 103 | 714 | 278 | 90 | 725 | 280 | 107 | 715 | 273 | 92 | 729 | 274 | 9855 |
| 9855 | 287 | 105 | 703 | 272 | 106 | 717 | 289 | 99 | 707 | 276 | 101 | 718 | 277 | 95 | 723 | 282 | 89 | 724 | 293 | 91 | 711 | 283 | 84 | 728 | 297 | 85 | 713 | 9855 |
| 9855 | 726 | 271 | 98 | 727 | 285 | 83 | 720 | 275 | 100 | 722 | 286 | 87 | 716 | 291 | 88 | 710 | 292 | 93 | 712 | 279 | 104 | 705 | 296 | 94 | 706 | 281 | 108 | 9855 |
| | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 |

In this case, the magic sum is $S_{27\times 27} := 9855$. All the 81 blocks of order 3 are semi-magic with equal semi-magic sums, $S_{3\times 3} := 1095$ (semi-magic in rows and columns).

#### 2.9.2 Second Approach: 9 Blocks of Order 9

**Example 2.25.** *A* ***pan diagonal*** *magic square of order 27 formed by 9 blocks of* ***pan diagonal*** *magic squares of order 9 with equal magic sums is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 |
| | 1 | 623 | 435 | 17 | 639 | 448 | 24 | 643 | 455 | 10 | 632 | 444 | 26 | 648 | 457 | 6 | 625 | 437 | 19 | 641 | 453 | 8 | 630 | 439 | 15 | 634 | 446 | 9855 |
| 9855 | 135 | 673 | 323 | 112 | 653 | 303 | 119 | 660 | 307 | 117 | 655 | 305 | 121 | 662 | 312 | 128 | 669 | 316 | 126 | 664 | 314 | 130 | 671 | 321 | 110 | 651 | 298 | 9855 |
| 9855 | 230 | 528 | 337 | 237 | 532 | 344 | 223 | 521 | 333 | 239 | 537 | 346 | 219 | 514 | 326 | 232 | 530 | 342 | 221 | 519 | 328 | 228 | 523 | 335 | 241 | 539 | 351 | 9855 |
| 9855 | 352 | 164 | 543 | 368 | 180 | 556 | 375 | 184 | 563 | 361 | 173 | 552 | 377 | 189 | 565 | 357 | 166 | 545 | 370 | 182 | 561 | 359 | 171 | 547 | 366 | 175 | 554 | 9855 |
| 9855 | 486 | 52 | 593 | 463 | 32 | 573 | 470 | 39 | 577 | 468 | 34 | 575 | 472 | 41 | 582 | 479 | 48 | 586 | 477 | 43 | 584 | 481 | 50 | 591 | 461 | 30 | 568 | 9855 |
| 9855 | 257 | 150 | 688 | 264 | 154 | 695 | 250 | 143 | 684 | 266 | 159 | 697 | 246 | 136 | 677 | 259 | 152 | 693 | 248 | 141 | 679 | 255 | 145 | 686 | 268 | 161 | 702 | 9855 |
| 9855 | 703 | 272 | 84 | 719 | 288 | 97 | 726 | 292 | 104 | 712 | 281 | 93 | 728 | 297 | 106 | 708 | 274 | 86 | 721 | 290 | 102 | 710 | 279 | 88 | 717 | 283 | 95 | 9855 |
| 9855 | 513 | 403 | 215 | 490 | 383 | 195 | 497 | 390 | 199 | 495 | 385 | 197 | 499 | 392 | 204 | 506 | 399 | 208 | 504 | 394 | 206 | 508 | 401 | 213 | 488 | 381 | 190 | 9855 |
| 9855 | 608 | 420 | 67 | 615 | 424 | 74 | 601 | 413 | 63 | 617 | 429 | 76 | 597 | 406 | 56 | 610 | 422 | 72 | 599 | 411 | 58 | 606 | 415 | 65 | 619 | 431 | 81 | 9855 |
| 9855 | 325 | 218 | 516 | 341 | 234 | 529 | 348 | 238 | 536 | 334 | 227 | 525 | 350 | 243 | 538 | 330 | 220 | 518 | 343 | 236 | 534 | 332 | 225 | 520 | 339 | 229 | 527 | 9855 |
| 9855 | 459 | 25 | 647 | 436 | 5 | 627 | 443 | 12 | 631 | 441 | 7 | 629 | 445 | 14 | 636 | 452 | 21 | 640 | 450 | 16 | 638 | 454 | 23 | 645 | 434 | 3 | 622 | 9855 |
| 9855 | 311 | 123 | 661 | 318 | 127 | 668 | 304 | 116 | 657 | 320 | 132 | 670 | 300 | 109 | 650 | 313 | 125 | 666 | 302 | 114 | 652 | 309 | 118 | 659 | 322 | 134 | 675 | 9855 |
| 9855 | 676 | 245 | 138 | 692 | 261 | 151 | 699 | 265 | 158 | 685 | 254 | 147 | 701 | 270 | 160 | 681 | 247 | 140 | 694 | 263 | 156 | 683 | 252 | 142 | 690 | 256 | 149 | 9855 |
| 9855 | 567 | 376 | 188 | 544 | 356 | 168 | 551 | 363 | 172 | 549 | 358 | 170 | 553 | 365 | 177 | 560 | 372 | 181 | 558 | 367 | 179 | 562 | 374 | 186 | 542 | 354 | 163 | 9855 |
| 9855 | 581 | 474 | 40 | 588 | 478 | 47 | 574 | 467 | 36 | 590 | 483 | 49 | 570 | 460 | 29 | 583 | 476 | 45 | 572 | 465 | 31 | 579 | 469 | 38 | 592 | 485 | 54 | 9855 |
| 9855 | 55 | 596 | 408 | 71 | 612 | 421 | 78 | 616 | 428 | 64 | 605 | 417 | 80 | 621 | 430 | 60 | 598 | 410 | 73 | 614 | 426 | 62 | 603 | 412 | 69 | 607 | 419 | 9855 |
| 9855 | 108 | 727 | 296 | 85 | 707 | 276 | 92 | 714 | 280 | 90 | 709 | 278 | 94 | 716 | 285 | 101 | 723 | 289 | 99 | 718 | 287 | 103 | 725 | 294 | 83 | 705 | 271 | 9855 |
| 9855 | 203 | 501 | 391 | 210 | 505 | 398 | 196 | 494 | 387 | 212 | 510 | 400 | 192 | 487 | 380 | 205 | 503 | 396 | 194 | 492 | 382 | 201 | 496 | 389 | 214 | 512 | 405 | 9855 |
| 9855 | 649 | 299 | 111 | 665 | 315 | 124 | 672 | 319 | 131 | 658 | 308 | 120 | 674 | 324 | 133 | 654 | 301 | 113 | 667 | 317 | 129 | 656 | 306 | 115 | 663 | 310 | 122 | 9855 |
| 9855 | 540 | 349 | 242 | 517 | 329 | 222 | 524 | 336 | 226 | 522 | 331 | 224 | 526 | 338 | 231 | 533 | 345 | 235 | 531 | 340 | 233 | 535 | 347 | 240 | 515 | 327 | 217 | 9855 |
| 9855 | 635 | 447 | 13 | 642 | 451 | 20 | 628 | 440 | 9 | 644 | 456 | 22 | 624 | 433 | 2 | 637 | 449 | 18 | 626 | 438 | 4 | 633 | 442 | 11 | 646 | 458 | 27 | 9855 |
| 9855 | 28 | 569 | 462 | 44 | 585 | 475 | 51 | 589 | 482 | 37 | 578 | 471 | 53 | 594 | 484 | 33 | 571 | 464 | 46 | 587 | 480 | 35 | 576 | 466 | 42 | 580 | 473 | 9855 |
| 9855 | 162 | 700 | 269 | 139 | 680 | 249 | 146 | 687 | 253 | 144 | 682 | 251 | 148 | 689 | 258 | 155 | 696 | 262 | 153 | 691 | 260 | 157 | 698 | 267 | 137 | 678 | 244 | 9855 |
| 9855 | 176 | 555 | 364 | 183 | 559 | 371 | 169 | 548 | 360 | 185 | 564 | 373 | 165 | 541 | 353 | 178 | 557 | 369 | 167 | 546 | 355 | 174 | 550 | 362 | 187 | 566 | 378 | 9855 |
| 9855 | 379 | 191 | 489 | 395 | 207 | 502 | 402 | 211 | 509 | 388 | 200 | 498 | 404 | 216 | 511 | 384 | 193 | 491 | 397 | 209 | 507 | 386 | 198 | 493 | 393 | 202 | 500 | 9855 |
| 9855 | 432 | 79 | 620 | 409 | 59 | 600 | 416 | 66 | 604 | 414 | 61 | 602 | 418 | 68 | 609 | 425 | 75 | 613 | 423 | 70 | 611 | 427 | 77 | 618 | 407 | 57 | 595 | 9855 |
| 9855 | 284 | 96 | 715 | 291 | 100 | 722 | 277 | 89 | 711 | 293 | 105 | 724 | 273 | 82 | 704 | 286 | 98 | 720 | 275 | 87 | 706 | 282 | 91 | 713 | 295 | 107 | 729 | 9855 |
| | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 | 9855 |

In this case, the magic sums is $S_{27\times 27} := 9855$. All the 9 blocks of order 9 are **pan diagonal** magic squares of equal magic sums, $S_{9\times 9} := 3285$. The construction of this magic square is due to Dwane [9].

### 2.9.3 Third Approach: 9 Blocks of Bimagic Squares of Order 9

**Example 2.26.** *A magic square of order 27 formed by 9 blocks of **bimagic** squares of order 9 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | *9855* |
| 28 | 133 | 202 | 292 | 361 | 439 | 529 | 598 | 694 | 57 | 162 | 231 | 321 | 390 | 468 | 558 | 627 | 723 | 2 | 107 | 176 | 266 | 335 | 413 | 503 | 572 | 668 | *9855* |
| 286 | 355 | 451 | 514 | 619 | 688 | 49 | 118 | 196 | 315 | 384 | 480 | 543 | 648 | 717 | 78 | 147 | 225 | 260 | 329 | 425 | 488 | 593 | 662 | 23 | 92 | 170 | *9855* |
| 535 | 604 | 682 | 43 | 112 | 208 | 271 | 376 | 445 | 564 | 633 | 711 | 72 | 141 | 237 | 300 | 405 | 474 | 509 | 578 | 656 | 17 | 86 | 182 | 245 | 350 | 419 | *9855* |
| 214 | 40 | 109 | 442 | 277 | 373 | 679 | 532 | 610 | 243 | 69 | 138 | 471 | 306 | 402 | 708 | 561 | 639 | 188 | 14 | 83 | 416 | 251 | 347 | 653 | 506 | 584 | *9855* |
| 436 | 289 | 367 | 700 | 526 | 595 | 199 | 34 | 130 | 465 | 318 | 396 | 729 | 555 | 624 | 228 | 63 | 159 | 410 | 263 | 341 | 674 | 500 | 569 | 173 | 8 | 104 | *9855* |
| 685 | 520 | 616 | 193 | 46 | 124 | 457 | 283 | 352 | 714 | 549 | 645 | 222 | 75 | 153 | 486 | 312 | 381 | 659 | 494 | 590 | 167 | 20 | 98 | 431 | 257 | 326 | *9855* |
| 121 | 190 | 52 | 358 | 454 | 280 | 613 | 691 | 517 | 150 | 219 | 81 | 387 | 483 | 309 | 642 | 720 | 546 | 95 | 164 | 26 | 332 | 428 | 254 | 587 | 665 | 491 | *9855* |
| 370 | 448 | 274 | 607 | 676 | 538 | 115 | 211 | 37 | 399 | 477 | 303 | 636 | 705 | 567 | 144 | 240 | 66 | 344 | 422 | 248 | 581 | 650 | 512 | 89 | 185 | 11 | *9855* |
| 601 | 697 | 523 | 127 | 205 | 31 | 364 | 433 | 295 | 630 | 726 | 552 | 156 | 234 | 60 | 393 | 462 | 324 | 575 | 671 | 497 | 101 | 179 | 5 | 338 | 407 | 269 | *9855* |
| 3 | 108 | 177 | 267 | 336 | 414 | 504 | 573 | 669 | 29 | 134 | 203 | 293 | 362 | 440 | 530 | 599 | 695 | 55 | 160 | 229 | 319 | 388 | 466 | 556 | 625 | 721 | *9855* |
| 261 | 330 | 426 | 489 | 594 | 663 | 24 | 93 | 171 | 287 | 356 | 452 | 515 | 620 | 689 | 50 | 119 | 197 | 313 | 382 | 478 | 541 | 646 | 715 | 76 | 145 | 223 | *9855* |
| 510 | 579 | 657 | 18 | 87 | 183 | 246 | 351 | 420 | 536 | 605 | 683 | 44 | 113 | 209 | 272 | 377 | 446 | 562 | 631 | 709 | 70 | 139 | 235 | 298 | 403 | 472 | *9855* |
| 189 | 15 | 84 | 417 | 252 | 348 | 654 | 507 | 585 | 215 | 41 | 110 | 443 | 278 | 374 | 680 | 533 | 611 | 241 | 67 | 136 | 469 | 304 | 400 | 706 | 559 | 637 | *9855* |
| 411 | 264 | 342 | 675 | 501 | 570 | 174 | 9 | 105 | 437 | 290 | 368 | 701 | 527 | 596 | 200 | 35 | 131 | 463 | 316 | 394 | 727 | 553 | 622 | 226 | 61 | 157 | *9855* |
| 660 | 495 | 591 | 168 | 21 | 99 | 432 | 258 | 327 | 686 | 521 | 617 | 194 | 47 | 125 | 458 | 284 | 353 | 712 | 547 | 643 | 220 | 73 | 151 | 484 | 310 | 379 | *9855* |
| 96 | 165 | 27 | 333 | 429 | 255 | 588 | 666 | 492 | 122 | 191 | 53 | 359 | 455 | 281 | 614 | 692 | 518 | 148 | 217 | 79 | 385 | 481 | 307 | 640 | 718 | 544 | *9855* |
| 345 | 423 | 249 | 582 | 651 | 513 | 90 | 186 | 12 | 371 | 449 | 275 | 608 | 677 | 539 | 116 | 212 | 38 | 397 | 475 | 301 | 634 | 703 | 565 | 142 | 238 | 64 | *9855* |
| 576 | 672 | 498 | 102 | 180 | 6 | 339 | 408 | 270 | 602 | 698 | 524 | 128 | 206 | 32 | 365 | 434 | 296 | 628 | 724 | 550 | 154 | 232 | 58 | 391 | 460 | 322 | *9855* |
| 56 | 161 | 230 | 320 | 389 | 467 | 557 | 626 | 722 | 1 | 106 | 175 | 265 | 334 | 412 | 502 | 571 | 667 | 30 | 135 | 204 | 294 | 363 | 441 | 531 | 600 | 696 | *9855* |
| 314 | 383 | 479 | 542 | 647 | 716 | 77 | 146 | 224 | 259 | 328 | 424 | 487 | 592 | 661 | 22 | 91 | 169 | 288 | 357 | 453 | 516 | 621 | 690 | 51 | 120 | 198 | *9855* |
| 563 | 632 | 710 | 71 | 140 | 236 | 299 | 404 | 473 | 508 | 577 | 655 | 16 | 85 | 181 | 244 | 349 | 418 | 537 | 606 | 684 | 45 | 114 | 210 | 273 | 378 | 447 | *9855* |
| 242 | 68 | 137 | 470 | 305 | 401 | 707 | 560 | 638 | 187 | 13 | 82 | 415 | 250 | 346 | 652 | 505 | 583 | 216 | 42 | 111 | 444 | 279 | 375 | 681 | 534 | 612 | *9855* |
| 464 | 317 | 395 | 728 | 554 | 623 | 227 | 62 | 158 | 409 | 262 | 340 | 673 | 499 | 568 | 172 | 7 | 103 | 438 | 291 | 369 | 702 | 528 | 597 | 201 | 36 | 132 | *9855* |
| 713 | 548 | 644 | 221 | 74 | 152 | 485 | 311 | 380 | 658 | 493 | 589 | 166 | 19 | 97 | 430 | 256 | 325 | 687 | 522 | 618 | 195 | 48 | 126 | 459 | 285 | 354 | *9855* |
| 149 | 218 | 80 | 386 | 482 | 308 | 641 | 719 | 545 | 94 | 163 | 25 | 331 | 427 | 253 | 586 | 664 | 490 | 123 | 192 | 54 | 360 | 456 | 282 | 615 | 693 | 519 | *9855* |
| 398 | 476 | 302 | 635 | 704 | 566 | 143 | 239 | 65 | 343 | 421 | 247 | 580 | 649 | 511 | 88 | 184 | 10 | 372 | 450 | 276 | 609 | 678 | 540 | 117 | 213 | 39 | *9855* |
| 629 | 725 | 551 | 155 | 233 | 59 | 392 | 461 | 323 | 574 | 670 | 496 | 100 | 178 | 4 | 337 | 406 | 268 | 603 | 699 | 525 | 129 | 207 | 33 | 366 | 435 | 297 | *9855* |
| *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* | *9855* |

In this case, the magic square of order 27 is not a **bimagic**, but it is formed by 9 blocks of different magic and **bimagic** sums of order 9 resulting in magic square of order 3 of magic square sums of order 9.

**Example 2.27.** *The magic square sums of order 9 give us a magic square of order 3 given by*

| | | | |
|---|---|---|---|
| | | | *9855* |
| *3276* | *3537* | *3042* | *9855* |
| *3051* | *3285* | *3519* | *9855* |
| *3528* | *3033* | *3294* | *9855* |
| *9855* | *9855* | *9855* | *9855* |

Unfortunately, still we don't have a proper block-wise **bimagic** or **semi-magic** square of order 27.

## 2.10 Magic Square of Order 28

In this subsection, we shall present three different ways of writing **pan diagonal** magic square of order 28. One as 4 × 7, i.e., magic square formed by 49 blocks of **pan diagonal** magic squares of order 4 with equal magic sums. Second as 7 × 4, i.e., magic square formed by 16 blocks of **pan diagonal** magic squares of order 7. Third as, 14 × 2, i.e., magic square formed by 4 blocks of equal magic sums of order 14. In the first two cases the magic squares of order 28 are **pan diagonal**, while in the third case, it is just a magic square.

### 2.10.1 First Approach: 49 Blocks of Order 4

**Example 2.28.** *A **pan diagonal** magic square of order 28 formed by 49 blocks of **pan diagonal** magic squares of order 4 constructed according to Example 1.2 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 |
| | 295 | 588 | 1 | 686 | 296 | 587 | 2 | 685 | 297 | 586 | 3 | 684 | 298 | 585 | 4 | 683 | 299 | 584 | 5 | 682 | 300 | 583 | 6 | 681 | 301 | 582 | 7 | 680 | 10990 |
| 10990 | 98 | 589 | 392 | 491 | 97 | 590 | 391 | 492 | 96 | 591 | 390 | 493 | 95 | 592 | 389 | 494 | 94 | 593 | 388 | 495 | 93 | 594 | 387 | 496 | 92 | 595 | 386 | 497 | 10990 |
| 10990 | 784 | 99 | 490 | 197 | 783 | 100 | 489 | 198 | 782 | 101 | 488 | 199 | 781 | 102 | 487 | 200 | 780 | 103 | 486 | 201 | 779 | 104 | 485 | 202 | 778 | 105 | 484 | 203 | 10990 |
| 10990 | 393 | 294 | 687 | 196 | 394 | 293 | 688 | 195 | 395 | 292 | 689 | 194 | 396 | 291 | 690 | 193 | 397 | 290 | 691 | 192 | 398 | 289 | 692 | 191 | 399 | 288 | 693 | 190 | 10990 |
| 10990 | 302 | 581 | 8 | 679 | 303 | 580 | 9 | 678 | 304 | 579 | 10 | 677 | 305 | 578 | 11 | 676 | 306 | 577 | 12 | 675 | 307 | 576 | 13 | 674 | 308 | 575 | 14 | 673 | 10990 |
| 10990 | 91 | 596 | 385 | 498 | 90 | 597 | 384 | 499 | 89 | 598 | 383 | 500 | 88 | 599 | 382 | 501 | 87 | 600 | 381 | 502 | 86 | 601 | 380 | 503 | 85 | 602 | 379 | 504 | 10990 |
| 10990 | 777 | 106 | 483 | 204 | 776 | 107 | 482 | 205 | 775 | 108 | 481 | 206 | 774 | 109 | 480 | 207 | 773 | 110 | 479 | 208 | 772 | 111 | 478 | 209 | 771 | 112 | 477 | 210 | 10990 |
| 10990 | 400 | 287 | 694 | 189 | 401 | 286 | 695 | 188 | 402 | 285 | 696 | 187 | 403 | 284 | 697 | 186 | 404 | 283 | 698 | 185 | 405 | 282 | 699 | 184 | 406 | 281 | 700 | 183 | 10990 |
| 10990 | 309 | 574 | 15 | 672 | 310 | 573 | 16 | 671 | 311 | 572 | 17 | 670 | 312 | 571 | 18 | 669 | 313 | 570 | 19 | 668 | 314 | 569 | 20 | 667 | 315 | 568 | 21 | 666 | 10990 |
| 10990 | 84 | 603 | 378 | 505 | 83 | 604 | 377 | 506 | 82 | 605 | 376 | 507 | 81 | 606 | 375 | 508 | 80 | 607 | 374 | 509 | 79 | 608 | 373 | 510 | 78 | 609 | 372 | 511 | 10990 |
| 10990 | 770 | 113 | 476 | 211 | 769 | 114 | 475 | 212 | 768 | 115 | 474 | 213 | 767 | 116 | 473 | 214 | 766 | 117 | 472 | 215 | 765 | 118 | 471 | 216 | 764 | 119 | 470 | 217 | 10990 |
| 10990 | 407 | 280 | 701 | 182 | 408 | 279 | 702 | 181 | 409 | 278 | 703 | 180 | 410 | 277 | 704 | 179 | 411 | 276 | 705 | 178 | 412 | 275 | 706 | 177 | 413 | 274 | 707 | 176 | 10990 |
| 10990 | 316 | 567 | 22 | 665 | 317 | 566 | 23 | 664 | 318 | 565 | 24 | 663 | 319 | 564 | 25 | 662 | 320 | 563 | 26 | 661 | 321 | 562 | 27 | 660 | 322 | 561 | 28 | 659 | 10990 |
| 10990 | 77 | 610 | 371 | 512 | 76 | 611 | 370 | 513 | 75 | 612 | 369 | 514 | 74 | 613 | 368 | 515 | 73 | 614 | 367 | 516 | 72 | 615 | 366 | 517 | 71 | 616 | 365 | 518 | 10990 |
| 10990 | 763 | 120 | 469 | 218 | 762 | 121 | 468 | 219 | 761 | 122 | 467 | 220 | 760 | 123 | 466 | 221 | 759 | 124 | 465 | 222 | 758 | 125 | 464 | 223 | 757 | 126 | 463 | 224 | 10990 |
| 10990 | 414 | 273 | 708 | 175 | 415 | 272 | 709 | 174 | 416 | 271 | 710 | 173 | 417 | 270 | 711 | 172 | 418 | 269 | 712 | 171 | 419 | 268 | 713 | 170 | 420 | 267 | 714 | 169 | 10990 |
| 10990 | 323 | 560 | 29 | 658 | 324 | 559 | 30 | 657 | 325 | 558 | 31 | 656 | 326 | 557 | 32 | 655 | 327 | 556 | 33 | 654 | 328 | 555 | 34 | 653 | 329 | 554 | 35 | 652 | 10990 |
| 10990 | 70 | 617 | 364 | 519 | 69 | 618 | 363 | 520 | 68 | 619 | 362 | 521 | 67 | 620 | 361 | 522 | 66 | 621 | 360 | 523 | 65 | 622 | 359 | 524 | 64 | 623 | 358 | 525 | 10990 |
| 10990 | 756 | 127 | 462 | 225 | 755 | 128 | 461 | 226 | 754 | 129 | 460 | 227 | 753 | 130 | 459 | 228 | 752 | 131 | 458 | 229 | 751 | 132 | 457 | 230 | 750 | 133 | 456 | 231 | 10990 |
| 10990 | 421 | 266 | 715 | 168 | 422 | 265 | 716 | 167 | 423 | 264 | 717 | 166 | 424 | 263 | 718 | 165 | 425 | 262 | 719 | 164 | 426 | 261 | 720 | 163 | 427 | 260 | 721 | 162 | 10990 |
| 10990 | 330 | 553 | 36 | 651 | 331 | 552 | 37 | 650 | 332 | 551 | 38 | 649 | 333 | 550 | 39 | 648 | 334 | 549 | 40 | 647 | 335 | 548 | 41 | 646 | 336 | 547 | 42 | 645 | 10990 |
| 10990 | 63 | 624 | 357 | 526 | 62 | 625 | 356 | 527 | 61 | 626 | 355 | 528 | 60 | 627 | 354 | 529 | 59 | 628 | 353 | 530 | 58 | 629 | 352 | 531 | 57 | 630 | 351 | 532 | 10990 |
| 10990 | 749 | 134 | 455 | 232 | 748 | 135 | 454 | 233 | 747 | 136 | 453 | 234 | 746 | 137 | 452 | 235 | 745 | 138 | 451 | 236 | 744 | 139 | 450 | 237 | 743 | 140 | 449 | 238 | 10990 |
| 10990 | 428 | 259 | 722 | 161 | 429 | 258 | 723 | 160 | 430 | 257 | 724 | 159 | 431 | 256 | 725 | 158 | 432 | 255 | 726 | 157 | 433 | 254 | 727 | 156 | 434 | 253 | 728 | 155 | 10990 |
| 10990 | 337 | 546 | 43 | 644 | 338 | 545 | 44 | 643 | 339 | 544 | 45 | 642 | 340 | 543 | 46 | 641 | 341 | 542 | 47 | 640 | 342 | 541 | 48 | 639 | 343 | 540 | 49 | 638 | 10990 |
| 10990 | 56 | 631 | 350 | 533 | 55 | 632 | 349 | 534 | 54 | 633 | 348 | 535 | 53 | 634 | 347 | 536 | 52 | 635 | 346 | 537 | 51 | 636 | 345 | 538 | 50 | 637 | 344 | 539 | 10990 |
| 10990 | 742 | 141 | 448 | 239 | 741 | 142 | 447 | 240 | 740 | 143 | 446 | 241 | 739 | 144 | 445 | 242 | 738 | 145 | 444 | 243 | 737 | 146 | 443 | 244 | 736 | 147 | 442 | 245 | 10990 |
| 10990 | 435 | 252 | 729 | 154 | 436 | 251 | 730 | 153 | 437 | 250 | 731 | 152 | 438 | 249 | 732 | 151 | 439 | 248 | 733 | 150 | 440 | 247 | 734 | 149 | 441 | 246 | 735 | 148 | 10990 |
| | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 |

In this case, all the 49 blocks of order 4 are **pan diagonal** magic squares with equal magic sums, $S_{4\times 4} :=$ 1570

### 2.10.2 Second Approach: 16 Blocks of Order 7

**Example 2.29.** *A **pan diagonal** magic square of order 28 formed by 16 blocks of **pan diagonal** magic squares of order 7 constructed according to Example 1.5 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 |
| | 7 | 135 | 263 | 391 | 519 | 647 | 775 | 12 | 140 | 268 | 396 | 524 | 652 | 780 | 1 | 129 | 257 | 385 | 513 | 641 | 769 | 14 | 142 | 270 | 398 | 526 | 654 | 782 | 10990 |
| 10990 | 631 | 759 | 103 | 119 | 247 | 375 | 503 | 636 | 764 | 108 | 124 | 252 | 380 | 508 | 625 | 753 | 97 | 113 | 241 | 369 | 497 | 638 | 766 | 110 | 126 | 254 | 382 | 510 | 10990 |
| 10990 | 359 | 487 | 615 | 743 | 87 | 215 | 231 | 364 | 492 | 620 | 748 | 92 | 220 | 236 | 353 | 481 | 609 | 737 | 81 | 209 | 225 | 366 | 494 | 622 | 750 | 94 | 222 | 238 | 10990 |
| 10990 | 199 | 327 | 343 | 471 | 599 | 727 | 71 | 204 | 332 | 348 | 476 | 604 | 732 | 76 | 193 | 321 | 337 | 465 | 593 | 721 | 65 | 206 | 334 | 350 | 478 | 606 | 734 | 78 | 10990 |
| 10990 | 711 | 55 | 183 | 311 | 439 | 455 | 583 | 716 | 60 | 188 | 316 | 444 | 460 | 588 | 705 | 49 | 177 | 305 | 433 | 449 | 577 | 718 | 62 | 190 | 318 | 446 | 462 | 590 | 10990 |
| 10990 | 551 | 567 | 695 | 39 | 167 | 295 | 423 | 556 | 572 | 700 | 44 | 172 | 300 | 428 | 545 | 561 | 689 | 33 | 161 | 289 | 417 | 558 | 574 | 702 | 46 | 174 | 302 | 430 | 10990 |
| 10990 | 279 | 407 | 535 | 663 | 679 | 23 | 151 | 284 | 412 | 540 | 668 | 684 | 28 | 156 | 273 | 401 | 529 | 657 | 673 | 17 | 145 | 286 | 414 | 542 | 670 | 686 | 30 | 158 | 10990 |
| 10990 | 2 | 130 | 258 | 386 | 514 | 642 | 770 | 13 | 141 | 269 | 397 | 525 | 653 | 781 | 8 | 136 | 264 | 392 | 520 | 648 | 776 | 11 | 139 | 267 | 395 | 523 | 651 | 779 | 10990 |
| 10990 | 626 | 754 | 98 | 114 | 242 | 370 | 498 | 637 | 765 | 109 | 125 | 253 | 381 | 509 | 632 | 760 | 104 | 120 | 248 | 376 | 504 | 635 | 763 | 107 | 123 | 251 | 379 | 507 | 10990 |
| 10990 | 354 | 482 | 610 | 738 | 82 | 210 | 226 | 365 | 493 | 621 | 749 | 93 | 221 | 237 | 360 | 488 | 616 | 744 | 88 | 216 | 232 | 363 | 491 | 619 | 747 | 91 | 219 | 235 | 10990 |
| 10990 | 194 | 322 | 338 | 466 | 594 | 722 | 66 | 205 | 333 | 349 | 477 | 605 | 733 | 77 | 200 | 328 | 344 | 472 | 600 | 728 | 72 | 203 | 331 | 347 | 475 | 603 | 731 | 75 | 10990 |
| 10990 | 706 | 50 | 178 | 306 | 434 | 450 | 578 | 717 | 61 | 189 | 317 | 445 | 461 | 589 | 712 | 56 | 184 | 312 | 440 | 456 | 584 | 715 | 59 | 187 | 315 | 443 | 459 | 587 | 10990 |
| 10990 | 546 | 562 | 690 | 34 | 162 | 290 | 418 | 557 | 573 | 701 | 45 | 173 | 301 | 429 | 552 | 568 | 696 | 40 | 168 | 296 | 424 | 555 | 571 | 699 | 43 | 171 | 299 | 427 | 10990 |
| 10990 | 274 | 402 | 530 | 658 | 674 | 18 | 146 | 285 | 413 | 541 | 669 | 685 | 29 | 157 | 280 | 408 | 536 | 664 | 680 | 24 | 152 | 283 | 411 | 539 | 667 | 683 | 27 | 155 | 10990 |
| 10990 | 16 | 144 | 272 | 400 | 528 | 656 | 784 | 3 | 131 | 259 | 387 | 515 | 643 | 771 | 10 | 138 | 266 | 394 | 522 | 650 | 778 | 5 | 133 | 261 | 389 | 517 | 645 | 773 | 10990 |
| 10990 | 640 | 768 | 112 | 128 | 256 | 384 | 512 | 627 | 755 | 99 | 115 | 243 | 371 | 499 | 634 | 762 | 106 | 122 | 250 | 378 | 506 | 629 | 757 | 101 | 117 | 245 | 373 | 501 | 10990 |
| 10990 | 368 | 496 | 624 | 752 | 96 | 224 | 240 | 355 | 483 | 611 | 739 | 83 | 211 | 227 | 362 | 490 | 618 | 746 | 90 | 218 | 234 | 357 | 485 | 613 | 741 | 85 | 213 | 229 | 10990 |
| 10990 | 208 | 336 | 352 | 480 | 608 | 736 | 80 | 195 | 323 | 339 | 467 | 595 | 723 | 67 | 202 | 330 | 346 | 474 | 602 | 730 | 74 | 197 | 325 | 341 | 469 | 597 | 725 | 69 | 10990 |
| 10990 | 720 | 64 | 192 | 320 | 448 | 464 | 592 | 707 | 51 | 179 | 307 | 435 | 451 | 579 | 714 | 58 | 186 | 314 | 442 | 458 | 586 | 709 | 53 | 181 | 309 | 437 | 453 | 581 | 10990 |
| 10990 | 560 | 576 | 704 | 48 | 176 | 304 | 432 | 547 | 563 | 691 | 35 | 163 | 291 | 419 | 554 | 570 | 698 | 42 | 170 | 298 | 426 | 549 | 565 | 693 | 37 | 165 | 293 | 421 | 10990 |
| 10990 | 288 | 416 | 544 | 672 | 688 | 32 | 160 | 275 | 403 | 531 | 659 | 675 | 19 | 147 | 282 | 410 | 538 | 666 | 682 | 26 | 154 | 277 | 405 | 533 | 661 | 677 | 21 | 149 | 10990 |
| 10990 | 9 | 137 | 265 | 393 | 521 | 649 | 777 | 6 | 134 | 262 | 390 | 518 | 646 | 774 | 15 | 143 | 271 | 399 | 527 | 655 | 783 | 4 | 132 | 260 | 388 | 516 | 644 | 772 | 10990 |
| 10990 | 633 | 761 | 105 | 121 | 249 | 377 | 505 | 630 | 758 | 102 | 118 | 246 | 374 | 502 | 639 | 767 | 111 | 127 | 255 | 383 | 511 | 628 | 756 | 100 | 116 | 244 | 372 | 500 | 10990 |
| 10990 | 361 | 489 | 617 | 745 | 89 | 217 | 233 | 358 | 486 | 614 | 742 | 86 | 214 | 230 | 367 | 495 | 623 | 751 | 95 | 223 | 239 | 356 | 484 | 612 | 740 | 84 | 212 | 228 | 10990 |
| 10990 | 201 | 329 | 345 | 473 | 601 | 729 | 73 | 198 | 326 | 342 | 470 | 598 | 726 | 70 | 207 | 335 | 351 | 479 | 607 | 735 | 79 | 196 | 324 | 340 | 468 | 596 | 724 | 68 | 10990 |
| 10990 | 713 | 57 | 185 | 313 | 441 | 457 | 585 | 710 | 54 | 182 | 310 | 438 | 454 | 582 | 719 | 63 | 191 | 319 | 447 | 463 | 591 | 708 | 52 | 180 | 308 | 436 | 452 | 580 | 10990 |
| 10990 | 553 | 569 | 697 | 41 | 169 | 297 | 425 | 550 | 566 | 694 | 38 | 166 | 294 | 422 | 559 | 575 | 703 | 47 | 175 | 303 | 431 | 548 | 564 | 692 | 36 | 164 | 292 | 420 | 10990 |
| 10990 | 281 | 409 | 537 | 665 | 681 | 25 | 153 | 278 | 406 | 534 | 662 | 678 | 22 | 150 | 287 | 415 | 543 | 671 | 687 | 31 | 159 | 276 | 404 | 532 | 660 | 676 | 20 | 148 | 10990 |
| | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 |

We observe that it is not possible to have equal magic sums of order 7 as it is impossible to divide magic square sum of order 28 by 4, i.e., 10990/4 := 2747.5. In this case, all the 16 blocks of order 7 are **pan diagonal** magic squares with different magic sums resulting in a **pan diagonal** magic square of order 4 given in example below.

**Example 2.30.** *A **pan diagonal** magic square of order 4, formed by 16 magic square sums of each* $7\times 7$ *block is given by.*

| | | | | | |
|---|---|---|---|---|---|
| | | 10990 | 10990 | 10990 | 10990 |
| | 2737 | 2772 | 2695 | 2786 | 10990 |
| 10990 | 2702 | 2779 | 2744 | 2765 | 10990 |
| 10990 | 2800 | 2709 | 2758 | 2723 | 10990 |
| 10990 | 2751 | 2730 | 2793 | 2716 | 10990 |
| | 10990 | 10990 | 10990 | 10990 | 10990 |

### 2.10.3 Third Approach: 4 Blocks of Order 14

**Example 2.31.** *A **pan diagonal** magic square of order 28 formed by 4 blocks of magic squares of order 14 constructed according to Example 1.12 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | 10990 |
| 1 | 377 | 761 | 448 | 552 | 312 | 232 | 128 | 65 | 633 | 584 | 712 | 497 | 193 | 2 | 378 | 762 | 447 | 551 | 311 | 231 | 127 | 66 | 634 | 583 | 711 | 498 | 194 | 10990 |
| 585 | 64 | 440 | 704 | 752 | 505 | 345 | 465 | 16 | 209 | 280 | 329 | 152 | 649 | 586 | 63 | 439 | 703 | 751 | 506 | 346 | 466 | 15 | 210 | 279 | 330 | 151 | 650 | 10990 |
| 504 | 648 | 121 | 545 | 688 | 249 | 320 | 24 | 185 | 104 | 385 | 393 | 593 | 736 | 503 | 647 | 122 | 546 | 687 | 250 | 319 | 23 | 186 | 103 | 386 | 394 | 594 | 735 | 10990 |
| 760 | 441 | 600 | 184 | 617 | 457 | 17 | 305 | 136 | 680 | 89 | 384 | 560 | 265 | 759 | 442 | 599 | 183 | 618 | 458 | 18 | 306 | 135 | 679 | 90 | 383 | 559 | 266 | 10990 |
| 656 | 336 | 217 | 48 | 481 | 344 | 113 | 401 | 529 | 248 | 689 | 97 | 744 | 592 | 655 | 335 | 218 | 47 | 482 | 343 | 114 | 402 | 530 | 247 | 690 | 98 | 743 | 591 | 10990 |
| 416 | 40 | 353 | 145 | 81 | 664 | 713 | 616 | 777 | 296 | 176 | 536 | 233 | 449 | 415 | 39 | 354 | 146 | 82 | 663 | 714 | 615 | 778 | 295 | 175 | 535 | 234 | 450 | 10990 |
| 73 | 137 | 528 | 225 | 9 | 609 | 784 | 657 | 720 | 368 | 488 | 201 | 288 | 408 | 74 | 138 | 527 | 226 | 10 | 610 | 783 | 658 | 719 | 367 | 487 | 202 | 287 | 407 | 10990 |
| 177 | 257 | 281 | 360 | 144 | 769 | 608 | 721 | 672 | 473 | 520 | 8 | 409 | 96 | 178 | 258 | 282 | 359 | 143 | 770 | 607 | 722 | 671 | 474 | 519 | 7 | 410 | 95 | 10990 |
| 120 | 169 | 240 | 289 | 376 | 728 | 665 | 776 | 601 | 425 | 25 | 472 | 88 | 521 | 119 | 170 | 239 | 290 | 375 | 727 | 666 | 775 | 602 | 426 | 26 | 471 | 87 | 522 | 10990 |
| 272 | 576 | 624 | 105 | 297 | 192 | 417 | 369 | 480 | 544 | 729 | 168 | 41 | 681 | 271 | 575 | 623 | 106 | 298 | 191 | 418 | 370 | 479 | 543 | 730 | 167 | 42 | 682 | 10990 |
| 321 | 745 | 697 | 456 | 273 | 72 | 537 | 208 | 337 | 56 | 424 | 569 | 640 | 160 | 322 | 746 | 698 | 455 | 274 | 71 | 538 | 207 | 338 | 55 | 423 | 570 | 639 | 159 | 10990 |
| 553 | 696 | 32 | 641 | 568 | 432 | 464 | 57 | 313 | 737 | 153 | 241 | 216 | 392 | 554 | 695 | 31 | 642 | 567 | 431 | 463 | 58 | 314 | 738 | 154 | 242 | 215 | 391 | 10990 |
| 705 | 513 | 489 | 768 | 224 | 129 | 80 | 256 | 400 | 561 | 328 | 632 | 361 | 49 | 706 | 514 | 490 | 767 | 223 | 130 | 79 | 255 | 399 | 562 | 327 | 631 | 362 | 50 | 10990 |
| 352 | 496 | 112 | 577 | 433 | 33 | 200 | 512 | 264 | 161 | 625 | 753 | 673 | 304 | 351 | 495 | 111 | 578 | 434 | 34 | 199 | 511 | 263 | 162 | 626 | 754 | 674 | 303 | 10990 |
| 3 | 379 | 763 | 446 | 550 | 310 | 230 | 126 | 67 | 635 | 582 | 710 | 499 | 195 | 4 | 380 | 764 | 445 | 549 | 309 | 229 | 125 | 68 | 636 | 581 | 709 | 500 | 196 | 10990 |
| 587 | 62 | 438 | 702 | 750 | 507 | 347 | 467 | 14 | 211 | 278 | 331 | 150 | 651 | 588 | 61 | 437 | 701 | 749 | 508 | 348 | 468 | 13 | 212 | 277 | 332 | 149 | 652 | 10990 |
| 502 | 646 | 123 | 547 | 686 | 251 | 318 | 22 | 187 | 102 | 387 | 395 | 595 | 734 | 501 | 645 | 124 | 548 | 685 | 252 | 317 | 21 | 188 | 101 | 388 | 396 | 596 | 733 | 10990 |
| 758 | 443 | 598 | 182 | 619 | 459 | 19 | 307 | 134 | 678 | 91 | 382 | 558 | 267 | 757 | 444 | 597 | 181 | 620 | 460 | 20 | 308 | 133 | 677 | 92 | 381 | 557 | 268 | 10990 |
| 654 | 334 | 219 | 46 | 483 | 342 | 115 | 403 | 531 | 246 | 691 | 99 | 742 | 590 | 653 | 333 | 220 | 45 | 484 | 341 | 116 | 404 | 532 | 245 | 692 | 100 | 741 | 589 | 10990 |
| 414 | 38 | 355 | 147 | 83 | 662 | 715 | 614 | 779 | 294 | 174 | 534 | 235 | 451 | 413 | 37 | 356 | 148 | 84 | 661 | 716 | 613 | 780 | 293 | 173 | 533 | 236 | 452 | 10990 |
| 75 | 139 | 526 | 227 | 11 | 611 | 782 | 659 | 718 | 366 | 486 | 203 | 286 | 406 | 76 | 140 | 525 | 228 | 12 | 612 | 781 | 660 | 717 | 365 | 485 | 204 | 285 | 405 | 10990 |
| 179 | 259 | 283 | 358 | 142 | 771 | 606 | 723 | 670 | 475 | 518 | 6 | 411 | 94 | 180 | 260 | 284 | 357 | 141 | 772 | 605 | 724 | 669 | 476 | 517 | 5 | 412 | 93 | 10990 |
| 118 | 171 | 238 | 291 | 374 | 726 | 667 | 774 | 603 | 427 | 27 | 470 | 86 | 523 | 117 | 172 | 237 | 292 | 373 | 725 | 668 | 773 | 604 | 428 | 28 | 469 | 85 | 524 | 10990 |
| 270 | 574 | 622 | 107 | 299 | 190 | 419 | 371 | 478 | 542 | 731 | 166 | 43 | 683 | 269 | 573 | 621 | 108 | 300 | 189 | 420 | 372 | 477 | 541 | 732 | 165 | 44 | 684 | 10990 |
| 323 | 747 | 699 | 454 | 275 | 70 | 539 | 206 | 339 | 54 | 422 | 571 | 638 | 158 | 324 | 748 | 700 | 453 | 276 | 69 | 540 | 205 | 340 | 53 | 421 | 572 | 637 | 157 | 10990 |
| 555 | 694 | 30 | 643 | 566 | 430 | 462 | 59 | 315 | 739 | 155 | 243 | 214 | 390 | 556 | 693 | 29 | 644 | 565 | 429 | 461 | 60 | 316 | 740 | 156 | 244 | 213 | 389 | 10990 |
| 707 | 515 | 491 | 766 | 222 | 131 | 78 | 254 | 398 | 563 | 326 | 630 | 363 | 51 | 708 | 516 | 492 | 765 | 221 | 132 | 77 | 253 | 397 | 564 | 325 | 629 | 364 | 52 | 10990 |
| 350 | 494 | 110 | 579 | 435 | 35 | 198 | 510 | 262 | 163 | 627 | 755 | 675 | 302 | 349 | 493 | 109 | 580 | 436 | 36 | 197 | 509 | 261 | 164 | 628 | 756 | 676 | 301 | 10990 |
| 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 | 10990 |

In this case, all the 4 blocks of order 14 are magic squares with equal magic sums $S_{14\times 14} := 5495$. The middle blocks in all the magic squares of order 14 are **pan diagonal** magic squares of order 4 with equal magic sums $S_{4\times 4} := 2770$.

### 2.11 Magic Square of Order 30

In this subsection, we shall present four different ways of writing block-wise magic square of order 30. One as $5\times 6$, i.e., magic square formed by 36 blocks of **pan diagonal** magic squares of order 5. Second as $6\times 5$, i.e., magic square formed by 25 blocks of magic squares of order 6. The third as $10\times 3$, i.e., magic square formed by 9 blocks of magic squares of order 10. The forth way is $3\times 10$, i.e., magic square formed by 100 blocks of magic squares of order 3 with different magic sums.

#### 2.11.1 First Approach: 36 Blocks of Order 5

**Example 2.32.** *A magic square of order 30 formed by 36 blocks of* ***pan diagonal*** *magic squares of order 5 constructed according to Example 1.3 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 13515 |
| 1 | 289 | 397 | 685 | 793 | 23 | 311 | 419 | 707 | 815 | 28 | 316 | 424 | 712 | 820 | 34 | 322 | 430 | 718 | 826 | 17 | 305 | 413 | 701 | 809 | 8 | 296 | 404 | 692 | 800 | 13515 |
| 577 | 865 | 73 | 181 | 469 | 599 | 887 | 95 | 203 | 491 | 604 | 892 | 100 | 208 | 496 | 610 | 898 | 106 | 214 | 502 | 593 | 881 | 89 | 197 | 485 | 584 | 872 | 80 | 188 | 476 | 13515 |
| 253 | 361 | 649 | 757 | 145 | 275 | 383 | 671 | 779 | 167 | 280 | 388 | 676 | 784 | 172 | 286 | 394 | 682 | 790 | 178 | 269 | 377 | 665 | 773 | 161 | 260 | 368 | 656 | 764 | 152 | 13515 |
| 829 | 37 | 325 | 433 | 541 | 851 | 59 | 347 | 455 | 563 | 856 | 64 | 352 | 460 | 568 | 862 | 70 | 358 | 466 | 574 | 845 | 53 | 341 | 449 | 557 | 836 | 44 | 332 | 440 | 548 | 13515 |
| 505 | 613 | 721 | 109 | 217 | 527 | 635 | 743 | 131 | 239 | 532 | 640 | 748 | 136 | 244 | 538 | 646 | 754 | 142 | 250 | 521 | 629 | 737 | 125 | 233 | 512 | 620 | 728 | 116 | 224 | 13515 |
| 29 | 317 | 425 | 713 | 821 | 7 | 295 | 403 | 691 | 799 | 35 | 323 | 431 | 719 | 827 | 14 | 302 | 410 | 698 | 806 | 21 | 309 | 417 | 705 | 813 | 5 | 293 | 401 | 689 | 797 | 13515 |
| 605 | 893 | 101 | 209 | 497 | 583 | 871 | 79 | 187 | 475 | 611 | 899 | 107 | 215 | 503 | 590 | 878 | 86 | 194 | 482 | 597 | 885 | 93 | 201 | 489 | 581 | 869 | 77 | 185 | 473 | 13515 |
| 281 | 389 | 677 | 785 | 173 | 259 | 367 | 655 | 763 | 151 | 287 | 395 | 683 | 791 | 179 | 266 | 374 | 662 | 770 | 158 | 273 | 381 | 669 | 777 | 165 | 257 | 365 | 653 | 761 | 149 | 13515 |
| 857 | 65 | 353 | 461 | 569 | 835 | 43 | 331 | 439 | 547 | 863 | 71 | 359 | 467 | 575 | 842 | 50 | 338 | 446 | 554 | 849 | 57 | 345 | 453 | 561 | 833 | 41 | 329 | 437 | 545 | 13515 |
| 533 | 641 | 749 | 137 | 245 | 511 | 619 | 727 | 115 | 223 | 539 | 647 | 755 | 143 | 251 | 518 | 626 | 734 | 122 | 230 | 525 | 633 | 741 | 129 | 237 | 509 | 617 | 725 | 113 | 221 | 13515 |
| 12 | 300 | 408 | 696 | 804 | 6 | 294 | 402 | 690 | 798 | 13 | 301 | 409 | 697 | 805 | 27 | 315 | 423 | 711 | 819 | 31 | 319 | 427 | 715 | 823 | 22 | 310 | 418 | 706 | 814 | 13515 |
| 588 | 876 | 84 | 192 | 480 | 582 | 870 | 78 | 186 | 474 | 589 | 877 | 85 | 193 | 481 | 603 | 891 | 99 | 207 | 495 | 607 | 895 | 103 | 211 | 499 | 598 | 886 | 94 | 202 | 490 | 13515 |
| 264 | 372 | 660 | 768 | 156 | 258 | 366 | 654 | 762 | 150 | 265 | 373 | 661 | 769 | 157 | 279 | 387 | 675 | 783 | 171 | 283 | 391 | 679 | 787 | 175 | 274 | 382 | 670 | 778 | 166 | 13515 |
| 840 | 48 | 336 | 444 | 552 | 834 | 42 | 330 | 438 | 546 | 841 | 49 | 337 | 445 | 553 | 855 | 63 | 351 | 459 | 567 | 859 | 67 | 355 | 463 | 571 | 850 | 58 | 346 | 454 | 562 | 13515 |
| 516 | 624 | 732 | 120 | 228 | 510 | 618 | 726 | 114 | 222 | 517 | 625 | 733 | 121 | 229 | 531 | 639 | 747 | 135 | 243 | 535 | 643 | 751 | 139 | 247 | 526 | 634 | 742 | 130 | 238 | 13515 |
| 32 | 320 | 428 | 716 | 824 | 16 | 304 | 412 | 700 | 808 | 4 | 292 | 400 | 688 | 796 | 24 | 312 | 420 | 708 | 816 | 10 | 298 | 406 | 694 | 802 | 25 | 313 | 421 | 709 | 817 | 13515 |
| 608 | 896 | 104 | 212 | 500 | 592 | 880 | 88 | 196 | 484 | 580 | 868 | 76 | 184 | 472 | 600 | 888 | 96 | 204 | 492 | 586 | 874 | 82 | 190 | 478 | 601 | 889 | 97 | 205 | 493 | 13515 |
| 284 | 392 | 680 | 788 | 176 | 268 | 376 | 664 | 772 | 160 | 256 | 364 | 652 | 760 | 148 | 276 | 384 | 672 | 780 | 168 | 262 | 370 | 658 | 766 | 154 | 277 | 385 | 673 | 781 | 169 | 13515 |
| 860 | 68 | 356 | 464 | 572 | 844 | 52 | 340 | 448 | 556 | 832 | 40 | 328 | 436 | 544 | 852 | 60 | 348 | 456 | 564 | 838 | 46 | 334 | 442 | 550 | 853 | 61 | 349 | 457 | 565 | 13515 |
| 536 | 644 | 752 | 140 | 248 | 520 | 628 | 736 | 124 | 232 | 508 | 616 | 724 | 112 | 220 | 528 | 636 | 744 | 132 | 240 | 514 | 622 | 730 | 118 | 226 | 529 | 637 | 745 | 133 | 241 | 13515 |
| 19 | 307 | 415 | 703 | 811 | 33 | 321 | 429 | 717 | 825 | 11 | 299 | 407 | 695 | 803 | 3 | 291 | 399 | 687 | 795 | 30 | 318 | 426 | 714 | 822 | 15 | 303 | 411 | 699 | 807 | 13515 |
| 595 | 883 | 91 | 199 | 487 | 609 | 897 | 105 | 213 | 501 | 587 | 875 | 83 | 191 | 479 | 579 | 867 | 75 | 183 | 471 | 606 | 894 | 102 | 210 | 498 | 591 | 879 | 87 | 195 | 483 | 13515 |
| 271 | 379 | 667 | 775 | 163 | 285 | 393 | 681 | 789 | 177 | 263 | 371 | 659 | 767 | 155 | 255 | 363 | 651 | 759 | 147 | 282 | 390 | 678 | 786 | 174 | 267 | 375 | 663 | 771 | 159 | 13515 |
| 847 | 55 | 343 | 451 | 559 | 861 | 69 | 357 | 465 | 573 | 839 | 47 | 335 | 443 | 551 | 831 | 39 | 327 | 435 | 543 | 858 | 66 | 354 | 462 | 570 | 843 | 51 | 339 | 447 | 555 | 13515 |
| 523 | 631 | 739 | 127 | 235 | 537 | 645 | 753 | 141 | 249 | 515 | 623 | 731 | 119 | 227 | 507 | 615 | 723 | 111 | 219 | 534 | 642 | 750 | 138 | 246 | 519 | 627 | 735 | 123 | 231 | 13515 |
| 18 | 306 | 414 | 702 | 810 | 26 | 314 | 422 | 710 | 818 | 20 | 308 | 416 | 704 | 812 | 9 | 297 | 405 | 693 | 801 | 2 | 290 | 398 | 686 | 794 | 36 | 324 | 432 | 720 | 828 | 13515 |
| 594 | 882 | 90 | 198 | 486 | 602 | 890 | 98 | 206 | 494 | 596 | 884 | 92 | 200 | 488 | 585 | 873 | 81 | 189 | 477 | 578 | 866 | 74 | 182 | 470 | 612 | 900 | 108 | 216 | 504 | 13515 |
| 270 | 378 | 666 | 774 | 162 | 278 | 386 | 674 | 782 | 170 | 272 | 380 | 668 | 776 | 164 | 261 | 369 | 657 | 765 | 153 | 254 | 362 | 650 | 758 | 146 | 288 | 396 | 684 | 792 | 180 | 13515 |
| 846 | 54 | 342 | 450 | 558 | 854 | 62 | 350 | 458 | 566 | 848 | 56 | 344 | 452 | 560 | 837 | 45 | 333 | 441 | 549 | 830 | 38 | 326 | 434 | 542 | 864 | 72 | 360 | 468 | 576 | 13515 |
| 522 | 630 | 738 | 126 | 234 | 530 | 638 | 746 | 134 | 242 | 524 | 632 | 740 | 128 | 236 | 513 | 621 | 729 | 117 | 225 | 506 | 614 | 722 | 110 | 218 | 540 | 648 | 756 | 144 | 252 | 13515 |
| 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 |

We observe that it is not possible to have equal magic sums of order 5 as it is impossible to divide magic square sum of order 30 by 6, i.e., 13515/6 := 2252.5. In this case, all the 36 blocks are **pan diagonal** magic squares of order 5 with different magic sums, resulting in a magic square of order 6 given in example below.

**Example 2.33.** *Magic sums of 36 blocks of* ***pan diagonal*** *magic squares of order 5 lead us to a magic square of order 6 given by*

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | 13515 |
| 2165 | 2275 | 2300 | 2330 | 2245 | 2200 | 13515 |
| 2305 | 2195 | 2335 | 2230 | 2265 | 2185 | 13515 |
| 2220 | 2190 | 2225 | 2295 | 2315 | 2270 | 13515 |
| 2320 | 2240 | 2180 | 2280 | 2210 | 2285 | 13515 |
| 2255 | 2325 | 2215 | 2175 | 2310 | 2235 | 13515 |
| 2250 | 2290 | 2260 | 2205 | 2170 | 2340 | 13515 |
| 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 |

### 2.11.2 Second Approach: 25 Blocks of Order 6

**Example 2.34.** *A magic square of order 30 formed by 25 blocks of magic squares of order 6 constructed according to Example 1.4 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 13515 |
| 1 | 851 | 850 | 801 | 50 | 150 | 2 | 852 | 849 | 802 | 49 | 149 | 3 | 853 | 848 | 803 | 48 | 148 | 4 | 854 | 847 | 804 | 47 | 147 | 5 | 855 | 846 | 805 | 46 | 146 | 13515 |
| 750 | 200 | 700 | 201 | 251 | 601 | 749 | 199 | 699 | 202 | 252 | 602 | 748 | 198 | 698 | 203 | 253 | 603 | 747 | 197 | 697 | 204 | 254 | 604 | 746 | 196 | 696 | 205 | 255 | 605 | 13515 |
| 600 | 551 | 351 | 400 | 500 | 301 | 599 | 552 | 352 | 399 | 499 | 302 | 598 | 553 | 353 | 398 | 498 | 303 | 597 | 554 | 354 | 397 | 497 | 304 | 596 | 555 | 355 | 396 | 496 | 305 | 13515 |
| 450 | 350 | 501 | 550 | 401 | 451 | 449 | 349 | 502 | 549 | 402 | 452 | 448 | 348 | 503 | 548 | 403 | 453 | 447 | 347 | 504 | 547 | 404 | 454 | 446 | 346 | 505 | 546 | 405 | 455 | 13515 |
| 151 | 650 | 250 | 651 | 701 | 300 | 152 | 649 | 249 | 652 | 702 | 299 | 153 | 648 | 248 | 653 | 703 | 298 | 154 | 647 | 247 | 654 | 704 | 297 | 155 | 646 | 246 | 655 | 705 | 296 | 13515 |
| 751 | 101 | 51 | 100 | 800 | 900 | 752 | 102 | 52 | 99 | 799 | 899 | 753 | 103 | 53 | 98 | 798 | 898 | 754 | 104 | 54 | 97 | 797 | 897 | 755 | 105 | 55 | 96 | 796 | 896 | 13515 |
| 6 | 856 | 845 | 806 | 45 | 145 | 7 | 857 | 844 | 807 | 44 | 144 | 8 | 858 | 843 | 808 | 43 | 143 | 9 | 859 | 842 | 809 | 42 | 142 | 10 | 860 | 841 | 810 | 41 | 141 | 13515 |
| 745 | 195 | 695 | 206 | 256 | 606 | 744 | 194 | 694 | 207 | 257 | 607 | 743 | 193 | 693 | 208 | 258 | 608 | 742 | 192 | 692 | 209 | 259 | 609 | 741 | 191 | 691 | 210 | 260 | 610 | 13515 |
| 595 | 556 | 356 | 395 | 495 | 306 | 594 | 557 | 357 | 394 | 494 | 307 | 593 | 558 | 358 | 393 | 493 | 308 | 592 | 559 | 359 | 392 | 492 | 309 | 591 | 560 | 360 | 391 | 491 | 310 | 13515 |
| 445 | 345 | 506 | 545 | 406 | 456 | 444 | 344 | 507 | 544 | 407 | 457 | 443 | 343 | 508 | 543 | 408 | 458 | 442 | 342 | 509 | 542 | 409 | 459 | 441 | 341 | 510 | 541 | 410 | 460 | 13515 |
| 156 | 645 | 245 | 656 | 706 | 295 | 157 | 644 | 244 | 657 | 707 | 294 | 158 | 643 | 243 | 658 | 708 | 293 | 159 | 642 | 242 | 659 | 709 | 292 | 160 | 641 | 241 | 660 | 710 | 291 | 13515 |
| 756 | 106 | 56 | 95 | 795 | 895 | 757 | 107 | 57 | 94 | 794 | 894 | 758 | 108 | 58 | 93 | 793 | 893 | 759 | 109 | 59 | 92 | 792 | 892 | 760 | 110 | 60 | 91 | 791 | 891 | 13515 |
| 11 | 861 | 840 | 811 | 40 | 140 | 12 | 862 | 839 | 812 | 39 | 139 | 13 | 863 | 838 | 813 | 38 | 138 | 14 | 864 | 837 | 814 | 37 | 137 | 15 | 865 | 836 | 815 | 36 | 136 | 13515 |
| 740 | 190 | 690 | 211 | 261 | 611 | 739 | 189 | 689 | 212 | 262 | 612 | 738 | 188 | 688 | 213 | 263 | 613 | 737 | 187 | 687 | 214 | 264 | 614 | 736 | 186 | 686 | 215 | 265 | 615 | 13515 |
| 590 | 561 | 361 | 390 | 490 | 311 | 589 | 562 | 362 | 389 | 489 | 312 | 588 | 563 | 363 | 388 | 488 | 313 | 587 | 564 | 364 | 387 | 487 | 314 | 586 | 565 | 365 | 386 | 486 | 315 | 13515 |
| 440 | 340 | 511 | 540 | 411 | 461 | 439 | 339 | 512 | 539 | 412 | 462 | 438 | 338 | 513 | 538 | 413 | 463 | 437 | 337 | 514 | 537 | 414 | 464 | 436 | 336 | 515 | 536 | 415 | 465 | 13515 |
| 161 | 640 | 240 | 661 | 711 | 290 | 162 | 639 | 239 | 662 | 712 | 289 | 163 | 638 | 238 | 663 | 713 | 288 | 164 | 637 | 237 | 664 | 714 | 287 | 165 | 636 | 236 | 665 | 715 | 286 | 13515 |
| 761 | 111 | 61 | 90 | 790 | 890 | 762 | 112 | 62 | 89 | 789 | 889 | 763 | 113 | 63 | 88 | 788 | 888 | 764 | 114 | 64 | 87 | 787 | 887 | 765 | 115 | 65 | 86 | 786 | 886 | 13515 |
| 16 | 866 | 835 | 816 | 35 | 135 | 17 | 867 | 834 | 817 | 34 | 134 | 18 | 868 | 833 | 818 | 33 | 133 | 19 | 869 | 832 | 819 | 32 | 132 | 20 | 870 | 831 | 820 | 31 | 131 | 13515 |
| 735 | 185 | 685 | 216 | 266 | 616 | 734 | 184 | 684 | 217 | 267 | 617 | 733 | 183 | 683 | 218 | 268 | 618 | 732 | 182 | 682 | 219 | 269 | 619 | 731 | 181 | 681 | 220 | 270 | 620 | 13515 |
| 585 | 566 | 366 | 385 | 485 | 316 | 584 | 567 | 367 | 384 | 484 | 317 | 583 | 568 | 368 | 383 | 483 | 318 | 582 | 569 | 369 | 382 | 482 | 319 | 581 | 570 | 370 | 381 | 481 | 320 | 13515 |
| 435 | 335 | 516 | 535 | 416 | 466 | 434 | 334 | 517 | 534 | 417 | 467 | 433 | 333 | 518 | 533 | 418 | 468 | 432 | 332 | 519 | 532 | 419 | 469 | 431 | 331 | 520 | 531 | 420 | 470 | 13515 |
| 166 | 635 | 235 | 666 | 716 | 285 | 167 | 634 | 234 | 667 | 717 | 284 | 168 | 633 | 233 | 668 | 718 | 283 | 169 | 632 | 232 | 669 | 719 | 282 | 170 | 631 | 231 | 670 | 720 | 281 | 13515 |
| 766 | 116 | 66 | 85 | 785 | 885 | 767 | 117 | 67 | 84 | 784 | 884 | 768 | 118 | 68 | 83 | 783 | 883 | 769 | 119 | 69 | 82 | 782 | 882 | 770 | 120 | 70 | 81 | 781 | 881 | 13515 |
| 21 | 871 | 830 | 821 | 30 | 130 | 22 | 872 | 829 | 822 | 29 | 129 | 23 | 873 | 828 | 823 | 28 | 128 | 24 | 874 | 827 | 824 | 27 | 127 | 25 | 875 | 826 | 825 | 26 | 126 | 13515 |
| 730 | 180 | 680 | 221 | 271 | 621 | 729 | 179 | 679 | 222 | 272 | 622 | 728 | 178 | 678 | 223 | 273 | 623 | 727 | 177 | 677 | 224 | 274 | 624 | 726 | 176 | 676 | 225 | 275 | 625 | 13515 |
| 580 | 571 | 371 | 380 | 480 | 321 | 579 | 572 | 372 | 379 | 479 | 322 | 578 | 573 | 373 | 378 | 478 | 323 | 577 | 574 | 374 | 377 | 477 | 324 | 576 | 575 | 375 | 376 | 476 | 325 | 13515 |
| 430 | 330 | 521 | 530 | 421 | 471 | 429 | 329 | 522 | 529 | 422 | 472 | 428 | 328 | 523 | 528 | 423 | 473 | 427 | 327 | 524 | 527 | 424 | 474 | 426 | 326 | 525 | 526 | 425 | 475 | 13515 |
| 171 | 630 | 230 | 671 | 721 | 280 | 172 | 629 | 229 | 672 | 722 | 279 | 173 | 628 | 228 | 673 | 723 | 278 | 174 | 627 | 227 | 674 | 724 | 277 | 175 | 626 | 226 | 675 | 725 | 276 | 13515 |
| 771 | 121 | 71 | 80 | 780 | 880 | 772 | 122 | 72 | 79 | 779 | 879 | 773 | 123 | 73 | 78 | 778 | 878 | 774 | 124 | 74 | 77 | 777 | 877 | 775 | 125 | 75 | 76 | 776 | 876 | 13515 |
| 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 |

In this case, all the 25 blocks of order 6 are magic squares with equal magic sums, $S_{6\times 6} := 2703$.

#### 2.11.3 Third Approach: 9 Blocks of Order 10

**Example 2.35.** *A magic square of order 30 formed by 9 blocks of magic squares of order 10 constructed according to Example 1.10 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 13515 |
| 1 | 720 | 577 | 865 | 343 | 198 | 432 | 774 | 469 | 126 | 2 | 719 | 578 | 866 | 344 | 197 | 431 | 773 | 470 | 125 | 3 | 718 | 579 | 867 | 345 | 196 | 430 | 772 | 471 | 124 | 13515 |
| 882 | 108 | 73 | 594 | 810 | 666 | 487 | 289 | 361 | 235 | 881 | 107 | 74 | 593 | 809 | 665 | 488 | 290 | 362 | 236 | 880 | 106 | 75 | 592 | 808 | 664 | 489 | 291 | 363 | 237 | 13515 |
| 415 | 721 | 199 | 703 | 144 | 307 | 846 | 540 | 558 | 72 | 416 | 722 | 200 | 704 | 143 | 308 | 845 | 539 | 557 | 71 | 417 | 723 | 201 | 705 | 142 | 309 | 844 | 538 | 556 | 70 | 13515 |
| 630 | 505 | 792 | 306 | 18 | 811 | 253 | 127 | 684 | 379 | 629 | 506 | 791 | 305 | 17 | 812 | 254 | 128 | 683 | 380 | 628 | 507 | 790 | 304 | 16 | 813 | 255 | 129 | 682 | 381 | 13515 |
| 756 | 883 | 468 | 91 | 397 | 612 | 649 | 55 | 270 | 324 | 755 | 884 | 467 | 92 | 398 | 611 | 650 | 56 | 269 | 323 | 754 | 885 | 466 | 93 | 399 | 610 | 651 | 57 | 268 | 322 | 13515 |
| 109 | 342 | 396 | 90 | 685 | 504 | 738 | 181 | 847 | 613 | 110 | 341 | 395 | 89 | 686 | 503 | 737 | 182 | 848 | 614 | 111 | 340 | 394 | 88 | 687 | 502 | 736 | 183 | 849 | 615 | 13515 |
| 667 | 414 | 360 | 739 | 252 | 163 | 595 | 828 | 36 | 451 | 668 | 413 | 359 | 740 | 251 | 164 | 596 | 827 | 35 | 452 | 669 | 412 | 358 | 741 | 250 | 165 | 597 | 826 | 34 | 453 | 13515 |
| 523 | 216 | 864 | 378 | 541 | 19 | 180 | 702 | 325 | 757 | 524 | 215 | 863 | 377 | 542 | 20 | 179 | 701 | 326 | 758 | 525 | 214 | 862 | 376 | 543 | 21 | 178 | 700 | 327 | 759 | 13515 |
| 234 | 37 | 145 | 522 | 829 | 450 | 271 | 576 | 793 | 648 | 233 | 38 | 146 | 521 | 830 | 449 | 272 | 575 | 794 | 647 | 232 | 39 | 147 | 520 | 831 | 448 | 273 | 574 | 795 | 646 | 13515 |
| 288 | 559 | 631 | 217 | 486 | 775 | 54 | 433 | 162 | 900 | 287 | 560 | 632 | 218 | 485 | 776 | 53 | 434 | 161 | 899 | 286 | 561 | 633 | 219 | 484 | 777 | 52 | 435 | 160 | 898 | 13515 |
| 4 | 717 | 580 | 868 | 346 | 195 | 429 | 771 | 472 | 123 | 5 | 716 | 581 | 869 | 347 | 194 | 428 | 770 | 473 | 122 | 6 | 715 | 582 | 870 | 348 | 193 | 427 | 769 | 474 | 121 | 13515 |
| 879 | 105 | 76 | 591 | 807 | 663 | 490 | 292 | 364 | 238 | 878 | 104 | 77 | 590 | 806 | 662 | 491 | 293 | 365 | 239 | 877 | 103 | 78 | 589 | 805 | 661 | 492 | 294 | 366 | 240 | 13515 |
| 418 | 724 | 202 | 706 | 141 | 310 | 843 | 537 | 555 | 69 | 419 | 725 | 203 | 707 | 140 | 311 | 842 | 536 | 554 | 68 | 420 | 726 | 204 | 708 | 139 | 312 | 841 | 535 | 553 | 67 | 13515 |
| 627 | 508 | 789 | 303 | 15 | 814 | 256 | 130 | 681 | 382 | 626 | 509 | 788 | 302 | 14 | 815 | 257 | 131 | 680 | 383 | 625 | 510 | 787 | 301 | 13 | 816 | 258 | 132 | 679 | 384 | 13515 |
| 753 | 886 | 465 | 94 | 400 | 609 | 652 | 58 | 267 | 321 | 752 | 887 | 464 | 95 | 401 | 608 | 653 | 59 | 266 | 320 | 751 | 888 | 463 | 96 | 402 | 607 | 654 | 60 | 265 | 319 | 13515 |
| 112 | 339 | 393 | 87 | 688 | 501 | 735 | 184 | 850 | 616 | 113 | 338 | 392 | 86 | 689 | 500 | 734 | 185 | 851 | 617 | 114 | 337 | 391 | 85 | 690 | 499 | 733 | 186 | 852 | 618 | 13515 |
| 670 | 411 | 357 | 742 | 249 | 166 | 598 | 825 | 33 | 454 | 671 | 410 | 356 | 743 | 248 | 167 | 599 | 824 | 32 | 455 | 672 | 409 | 355 | 744 | 247 | 168 | 600 | 823 | 31 | 456 | 13515 |
| 526 | 213 | 861 | 375 | 544 | 22 | 177 | 699 | 328 | 760 | 527 | 212 | 860 | 374 | 545 | 23 | 176 | 698 | 329 | 761 | 528 | 211 | 859 | 373 | 546 | 24 | 175 | 697 | 330 | 762 | 13515 |
| 231 | 40 | 148 | 519 | 832 | 447 | 274 | 573 | 796 | 645 | 230 | 41 | 149 | 518 | 833 | 446 | 275 | 572 | 797 | 644 | 229 | 42 | 150 | 517 | 834 | 445 | 276 | 571 | 798 | 643 | 13515 |
| 285 | 562 | 634 | 220 | 483 | 778 | 51 | 436 | 159 | 897 | 284 | 563 | 635 | 221 | 482 | 779 | 50 | 437 | 158 | 896 | 283 | 564 | 636 | 222 | 481 | 780 | 49 | 438 | 157 | 895 | 13515 |
| 7 | 714 | 583 | 871 | 349 | 192 | 426 | 768 | 475 | 120 | 8 | 713 | 584 | 872 | 350 | 191 | 425 | 767 | 476 | 119 | 9 | 712 | 585 | 873 | 351 | 190 | 424 | 766 | 477 | 118 | 13515 |
| 876 | 102 | 79 | 588 | 804 | 660 | 493 | 295 | 367 | 241 | 875 | 101 | 80 | 587 | 803 | 659 | 494 | 296 | 368 | 242 | 874 | 100 | 81 | 586 | 802 | 658 | 495 | 297 | 369 | 243 | 13515 |
| 421 | 727 | 205 | 709 | 138 | 313 | 840 | 534 | 552 | 66 | 422 | 728 | 206 | 710 | 137 | 314 | 839 | 533 | 551 | 65 | 423 | 729 | 207 | 711 | 136 | 315 | 838 | 532 | 550 | 64 | 13515 |
| 624 | 511 | 786 | 300 | 12 | 817 | 259 | 133 | 678 | 385 | 623 | 512 | 785 | 299 | 11 | 818 | 260 | 134 | 677 | 386 | 622 | 513 | 784 | 298 | 10 | 819 | 261 | 135 | 676 | 387 | 13515 |
| 750 | 889 | 462 | 97 | 403 | 606 | 655 | 61 | 264 | 318 | 749 | 890 | 461 | 98 | 404 | 605 | 656 | 62 | 263 | 317 | 748 | 891 | 460 | 99 | 405 | 604 | 657 | 63 | 262 | 316 | 13515 |
| 115 | 336 | 390 | 84 | 691 | 498 | 732 | 187 | 853 | 619 | 116 | 335 | 389 | 83 | 692 | 497 | 731 | 188 | 854 | 620 | 117 | 334 | 388 | 82 | 693 | 496 | 730 | 189 | 855 | 621 | 13515 |
| 673 | 408 | 354 | 745 | 246 | 169 | 601 | 822 | 30 | 457 | 674 | 407 | 353 | 746 | 245 | 170 | 602 | 821 | 29 | 458 | 675 | 406 | 352 | 747 | 244 | 171 | 603 | 820 | 28 | 459 | 13515 |
| 529 | 210 | 858 | 372 | 547 | 25 | 174 | 696 | 331 | 763 | 530 | 209 | 857 | 371 | 548 | 26 | 173 | 695 | 332 | 764 | 531 | 208 | 856 | 370 | 549 | 27 | 172 | 694 | 333 | 765 | 13515 |
| 228 | 43 | 151 | 516 | 835 | 444 | 277 | 570 | 799 | 642 | 227 | 44 | 152 | 515 | 836 | 443 | 278 | 569 | 800 | 641 | 226 | 45 | 153 | 514 | 837 | 442 | 279 | 568 | 801 | 640 | 13515 |
| 282 | 565 | 637 | 223 | 480 | 781 | 48 | 439 | 156 | 894 | 281 | 566 | 638 | 224 | 479 | 782 | 47 | 440 | 155 | 893 | 280 | 567 | 639 | 225 | 478 | 783 | 46 | 441 | 154 | 892 | 13515 |
| 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 |

In this case, all the 9 blocks of order 10 are magic squares with equal magic sums, $S_{10\times 10} := 4505$.

#### 2.11.4 Forth Approach: 100 Blocks of Order 3

**Example 2.36.** *A magic square of order 30 formed by 100 blocks of magic squares of order 3 constructed according to Example 1.1 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 13515 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 31 | 63 | 2 | 865 | 837 | 896 | 408 | 436 | 377 | 576 | 604 | 545 | 144 | 112 | 173 | 672 | 640 | 701 | 493 | 465 | 524 | 769 | 801 | 740 | 240 | 268 | 209 | 307 | 279 | 338 | 13515 |
| 3 | 32 | 61 | 897 | 866 | 835 | 376 | 407 | 438 | 544 | 575 | 606 | 172 | 143 | 114 | 700 | 671 | 642 | 525 | 494 | 463 | 741 | 770 | 799 | 208 | 239 | 270 | 339 | 308 | 277 | 13515 |
| 62 | 1 | 33 | 836 | 895 | 867 | 437 | 378 | 406 | 605 | 546 | 574 | 113 | 174 | 142 | 641 | 702 | 670 | 464 | 523 | 495 | 800 | 739 | 771 | 269 | 210 | 238 | 278 | 337 | 309 | 13515 |
| 678 | 646 | 707 | 126 | 94 | 155 | 864 | 832 | 893 | 397 | 429 | 368 | 510 | 478 | 539 | 325 | 297 | 356 | 751 | 783 | 722 | 42 | 70 | 11 | 583 | 615 | 554 | 229 | 261 | 200 | 13515 |
| 706 | 677 | 648 | 154 | 125 | 96 | 892 | 863 | 834 | 369 | 398 | 427 | 538 | 509 | 480 | 357 | 326 | 295 | 723 | 752 | 781 | 10 | 41 | 72 | 555 | 584 | 613 | 201 | 230 | 259 | 13515 |
| 647 | 708 | 676 | 95 | 156 | 124 | 833 | 894 | 862 | 428 | 367 | 399 | 479 | 540 | 508 | 296 | 355 | 327 | 782 | 721 | 753 | 71 | 12 | 40 | 614 | 553 | 585 | 260 | 199 | 231 | 13515 |
| 396 | 424 | 365 | 594 | 622 | 563 | 217 | 249 | 188 | 780 | 808 | 749 | 318 | 286 | 347 | 139 | 111 | 170 | 685 | 657 | 716 | 481 | 453 | 512 | 852 | 820 | 881 | 43 | 75 | 14 | 13515 |
| 364 | 395 | 426 | 562 | 593 | 624 | 189 | 218 | 247 | 748 | 779 | 810 | 346 | 317 | 288 | 171 | 140 | 109 | 717 | 686 | 655 | 513 | 482 | 451 | 880 | 851 | 822 | 15 | 44 | 73 | 13515 |
| 425 | 366 | 394 | 623 | 564 | 592 | 248 | 187 | 219 | 809 | 750 | 778 | 287 | 348 | 316 | 110 | 169 | 141 | 656 | 715 | 687 | 452 | 511 | 483 | 821 | 882 | 850 | 74 | 13 | 45 | 13515 |
| 600 | 628 | 569 | 49 | 81 | 20 | 673 | 645 | 704 | 312 | 280 | 341 | 211 | 243 | 182 | 757 | 789 | 728 | 414 | 442 | 383 | 846 | 814 | 875 | 138 | 106 | 167 | 505 | 477 | 536 | 13515 |
| 568 | 599 | 630 | 21 | 50 | 79 | 705 | 674 | 643 | 340 | 311 | 282 | 183 | 212 | 241 | 729 | 758 | 787 | 382 | 413 | 444 | 874 | 845 | 816 | 166 | 137 | 108 | 537 | 506 | 475 | 13515 |
| 629 | 570 | 598 | 80 | 19 | 51 | 644 | 703 | 675 | 281 | 342 | 310 | 242 | 181 | 213 | 788 | 727 | 759 | 443 | 384 | 412 | 815 | 876 | 844 | 107 | 168 | 136 | 476 | 535 | 507 | 13515 |
| 492 | 460 | 521 | 301 | 273 | 332 | 595 | 627 | 566 | 679 | 651 | 710 | 403 | 435 | 374 | 216 | 244 | 185 | 858 | 826 | 887 | 150 | 118 | 179 | 37 | 69 | 8 | 774 | 802 | 743 | 13515 |
| 520 | 491 | 462 | 333 | 302 | 271 | 567 | 596 | 625 | 711 | 680 | 649 | 375 | 404 | 433 | 184 | 215 | 246 | 886 | 857 | 828 | 178 | 149 | 120 | 9 | 38 | 67 | 742 | 773 | 804 | 13515 |
| 461 | 522 | 490 | 272 | 331 | 303 | 626 | 565 | 597 | 650 | 709 | 681 | 434 | 373 | 405 | 245 | 186 | 214 | 827 | 888 | 856 | 119 | 180 | 148 | 68 | 7 | 39 | 803 | 744 | 772 | 13515 |
| 763 | 795 | 734 | 402 | 430 | 371 | 121 | 93 | 152 | 235 | 267 | 206 | 859 | 831 | 890 | 498 | 466 | 527 | 60 | 88 | 29 | 577 | 609 | 548 | 324 | 292 | 353 | 666 | 634 | 695 | 13515 |
| 735 | 764 | 793 | 370 | 401 | 432 | 153 | 122 | 91 | 207 | 236 | 265 | 891 | 860 | 829 | 526 | 497 | 468 | 28 | 59 | 90 | 549 | 578 | 607 | 352 | 323 | 294 | 694 | 665 | 636 | 13515 |
| 794 | 733 | 765 | 431 | 372 | 400 | 92 | 151 | 123 | 266 | 205 | 237 | 830 | 889 | 861 | 467 | 528 | 496 | 89 | 30 | 58 | 608 | 547 | 579 | 293 | 354 | 322 | 635 | 696 | 664 | 13515 |
| 234 | 262 | 203 | 487 | 459 | 518 | 330 | 298 | 359 | 48 | 76 | 17 | 756 | 784 | 725 | 853 | 825 | 884 | 589 | 621 | 560 | 415 | 447 | 386 | 661 | 633 | 692 | 132 | 100 | 161 | 13515 |
| 202 | 233 | 264 | 519 | 488 | 457 | 358 | 329 | 300 | 16 | 47 | 78 | 724 | 755 | 786 | 885 | 854 | 823 | 561 | 590 | 619 | 387 | 416 | 445 | 693 | 662 | 631 | 160 | 131 | 102 | 13515 |
| 263 | 204 | 232 | 458 | 517 | 489 | 299 | 360 | 328 | 77 | 18 | 46 | 785 | 726 | 754 | 824 | 883 | 855 | 620 | 559 | 591 | 446 | 385 | 417 | 632 | 691 | 663 | 101 | 162 | 130 | 13515 |
| 319 | 291 | 350 | 223 | 255 | 194 | 762 | 790 | 731 | 841 | 813 | 872 | 55 | 87 | 26 | 420 | 448 | 389 | 127 | 99 | 158 | 684 | 652 | 713 | 486 | 454 | 515 | 588 | 616 | 557 | 13515 |
| 351 | 320 | 289 | 195 | 224 | 253 | 730 | 761 | 792 | 873 | 842 | 811 | 27 | 56 | 85 | 388 | 419 | 450 | 159 | 128 | 97 | 712 | 683 | 654 | 514 | 485 | 456 | 556 | 587 | 618 | 13515 |
| 290 | 349 | 321 | 254 | 193 | 225 | 791 | 732 | 760 | 812 | 871 | 843 | 86 | 25 | 57 | 449 | 390 | 418 | 98 | 157 | 129 | 653 | 714 | 682 | 455 | 516 | 484 | 617 | 558 | 586 | 13515 |
| 847 | 819 | 878 | 690 | 658 | 719 | 499 | 471 | 530 | 133 | 105 | 164 | 582 | 610 | 551 | 54 | 82 | 23 | 306 | 274 | 335 | 228 | 256 | 197 | 775 | 807 | 746 | 391 | 423 | 362 | 13515 |
| 879 | 848 | 817 | 718 | 689 | 660 | 531 | 500 | 469 | 165 | 134 | 103 | 550 | 581 | 612 | 22 | 53 | 84 | 334 | 305 | 276 | 196 | 227 | 258 | 747 | 776 | 805 | 363 | 392 | 421 | 13515 |
| 818 | 877 | 849 | 659 | 720 | 688 | 470 | 529 | 501 | 104 | 163 | 135 | 611 | 552 | 580 | 83 | 24 | 52 | 275 | 336 | 304 | 257 | 198 | 226 | 806 | 745 | 777 | 422 | 361 | 393 | 13515 |
| 145 | 117 | 176 | 768 | 796 | 737 | 36 | 64 | 5 | 504 | 472 | 533 | 667 | 639 | 698 | 571 | 603 | 542 | 222 | 250 | 191 | 313 | 285 | 344 | 409 | 441 | 380 | 870 | 838 | 899 | 13515 |
| 177 | 146 | 115 | 736 | 767 | 798 | 4 | 35 | 66 | 532 | 503 | 474 | 699 | 668 | 637 | 543 | 572 | 601 | 190 | 221 | 252 | 345 | 314 | 283 | 381 | 410 | 439 | 898 | 869 | 840 | 13515 |
| 116 | 175 | 147 | 797 | 738 | 766 | 65 | 6 | 34 | 473 | 534 | 502 | 638 | 697 | 669 | 602 | 541 | 573 | 251 | 192 | 220 | 284 | 343 | 315 | 440 | 379 | 411 | 839 | 900 | 868 | 13515 |
| 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 |

We observe that it is not possible to have equal magic sums of order 3 as it is impossible to divide magic square sum of order 30 by 10, i.e., 13515/10 := 1351.5. In this case, all the 100 blocks are magic squares of order 3 with different magic sums resulting in a magic square of order 10:

**Example 2.37.** *Magic sums of 100 blocks of magic squares of order 3 lead us to a magic square of order 10 is given by*

| | | | | | | | | | | 13515 |
|---|---|---|---|---|---|---|---|---|---|---|
| 96 | 2598 | 1221 | 1725 | 429 | 2013 | 1482 | 2310 | 717 | 924 | 13515 |
| 2031 | 375 | 2589 | 1194 | 1527 | 978 | 2256 | 123 | 1752 | 690 | 13515 |
| 1185 | 1779 | 654 | 2337 | 951 | 420 | 2058 | 1446 | 2553 | 132 | 13515 |
| 1797 | 150 | 2022 | 933 | 636 | 2274 | 1239 | 2535 | 411 | 1518 | 13515 |
| 1473 | 906 | 1788 | 2040 | 1212 | 645 | 2571 | 447 | 114 | 2319 | 13515 |
| 2292 | 1203 | 366 | 708 | 2580 | 1491 | 177 | 1734 | 969 | 1995 | 13515 |
| 699 | 1464 | 987 | 141 | 2265 | 2562 | 1770 | 1248 | 1986 | 393 | 13515 |
| 960 | 672 | 2283 | 2526 | 168 | 1257 | 384 | 2049 | 1455 | 1761 | 13515 |
| 2544 | 2067 | 1500 | 402 | 1743 | 159 | 915 | 681 | 2328 | 1176 | 13515 |
| 438 | 2301 | 105 | 1509 | 2004 | 1716 | 663 | 942 | 1230 | 2607 | 13515 |
| 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 | 13515 |

### 2.12 Magic and Bimagic Squares of Order 32

In this subsection, we shall present two ways of writing block-wise magic square of order 32. One as $4 \times 8$, i.e., , magic square formed by 64 blocks of **pan diagonal** magic squares of order 4 with equal magic sums. Second as $8 \times 4$, i.e., 16 blocks of **bimagic** squares of order 8 with equal magic sums resulting in **bimagic** square of order 32.

### 2.12.1 First Approach: 64 Blocks of Order 4

**Example 2.38.** *A **pan diagonal** magic square of order 32 formed by 64 blocks of **pan diagonal** magic squares of order 4 constructed according to Example 1.2 is given by*

| | | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 385 | 768 | 1 | 896 | 386 | 767 | 2 | 895 | 387 | 766 | 3 | 894 | 388 | 765 | 4 | 893 | 389 | 764 | 5 | 892 | 390 | 763 | 6 | 891 | 391 | 762 | 7 | 890 | 392 | 761 | 8 | 889 | 16400 |
| 16400 | 128 | 769 | 512 | 641 | 127 | 770 | 511 | 642 | 126 | 771 | 510 | 643 | 125 | 772 | 509 | 644 | 124 | 773 | 508 | 645 | 123 | 774 | 507 | 646 | 122 | 775 | 506 | 647 | 121 | 776 | 505 | 648 | 16400 |
| 16400 | 1024 | 129 | 640 | 257 | 1023 | 130 | 639 | 258 | 1022 | 131 | 638 | 259 | 1021 | 132 | 637 | 260 | 1020 | 133 | 636 | 261 | 1019 | 134 | 635 | 262 | 1018 | 135 | 634 | 263 | 1017 | 136 | 633 | 264 | 16400 |
| 16400 | 513 | 384 | 897 | 256 | 514 | 383 | 898 | 255 | 515 | 382 | 899 | 254 | 516 | 381 | 900 | 253 | 517 | 380 | 901 | 252 | 518 | 379 | 902 | 251 | 519 | 378 | 903 | 250 | 520 | 377 | 904 | 249 | 16400 |
| 16400 | 393 | 760 | 9 | 888 | 394 | 759 | 10 | 887 | 395 | 758 | 11 | 886 | 396 | 757 | 12 | 885 | 397 | 756 | 13 | 884 | 398 | 755 | 14 | 883 | 399 | 754 | 15 | 882 | 400 | 753 | 16 | 881 | 16400 |
| 16400 | 120 | 777 | 504 | 649 | 119 | 778 | 503 | 650 | 118 | 779 | 502 | 651 | 117 | 780 | 501 | 652 | 116 | 781 | 500 | 653 | 115 | 782 | 499 | 654 | 114 | 783 | 498 | 655 | 113 | 784 | 497 | 656 | 16400 |
| 16400 | 1016 | 137 | 632 | 265 | 1015 | 138 | 631 | 266 | 1014 | 139 | 630 | 267 | 1013 | 140 | 629 | 268 | 1012 | 141 | 628 | 269 | 1011 | 142 | 627 | 270 | 1010 | 143 | 626 | 271 | 1009 | 144 | 625 | 272 | 16400 |
| 16400 | 521 | 376 | 905 | 248 | 522 | 375 | 906 | 247 | 523 | 374 | 907 | 246 | 524 | 373 | 908 | 245 | 525 | 372 | 909 | 244 | 526 | 371 | 910 | 243 | 527 | 370 | 911 | 242 | 528 | 369 | 912 | 241 | 16400 |
| 16400 | 401 | 752 | 17 | 880 | 402 | 751 | 18 | 879 | 403 | 750 | 19 | 878 | 404 | 749 | 20 | 877 | 405 | 748 | 21 | 876 | 406 | 747 | 22 | 875 | 407 | 746 | 23 | 874 | 408 | 745 | 24 | 873 | 16400 |
| 16400 | 112 | 785 | 496 | 657 | 111 | 786 | 495 | 658 | 110 | 787 | 494 | 659 | 109 | 788 | 493 | 660 | 108 | 789 | 492 | 661 | 107 | 790 | 491 | 662 | 106 | 791 | 490 | 663 | 105 | 792 | 489 | 664 | 16400 |
| 16400 | 1008 | 145 | 624 | 273 | 1007 | 146 | 623 | 274 | 1006 | 147 | 622 | 275 | 1005 | 148 | 621 | 276 | 1004 | 149 | 620 | 277 | 1003 | 150 | 619 | 278 | 1002 | 151 | 618 | 279 | 1001 | 152 | 617 | 280 | 16400 |
| 16400 | 529 | 368 | 913 | 240 | 530 | 367 | 914 | 239 | 531 | 366 | 915 | 238 | 532 | 365 | 916 | 237 | 533 | 364 | 917 | 236 | 534 | 363 | 918 | 235 | 535 | 362 | 919 | 234 | 536 | 361 | 920 | 233 | 16400 |
| 16400 | 409 | 744 | 25 | 872 | 410 | 743 | 26 | 871 | 411 | 742 | 27 | 870 | 412 | 741 | 28 | 869 | 413 | 740 | 29 | 868 | 414 | 739 | 30 | 867 | 415 | 738 | 31 | 866 | 416 | 737 | 32 | 865 | 16400 |
| 16400 | 104 | 793 | 488 | 665 | 103 | 794 | 487 | 666 | 102 | 795 | 486 | 667 | 101 | 796 | 485 | 668 | 100 | 797 | 484 | 669 | 99 | 798 | 483 | 670 | 98 | 799 | 482 | 671 | 97 | 800 | 481 | 672 | 16400 |
| 16400 | 1000 | 153 | 616 | 281 | 999 | 154 | 615 | 282 | 998 | 155 | 614 | 283 | 997 | 156 | 613 | 284 | 996 | 157 | 612 | 285 | 995 | 158 | 611 | 286 | 994 | 159 | 610 | 287 | 993 | 160 | 609 | 288 | 16400 |
| 16400 | 537 | 360 | 921 | 232 | 538 | 359 | 922 | 231 | 539 | 358 | 923 | 230 | 540 | 357 | 924 | 229 | 541 | 356 | 925 | 228 | 542 | 355 | 926 | 227 | 543 | 354 | 927 | 226 | 544 | 353 | 928 | 225 | 16400 |
| 16400 | 417 | 736 | 33 | 864 | 418 | 735 | 34 | 863 | 419 | 734 | 35 | 862 | 420 | 733 | 36 | 861 | 421 | 732 | 37 | 860 | 422 | 731 | 38 | 859 | 423 | 730 | 39 | 858 | 424 | 729 | 40 | 857 | 16400 |
| 16400 | 96 | 801 | 480 | 673 | 95 | 802 | 479 | 674 | 94 | 803 | 478 | 675 | 93 | 804 | 477 | 676 | 92 | 805 | 476 | 677 | 91 | 806 | 475 | 678 | 90 | 807 | 474 | 679 | 89 | 808 | 473 | 680 | 16400 |
| 16400 | 992 | 161 | 608 | 289 | 991 | 162 | 607 | 290 | 990 | 163 | 606 | 291 | 989 | 164 | 605 | 292 | 988 | 165 | 604 | 293 | 987 | 166 | 603 | 294 | 986 | 167 | 602 | 295 | 985 | 168 | 601 | 296 | 16400 |
| 16400 | 545 | 352 | 929 | 224 | 546 | 351 | 930 | 223 | 547 | 350 | 931 | 222 | 548 | 349 | 932 | 221 | 549 | 348 | 933 | 220 | 550 | 347 | 934 | 219 | 551 | 346 | 935 | 218 | 552 | 345 | 936 | 217 | 16400 |
| 16400 | 425 | 728 | 41 | 856 | 426 | 727 | 42 | 855 | 427 | 726 | 43 | 854 | 428 | 725 | 44 | 853 | 429 | 724 | 45 | 852 | 430 | 723 | 46 | 851 | 431 | 722 | 47 | 850 | 432 | 721 | 48 | 849 | 16400 |
| 16400 | 88 | 809 | 472 | 681 | 87 | 810 | 471 | 682 | 86 | 811 | 470 | 683 | 85 | 812 | 469 | 684 | 84 | 813 | 468 | 685 | 83 | 814 | 467 | 686 | 82 | 815 | 466 | 687 | 81 | 816 | 465 | 688 | 16400 |
| 16400 | 984 | 169 | 600 | 297 | 983 | 170 | 599 | 298 | 982 | 171 | 598 | 299 | 981 | 172 | 597 | 300 | 980 | 173 | 596 | 301 | 979 | 174 | 595 | 302 | 978 | 175 | 594 | 303 | 977 | 176 | 593 | 304 | 16400 |
| 16400 | 553 | 344 | 937 | 216 | 554 | 343 | 938 | 215 | 555 | 342 | 939 | 214 | 556 | 341 | 940 | 213 | 557 | 340 | 941 | 212 | 558 | 339 | 942 | 211 | 559 | 338 | 943 | 210 | 560 | 337 | 944 | 209 | 16400 |
| 16400 | 433 | 720 | 49 | 848 | 434 | 719 | 50 | 847 | 435 | 718 | 51 | 846 | 436 | 717 | 52 | 845 | 437 | 716 | 53 | 844 | 438 | 715 | 54 | 843 | 439 | 714 | 55 | 842 | 440 | 713 | 56 | 841 | 16400 |
| 16400 | 80 | 817 | 464 | 689 | 79 | 818 | 463 | 690 | 78 | 819 | 462 | 691 | 77 | 820 | 461 | 692 | 76 | 821 | 460 | 693 | 75 | 822 | 459 | 694 | 74 | 823 | 458 | 695 | 73 | 824 | 457 | 696 | 16400 |
| 16400 | 976 | 177 | 592 | 305 | 975 | 178 | 591 | 306 | 974 | 179 | 590 | 307 | 973 | 180 | 589 | 308 | 972 | 181 | 588 | 309 | 971 | 182 | 587 | 310 | 970 | 183 | 586 | 311 | 969 | 184 | 585 | 312 | 16400 |
| 16400 | 561 | 336 | 945 | 208 | 562 | 335 | 946 | 207 | 563 | 334 | 947 | 206 | 564 | 333 | 948 | 205 | 565 | 332 | 949 | 204 | 566 | 331 | 950 | 203 | 567 | 330 | 951 | 202 | 568 | 329 | 952 | 201 | 16400 |
| 16400 | 441 | 712 | 57 | 840 | 442 | 711 | 58 | 839 | 443 | 710 | 59 | 838 | 444 | 709 | 60 | 837 | 445 | 708 | 61 | 836 | 446 | 707 | 62 | 835 | 447 | 706 | 63 | 834 | 448 | 705 | 64 | 833 | 16400 |
| 16400 | 72 | 825 | 456 | 697 | 71 | 826 | 455 | 698 | 70 | 827 | 454 | 699 | 69 | 828 | 453 | 700 | 68 | 829 | 452 | 701 | 67 | 830 | 451 | 702 | 66 | 831 | 450 | 703 | 65 | 832 | 449 | 704 | 16400 |
| 16400 | 968 | 185 | 584 | 313 | 967 | 186 | 583 | 314 | 966 | 187 | 582 | 315 | 965 | 188 | 581 | 316 | 964 | 189 | 580 | 317 | 963 | 190 | 579 | 318 | 962 | 191 | 578 | 319 | 961 | 192 | 577 | 320 | 16400 |
| 16400 | 569 | 328 | 953 | 200 | 570 | 327 | 954 | 199 | 571 | 326 | 955 | 198 | 572 | 325 | 956 | 197 | 573 | 324 | 957 | 196 | 574 | 323 | 958 | 195 | 575 | 322 | 959 | 194 | 576 | 321 | 960 | 193 | 16400 |
| | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 |

In this case, all the magic squares of order 4 are **pan diagonal** and with equal magic sums, $S_{4\times4} := 2050$

### 2.12.2 Second Approach: 16 Blocks of Order 8

**Example 2.39.** *A **bimagic** square of order 32 formed by 16 blocks of magic squares of order 8 constructed according to Example 2.20 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 |
| | 222 | 707 | 558 | 51 | 459 | 982 | 827 | 294 | 189 | 676 | 589 | 84 | 428 | 949 | 860 | 325 | 256 | 737 | 528 | 17 | 489 | 1016 | 793 | 264 | 159 | 642 | 623 | 114 | 394 | 919 | 890 | 359 | 16400 |
| 16400 | 454 | 987 | 822 | 299 | 211 | 718 | 547 | 62 | 421 | 956 | 853 | 332 | 180 | 685 | 580 | 93 | 488 | 1017 | 792 | 265 | 241 | 752 | 513 | 32 | 391 | 922 | 887 | 362 | 146 | 655 | 610 | 127 | 16400 |
| 16400 | 35 | 574 | 723 | 206 | 310 | 811 | 966 | 475 | 68 | 605 | 692 | 173 | 341 | 844 | 933 | 444 | 1 | 544 | 753 | 240 | 280 | 777 | 1000 | 505 | 98 | 639 | 658 | 143 | 375 | 874 | 903 | 410 | 16400 |
| 16400 | 315 | 806 | 971 | 470 | 46 | 563 | 734 | 195 | 348 | 837 | 940 | 437 | 77 | 596 | 701 | 164 | 281 | 776 | 1001 | 504 | 16 | 529 | 768 | 225 | 378 | 871 | 906 | 407 | 111 | 626 | 671 | 130 | 16400 |
| 16400 | 566 | 43 | 198 | 731 | 803 | 318 | 467 | 974 | 597 | 76 | 165 | 700 | 836 | 349 | 436 | 941 | 536 | 9 | 232 | 761 | 769 | 288 | 497 | 1008 | 631 | 106 | 135 | 666 | 866 | 383 | 402 | 911 | 16400 |
| 16400 | 814 | 307 | 478 | 963 | 571 | 38 | 203 | 726 | 845 | 340 | 445 | 932 | 604 | 69 | 172 | 693 | 784 | 273 | 512 | 993 | 537 | 8 | 233 | 760 | 879 | 370 | 415 | 898 | 634 | 103 | 138 | 663 | 16400 |
| 16400 | 715 | 214 | 59 | 550 | 990 | 451 | 302 | 819 | 684 | 181 | 92 | 581 | 957 | 420 | 333 | 852 | 745 | 248 | 25 | 520 | 1024 | 481 | 272 | 785 | 650 | 151 | 122 | 615 | 927 | 386 | 367 | 882 | 16400 |
| 16400 | 979 | 462 | 291 | 830 | 710 | 219 | 54 | 555 | 948 | 429 | 324 | 861 | 677 | 188 | 85 | 588 | 1009 | 496 | 257 | 800 | 744 | 249 | 24 | 521 | 914 | 399 | 354 | 895 | 647 | 154 | 119 | 618 | 16400 |
| 16400 | 255 | 738 | 527 | 18 | 490 | 1015 | 794 | 263 | 160 | 641 | 624 | 113 | 393 | 920 | 889 | 360 | 221 | 708 | 557 | 52 | 460 | 981 | 828 | 293 | 190 | 675 | 590 | 83 | 427 | 950 | 859 | 326 | 16400 |
| 16400 | 487 | 1018 | 791 | 266 | 242 | 751 | 514 | 31 | 392 | 921 | 888 | 361 | 145 | 656 | 609 | 128 | 453 | 988 | 821 | 300 | 212 | 717 | 548 | 61 | 422 | 955 | 854 | 331 | 179 | 686 | 579 | 94 | 16400 |
| 16400 | 2 | 543 | 754 | 239 | 279 | 778 | 999 | 506 | 97 | 640 | 657 | 144 | 376 | 873 | 904 | 409 | 36 | 573 | 724 | 205 | 309 | 812 | 965 | 476 | 67 | 606 | 691 | 174 | 342 | 843 | 934 | 443 | 16400 |
| 16400 | 282 | 775 | 1002 | 503 | 15 | 530 | 767 | 226 | 377 | 872 | 905 | 408 | 112 | 625 | 672 | 129 | 316 | 805 | 972 | 469 | 45 | 564 | 733 | 196 | 347 | 838 | 939 | 438 | 78 | 595 | 702 | 163 | 16400 |
| 16400 | 535 | 10 | 231 | 762 | 770 | 287 | 498 | 1007 | 632 | 105 | 136 | 665 | 865 | 384 | 401 | 912 | 565 | 44 | 197 | 732 | 804 | 317 | 468 | 973 | 598 | 75 | 166 | 699 | 835 | 350 | 435 | 942 | 16400 |
| 16400 | 783 | 274 | 511 | 994 | 538 | 7 | 234 | 759 | 880 | 369 | 416 | 897 | 633 | 104 | 137 | 664 | 813 | 308 | 477 | 964 | 572 | 37 | 204 | 725 | 846 | 339 | 446 | 931 | 603 | 70 | 171 | 694 | 16400 |
| 16400 | 746 | 247 | 26 | 519 | 1023 | 482 | 271 | 786 | 649 | 152 | 121 | 616 | 928 | 385 | 368 | 881 | 716 | 213 | 60 | 549 | 989 | 452 | 301 | 820 | 683 | 182 | 91 | 582 | 958 | 419 | 334 | 851 | 16400 |
| 16400 | 1010 | 495 | 258 | 799 | 743 | 250 | 23 | 522 | 913 | 400 | 353 | 896 | 648 | 153 | 120 | 617 | 980 | 461 | 292 | 829 | 709 | 220 | 53 | 556 | 947 | 430 | 323 | 862 | 678 | 187 | 86 | 587 | 16400 |
| 16400 | 157 | 644 | 621 | 116 | 396 | 917 | 892 | 357 | 254 | 739 | 526 | 19 | 491 | 1014 | 795 | 262 | 191 | 674 | 591 | 82 | 426 | 951 | 858 | 327 | 224 | 705 | 560 | 49 | 457 | 984 | 825 | 296 | 16400 |
| 16400 | 389 | 924 | 885 | 364 | 148 | 653 | 612 | 125 | 486 | 1019 | 790 | 267 | 243 | 750 | 515 | 30 | 423 | 954 | 855 | 330 | 178 | 687 | 578 | 95 | 456 | 985 | 824 | 297 | 209 | 720 | 545 | 64 | 16400 |
| 16400 | 100 | 637 | 660 | 141 | 373 | 876 | 901 | 412 | 3 | 542 | 755 | 238 | 278 | 779 | 998 | 507 | 66 | 607 | 690 | 175 | 343 | 842 | 935 | 442 | 33 | 576 | 721 | 208 | 312 | 809 | 968 | 473 | 16400 |
| 16400 | 380 | 869 | 908 | 405 | 109 | 628 | 669 | 132 | 283 | 774 | 1003 | 502 | 14 | 531 | 766 | 227 | 346 | 839 | 938 | 439 | 79 | 594 | 703 | 162 | 313 | 808 | 969 | 472 | 48 | 561 | 736 | 193 | 16400 |
| 16400 | 629 | 108 | 133 | 668 | 868 | 381 | 404 | 909 | 534 | 11 | 230 | 763 | 771 | 286 | 499 | 1006 | 599 | 74 | 167 | 698 | 834 | 351 | 434 | 943 | 568 | 41 | 200 | 729 | 801 | 320 | 465 | 976 | 16400 |
| 16400 | 877 | 372 | 413 | 900 | 636 | 101 | 140 | 661 | 782 | 275 | 510 | 995 | 539 | 6 | 235 | 758 | 847 | 338 | 447 | 930 | 602 | 71 | 170 | 695 | 816 | 305 | 480 | 961 | 569 | 40 | 201 | 728 | 16400 |
| 16400 | 652 | 149 | 124 | 613 | 925 | 388 | 365 | 884 | 747 | 246 | 27 | 518 | 1022 | 483 | 270 | 787 | 682 | 183 | 90 | 583 | 959 | 418 | 335 | 850 | 713 | 216 | 57 | 552 | 992 | 449 | 304 | 817 | 16400 |
| 16400 | 916 | 397 | 356 | 893 | 645 | 156 | 117 | 620 | 1011 | 494 | 259 | 798 | 742 | 251 | 22 | 523 | 946 | 431 | 322 | 863 | 679 | 186 | 87 | 586 | 977 | 464 | 289 | 832 | 712 | 217 | 56 | 553 | 16400 |
| 16400 | 192 | 673 | 592 | 81 | 425 | 952 | 857 | 328 | 223 | 706 | 559 | 50 | 458 | 983 | 826 | 295 | 158 | 643 | 622 | 115 | 395 | 918 | 891 | 358 | 253 | 740 | 525 | 20 | 492 | 1013 | 796 | 261 | 16400 |
| 16400 | 424 | 953 | 856 | 329 | 177 | 688 | 577 | 96 | 455 | 986 | 823 | 298 | 210 | 719 | 546 | 63 | 390 | 923 | 886 | 363 | 147 | 654 | 611 | 126 | 485 | 1020 | 789 | 268 | 244 | 749 | 516 | 29 | 16400 |
| 16400 | 65 | 608 | 689 | 176 | 344 | 841 | 936 | 441 | 34 | 575 | 722 | 207 | 311 | 810 | 967 | 474 | 99 | 638 | 659 | 142 | 374 | 875 | 902 | 411 | 4 | 541 | 756 | 237 | 277 | 780 | 997 | 508 | 16400 |
| 16400 | 345 | 840 | 937 | 440 | 80 | 593 | 704 | 161 | 314 | 807 | 970 | 471 | 47 | 562 | 735 | 194 | 379 | 870 | 907 | 406 | 110 | 627 | 670 | 131 | 284 | 773 | 1004 | 501 | 13 | 532 | 765 | 228 | 16400 |
| 16400 | 600 | 73 | 168 | 697 | 833 | 352 | 433 | 944 | 567 | 42 | 199 | 730 | 802 | 319 | 466 | 975 | 630 | 107 | 134 | 667 | 867 | 382 | 403 | 910 | 533 | 12 | 229 | 764 | 772 | 285 | 500 | 1005 | 16400 |
| 16400 | 848 | 337 | 448 | 929 | 601 | 72 | 169 | 696 | 815 | 306 | 479 | 962 | 570 | 39 | 202 | 727 | 878 | 371 | 414 | 899 | 635 | 102 | 139 | 662 | 781 | 276 | 509 | 996 | 540 | 5 | 236 | 757 | 16400 |
| 16400 | 681 | 184 | 89 | 584 | 960 | 417 | 336 | 849 | 714 | 215 | 58 | 551 | 991 | 450 | 303 | 818 | 651 | 150 | 123 | 614 | 926 | 387 | 366 | 883 | 748 | 245 | 28 | 517 | 1021 | 484 | 269 | 788 | 16400 |
| 16400 | 945 | 432 | 321 | 864 | 680 | 185 | 88 | 585 | 978 | 463 | 290 | 831 | 711 | 218 | 55 | 554 | 915 | 398 | 355 | 894 | 646 | 155 | 118 | 619 | 1012 | 493 | 260 | 797 | 741 | 252 | 21 | 524 | 16400 |
| | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 | 16400 |

In this case, the magic square is **pan diagonal bimagic** with magic and **bimagic** sums, $S_{32\times 32} := 16400$ and $Sb_{32\times 32} := 11201200$ respectively. All the block of order 8 are magic squares with equal magic sums, $S_{8\times 8} := 4100$. Moreover each block of order 8 is **semi-bimagic** resulting in a **pan diagonal** magic square of order 4:

**Example 2.40.** *A **pan diagonal** magic square of order 4 formed by 16 **semi-bimagic** sums of order 8 is given by*

| | | | | | |
|---|---|---|---|---|---|
| | | *11201200* | *11201200* | *11201200* | *11201200* |
| | *2808468* | *2775684* | *2857700* | *2759348* | *11201200* |
| *11201200* | *2857652* | *2759396* | *2808452* | *2775700* | *11201200* |
| *11201200* | *2759300* | *2857620* | *2775732* | *2808548* | *11201200* |
| *11201200* | *2775780* | *2808500* | *2759316* | *2857604* | *11201200* |
| | *11201200* | *11201200* | *11201200* | *11201200* | *11201200* |

Sum of semi-diagonals also forms a **pan diagonal** magic square of order 4.

## 2.13 Magic Squares of Order 33

In this subsection, we shall present two different ways of writing block-wise magic square of order 33. One as 11x3, i.e., magic square formed by 9 blocks of **pan diagonal** magic squares of order 11 with equal magic sums. Second as 3 × 11, i.e., 121 blocks of magic squares of order 3 with equal magic sums.

### 2.13.1 First Approach: 9 Blocks of Order 11

**Example 2.41.** *A **pan diagonal**magic square of order 33 formed by 9 blocks of **pan diagonal** magic squares of order 11 constructed according to Example 1.11 is given by*

| | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 171 | 239 | 375 | 409 | 579 | 613 | 783 | 817 | 987 | 1021 | 3 | 170 | 238 | 374 | 410 | 578 | 614 | 782 | 818 | 986 | 1022 | 2 | 169 | 240 | 373 | 411 | 577 | 615 | 781 | 819 | 985 | 1023 | 17985 |
| 17985 | 982 | 1020 | 31 | 166 | 237 | 371 | 408 | 574 | 612 | 778 | 816 | 983 | 1019 | 32 | 168 | 236 | 370 | 407 | 575 | 611 | 779 | 815 | 984 | 1018 | 33 | 167 | 235 | 372 | 406 | 576 | 610 | 780 | 814 | 17985 |
| 17985 | 777 | 811 | 981 | 1015 | 30 | 196 | 232 | 369 | 404 | 573 | 607 | 776 | 812 | 980 | 1016 | 29 | 197 | 234 | 368 | 403 | 572 | 608 | 775 | 813 | 979 | 1017 | 28 | 198 | 233 | 367 | 405 | 571 | 609 | 17985 |
| 17985 | 569 | 606 | 772 | 810 | 976 | 1014 | 25 | 195 | 262 | 364 | 402 | 568 | 605 | 773 | 809 | 977 | 1013 | 26 | 194 | 263 | 366 | 401 | 570 | 604 | 774 | 808 | 978 | 1012 | 27 | 193 | 264 | 365 | 400 | 17985 |
| 17985 | 394 | 397 | 567 | 602 | 771 | 805 | 975 | 1009 | 24 | 190 | 261 | 395 | 399 | 566 | 601 | 770 | 806 | 974 | 1010 | 23 | 191 | 260 | 396 | 398 | 565 | 603 | 769 | 807 | 973 | 1011 | 22 | 192 | 259 | 17985 |
| 17985 | 189 | 256 | 393 | 427 | 562 | 600 | 767 | 804 | 970 | 1008 | 19 | 188 | 257 | 392 | 428 | 564 | 599 | 766 | 803 | 971 | 1007 | 20 | 187 | 258 | 391 | 429 | 563 | 598 | 768 | 802 | 972 | 1006 | 21 | 17985 |
| 17985 | 1003 | 18 | 184 | 255 | 388 | 426 | 592 | 595 | 765 | 800 | 969 | 1004 | 17 | 185 | 254 | 389 | 425 | 593 | 597 | 764 | 799 | 968 | 1005 | 16 | 186 | 253 | 390 | 424 | 594 | 596 | 763 | 801 | 967 | 17985 |
| 17985 | 798 | 965 | 1002 | 13 | 183 | 250 | 387 | 421 | 591 | 625 | 760 | 797 | 964 | 1001 | 14 | 182 | 251 | 386 | 422 | 590 | 626 | 762 | 796 | 966 | 1000 | 15 | 181 | 252 | 385 | 423 | 589 | 627 | 761 | 17985 |
| 17985 | 624 | 790 | 793 | 963 | 998 | 12 | 178 | 249 | 382 | 420 | 586 | 623 | 791 | 795 | 962 | 997 | 11 | 179 | 248 | 383 | 419 | 587 | 622 | 792 | 794 | 961 | 999 | 10 | 180 | 247 | 384 | 418 | 588 | 17985 |
| 17985 | 415 | 585 | 619 | 789 | 823 | 958 | 996 | 8 | 177 | 244 | 381 | 416 | 584 | 620 | 788 | 824 | 960 | 995 | 7 | 176 | 245 | 380 | 417 | 583 | 621 | 787 | 825 | 959 | 994 | 9 | 175 | 246 | 379 | 17985 |
| 17985 | 243 | 376 | 414 | 580 | 618 | 784 | 822 | 988 | 991 | 6 | 173 | 242 | 377 | 413 | 581 | 617 | 785 | 821 | 989 | 993 | 5 | 172 | 241 | 378 | 412 | 582 | 616 | 786 | 820 | 990 | 992 | 4 | 174 | 17985 |
| 17985 | 67 | 138 | 206 | 342 | 442 | 546 | 646 | 750 | 850 | 954 | 1054 | 69 | 137 | 205 | 341 | 443 | 545 | 647 | 749 | 851 | 953 | 1055 | 68 | 136 | 207 | 340 | 444 | 544 | 648 | 748 | 852 | 952 | 1056 | 17985 |
| 17985 | 949 | 1053 | 97 | 133 | 204 | 338 | 441 | 541 | 645 | 745 | 849 | 950 | 1052 | 98 | 135 | 203 | 337 | 440 | 542 | 644 | 746 | 848 | 951 | 1051 | 99 | 134 | 202 | 339 | 439 | 543 | 643 | 747 | 847 | 17985 |
| 17985 | 744 | 844 | 948 | 1048 | 96 | 163 | 199 | 336 | 437 | 540 | 640 | 743 | 845 | 947 | 1049 | 95 | 164 | 201 | 335 | 436 | 539 | 641 | 742 | 846 | 946 | 1050 | 94 | 165 | 200 | 334 | 438 | 538 | 642 | 17985 |
| 17985 | 536 | 639 | 739 | 843 | 943 | 1047 | 91 | 162 | 229 | 331 | 435 | 535 | 638 | 740 | 842 | 944 | 1046 | 92 | 161 | 230 | 333 | 434 | 537 | 637 | 741 | 841 | 945 | 1045 | 93 | 160 | 231 | 332 | 433 | 17985 |
| 17985 | 361 | 430 | 534 | 635 | 738 | 838 | 942 | 1042 | 90 | 157 | 228 | 362 | 432 | 533 | 634 | 737 | 839 | 941 | 1043 | 89 | 158 | 227 | 363 | 431 | 532 | 636 | 736 | 840 | 940 | 1044 | 88 | 159 | 226 | 17985 |
| 17985 | 156 | 223 | 360 | 460 | 529 | 633 | 734 | 837 | 937 | 1041 | 85 | 155 | 224 | 359 | 461 | 531 | 632 | 733 | 836 | 938 | 1040 | 86 | 154 | 225 | 358 | 462 | 530 | 631 | 735 | 835 | 939 | 1039 | 87 | 17985 |
| 17985 | 1036 | 84 | 151 | 222 | 355 | 459 | 559 | 628 | 732 | 833 | 936 | 1037 | 83 | 152 | 221 | 356 | 458 | 560 | 630 | 731 | 832 | 935 | 1038 | 82 | 153 | 220 | 357 | 457 | 561 | 629 | 730 | 834 | 934 | 17985 |
| 17985 | 831 | 932 | 1035 | 79 | 150 | 217 | 354 | 454 | 558 | 658 | 727 | 830 | 931 | 1034 | 80 | 149 | 218 | 353 | 455 | 557 | 659 | 729 | 829 | 933 | 1033 | 81 | 148 | 219 | 352 | 456 | 556 | 660 | 728 | 17985 |
| 17985 | 657 | 757 | 826 | 930 | 1031 | 78 | 145 | 216 | 349 | 453 | 553 | 656 | 758 | 828 | 929 | 1030 | 77 | 146 | 215 | 350 | 452 | 554 | 655 | 759 | 827 | 928 | 1032 | 76 | 147 | 214 | 351 | 451 | 555 | 17985 |
| 17985 | 448 | 552 | 652 | 756 | 856 | 925 | 1029 | 74 | 144 | 211 | 348 | 449 | 551 | 653 | 755 | 857 | 927 | 1028 | 73 | 143 | 212 | 347 | 450 | 550 | 654 | 754 | 858 | 926 | 1027 | 75 | 142 | 213 | 346 | 17985 |
| 17985 | 210 | 343 | 447 | 547 | 651 | 751 | 855 | 955 | 1024 | 72 | 140 | 209 | 344 | 446 | 548 | 650 | 752 | 854 | 956 | 1026 | 71 | 139 | 208 | 345 | 445 | 549 | 649 | 753 | 853 | 957 | 1025 | 70 | 141 | 17985 |
| 17985 | 34 | 105 | 272 | 309 | 475 | 513 | 679 | 717 | 883 | 921 | 1087 | 36 | 104 | 271 | 308 | 476 | 512 | 680 | 716 | 884 | 920 | 1088 | 35 | 103 | 273 | 307 | 477 | 511 | 681 | 715 | 885 | 919 | 1089 | 17985 |
| 17985 | 916 | 1086 | 64 | 100 | 270 | 305 | 474 | 508 | 678 | 712 | 882 | 917 | 1085 | 65 | 102 | 269 | 304 | 473 | 509 | 677 | 713 | 881 | 918 | 1084 | 66 | 101 | 268 | 306 | 472 | 510 | 676 | 714 | 880 | 17985 |
| 17985 | 711 | 877 | 915 | 1081 | 63 | 130 | 265 | 303 | 470 | 507 | 673 | 710 | 878 | 914 | 1082 | 62 | 131 | 267 | 302 | 469 | 506 | 674 | 709 | 879 | 913 | 1083 | 61 | 132 | 266 | 301 | 471 | 505 | 675 | 17985 |
| 17985 | 503 | 672 | 706 | 876 | 910 | 1080 | 58 | 129 | 295 | 298 | 468 | 502 | 671 | 707 | 875 | 911 | 1079 | 59 | 128 | 296 | 300 | 467 | 504 | 670 | 708 | 874 | 912 | 1078 | 60 | 127 | 297 | 299 | 466 | 17985 |
| 17985 | 328 | 463 | 501 | 668 | 705 | 871 | 909 | 1075 | 57 | 124 | 294 | 329 | 465 | 500 | 667 | 704 | 872 | 908 | 1076 | 56 | 125 | 293 | 330 | 464 | 499 | 669 | 703 | 873 | 907 | 1077 | 55 | 126 | 292 | 17985 |
| 17985 | 123 | 289 | 327 | 493 | 496 | 666 | 701 | 870 | 904 | 1074 | 52 | 122 | 290 | 326 | 494 | 498 | 665 | 700 | 869 | 905 | 1073 | 53 | 121 | 291 | 325 | 495 | 497 | 664 | 702 | 868 | 906 | 1072 | 54 | 17985 |
| 17985 | 1069 | 51 | 118 | 288 | 322 | 492 | 526 | 661 | 699 | 866 | 903 | 1070 | 50 | 119 | 287 | 323 | 491 | 527 | 663 | 698 | 865 | 902 | 1071 | 49 | 120 | 286 | 324 | 490 | 528 | 662 | 697 | 867 | 901 | 17985 |
| 17985 | 864 | 899 | 1068 | 46 | 117 | 283 | 321 | 487 | 525 | 691 | 694 | 863 | 898 | 1067 | 47 | 116 | 284 | 320 | 488 | 524 | 692 | 696 | 862 | 900 | 1066 | 48 | 115 | 285 | 319 | 489 | 523 | 693 | 695 | 17985 |
| 17985 | 690 | 724 | 859 | 897 | 1064 | 45 | 112 | 282 | 316 | 486 | 520 | 689 | 725 | 861 | 896 | 1063 | 44 | 113 | 281 | 317 | 485 | 521 | 688 | 726 | 860 | 895 | 1065 | 43 | 114 | 280 | 318 | 484 | 522 | 17985 |
| 17985 | 481 | 519 | 685 | 723 | 889 | 892 | 1062 | 41 | 111 | 277 | 315 | 482 | 518 | 686 | 722 | 890 | 894 | 1061 | 40 | 110 | 278 | 314 | 483 | 517 | 687 | 721 | 891 | 893 | 1060 | 42 | 109 | 279 | 313 | 17985 |
| 17985 | 276 | 310 | 480 | 514 | 684 | 718 | 888 | 922 | 1057 | 39 | 107 | 275 | 311 | 479 | 515 | 683 | 719 | 887 | 923 | 1059 | 38 | 106 | 274 | 312 | 478 | 516 | 682 | 720 | 886 | 924 | 1058 | 37 | 108 | 17985 |
| | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 |

In this case all the 9 blocks are of **pan diagonal** magic squares of order 11 with equal magic sums, $S_{11\times 11} := 5995$

### 2.13.2 Second Approach: 121 Blocks of Order 3

**Example 2.42.** *A **pan diagonal**magic square of order 33 formed by 121 blocks of **semi-magic** squares of order 3 constructed according to Example 1.1 is given by*

| | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 22 | 919 | 694 | 656 | 35 | 944 | 960 | 579 | 96 | 733 | 681 | 221 | 618 | 239 | 778 | 537 | 817 | 281 | 488 | 307 | 840 | 343 | 885 | 407 | 115 | 1021 | 499 | 461 | 137 | 1037 | 1062 | 375 | 198 | 17985 |
| 17985 | 721 | 1 | 913 | 926 | 647 | 62 | 84 | 987 | 564 | 219 | 749 | 667 | 767 | 613 | 255 | 289 | 545 | 801 | 835 | 477 | 323 | 423 | 341 | 871 | 526 | 103 | 1006 | 1028 | 443 | 164 | 177 | 1089 | 369 | 17985 |
| 17985 | 892 | 715 | 28 | 53 | 953 | 629 | 591 | 69 | 975 | 683 | 205 | 747 | 250 | 783 | 602 | 809 | 273 | 553 | 312 | 851 | 472 | 869 | 409 | 357 | 994 | 511 | 130 | 146 | 1055 | 434 | 396 | 171 | 1068 | 17985 |
| 17985 | 445 | 163 | 1027 | 1088 | 368 | 179 | 6 | 903 | 726 | 649 | 61 | 925 | 986 | 563 | 86 | 729 | 678 | 228 | 601 | 252 | 782 | 552 | 800 | 283 | 471 | 322 | 842 | 356 | 868 | 411 | 112 | 1017 | 506 | 17985 |
| 17985 | 1054 | 433 | 148 | 170 | 1070 | 395 | 705 | 33 | 897 | 952 | 628 | 55 | 68 | 977 | 590 | 216 | 756 | 663 | 780 | 617 | 238 | 272 | 547 | 816 | 850 | 479 | 306 | 406 | 345 | 884 | 522 | 110 | 1003 | 17985 |
| 17985 | 136 | 1039 | 460 | 377 | 197 | 1061 | 924 | 699 | 12 | 34 | 946 | 655 | 581 | 95 | 959 | 690 | 201 | 744 | 254 | 766 | 615 | 811 | 288 | 536 | 314 | 834 | 487 | 873 | 422 | 340 | 1001 | 508 | 126 | 17985 |
| 17985 | 339 | 883 | 413 | 125 | 1000 | 510 | 442 | 159 | 1034 | 1072 | 394 | 169 | 32 | 896 | 707 | 633 | 45 | 957 | 979 | 589 | 67 | 755 | 662 | 218 | 597 | 249 | 789 | 535 | 813 | 287 | 486 | 305 | 844 | 17985 |
| 17985 | 421 | 347 | 867 | 505 | 114 | 1016 | 1050 | 440 | 145 | 196 | 1060 | 379 | 698 | 14 | 923 | 936 | 660 | 39 | 94 | 958 | 583 | 200 | 746 | 689 | 777 | 624 | 234 | 285 | 551 | 799 | 833 | 481 | 321 | 17985 |
| 17985 | 875 | 405 | 355 | 1005 | 521 | 109 | 143 | 1036 | 456 | 367 | 181 | 1087 | 905 | 725 | 5 | 66 | 930 | 639 | 562 | 88 | 985 | 680 | 227 | 728 | 261 | 762 | 612 | 815 | 271 | 549 | 316 | 849 | 470 | 17985 |
| 17985 | 531 | 810 | 294 | 469 | 318 | 848 | 354 | 866 | 415 | 108 | 1015 | 512 | 455 | 142 | 1038 | 1069 | 390 | 176 | 16 | 922 | 697 | 659 | 38 | 938 | 963 | 573 | 99 | 748 | 688 | 199 | 623 | 233 | 779 | 17985 |
| 17985 | 282 | 558 | 795 | 846 | 485 | 304 | 404 | 349 | 882 | 520 | 116 | 999 | 1033 | 444 | 158 | 192 | 1067 | 376 | 724 | 4 | 907 | 929 | 641 | 65 | 78 | 990 | 567 | 226 | 727 | 682 | 761 | 614 | 260 | 17985 |
| 17985 | 822 | 267 | 546 | 320 | 832 | 483 | 877 | 420 | 338 | 1007 | 504 | 124 | 147 | 1049 | 439 | 374 | 178 | 1083 | 895 | 709 | 31 | 47 | 956 | 632 | 594 | 72 | 969 | 661 | 220 | 754 | 251 | 788 | 596 | 17985 |
| 17985 | 732 | 672 | 231 | 616 | 259 | 760 | 557 | 794 | 284 | 465 | 315 | 855 | 337 | 879 | 419 | 123 | 998 | 514 | 438 | 157 | 1040 | 1082 | 373 | 180 | 13 | 918 | 704 | 643 | 64 | 928 | 989 | 566 | 80 | 17985 |
| 17985 | 210 | 759 | 666 | 787 | 595 | 253 | 266 | 548 | 821 | 843 | 492 | 300 | 417 | 353 | 865 | 503 | 118 | 1014 | 1048 | 446 | 141 | 175 | 1071 | 389 | 720 | 11 | 904 | 955 | 631 | 49 | 71 | 971 | 593 | 17985 |
| 17985 | 693 | 204 | 738 | 232 | 781 | 622 | 812 | 293 | 530 | 327 | 828 | 480 | 881 | 403 | 351 | 1009 | 519 | 107 | 149 | 1032 | 454 | 378 | 191 | 1066 | 902 | 706 | 27 | 37 | 940 | 658 | 575 | 98 | 962 | 17985 |
| 17985 | 640 | 60 | 935 | 973 | 592 | 70 | 758 | 665 | 212 | 600 | 243 | 792 | 550 | 820 | 265 | 491 | 299 | 845 | 333 | 876 | 426 | 106 | 1011 | 518 | 453 | 140 | 1042 | 1065 | 388 | 182 | 26 | 901 | 708 | 17985 |
| 17985 | 951 | 638 | 46 | 97 | 961 | 577 | 203 | 740 | 692 | 771 | 627 | 237 | 292 | 529 | 814 | 827 | 482 | 326 | 414 | 360 | 861 | 516 | 122 | 997 | 1031 | 448 | 156 | 190 | 1073 | 372 | 703 | 15 | 917 | 17985 |
| 17985 | 44 | 937 | 654 | 565 | 82 | 988 | 674 | 230 | 731 | 264 | 765 | 606 | 793 | 286 | 556 | 317 | 854 | 464 | 888 | 399 | 348 | 1013 | 502 | 120 | 151 | 1047 | 437 | 380 | 174 | 1081 | 906 | 719 | 10 | 17985 |
| 17985 | 1080 | 371 | 184 | 9 | 916 | 710 | 653 | 43 | 939 | 970 | 588 | 77 | 742 | 691 | 202 | 626 | 236 | 773 | 534 | 804 | 297 | 484 | 325 | 826 | 359 | 860 | 416 | 102 | 1008 | 525 | 436 | 153 | 1046 | 17985 |
| 17985 | 173 | 1075 | 387 | 718 | 17 | 900 | 934 | 642 | 59 | 93 | 968 | 574 | 229 | 730 | 676 | 764 | 608 | 263 | 276 | 561 | 798 | 853 | 463 | 319 | 398 | 350 | 887 | 513 | 129 | 993 | 1044 | 452 | 139 | 17985 |
| 17985 | 382 | 189 | 1064 | 908 | 702 | 25 | 48 | 950 | 637 | 572 | 79 | 984 | 664 | 214 | 757 | 245 | 791 | 599 | 825 | 270 | 540 | 298 | 847 | 490 | 878 | 425 | 332 | 1020 | 498 | 117 | 155 | 1030 | 450 | 17985 |
| 17985 | 128 | 992 | 515 | 432 | 150 | 1053 | 1063 | 384 | 188 | 24 | 899 | 712 | 636 | 58 | 941 | 983 | 571 | 81 | 739 | 687 | 209 | 610 | 262 | 763 | 560 | 797 | 278 | 468 | 309 | 858 | 352 | 886 | 397 | 17985 |
| 17985 | 497 | 119 | 1019 | 1041 | 459 | 135 | 186 | 1079 | 370 | 701 | 19 | 915 | 949 | 644 | 42 | 76 | 972 | 587 | 225 | 737 | 673 | 790 | 598 | 247 | 269 | 542 | 824 | 837 | 495 | 303 | 424 | 331 | 880 | 17985 |
| 17985 | 1010 | 524 | 101 | 162 | 1026 | 447 | 386 | 172 | 1077 | 910 | 717 | 8 | 50 | 933 | 652 | 576 | 92 | 967 | 671 | 211 | 753 | 235 | 775 | 625 | 806 | 296 | 533 | 330 | 831 | 474 | 859 | 418 | 358 | 17985 |
| 17985 | 494 | 302 | 839 | 336 | 870 | 429 | 121 | 1018 | 496 | 458 | 134 | 1043 | 1059 | 381 | 195 | 7 | 912 | 716 | 651 | 41 | 943 | 966 | 586 | 83 | 752 | 670 | 213 | 607 | 258 | 770 | 544 | 823 | 268 | 17985 |
| 17985 | 830 | 476 | 329 | 408 | 363 | 864 | 523 | 100 | 1012 | 1025 | 449 | 161 | 183 | 1086 | 366 | 714 | 23 | 898 | 932 | 646 | 57 | 91 | 974 | 570 | 208 | 741 | 686 | 786 | 605 | 244 | 295 | 532 | 808 | 17985 |
| 17985 | 311 | 857 | 467 | 891 | 402 | 342 | 991 | 517 | 127 | 152 | 1052 | 431 | 393 | 168 | 1074 | 914 | 700 | 21 | 52 | 948 | 635 | 578 | 75 | 982 | 675 | 224 | 736 | 242 | 772 | 621 | 796 | 280 | 559 | 17985 |
| 17985 | 620 | 241 | 774 | 541 | 819 | 275 | 478 | 328 | 829 | 362 | 863 | 410 | 105 | 1002 | 528 | 451 | 160 | 1024 | 1085 | 365 | 185 | 3 | 909 | 723 | 634 | 54 | 947 | 981 | 569 | 85 | 735 | 685 | 215 | 17985 |
| 17985 | 769 | 609 | 257 | 291 | 539 | 805 | 856 | 466 | 313 | 401 | 344 | 890 | 507 | 132 | 996 | 1051 | 430 | 154 | 167 | 1076 | 392 | 711 | 30 | 894 | 945 | 650 | 40 | 74 | 976 | 585 | 223 | 743 | 669 | 17985 |
| 17985 | 246 | 785 | 604 | 803 | 277 | 555 | 301 | 841 | 493 | 872 | 428 | 335 | 1023 | 501 | 111 | 133 | 1045 | 457 | 383 | 194 | 1058 | 921 | 696 | 18 | 56 | 931 | 648 | 580 | 90 | 965 | 677 | 207 | 751 | 17985 |
| 17985 | 964 | 582 | 89 | 750 | 668 | 217 | 603 | 256 | 776 | 554 | 802 | 279 | 475 | 324 | 836 | 346 | 889 | 400 | 131 | 995 | 509 | 435 | 144 | 1056 | 1078 | 391 | 166 | 29 | 893 | 713 | 630 | 51 | 954 | 17985 |
| 17985 | 87 | 980 | 568 | 206 | 745 | 684 | 784 | 611 | 240 | 274 | 543 | 818 | 852 | 473 | 310 | 427 | 334 | 874 | 500 | 113 | 1022 | 1035 | 462 | 138 | 193 | 1057 | 385 | 695 | 20 | 920 | 942 | 657 | 36 | 17985 |
| 17985 | 584 | 73 | 978 | 679 | 222 | 734 | 248 | 768 | 619 | 807 | 290 | 538 | 308 | 838 | 489 | 862 | 412 | 361 | 1004 | 527 | 104 | 165 | 1029 | 441 | 364 | 187 | 1084 | 911 | 722 | 2 | 63 | 927 | 645 | 17985 |
| | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 | 17985 |

In this case all the 121 blocks are of **semi-magic** squares of order 3 with equal **semi-magic** sums, $S_{3\times3} := 1635$ (in rows and columns).

### 2.14 Magic Squares of Order 35

In this subsection, we shall present two different ways of writing block-wise magic square of order 35. One as 5 × 7, i.e., magic square formed by 49 blocks of **pan diagonal** magic squares of order 7 with equal magic sums. Second as 7 × 5, i.e., magic square formed by 25 blocks of **pan diagonal** magic squares of order 5 with equal magic sums. In both the cases, the magic square of order 35 is **pan diagonal**.

#### 2.14.1 First Approach: 49 Blocks of Order 5

**Example 2.43.** *A* ***pan diagonal****magic square of order 35 formed by 49 blocks of* ***pan diagonal*** *magic squares of order 5 constructed according to Example 1.3 is given by*

| | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 901 | 674 | 90 | 312 | 1088 | 649 | 210 | 216 | 1213 | 777 | 253 | 1177 | 524 | 746 | 365 | 1081 | 817 | 619 | 390 | 158 | 865 | 457 | 714 | 63 | 966 | 433 | 30 | 981 | 1032 | 589 | 109 | 926 | 1147 | 535 | 348 | 21455 |
| 21455 | 300 | 1117 | 878 | 691 | 79 | 1196 | 793 | 637 | 194 | 245 | 769 | 361 | 260 | 1163 | 512 | 409 | 145 | 1068 | 836 | 607 | 49 | 973 | 861 | 480 | 702 | 1016 | 577 | 449 | 13 | 1010 | 552 | 325 | 138 | 914 | 1136 | 21455 |
| 21455 | 668 | 96 | 289 | 1105 | 907 | 182 | 229 | 1225 | 776 | 653 | 1170 | 498 | 757 | 384 | 256 | 823 | 626 | 397 | 164 | 1055 | 476 | 725 | 37 | 959 | 868 | 29 | 993 | 1045 | 561 | 437 | 943 | 1124 | 541 | 342 | 115 | 21455 |
| 21455 | 1094 | 895 | 697 | 73 | 306 | 805 | 636 | 198 | 217 | 1209 | 372 | 279 | 1166 | 505 | 743 | 152 | 1074 | 810 | 613 | 416 | 947 | 854 | 483 | 721 | 60 | 590 | 421 | 17 | 1009 | 1028 | 331 | 132 | 920 | 1153 | 529 | 21455 |
| 21455 | 102 | 283 | 1111 | 884 | 685 | 233 | 1197 | 789 | 665 | 181 | 501 | 750 | 358 | 267 | 1189 | 600 | 403 | 171 | 1062 | 829 | 728 | 56 | 970 | 842 | 469 | 997 | 1044 | 573 | 450 | 1 | 1130 | 558 | 319 | 121 | 937 | 21455 |
| 21455 | 445 | 2 | 994 | 1043 | 581 | 118 | 940 | 1121 | 542 | 344 | 879 | 681 | 97 | 290 | 1118 | 656 | 184 | 230 | 1222 | 773 | 264 | 1190 | 496 | 758 | 357 | 1058 | 827 | 629 | 396 | 155 | 871 | 467 | 724 | 40 | 963 | 21455 |
| 21455 | 1029 | 588 | 441 | 25 | 982 | 526 | 332 | 134 | 923 | 1150 | 307 | 1095 | 908 | 669 | 86 | 1210 | 802 | 633 | 201 | 219 | 741 | 373 | 252 | 1174 | 525 | 419 | 151 | 1065 | 813 | 617 | 59 | 950 | 858 | 486 | 712 | 21455 |
| 21455 | 21 | 1005 | 1017 | 574 | 448 | 939 | 1133 | 555 | 316 | 122 | 698 | 74 | 296 | 1112 | 885 | 178 | 236 | 1199 | 790 | 662 | 1162 | 509 | 770 | 356 | 268 | 820 | 603 | 407 | 174 | 1061 | 473 | 731 | 47 | 969 | 845 | 21455 |
| 21455 | 562 | 434 | 28 | 1001 | 1040 | 345 | 106 | 927 | 1149 | 538 | 1101 | 902 | 675 | 103 | 284 | 779 | 650 | 207 | 213 | 1216 | 385 | 251 | 1178 | 497 | 754 | 162 | 1084 | 816 | 610 | 393 | 957 | 864 | 460 | 718 | 66 | 21455 |
| 21455 | 1008 | 1036 | 585 | 422 | 14 | 1137 | 554 | 328 | 135 | 911 | 80 | 313 | 1089 | 891 | 692 | 242 | 1193 | 796 | 639 | 195 | 513 | 742 | 369 | 280 | 1161 | 606 | 400 | 148 | 1072 | 839 | 705 | 53 | 976 | 852 | 479 | 21455 |
| 21455 | 1069 | 840 | 601 | 408 | 147 | 848 | 477 | 734 | 46 | 960 | 451 | 12 | 1004 | 1020 | 578 | 130 | 912 | 1134 | 553 | 336 | 888 | 695 | 71 | 297 | 1114 | 634 | 191 | 237 | 1200 | 803 | 271 | 1164 | 510 | 767 | 353 | 21455 |
| 21455 | 391 | 163 | 1057 | 824 | 630 | 69 | 956 | 855 | 463 | 722 | 1039 | 565 | 438 | 31 | 992 | 539 | 343 | 126 | 935 | 1122 | 281 | 1102 | 904 | 678 | 100 | 1217 | 780 | 663 | 179 | 226 | 755 | 382 | 248 | 1181 | 499 | 21455 |
| 21455 | 812 | 614 | 420 | 146 | 1073 | 470 | 708 | 57 | 979 | 851 | 18 | 1011 | 1027 | 584 | 425 | 931 | 1145 | 527 | 329 | 133 | 694 | 83 | 310 | 1086 | 892 | 208 | 214 | 1206 | 797 | 640 | 1158 | 516 | 744 | 370 | 277 | 21455 |
| 21455 | 175 | 1056 | 828 | 602 | 404 | 967 | 874 | 466 | 715 | 43 | 572 | 444 | 5 | 998 | 1046 | 317 | 119 | 938 | 1141 | 550 | 1115 | 876 | 682 | 99 | 293 | 786 | 657 | 185 | 243 | 1194 | 359 | 265 | 1187 | 493 | 761 | 21455 |
| 21455 | 618 | 392 | 159 | 1085 | 811 | 711 | 50 | 953 | 862 | 489 | 985 | 1033 | 591 | 432 | 24 | 1148 | 546 | 340 | 107 | 924 | 87 | 309 | 1098 | 905 | 666 | 220 | 1223 | 774 | 646 | 202 | 522 | 738 | 376 | 254 | 1175 | 21455 |
| 21455 | 643 | 205 | 211 | 1207 | 799 | 249 | 1171 | 517 | 745 | 383 | 1076 | 814 | 615 | 417 | 143 | 859 | 490 | 706 | 58 | 952 | 428 | 22 | 1014 | 1026 | 575 | 136 | 922 | 1144 | 530 | 333 | 900 | 667 | 84 | 308 | 1106 | 21455 |
| 21455 | 1191 | 787 | 659 | 188 | 240 | 762 | 360 | 278 | 1159 | 506 | 405 | 172 | 1053 | 831 | 604 | 41 | 968 | 847 | 474 | 735 | 1049 | 571 | 435 | 8 | 1002 | 549 | 320 | 123 | 941 | 1132 | 294 | 1113 | 896 | 690 | 72 | 21455 |
| 21455 | 204 | 223 | 1220 | 771 | 647 | 1188 | 494 | 751 | 377 | 255 | 808 | 621 | 394 | 160 | 1082 | 462 | 719 | 70 | 951 | 863 | 15 | 988 | 1037 | 594 | 431 | 928 | 1151 | 537 | 339 | 110 | 686 | 95 | 282 | 1099 | 903 | 21455 |
| 21455 | 800 | 631 | 192 | 239 | 1203 | 366 | 272 | 1165 | 523 | 739 | 149 | 1070 | 837 | 598 | 411 | 980 | 846 | 478 | 707 | 54 | 582 | 454 | 11 | 995 | 1023 | 327 | 129 | 915 | 1138 | 556 | 1087 | 889 | 693 | 91 | 305 | 21455 |
| 21455 | 227 | 1219 | 783 | 660 | 176 | 500 | 768 | 354 | 261 | 1182 | 627 | 388 | 166 | 1059 | 825 | 723 | 42 | 964 | 875 | 461 | 991 | 1030 | 568 | 442 | 34 | 1125 | 543 | 346 | 117 | 934 | 98 | 301 | 1110 | 877 | 679 | 21455 |
| 21455 | 113 | 932 | 1154 | 536 | 330 | 906 | 677 | 94 | 285 | 1103 | 655 | 177 | 224 | 1218 | 791 | 258 | 1185 | 491 | 752 | 379 | 1054 | 821 | 622 | 395 | 173 | 866 | 464 | 720 | 67 | 948 | 439 | 35 | 986 | 1038 | 567 | 21455 |
| 21455 | 559 | 326 | 120 | 918 | 1142 | 304 | 1090 | 893 | 696 | 82 | 1204 | 798 | 651 | 200 | 212 | 736 | 367 | 274 | 1168 | 520 | 412 | 150 | 1083 | 809 | 611 | 55 | 977 | 843 | 481 | 709 | 1021 | 583 | 427 | 19 | 1015 | 21455 |
| 21455 | 925 | 1128 | 547 | 349 | 116 | 683 | 101 | 292 | 1109 | 880 | 196 | 235 | 1192 | 784 | 658 | 1184 | 503 | 765 | 351 | 262 | 838 | 599 | 401 | 167 | 1060 | 458 | 726 | 44 | 965 | 872 | 7 | 999 | 1050 | 566 | 443 | 21455 |
| 21455 | 337 | 139 | 921 | 1135 | 533 | 1097 | 899 | 670 | 88 | 311 | 772 | 644 | 203 | 231 | 1215 | 380 | 246 | 1172 | 519 | 748 | 156 | 1077 | 815 | 628 | 389 | 954 | 860 | 487 | 703 | 61 | 595 | 426 | 23 | 987 | 1034 | 21455 |
| 21455 | 1131 | 540 | 323 | 127 | 944 | 75 | 298 | 1116 | 887 | 689 | 238 | 1211 | 795 | 632 | 189 | 507 | 764 | 363 | 275 | 1156 | 605 | 418 | 144 | 1066 | 832 | 732 | 38 | 971 | 849 | 475 | 1003 | 1022 | 579 | 455 | 6 | 21455 |
| 21455 | 844 | 471 | 727 | 45 | 978 | 446 | 9 | 1000 | 1047 | 563 | 124 | 945 | 1126 | 548 | 322 | 883 | 687 | 104 | 291 | 1100 | 661 | 187 | 234 | 1195 | 788 | 270 | 1157 | 504 | 763 | 371 | 1063 | 835 | 596 | 402 | 169 | 21455 |
| 21455 | 62 | 955 | 873 | 459 | 716 | 1035 | 592 | 423 | 26 | 989 | 531 | 338 | 112 | 929 | 1155 | 314 | 1096 | 890 | 673 | 92 | 1214 | 775 | 648 | 206 | 222 | 749 | 378 | 266 | 1180 | 492 | 386 | 157 | 1079 | 818 | 625 | 21455 |
| 21455 | 488 | 704 | 51 | 972 | 850 | 3 | 1006 | 1024 | 580 | 452 | 917 | 1139 | 560 | 321 | 128 | 680 | 78 | 302 | 1119 | 886 | 193 | 241 | 1202 | 794 | 635 | 1176 | 515 | 737 | 364 | 273 | 834 | 608 | 415 | 141 | 1067 | 21455 |
| 21455 | 961 | 867 | 465 | 733 | 39 | 569 | 440 | 32 | 983 | 1041 | 350 | 111 | 933 | 1127 | 544 | 1107 | 909 | 676 | 85 | 288 | 782 | 654 | 180 | 228 | 1221 | 352 | 259 | 1183 | 511 | 760 | 170 | 1051 | 822 | 624 | 398 | 21455 |
| 21455 | 710 | 68 | 949 | 856 | 482 | 1012 | 1018 | 586 | 429 | 20 | 1143 | 532 | 334 | 140 | 916 | 81 | 295 | 1093 | 897 | 699 | 215 | 1208 | 801 | 642 | 199 | 518 | 756 | 375 | 247 | 1169 | 612 | 414 | 153 | 1080 | 806 | 21455 |
| 21455 | 276 | 1167 | 514 | 740 | 368 | 1075 | 807 | 609 | 413 | 161 | 853 | 485 | 701 | 52 | 974 | 424 | 16 | 1007 | 1025 | 593 | 131 | 919 | 1140 | 557 | 318 | 894 | 700 | 76 | 303 | 1092 | 638 | 197 | 244 | 1201 | 785 | 21455 |
| 21455 | 759 | 355 | 263 | 1186 | 502 | 399 | 168 | 1071 | 830 | 597 | 36 | 962 | 869 | 468 | 730 | 1042 | 570 | 453 | 4 | 996 | 545 | 347 | 108 | 936 | 1129 | 286 | 1108 | 882 | 684 | 105 | 1224 | 781 | 645 | 183 | 232 | 21455 |
| 21455 | 1173 | 521 | 747 | 374 | 250 | 826 | 620 | 387 | 154 | 1078 | 484 | 713 | 65 | 946 | 857 | 33 | 984 | 1031 | 587 | 430 | 913 | 1146 | 534 | 335 | 137 | 672 | 89 | 315 | 1091 | 898 | 190 | 218 | 1212 | 804 | 641 | 21455 |
| 21455 | 362 | 269 | 1160 | 508 | 766 | 142 | 1064 | 833 | 616 | 410 | 975 | 841 | 472 | 729 | 48 | 576 | 447 | 10 | 1013 | 1019 | 324 | 125 | 942 | 1123 | 551 | 1120 | 881 | 688 | 77 | 299 | 792 | 664 | 186 | 225 | 1198 | 21455 |
| 21455 | 495 | 753 | 381 | 257 | 1179 | 623 | 406 | 165 | 1052 | 819 | 717 | 64 | 958 | 870 | 456 | 990 | 1048 | 564 | 436 | 27 | 1152 | 528 | 341 | 114 | 930 | 93 | 287 | 1104 | 910 | 671 | 221 | 1205 | 778 | 652 | 209 | 21455 |
| | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 |

In this case, all the 49 blocks are **pan diagonal** magic squares of order 5 with equal magic sums, $S_{5\times5} := 3065$

#### 2.14.2 Second Approach: 25 Blocks of Order 7

**Example 2.44.** *A* ***pan diagonal****magic square of order 35 formed by 25 blocks of* ***pan diagonal*** *magic squares of order 7 constructed according to Example 1.5 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 |
| | 901 | 649 | 253 | 1081 | 865 | 433 | 109 | 685 | 181 | 1189 | 829 | 469 | 1 | 937 | 73 | 217 | 505 | 613 | 721 | 1009 | 1153 | 289 | 1225 | 757 | 397 | 37 | 1045 | 541 | 1117 | 793 | 361 | 145 | 973 | 577 | 325 | 21455 |
| 21455 | 445 | 118 | 879 | 656 | 264 | 1058 | 871 | 14 | 911 | 692 | 195 | 1161 | 839 | 479 | 1001 | 1149 | 103 | 213 | 497 | 610 | 718 | 1017 | 555 | 296 | 1199 | 770 | 407 | 47 | 588 | 332 | 1095 | 802 | 373 | 151 | 950 | 21455 |
| 21455 | 1069 | 848 | 451 | 130 | 888 | 634 | 271 | 811 | 489 | 24 | 924 | 666 | 202 | 1175 | 602 | 715 | 998 | 1141 | 99 | 243 | 493 | 420 | 57 | 1027 | 527 | 310 | 1206 | 744 | 163 | 956 | 565 | 343 | 1102 | 780 | 382 | 21455 |
| 21455 | 643 | 249 | 1076 | 859 | 428 | 136 | 900 | 176 | 1182 | 825 | 461 | 34 | 934 | 679 | 239 | 523 | 598 | 707 | 995 | 1138 | 91 | 1220 | 751 | 394 | 70 | 1037 | 537 | 282 | 787 | 360 | 172 | 968 | 571 | 320 | 1113 | 21455 |
| 21455 | 113 | 906 | 655 | 258 | 1054 | 866 | 439 | 944 | 689 | 189 | 1156 | 832 | 475 | 6 | 1135 | 88 | 231 | 519 | 628 | 703 | 987 | 547 | 292 | 1192 | 765 | 401 | 44 | 1050 | 326 | 1090 | 798 | 367 | 150 | 977 | 583 | 21455 |
| 21455 | 844 | 446 | 124 | 883 | 661 | 270 | 1063 | 482 | 20 | 916 | 699 | 199 | 1169 | 806 | 733 | 983 | 1127 | 85 | 228 | 511 | 624 | 51 | 1024 | 560 | 302 | 1202 | 737 | 415 | 955 | 592 | 338 | 1096 | 775 | 378 | 157 | 21455 |
| 21455 | 276 | 1075 | 853 | 424 | 131 | 894 | 638 | 1179 | 819 | 456 | 27 | 930 | 671 | 209 | 508 | 616 | 729 | 1013 | 1123 | 77 | 225 | 747 | 387 | 65 | 1031 | 534 | 315 | 1212 | 355 | 168 | 962 | 570 | 347 | 1108 | 781 | 21455 |
| 21455 | 283 | 1197 | 750 | 403 | 56 | 1044 | 558 | 1094 | 805 | 372 | 152 | 947 | 590 | 331 | 907 | 653 | 256 | 1055 | 868 | 437 | 115 | 691 | 194 | 1163 | 836 | 480 | 13 | 914 | 90 | 216 | 524 | 619 | 714 | 981 | 1147 | 21455 |
| 21455 | 1036 | 554 | 313 | 1193 | 742 | 400 | 53 | 562 | 345 | 1101 | 779 | 385 | 162 | 957 | 448 | 122 | 885 | 662 | 268 | 1061 | 845 | 25 | 923 | 669 | 201 | 1174 | 813 | 486 | 994 | 1121 | 97 | 230 | 496 | 629 | 724 | 21455 |
| 21455 | 392 | 50 | 1033 | 546 | 309 | 1223 | 738 | 175 | 967 | 572 | 317 | 1115 | 786 | 359 | 1073 | 851 | 425 | 133 | 892 | 640 | 277 | 824 | 463 | 31 | 935 | 678 | 179 | 1181 | 601 | 734 | 1004 | 1134 | 71 | 237 | 510 | 21455 |
| 21455 | 1219 | 768 | 388 | 42 | 1030 | 543 | 301 | 800 | 366 | 149 | 980 | 582 | 327 | 1087 | 647 | 255 | 1082 | 863 | 431 | 110 | 903 | 188 | 1159 | 831 | 474 | 8 | 941 | 690 | 211 | 517 | 615 | 706 | 1014 | 1144 | 84 | 21455 |
| 21455 | 540 | 298 | 1211 | 764 | 418 | 38 | 1022 | 337 | 1097 | 772 | 380 | 156 | 954 | 595 | 116 | 880 | 658 | 262 | 1060 | 872 | 443 | 918 | 696 | 200 | 1168 | 809 | 481 | 19 | 1154 | 94 | 224 | 491 | 622 | 720 | 986 | 21455 |
| 21455 | 68 | 1018 | 532 | 295 | 1208 | 756 | 414 | 961 | 569 | 350 | 1107 | 782 | 352 | 170 | 850 | 452 | 128 | 886 | 635 | 273 | 1067 | 459 | 26 | 929 | 673 | 206 | 1180 | 818 | 727 | 1000 | 1126 | 104 | 234 | 504 | 596 | 21455 |
| 21455 | 753 | 406 | 64 | 1048 | 528 | 287 | 1205 | 362 | 142 | 975 | 576 | 324 | 1120 | 792 | 250 | 1078 | 857 | 430 | 137 | 898 | 641 | 1186 | 830 | 468 | 4 | 936 | 684 | 183 | 514 | 609 | 701 | 1007 | 1140 | 76 | 244 | 21455 |
| 21455 | 697 | 198 | 1166 | 810 | 483 | 17 | 920 | 96 | 229 | 498 | 626 | 725 | 993 | 1124 | 300 | 1196 | 769 | 409 | 49 | 1016 | 552 | 1088 | 777 | 365 | 158 | 966 | 589 | 348 | 884 | 665 | 267 | 1062 | 842 | 450 | 121 | 21455 |
| 21455 | 28 | 927 | 675 | 207 | 1178 | 816 | 460 | 1005 | 1133 | 74 | 236 | 509 | 603 | 731 | 1029 | 526 | 307 | 1210 | 741 | 419 | 59 | 581 | 344 | 1118 | 773 | 357 | 155 | 963 | 422 | 135 | 891 | 639 | 280 | 1072 | 852 | 21455 |
| 21455 | 828 | 466 | 5 | 938 | 682 | 185 | 1187 | 614 | 708 | 1011 | 1145 | 83 | 214 | 516 | 391 | 69 | 1039 | 539 | 281 | 1217 | 755 | 147 | 960 | 578 | 336 | 1114 | 803 | 353 | 1085 | 862 | 432 | 107 | 905 | 646 | 254 | 21455 |
| 21455 | 192 | 1165 | 837 | 478 | 11 | 915 | 693 | 223 | 494 | 621 | 719 | 988 | 1151 | 95 | 1191 | 762 | 405 | 41 | 1049 | 549 | 294 | 799 | 383 | 143 | 952 | 575 | 333 | 1106 | 660 | 261 | 1059 | 875 | 442 | 117 | 877 | 21455 |
| 21455 | 921 | 670 | 203 | 1172 | 815 | 487 | 23 | 1128 | 101 | 235 | 503 | 599 | 726 | 999 | 559 | 304 | 1204 | 736 | 412 | 55 | 1021 | 330 | 1103 | 791 | 379 | 173 | 948 | 567 | 127 | 887 | 632 | 275 | 1066 | 849 | 455 | 21455 |
| 21455 | 465 | 32 | 933 | 676 | 180 | 1183 | 822 | 704 | 1006 | 1139 | 78 | 241 | 515 | 608 | 62 | 1035 | 531 | 314 | 1214 | 749 | 386 | 978 | 563 | 322 | 1100 | 788 | 371 | 169 | 856 | 429 | 140 | 897 | 642 | 247 | 1080 | 21455 |
| 21455 | 1160 | 833 | 472 | 10 | 942 | 688 | 186 | 521 | 620 | 713 | 984 | 1146 | 89 | 218 | 759 | 399 | 36 | 1042 | 545 | 286 | 1224 | 368 | 161 | 974 | 593 | 318 | 1092 | 785 | 257 | 1052 | 870 | 436 | 114 | 910 | 652 | 21455 |
| 21455 | 1105 | 776 | 384 | 164 | 959 | 561 | 342 | 878 | 637 | 260 | 1068 | 861 | 449 | 138 | 674 | 210 | 1177 | 817 | 457 | 30 | 926 | 102 | 233 | 501 | 600 | 728 | 997 | 1130 | 306 | 1209 | 743 | 416 | 60 | 1028 | 529 | 21455 |
| 21455 | 574 | 316 | 1112 | 790 | 356 | 174 | 969 | 441 | 134 | 908 | 633 | 252 | 1065 | 858 | 2 | 940 | 681 | 184 | 1190 | 827 | 467 | 1008 | 1137 | 80 | 242 | 513 | 606 | 705 | 1040 | 538 | 284 | 1216 | 754 | 393 | 66 | 21455 |
| 21455 | 146 | 979 | 584 | 329 | 1086 | 797 | 370 | 1057 | 855 | 438 | 126 | 904 | 663 | 248 | 840 | 477 | 12 | 912 | 695 | 191 | 1164 | 618 | 711 | 985 | 1148 | 87 | 220 | 522 | 404 | 43 | 1046 | 550 | 293 | 1194 | 761 | 21455 |
| 21455 | 771 | 377 | 160 | 951 | 594 | 339 | 1099 | 659 | 278 | 1053 | 847 | 435 | 123 | 896 | 205 | 1171 | 814 | 490 | 22 | 922 | 667 | 227 | 500 | 627 | 723 | 991 | 1125 | 98 | 1203 | 739 | 411 | 54 | 1023 | 556 | 305 | 21455 |
| 21455 | 349 | 1109 | 784 | 351 | 167 | 965 | 566 | 120 | 893 | 651 | 274 | 1083 | 843 | 427 | 932 | 677 | 177 | 1185 | 821 | 464 | 35 | 1131 | 75 | 238 | 507 | 605 | 732 | 1003 | 533 | 311 | 1215 | 748 | 389 | 61 | 1034 | 21455 |
| 21455 | 972 | 580 | 321 | 1119 | 794 | 364 | 141 | 873 | 423 | 112 | 890 | 648 | 266 | 1079 | 471 | 9 | 945 | 687 | 187 | 1157 | 835 | 710 | 1012 | 1143 | 81 | 215 | 518 | 612 | 39 | 1041 | 544 | 288 | 1221 | 760 | 398 | 21455 |
| 21455 | 374 | 154 | 946 | 587 | 335 | 1091 | 804 | 263 | 1071 | 869 | 453 | 108 | 882 | 645 | 1167 | 807 | 485 | 16 | 919 | 700 | 197 | 495 | 623 | 717 | 990 | 1152 | 93 | 221 | 766 | 410 | 48 | 1019 | 551 | 299 | 1198 | 21455 |
| 21455 | 79 | 245 | 512 | 607 | 702 | 1010 | 1136 | 312 | 1213 | 746 | 390 | 63 | 1032 | 535 | 1111 | 789 | 358 | 171 | 970 | 573 | 319 | 895 | 636 | 279 | 1074 | 854 | 421 | 132 | 668 | 182 | 1170 | 823 | 476 | 29 | 943 | 21455 |
| 21455 | 982 | 1150 | 86 | 219 | 525 | 617 | 712 | 1043 | 542 | 290 | 1222 | 758 | 396 | 40 | 585 | 328 | 1089 | 796 | 369 | 148 | 976 | 434 | 106 | 902 | 650 | 251 | 1084 | 864 | 21 | 939 | 698 | 178 | 1162 | 820 | 473 | 21455 |
| 21455 | 630 | 722 | 992 | 1122 | 100 | 226 | 499 | 408 | 46 | 1020 | 553 | 297 | 1200 | 767 | 159 | 953 | 591 | 340 | 1098 | 774 | 376 | 1056 | 874 | 444 | 119 | 876 | 657 | 265 | 812 | 470 | 18 | 931 | 694 | 208 | 1158 | 21455 |
| 21455 | 240 | 506 | 604 | 735 | 1002 | 1132 | 72 | 1207 | 745 | 417 | 58 | 1026 | 530 | 308 | 783 | 354 | 166 | 964 | 568 | 346 | 1110 | 631 | 272 | 1070 | 846 | 454 | 129 | 889 | 204 | 1188 | 808 | 462 | 15 | 928 | 686 | 21455 |
| 21455 | 1142 | 82 | 212 | 520 | 611 | 709 | 1015 | 536 | 285 | 1218 | 752 | 395 | 67 | 1038 | 323 | 1116 | 795 | 363 | 144 | 971 | 579 | 139 | 899 | 644 | 246 | 1077 | 860 | 426 | 925 | 683 | 196 | 1184 | 838 | 458 | 7 | 21455 |
| 21455 | 716 | 989 | 1155 | 92 | 222 | 492 | 625 | 45 | 1047 | 548 | 291 | 1195 | 763 | 402 | 949 | 586 | 334 | 1093 | 801 | 375 | 153 | 867 | 440 | 111 | 909 | 654 | 259 | 1051 | 488 | 3 | 917 | 680 | 193 | 1176 | 834 | 21455 |
| 21455 | 502 | 597 | 730 | 996 | 1129 | 105 | 232 | 740 | 413 | 52 | 1025 | 557 | 303 | 1201 | 381 | 165 | 958 | 564 | 341 | 1104 | 778 | 269 | 1064 | 841 | 447 | 125 | 881 | 664 | 1173 | 826 | 484 | 33 | 913 | 672 | 190 | 21455 |
| | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 | 21455 |

In this case, all the 25 blocks are of **pan diagonal** magic squares of order 7 with equal magic sums, $S_{7\times7} :=$ 4291

## 2.15 Magic Squares of Order 36

In this subsection, we shall present six different ways of writing block-wise magic square of order 36. One as $4\times9$, i.e., magic square formed by 81 blocks of **pan diagonal** magic squares of order 4. Second as $12\times3$, i.e., magic square formed by 9 blocks of **pan diagonal** magic squares of order 12. Third, $9\times4$, i.e., magic square formed by 16 blocks of magic squares of order 3. Forth as, 3x12, i.e., magic square formed by 144 blocks of magic squares of order 3. Fifth as, $6\times6$, i.e., magic square formed by 36 blocks of magic squares of order 6 with equal magic sums. Sixed as $4\times9$, i.e., 16 blocks of bimagic squares of order 9. In the first four cases, the magic square of order 36 is **pan diagonal**, while in the fifht and sixth it is just a magic square of order 36.

### 2.15.1 First Approach: 81 Blocks of Order 4

**Example 2.45.** *A* ***pan diagonal*** *magic square of order 36 formed by 81 blocks of* ***pan diagonal*** *magic squares of order 4 constructed according to Example 1.2 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |
| | 487 | 972 | 1 | 1134 | 488 | 971 | 2 | 1133 | 489 | 970 | 3 | 1132 | 490 | 969 | 4 | 1131 | 491 | 968 | 5 | 1130 | 492 | 967 | 6 | 1129 | 493 | 966 | 7 | 1128 | 494 | 965 | 8 | 1127 | 495 | 964 | 9 | 1126 | 23346 |
| 23346 | 162 | 973 | 648 | 811 | 161 | 974 | 647 | 812 | 160 | 975 | 646 | 813 | 159 | 976 | 645 | 814 | 158 | 977 | 644 | 815 | 157 | 978 | 643 | 816 | 156 | 979 | 642 | 817 | 155 | 980 | 641 | 818 | 154 | 981 | 640 | 819 | 23346 |
| 23346 | 1296 | 163 | 810 | 325 | 1295 | 164 | 809 | 326 | 1294 | 165 | 808 | 327 | 1293 | 166 | 807 | 328 | 1292 | 167 | 806 | 329 | 1291 | 168 | 805 | 330 | 1290 | 169 | 804 | 331 | 1289 | 170 | 803 | 332 | 1288 | 171 | 802 | 333 | 23346 |
| 23346 | 649 | 486 | 1135 | 324 | 650 | 485 | 1136 | 323 | 651 | 484 | 1137 | 322 | 652 | 483 | 1138 | 321 | 653 | 482 | 1139 | 320 | 654 | 481 | 1140 | 319 | 655 | 480 | 1141 | 318 | 656 | 479 | 1142 | 317 | 657 | 478 | 1143 | 316 | 23346 |
| 23346 | 496 | 963 | 10 | 1125 | 497 | 962 | 11 | 1124 | 498 | 961 | 12 | 1123 | 499 | 960 | 13 | 1122 | 500 | 959 | 14 | 1121 | 501 | 958 | 15 | 1120 | 502 | 957 | 16 | 1119 | 503 | 956 | 17 | 1118 | 504 | 955 | 18 | 1117 | 23346 |
| 23346 | 153 | 982 | 639 | 820 | 152 | 983 | 638 | 821 | 151 | 984 | 637 | 822 | 150 | 985 | 636 | 823 | 149 | 986 | 635 | 824 | 148 | 987 | 634 | 825 | 147 | 988 | 633 | 826 | 146 | 989 | 632 | 827 | 145 | 990 | 631 | 828 | 23346 |
| 23346 | 1287 | 172 | 801 | 334 | 1286 | 173 | 800 | 335 | 1285 | 174 | 799 | 336 | 1284 | 175 | 798 | 337 | 1283 | 176 | 797 | 338 | 1282 | 177 | 796 | 339 | 1281 | 178 | 795 | 340 | 1280 | 179 | 794 | 341 | 1279 | 180 | 793 | 342 | 23346 |
| 23346 | 658 | 477 | 1144 | 315 | 659 | 476 | 1145 | 314 | 660 | 475 | 1146 | 313 | 661 | 474 | 1147 | 312 | 662 | 473 | 1148 | 311 | 663 | 472 | 1149 | 310 | 664 | 471 | 1150 | 309 | 665 | 470 | 1151 | 308 | 666 | 469 | 1152 | 307 | 23346 |
| 23346 | 505 | 954 | 19 | 1116 | 506 | 953 | 20 | 1115 | 507 | 952 | 21 | 1114 | 508 | 951 | 22 | 1113 | 509 | 950 | 23 | 1112 | 510 | 949 | 24 | 1111 | 511 | 948 | 25 | 1110 | 512 | 947 | 26 | 1109 | 513 | 946 | 27 | 1108 | 23346 |
| 23346 | 144 | 991 | 630 | 829 | 143 | 992 | 629 | 830 | 142 | 993 | 628 | 831 | 141 | 994 | 627 | 832 | 140 | 995 | 626 | 833 | 139 | 996 | 625 | 834 | 138 | 997 | 624 | 835 | 137 | 998 | 623 | 836 | 136 | 999 | 622 | 837 | 23346 |
| 23346 | 1278 | 181 | 792 | 343 | 1277 | 182 | 791 | 344 | 1276 | 183 | 790 | 345 | 1275 | 184 | 789 | 346 | 1274 | 185 | 788 | 347 | 1273 | 186 | 787 | 348 | 1272 | 187 | 786 | 349 | 1271 | 188 | 785 | 350 | 1270 | 189 | 784 | 351 | 23346 |
| 23346 | 667 | 468 | 1153 | 306 | 668 | 467 | 1154 | 305 | 669 | 466 | 1155 | 304 | 670 | 465 | 1156 | 303 | 671 | 464 | 1157 | 302 | 672 | 463 | 1158 | 301 | 673 | 462 | 1159 | 300 | 674 | 461 | 1160 | 299 | 675 | 460 | 1161 | 298 | 23346 |
| 23346 | 514 | 945 | 28 | 1107 | 515 | 944 | 29 | 1106 | 516 | 943 | 30 | 1105 | 517 | 942 | 31 | 1104 | 518 | 941 | 32 | 1103 | 519 | 940 | 33 | 1102 | 520 | 939 | 34 | 1101 | 521 | 938 | 35 | 1100 | 522 | 937 | 36 | 1099 | 23346 |
| 23346 | 135 | 1000 | 621 | 838 | 134 | 1001 | 620 | 839 | 133 | 1002 | 619 | 840 | 132 | 1003 | 618 | 841 | 131 | 1004 | 617 | 842 | 130 | 1005 | 616 | 843 | 129 | 1006 | 615 | 844 | 128 | 1007 | 614 | 845 | 127 | 1008 | 613 | 846 | 23346 |
| 23346 | 1269 | 190 | 783 | 352 | 1268 | 191 | 782 | 353 | 1267 | 192 | 781 | 354 | 1266 | 193 | 780 | 355 | 1265 | 194 | 779 | 356 | 1264 | 195 | 778 | 357 | 1263 | 196 | 777 | 358 | 1262 | 197 | 776 | 359 | 1261 | 198 | 775 | 360 | 23346 |
| 23346 | 676 | 459 | 1162 | 297 | 677 | 458 | 1163 | 296 | 678 | 457 | 1164 | 295 | 679 | 456 | 1165 | 294 | 680 | 455 | 1166 | 293 | 681 | 454 | 1167 | 292 | 682 | 453 | 1168 | 291 | 683 | 452 | 1169 | 290 | 684 | 451 | 1170 | 289 | 23346 |
| 23346 | 523 | 936 | 37 | 1098 | 524 | 935 | 38 | 1097 | 525 | 934 | 39 | 1096 | 526 | 933 | 40 | 1095 | 527 | 932 | 41 | 1094 | 528 | 931 | 42 | 1093 | 529 | 930 | 43 | 1092 | 530 | 929 | 44 | 1091 | 531 | 928 | 45 | 1090 | 23346 |
| 23346 | 126 | 1009 | 612 | 847 | 125 | 1010 | 611 | 848 | 124 | 1011 | 610 | 849 | 123 | 1012 | 609 | 850 | 122 | 1013 | 608 | 851 | 121 | 1014 | 607 | 852 | 120 | 1015 | 606 | 853 | 119 | 1016 | 605 | 854 | 118 | 1017 | 604 | 855 | 23346 |
| 23346 | 1260 | 199 | 774 | 361 | 1259 | 200 | 773 | 362 | 1258 | 201 | 772 | 363 | 1257 | 202 | 771 | 364 | 1256 | 203 | 770 | 365 | 1255 | 204 | 769 | 366 | 1254 | 205 | 768 | 367 | 1253 | 206 | 767 | 368 | 1252 | 207 | 766 | 369 | 23346 |
| 23346 | 685 | 450 | 1171 | 288 | 686 | 449 | 1172 | 287 | 687 | 448 | 1173 | 286 | 688 | 447 | 1174 | 285 | 689 | 446 | 1175 | 284 | 690 | 445 | 1176 | 283 | 691 | 444 | 1177 | 282 | 692 | 443 | 1178 | 281 | 693 | 442 | 1179 | 280 | 23346 |
| 23346 | 532 | 927 | 46 | 1089 | 533 | 926 | 47 | 1088 | 534 | 925 | 48 | 1087 | 535 | 924 | 49 | 1086 | 536 | 923 | 50 | 1085 | 537 | 922 | 51 | 1084 | 538 | 921 | 52 | 1083 | 539 | 920 | 53 | 1082 | 540 | 919 | 54 | 1081 | 23346 |
| 23346 | 117 | 1018 | 603 | 856 | 116 | 1019 | 602 | 857 | 115 | 1020 | 601 | 858 | 114 | 1021 | 600 | 859 | 113 | 1022 | 599 | 860 | 112 | 1023 | 598 | 861 | 111 | 1024 | 597 | 862 | 110 | 1025 | 596 | 863 | 109 | 1026 | 595 | 864 | 23346 |
| 23346 | 1251 | 208 | 765 | 370 | 1250 | 209 | 764 | 371 | 1249 | 210 | 763 | 372 | 1248 | 211 | 762 | 373 | 1247 | 212 | 761 | 374 | 1246 | 213 | 760 | 375 | 1245 | 214 | 759 | 376 | 1244 | 215 | 758 | 377 | 1243 | 216 | 757 | 378 | 23346 |
| 23346 | 694 | 441 | 1180 | 279 | 695 | 440 | 1181 | 278 | 696 | 439 | 1182 | 277 | 697 | 438 | 1183 | 276 | 698 | 437 | 1184 | 275 | 699 | 436 | 1185 | 274 | 700 | 435 | 1186 | 273 | 701 | 434 | 1187 | 272 | 702 | 433 | 1188 | 271 | 23346 |
| 23346 | 541 | 918 | 55 | 1080 | 542 | 917 | 56 | 1079 | 543 | 916 | 57 | 1078 | 544 | 915 | 58 | 1077 | 545 | 914 | 59 | 1076 | 546 | 913 | 60 | 1075 | 547 | 912 | 61 | 1074 | 548 | 911 | 62 | 1073 | 549 | 910 | 63 | 1072 | 23346 |
| 23346 | 108 | 1027 | 594 | 865 | 107 | 1028 | 593 | 866 | 106 | 1029 | 592 | 867 | 105 | 1030 | 591 | 868 | 104 | 1031 | 590 | 869 | 103 | 1032 | 589 | 870 | 102 | 1033 | 588 | 871 | 101 | 1034 | 587 | 872 | 100 | 1035 | 586 | 873 | 23346 |
| 23346 | 1242 | 217 | 756 | 379 | 1241 | 218 | 755 | 380 | 1240 | 219 | 754 | 381 | 1239 | 220 | 753 | 382 | 1238 | 221 | 752 | 383 | 1237 | 222 | 751 | 384 | 1236 | 223 | 750 | 385 | 1235 | 224 | 749 | 386 | 1234 | 225 | 748 | 387 | 23346 |
| 23346 | 703 | 432 | 1189 | 270 | 704 | 431 | 1190 | 269 | 705 | 430 | 1191 | 268 | 706 | 429 | 1192 | 267 | 707 | 428 | 1193 | 266 | 708 | 427 | 1194 | 265 | 709 | 426 | 1195 | 264 | 710 | 425 | 1196 | 263 | 711 | 424 | 1197 | 262 | 23346 |
| 23346 | 550 | 909 | 64 | 1071 | 551 | 908 | 65 | 1070 | 552 | 907 | 66 | 1069 | 553 | 906 | 67 | 1068 | 554 | 905 | 68 | 1067 | 555 | 904 | 69 | 1066 | 556 | 903 | 70 | 1065 | 557 | 902 | 71 | 1064 | 558 | 901 | 72 | 1063 | 23346 |
| 23346 | 99 | 1036 | 585 | 874 | 98 | 1037 | 584 | 875 | 97 | 1038 | 583 | 876 | 96 | 1039 | 582 | 877 | 95 | 1040 | 581 | 878 | 94 | 1041 | 580 | 879 | 93 | 1042 | 579 | 880 | 92 | 1043 | 578 | 881 | 91 | 1044 | 577 | 882 | 23346 |
| 23346 | 1233 | 226 | 747 | 388 | 1232 | 227 | 746 | 389 | 1231 | 228 | 745 | 390 | 1230 | 229 | 744 | 391 | 1229 | 230 | 743 | 392 | 1228 | 231 | 742 | 393 | 1227 | 232 | 741 | 394 | 1226 | 233 | 740 | 395 | 1225 | 234 | 739 | 396 | 23346 |
| 23346 | 712 | 423 | 1198 | 261 | 713 | 422 | 1199 | 260 | 714 | 421 | 1200 | 259 | 715 | 420 | 1201 | 258 | 716 | 419 | 1202 | 257 | 717 | 418 | 1203 | 256 | 718 | 417 | 1204 | 255 | 719 | 416 | 1205 | 254 | 720 | 415 | 1206 | 253 | 23346 |
| 23346 | 559 | 900 | 73 | 1062 | 560 | 899 | 74 | 1061 | 561 | 898 | 75 | 1060 | 562 | 897 | 76 | 1059 | 563 | 896 | 77 | 1058 | 564 | 895 | 78 | 1057 | 565 | 894 | 79 | 1056 | 566 | 893 | 80 | 1055 | 567 | 892 | 81 | 1054 | 23346 |
| 23346 | 90 | 1045 | 576 | 883 | 89 | 1046 | 575 | 884 | 88 | 1047 | 574 | 885 | 87 | 1048 | 573 | 886 | 86 | 1049 | 572 | 887 | 85 | 1050 | 571 | 888 | 84 | 1051 | 570 | 889 | 83 | 1052 | 569 | 890 | 82 | 1053 | 568 | 891 | 23346 |
| 23346 | 1224 | 235 | 738 | 397 | 1223 | 236 | 737 | 398 | 1222 | 237 | 736 | 399 | 1221 | 238 | 735 | 400 | 1220 | 239 | 734 | 401 | 1219 | 240 | 733 | 402 | 1218 | 241 | 732 | 403 | 1217 | 242 | 731 | 404 | 1216 | 243 | 730 | 405 | 23346 |
| 23346 | 721 | 414 | 1207 | 252 | 722 | 413 | 1208 | 251 | 723 | 412 | 1209 | 250 | 724 | 411 | 1210 | 249 | 725 | 410 | 1211 | 248 | 726 | 409 | 1212 | 247 | 727 | 408 | 1213 | 246 | 728 | 407 | 1214 | 245 | 729 | 406 | 1215 | 244 | 23346 |
| | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |

In this case, all the blocks of order 4 are **pan diagonal** magic square with equal magic sums, $S_{4\times4} := 2594$.

#### 2.15.2 Second Approach: 9 Blocks of Order 12

**Example 2.46.** *A* ***pan diagonal*** *magic square of order 36 formed by 9 blocks of* ***pan diagonal*** *magic squares of order 12 constructed according to Example 2.2 is given by*

| | | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 451 | 421 | 636 | 864 | 894 | 679 | 181 | 223 | 6 | 1098 | 1056 | 1273 | 452 | 422 | 635 | 863 | 893 | 680 | 182 | 224 | 5 | 1097 | 1055 | 1274 | 453 | 423 | 634 | 862 | 892 | 681 | 183 | 225 | 4 | 1096 | 1054 | 1275 | 23346 |
| 23346 | 637 | 456 | 415 | 678 | 859 | 900 | 7 | 186 | 217 | 1272 | 1093 | 1062 | 638 | 455 | 416 | 677 | 860 | 899 | 8 | 185 | 218 | 1271 | 1094 | 1061 | 639 | 454 | 417 | 676 | 861 | 898 | 9 | 184 | 219 | 1270 | 1095 | 1060 | 23346 |
| 23346 | 420 | 631 | 457 | 895 | 684 | 858 | 222 | 1 | 187 | 1057 | 1278 | 1092 | 419 | 632 | 458 | 896 | 683 | 857 | 221 | 2 | 188 | 1058 | 1277 | 1091 | 418 | 633 | 459 | 897 | 682 | 856 | 220 | 3 | 189 | 1059 | 1276 | 1090 | 23346 |
| 23346 | 198 | 228 | 13 | 1081 | 1051 | 1266 | 468 | 426 | 643 | 847 | 889 | 672 | 197 | 227 | 14 | 1082 | 1052 | 1265 | 467 | 425 | 644 | 848 | 890 | 671 | 196 | 226 | 15 | 1083 | 1053 | 1264 | 466 | 424 | 645 | 849 | 891 | 670 | 23346 |
| 23346 | 12 | 193 | 234 | 1267 | 1086 | 1045 | 642 | 463 | 432 | 673 | 852 | 883 | 11 | 194 | 233 | 1268 | 1085 | 1046 | 641 | 464 | 431 | 674 | 851 | 884 | 10 | 195 | 232 | 1269 | 1084 | 1047 | 640 | 465 | 430 | 675 | 850 | 885 | 23346 |
| 23346 | 229 | 18 | 192 | 1050 | 1261 | 1087 | 427 | 648 | 462 | 888 | 667 | 853 | 230 | 17 | 191 | 1049 | 1262 | 1088 | 428 | 647 | 461 | 887 | 668 | 854 | 231 | 16 | 190 | 1048 | 1263 | 1089 | 429 | 646 | 460 | 886 | 669 | 855 | 23346 |
| 23346 | 1116 | 1074 | 1291 | 199 | 241 | 24 | 846 | 876 | 661 | 433 | 403 | 618 | 1115 | 1073 | 1292 | 200 | 242 | 23 | 845 | 875 | 662 | 434 | 404 | 617 | 1114 | 1072 | 1293 | 201 | 243 | 22 | 844 | 874 | 663 | 435 | 405 | 616 | 23346 |
| 23346 | 1290 | 1111 | 1080 | 25 | 204 | 235 | 660 | 841 | 882 | 619 | 438 | 397 | 1289 | 1112 | 1079 | 26 | 203 | 236 | 659 | 842 | 881 | 620 | 437 | 398 | 1288 | 1113 | 1078 | 27 | 202 | 237 | 658 | 843 | 880 | 621 | 436 | 399 | 23346 |
| 23346 | 1075 | 1296 | 1110 | 240 | 19 | 205 | 877 | 666 | 840 | 402 | 613 | 439 | 1076 | 1295 | 1109 | 239 | 20 | 206 | 878 | 665 | 839 | 401 | 614 | 440 | 1077 | 1294 | 1108 | 238 | 21 | 207 | 879 | 664 | 838 | 400 | 615 | 441 | 23346 |
| 23346 | 829 | 871 | 654 | 450 | 408 | 625 | 1099 | 1069 | 1284 | 216 | 246 | 31 | 830 | 872 | 653 | 449 | 407 | 626 | 1100 | 1070 | 1283 | 215 | 245 | 32 | 831 | 873 | 652 | 448 | 406 | 627 | 1101 | 1071 | 1282 | 214 | 244 | 33 | 23346 |
| 23346 | 655 | 834 | 865 | 624 | 445 | 414 | 1285 | 1104 | 1063 | 30 | 211 | 252 | 656 | 833 | 866 | 623 | 446 | 413 | 1286 | 1103 | 1064 | 29 | 212 | 251 | 657 | 832 | 867 | 622 | 447 | 412 | 1287 | 1102 | 1065 | 28 | 213 | 250 | 23346 |
| 23346 | 870 | 649 | 835 | 409 | 630 | 444 | 1068 | 1279 | 1105 | 247 | 36 | 210 | 869 | 650 | 836 | 410 | 629 | 443 | 1067 | 1280 | 1106 | 248 | 35 | 209 | 868 | 651 | 837 | 411 | 628 | 442 | 1066 | 1281 | 1107 | 249 | 34 | 208 | 23346 |
| 23346 | 487 | 385 | 600 | 828 | 930 | 715 | 145 | 259 | 42 | 1134 | 1020 | 1237 | 488 | 386 | 599 | 827 | 929 | 716 | 146 | 260 | 41 | 1133 | 1019 | 1238 | 489 | 387 | 598 | 826 | 928 | 717 | 147 | 261 | 40 | 1132 | 1018 | 1239 | 23346 |
| 23346 | 601 | 492 | 379 | 714 | 823 | 936 | 43 | 150 | 253 | 1236 | 1129 | 1026 | 602 | 491 | 380 | 713 | 824 | 935 | 44 | 149 | 254 | 1235 | 1130 | 1025 | 603 | 490 | 381 | 712 | 825 | 934 | 45 | 148 | 255 | 1234 | 1131 | 1024 | 23346 |
| 23346 | 384 | 595 | 493 | 931 | 720 | 822 | 258 | 37 | 151 | 1021 | 1242 | 1128 | 383 | 596 | 494 | 932 | 719 | 821 | 257 | 38 | 152 | 1022 | 1241 | 1127 | 382 | 597 | 495 | 933 | 718 | 820 | 256 | 39 | 153 | 1023 | 1240 | 1126 | 23346 |
| 23346 | 162 | 264 | 49 | 1117 | 1015 | 1230 | 504 | 390 | 607 | 811 | 925 | 708 | 161 | 263 | 50 | 1118 | 1016 | 1229 | 503 | 389 | 608 | 812 | 926 | 707 | 160 | 262 | 51 | 1119 | 1017 | 1228 | 502 | 388 | 609 | 813 | 927 | 706 | 23346 |
| 23346 | 48 | 157 | 270 | 1231 | 1122 | 1009 | 606 | 499 | 396 | 709 | 816 | 919 | 47 | 158 | 269 | 1232 | 1121 | 1010 | 605 | 500 | 395 | 710 | 815 | 920 | 46 | 159 | 268 | 1233 | 1120 | 1011 | 604 | 501 | 394 | 711 | 814 | 921 | 23346 |
| 23346 | 265 | 54 | 156 | 1014 | 1225 | 1123 | 391 | 612 | 498 | 924 | 703 | 817 | 266 | 53 | 155 | 1013 | 1226 | 1124 | 392 | 611 | 497 | 923 | 704 | 818 | 267 | 52 | 154 | 1012 | 1227 | 1125 | 393 | 610 | 496 | 922 | 705 | 819 | 23346 |
| 23346 | 1152 | 1038 | 1255 | 163 | 277 | 60 | 810 | 912 | 697 | 469 | 367 | 582 | 1151 | 1037 | 1256 | 164 | 278 | 59 | 809 | 911 | 698 | 470 | 368 | 581 | 1150 | 1036 | 1257 | 165 | 279 | 58 | 808 | 910 | 699 | 471 | 369 | 580 | 23346 |
| 23346 | 1254 | 1147 | 1044 | 61 | 168 | 271 | 696 | 805 | 918 | 583 | 474 | 361 | 1253 | 1148 | 1043 | 62 | 167 | 272 | 695 | 806 | 917 | 584 | 473 | 362 | 1252 | 1149 | 1042 | 63 | 166 | 273 | 694 | 807 | 916 | 585 | 472 | 363 | 23346 |
| 23346 | 1039 | 1260 | 1146 | 276 | 55 | 169 | 913 | 702 | 804 | 366 | 577 | 475 | 1040 | 1259 | 1145 | 275 | 56 | 170 | 914 | 701 | 803 | 365 | 578 | 476 | 1041 | 1258 | 1144 | 274 | 57 | 171 | 915 | 700 | 802 | 364 | 579 | 477 | 23346 |
| 23346 | 793 | 907 | 690 | 486 | 372 | 589 | 1135 | 1033 | 1248 | 180 | 282 | 67 | 794 | 908 | 689 | 485 | 371 | 590 | 1136 | 1034 | 1247 | 179 | 281 | 68 | 795 | 909 | 688 | 484 | 370 | 591 | 1137 | 1035 | 1246 | 178 | 280 | 69 | 23346 |
| 23346 | 691 | 798 | 901 | 588 | 481 | 378 | 1249 | 1140 | 1027 | 66 | 175 | 288 | 692 | 797 | 902 | 587 | 482 | 377 | 1250 | 1139 | 1028 | 65 | 176 | 287 | 693 | 796 | 903 | 586 | 483 | 376 | 1251 | 1138 | 1029 | 64 | 177 | 286 | 23346 |
| 23346 | 906 | 685 | 799 | 373 | 594 | 480 | 1032 | 1243 | 1141 | 283 | 72 | 174 | 905 | 686 | 800 | 374 | 593 | 479 | 1031 | 1244 | 1142 | 284 | 71 | 173 | 904 | 687 | 801 | 375 | 592 | 478 | 1030 | 1245 | 1143 | 285 | 70 | 172 | 23346 |
| 23346 | 523 | 349 | 564 | 792 | 966 | 751 | 109 | 295 | 78 | 1170 | 984 | 1201 | 524 | 350 | 563 | 791 | 965 | 752 | 110 | 296 | 77 | 1169 | 983 | 1202 | 525 | 351 | 562 | 790 | 964 | 753 | 111 | 297 | 76 | 1168 | 982 | 1203 | 23346 |
| 23346 | 565 | 528 | 343 | 750 | 787 | 972 | 79 | 114 | 289 | 1200 | 1165 | 990 | 566 | 527 | 344 | 749 | 788 | 971 | 80 | 113 | 290 | 1199 | 1166 | 989 | 567 | 526 | 345 | 748 | 789 | 970 | 81 | 112 | 291 | 1198 | 1167 | 988 | 23346 |
| 23346 | 348 | 559 | 529 | 967 | 756 | 786 | 294 | 73 | 115 | 985 | 1206 | 1164 | 347 | 560 | 530 | 968 | 755 | 785 | 293 | 74 | 116 | 986 | 1205 | 1163 | 346 | 561 | 531 | 969 | 754 | 784 | 292 | 75 | 117 | 987 | 1204 | 1162 | 23346 |
| 23346 | 126 | 300 | 85 | 1153 | 979 | 1194 | 540 | 354 | 571 | 775 | 961 | 744 | 125 | 299 | 86 | 1154 | 980 | 1193 | 539 | 353 | 572 | 776 | 962 | 743 | 124 | 298 | 87 | 1155 | 981 | 1192 | 538 | 352 | 573 | 777 | 963 | 742 | 23346 |
| 23346 | 84 | 121 | 306 | 1195 | 1158 | 973 | 570 | 535 | 360 | 745 | 780 | 955 | 83 | 122 | 305 | 1196 | 1157 | 974 | 569 | 536 | 359 | 746 | 779 | 956 | 82 | 123 | 304 | 1197 | 1156 | 975 | 568 | 537 | 358 | 747 | 778 | 957 | 23346 |
| 23346 | 301 | 90 | 120 | 978 | 1189 | 1159 | 355 | 576 | 534 | 960 | 739 | 781 | 302 | 89 | 119 | 977 | 1190 | 1160 | 356 | 575 | 533 | 959 | 740 | 782 | 303 | 88 | 118 | 976 | 1191 | 1161 | 357 | 574 | 532 | 958 | 741 | 783 | 23346 |
| 23346 | 1188 | 1002 | 1219 | 127 | 313 | 96 | 774 | 948 | 733 | 505 | 331 | 546 | 1187 | 1001 | 1220 | 128 | 314 | 95 | 773 | 947 | 734 | 506 | 332 | 545 | 1186 | 1000 | 1221 | 129 | 315 | 94 | 772 | 946 | 735 | 507 | 333 | 544 | 23346 |
| 23346 | 1218 | 1183 | 1008 | 97 | 132 | 307 | 732 | 769 | 954 | 547 | 510 | 325 | 1217 | 1184 | 1007 | 98 | 131 | 308 | 731 | 770 | 953 | 548 | 509 | 326 | 1216 | 1185 | 1006 | 99 | 130 | 309 | 730 | 771 | 952 | 549 | 508 | 327 | 23346 |
| 23346 | 1003 | 1224 | 1182 | 312 | 91 | 133 | 949 | 738 | 768 | 330 | 541 | 511 | 1004 | 1223 | 1181 | 311 | 92 | 134 | 950 | 737 | 767 | 329 | 542 | 512 | 1005 | 1222 | 1180 | 310 | 93 | 135 | 951 | 736 | 766 | 328 | 543 | 513 | 23346 |
| 23346 | 757 | 943 | 726 | 522 | 336 | 553 | 1171 | 997 | 1212 | 144 | 318 | 103 | 758 | 944 | 725 | 521 | 335 | 554 | 1172 | 998 | 1211 | 143 | 317 | 104 | 759 | 945 | 724 | 520 | 334 | 555 | 1173 | 999 | 1210 | 142 | 316 | 105 | 23346 |
| 23346 | 727 | 762 | 937 | 552 | 517 | 342 | 1213 | 1176 | 991 | 102 | 139 | 324 | 728 | 761 | 938 | 551 | 518 | 341 | 1214 | 1175 | 992 | 101 | 140 | 323 | 729 | 760 | 939 | 550 | 519 | 340 | 1215 | 1174 | 993 | 100 | 141 | 322 | 23346 |
| 23346 | 942 | 721 | 763 | 337 | 558 | 516 | 996 | 1207 | 1177 | 319 | 108 | 138 | 941 | 722 | 764 | 338 | 557 | 515 | 995 | 1208 | 1178 | 320 | 107 | 137 | 940 | 723 | 765 | 339 | 556 | 514 | 994 | 1209 | 1179 | 321 | 106 | 136 | 23346 |
| | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |

In this case, all the blocks of order 12 are **pan diagonal** magic squares with equal magic sums, $S_{12\times 12} := 7782$. The $3\times 3$ blocks are **semi-magic** squares with different **semi-magic** sums of order 3 (in rows and columns).

### 2.15.3 Third Approach: 16 Blocks of Order 9

**Example 2.47.** *A **pan diagonal** magic square of order 36 formed by 16 blocks of magic squares of order 9 constructed according to Example 1.8 is given by*

| | | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 7 | 695 | 1239 | 23 | 711 | 1207 | 39 | 679 | 1223 | 12 | 700 | 1244 | 28 | 716 | 1212 | 44 | 684 | 1228 | 1 | 689 | 1233 | 17 | 705 | 1201 | 33 | 673 | 1217 | 14 | 702 | 1246 | 30 | 718 | 1214 | 46 | 686 | 1230 | 23346 |
| 23346 | 647 | 1191 | 103 | 663 | 1159 | 119 | 631 | 1175 | 135 | 652 | 1196 | 108 | 668 | 1164 | 124 | 636 | 1180 | 140 | 641 | 1185 | 97 | 657 | 1153 | 113 | 625 | 1169 | 129 | 654 | 1198 | 110 | 670 | 1166 | 126 | 638 | 1182 | 142 | 23346 |
| 23346 | 1287 | 55 | 599 | 1255 | 71 | 615 | 1271 | 87 | 583 | 1292 | 60 | 604 | 1260 | 76 | 620 | 1276 | 92 | 588 | 1281 | 49 | 593 | 1249 | 65 | 609 | 1265 | 81 | 577 | 1294 | 62 | 606 | 1262 | 78 | 622 | 1278 | 94 | 590 | 23346 |
| 23346 | 775 | 887 | 279 | 791 | 903 | 247 | 807 | 871 | 263 | 780 | 892 | 284 | 796 | 908 | 252 | 812 | 876 | 268 | 769 | 881 | 273 | 785 | 897 | 241 | 801 | 865 | 257 | 782 | 894 | 286 | 798 | 910 | 254 | 814 | 878 | 270 | 23346 |
| 23346 | 983 | 231 | 727 | 999 | 199 | 743 | 967 | 215 | 759 | 988 | 236 | 732 | 1004 | 204 | 748 | 972 | 220 | 764 | 977 | 225 | 721 | 993 | 193 | 737 | 961 | 209 | 753 | 990 | 238 | 734 | 1006 | 206 | 750 | 974 | 222 | 766 | 23346 |
| 23346 | 183 | 823 | 935 | 151 | 839 | 951 | 167 | 855 | 919 | 188 | 828 | 940 | 156 | 844 | 956 | 172 | 860 | 924 | 177 | 817 | 929 | 145 | 833 | 945 | 161 | 849 | 913 | 190 | 830 | 942 | 158 | 846 | 958 | 174 | 862 | 926 | 23346 |
| 23346 | 1111 | 359 | 471 | 1127 | 375 | 439 | 1143 | 343 | 455 | 1116 | 364 | 476 | 1132 | 380 | 444 | 1148 | 348 | 460 | 1105 | 353 | 465 | 1121 | 369 | 433 | 1137 | 337 | 449 | 1118 | 366 | 478 | 1134 | 382 | 446 | 1150 | 350 | 462 | 23346 |
| 23346 | 311 | 567 | 1063 | 327 | 535 | 1079 | 295 | 551 | 1095 | 316 | 572 | 1068 | 332 | 540 | 1084 | 300 | 556 | 1100 | 305 | 561 | 1057 | 321 | 529 | 1073 | 289 | 545 | 1089 | 318 | 574 | 1070 | 334 | 542 | 1086 | 302 | 558 | 1102 | 23346 |
| 23346 | 519 | 1015 | 407 | 487 | 1031 | 423 | 503 | 1047 | 391 | 524 | 1020 | 412 | 492 | 1036 | 428 | 508 | 1052 | 396 | 513 | 1009 | 401 | 481 | 1025 | 417 | 497 | 1041 | 385 | 526 | 1022 | 414 | 494 | 1038 | 430 | 510 | 1054 | 398 | 23346 |
| 23346 | 2 | 690 | 1234 | 18 | 706 | 1202 | 34 | 674 | 1218 | 13 | 701 | 1245 | 29 | 717 | 1213 | 45 | 685 | 1229 | 8 | 696 | 1240 | 24 | 712 | 1208 | 40 | 680 | 1224 | 11 | 699 | 1243 | 27 | 715 | 1211 | 43 | 683 | 1227 | 23346 |
| 23346 | 642 | 1186 | 98 | 658 | 1154 | 114 | 626 | 1170 | 130 | 653 | 1197 | 109 | 669 | 1165 | 125 | 637 | 1181 | 141 | 648 | 1192 | 104 | 664 | 1160 | 120 | 632 | 1176 | 136 | 651 | 1195 | 107 | 667 | 1163 | 123 | 635 | 1179 | 139 | 23346 |
| 23346 | 1282 | 50 | 594 | 1250 | 66 | 610 | 1266 | 82 | 578 | 1293 | 61 | 605 | 1261 | 77 | 621 | 1277 | 93 | 589 | 1288 | 56 | 600 | 1256 | 72 | 616 | 1272 | 88 | 584 | 1291 | 59 | 603 | 1259 | 75 | 619 | 1275 | 91 | 587 | 23346 |
| 23346 | 770 | 882 | 274 | 786 | 898 | 242 | 802 | 866 | 258 | 781 | 893 | 285 | 797 | 909 | 253 | 813 | 877 | 269 | 776 | 888 | 280 | 792 | 904 | 248 | 808 | 872 | 264 | 779 | 891 | 283 | 795 | 907 | 251 | 811 | 875 | 267 | 23346 |
| 23346 | 978 | 226 | 722 | 994 | 194 | 738 | 962 | 210 | 754 | 989 | 237 | 733 | 1005 | 205 | 749 | 973 | 221 | 765 | 984 | 232 | 728 | 1000 | 200 | 744 | 968 | 216 | 760 | 987 | 235 | 731 | 1003 | 203 | 747 | 971 | 219 | 763 | 23346 |
| 23346 | 178 | 818 | 930 | 146 | 834 | 946 | 162 | 850 | 914 | 189 | 829 | 941 | 157 | 845 | 957 | 173 | 861 | 925 | 184 | 824 | 936 | 152 | 840 | 952 | 168 | 856 | 920 | 187 | 827 | 939 | 155 | 843 | 955 | 171 | 859 | 923 | 23346 |
| 23346 | 1106 | 354 | 466 | 1122 | 370 | 434 | 1138 | 338 | 450 | 1117 | 365 | 477 | 1133 | 381 | 445 | 1149 | 349 | 461 | 1112 | 360 | 472 | 1128 | 376 | 440 | 1144 | 344 | 456 | 1115 | 363 | 475 | 1131 | 379 | 443 | 1147 | 347 | 459 | 23346 |
| 23346 | 306 | 562 | 1058 | 322 | 530 | 1074 | 290 | 546 | 1090 | 317 | 573 | 1069 | 333 | 541 | 1085 | 301 | 557 | 1101 | 312 | 568 | 1064 | 328 | 536 | 1080 | 296 | 552 | 1096 | 315 | 571 | 1067 | 331 | 539 | 1083 | 299 | 555 | 1099 | 23346 |
| 23346 | 514 | 1010 | 402 | 482 | 1026 | 418 | 498 | 1042 | 386 | 525 | 1021 | 413 | 493 | 1037 | 429 | 509 | 1053 | 397 | 520 | 1016 | 408 | 488 | 1032 | 424 | 504 | 1048 | 392 | 523 | 1019 | 411 | 491 | 1035 | 427 | 507 | 1051 | 395 | 23346 |
| 23346 | 16 | 704 | 1248 | 32 | 720 | 1216 | 48 | 688 | 1232 | 3 | 691 | 1235 | 19 | 707 | 1203 | 35 | 675 | 1219 | 10 | 698 | 1242 | 26 | 714 | 1210 | 42 | 682 | 1226 | 5 | 693 | 1237 | 21 | 709 | 1205 | 37 | 677 | 1221 | 23346 |
| 23346 | 656 | 1200 | 112 | 672 | 1168 | 128 | 640 | 1184 | 144 | 643 | 1187 | 99 | 659 | 1155 | 115 | 627 | 1171 | 131 | 650 | 1194 | 106 | 666 | 1162 | 122 | 634 | 1178 | 138 | 645 | 1189 | 101 | 661 | 1157 | 117 | 629 | 1173 | 133 | 23346 |
| 23346 | 1296 | 64 | 608 | 1264 | 80 | 624 | 1280 | 96 | 592 | 1283 | 51 | 595 | 1251 | 67 | 611 | 1267 | 83 | 579 | 1290 | 58 | 602 | 1258 | 74 | 618 | 1274 | 90 | 586 | 1285 | 53 | 597 | 1253 | 69 | 613 | 1269 | 85 | 581 | 23346 |
| 23346 | 784 | 896 | 288 | 800 | 912 | 256 | 816 | 880 | 272 | 771 | 883 | 275 | 787 | 899 | 243 | 803 | 867 | 259 | 778 | 890 | 282 | 794 | 906 | 250 | 810 | 874 | 266 | 773 | 885 | 277 | 789 | 901 | 245 | 805 | 869 | 261 | 23346 |
| 23346 | 992 | 240 | 736 | 1008 | 208 | 752 | 976 | 224 | 768 | 979 | 227 | 723 | 995 | 195 | 739 | 963 | 211 | 755 | 986 | 234 | 730 | 1002 | 202 | 746 | 970 | 218 | 762 | 981 | 229 | 725 | 997 | 197 | 741 | 965 | 213 | 757 | 23346 |
| 23346 | 192 | 832 | 944 | 160 | 848 | 960 | 176 | 864 | 928 | 179 | 819 | 931 | 147 | 835 | 947 | 163 | 851 | 915 | 186 | 826 | 938 | 154 | 842 | 954 | 170 | 858 | 922 | 181 | 821 | 933 | 149 | 837 | 949 | 165 | 853 | 917 | 23346 |
| 23346 | 1120 | 368 | 480 | 1136 | 384 | 448 | 1152 | 352 | 464 | 1107 | 355 | 467 | 1123 | 371 | 435 | 1139 | 339 | 451 | 1114 | 362 | 474 | 1130 | 378 | 442 | 1146 | 346 | 458 | 1109 | 357 | 469 | 1125 | 373 | 437 | 1141 | 341 | 453 | 23346 |
| 23346 | 320 | 576 | 1072 | 336 | 544 | 1088 | 304 | 560 | 1104 | 307 | 563 | 1059 | 323 | 531 | 1075 | 291 | 547 | 1091 | 314 | 570 | 1066 | 330 | 538 | 1082 | 298 | 554 | 1098 | 309 | 565 | 1061 | 325 | 533 | 1077 | 293 | 549 | 1093 | 23346 |
| 23346 | 528 | 1024 | 416 | 496 | 1040 | 432 | 512 | 1056 | 400 | 515 | 1011 | 403 | 483 | 1027 | 419 | 499 | 1043 | 387 | 522 | 1018 | 410 | 490 | 1034 | 426 | 506 | 1050 | 394 | 517 | 1013 | 405 | 485 | 1029 | 421 | 501 | 1045 | 389 | 23346 |
| 23346 | 9 | 697 | 1241 | 25 | 713 | 1209 | 41 | 681 | 1225 | 6 | 694 | 1238 | 22 | 710 | 1206 | 38 | 678 | 1222 | 15 | 703 | 1247 | 31 | 719 | 1215 | 47 | 687 | 1231 | 4 | 692 | 1236 | 20 | 708 | 1204 | 36 | 676 | 1220 | 23346 |
| 23346 | 649 | 1193 | 105 | 665 | 1161 | 121 | 633 | 1177 | 137 | 646 | 1190 | 102 | 662 | 1158 | 118 | 630 | 1174 | 134 | 655 | 1199 | 111 | 671 | 1167 | 127 | 639 | 1183 | 143 | 644 | 1188 | 100 | 660 | 1156 | 116 | 628 | 1172 | 132 | 23346 |
| 23346 | 1289 | 57 | 601 | 1257 | 73 | 617 | 1273 | 89 | 585 | 1286 | 54 | 598 | 1254 | 70 | 614 | 1270 | 86 | 582 | 1295 | 63 | 607 | 1263 | 79 | 623 | 1279 | 95 | 591 | 1284 | 52 | 596 | 1252 | 68 | 612 | 1268 | 84 | 580 | 23346 |
| 23346 | 777 | 889 | 281 | 793 | 905 | 249 | 809 | 873 | 265 | 774 | 886 | 278 | 790 | 902 | 246 | 806 | 870 | 262 | 783 | 895 | 287 | 799 | 911 | 255 | 815 | 879 | 271 | 772 | 884 | 276 | 788 | 900 | 244 | 804 | 868 | 260 | 23346 |
| 23346 | 985 | 233 | 729 | 1001 | 201 | 745 | 969 | 217 | 761 | 982 | 230 | 726 | 998 | 198 | 742 | 966 | 214 | 758 | 991 | 239 | 735 | 1007 | 207 | 751 | 975 | 223 | 767 | 980 | 228 | 724 | 996 | 196 | 740 | 964 | 212 | 756 | 23346 |
| 23346 | 185 | 825 | 937 | 153 | 841 | 953 | 169 | 857 | 921 | 182 | 822 | 934 | 150 | 838 | 950 | 166 | 854 | 918 | 191 | 831 | 943 | 159 | 847 | 959 | 175 | 863 | 927 | 180 | 820 | 932 | 148 | 836 | 948 | 164 | 852 | 916 | 23346 |
| 23346 | 1113 | 361 | 473 | 1129 | 377 | 441 | 1145 | 345 | 457 | 1110 | 358 | 470 | 1126 | 374 | 438 | 1142 | 342 | 454 | 1119 | 367 | 479 | 1135 | 383 | 447 | 1151 | 351 | 463 | 1108 | 356 | 468 | 1124 | 372 | 436 | 1140 | 340 | 452 | 23346 |
| 23346 | 313 | 569 | 1065 | 329 | 537 | 1081 | 297 | 553 | 1097 | 310 | 566 | 1062 | 326 | 534 | 1078 | 294 | 550 | 1094 | 319 | 575 | 1071 | 335 | 543 | 1087 | 303 | 559 | 1103 | 308 | 564 | 1060 | 324 | 532 | 1076 | 292 | 548 | 1092 | 23346 |
| 23346 | 521 | 1017 | 409 | 489 | 1033 | 425 | 505 | 1049 | 393 | 518 | 1014 | 406 | 486 | 1030 | 422 | 502 | 1046 | 390 | 527 | 1023 | 415 | 495 | 1039 | 431 | 511 | 1055 | 399 | 516 | 1012 | 404 | 484 | 1028 | 420 | 500 | 1044 | 388 | 23346 |
| | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |

In this case, all the 9 × 9 blocks are **pan diagonal** magic squares of order 9 with different magic square sums. In each case, the 3 × 3 blocks are **semi-magic** squares of order 3 with equal **semi-magic** sums (in rows and columns). The magic sums of order 9 **pan diagonal** magic squares lead us to a **pan diagonal** magic square of order 4 given in example below:

**Example 2.48.** *Magic sums of* ***pan diagonal*** *magic squares of order 9 lead us to a* ***pan diagonal*** *magic of order 4 given by*

| | | | | | |
|---|---|---|---|---|---|
| | | 23346 | 23346 | 23346 | 23346 |
| | 5823 | 5868 | 5769 | 5886 | 23346 |
| 23346 | 5778 | 5877 | 5832 | 5859 | 23346 |
| 23346 | 5904 | 5787 | 5850 | 5805 | 23346 |
| 23346 | 5841 | 5814 | 5895 | 5796 | 23346 |
| | 23346 | 23346 | 23346 | 23346 | 23346 |

### 2.15.4 Forth Approach: 144 Blocks of Order 3

We observe that in the Examples 2.46 and 2.47 that the 3 × 3 blocks are not magic squares of order 3. In Example 2.47, the 3 × 3 blocks are semi-magic squares in each block of order 9 × 9. On the other side, it is impossible to have all the 3 × 3 blocks of equal sums as 23346 is not divisible by 12, i.e., 23346/12 := 1945.5. Below is a example of a **pan diagonal** magic square of order 36 where each block of order 3 is a magic square giving total 144 blocks of magic squares of order 3:

**Example 2.49.** *A* ***pan diagonal*** *magic square of order 36 formed by 144 blocks of magic squares of order 3 constructed according to Example 1.1 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |
| | 487 | 525 | 452 | 385 | 351 | 422 | 600 | 562 | 635 | 828 | 790 | 863 | 930 | 964 | 893 | 715 | 753 | 680 | 145 | 111 | 182 | 259 | 297 | 224 | 42 | 76 | 5 | 1134 | 1168 | 1097 | 1020 | 982 | 1055 | 1237 | 1203 | 1274 | 23346 |
| 23346 | 453 | 488 | 523 | 423 | 386 | 349 | 634 | 599 | 564 | 862 | 827 | 792 | 892 | 929 | 966 | 681 | 716 | 751 | 183 | 146 | 109 | 225 | 260 | 295 | 4 | 41 | 78 | 1096 | 1133 | 1170 | 1054 | 1019 | 984 | 1275 | 1238 | 1201 | 23346 |
| 23346 | 524 | 451 | 489 | 350 | 421 | 387 | 563 | 636 | 598 | 791 | 864 | 826 | 965 | 894 | 928 | 752 | 679 | 717 | 110 | 181 | 147 | 296 | 223 | 261 | 77 | 6 | 40 | 1169 | 1098 | 1132 | 983 | 1056 | 1018 | 1202 | 1273 | 1239 | 23346 |
| 23346 | 601 | 567 | 638 | 492 | 526 | 455 | 379 | 345 | 416 | 714 | 748 | 677 | 823 | 789 | 860 | 936 | 970 | 899 | 43 | 81 | 8 | 150 | 112 | 185 | 253 | 291 | 218 | 1236 | 1198 | 1271 | 1129 | 1167 | 1094 | 1026 | 988 | 1061 | 23346 |
| 23346 | 639 | 602 | 565 | 454 | 491 | 528 | 417 | 380 | 343 | 676 | 713 | 750 | 861 | 824 | 787 | 898 | 935 | 972 | 9 | 44 | 79 | 184 | 149 | 114 | 219 | 254 | 289 | 1270 | 1235 | 1200 | 1095 | 1130 | 1165 | 1060 | 1025 | 990 | 23346 |
| 23346 | 566 | 637 | 603 | 527 | 456 | 490 | 344 | 415 | 381 | 749 | 678 | 712 | 788 | 859 | 825 | 971 | 900 | 934 | 80 | 7 | 45 | 113 | 186 | 148 | 290 | 217 | 255 | 1199 | 1272 | 1234 | 1166 | 1093 | 1131 | 989 | 1062 | 1024 | 23346 |
| 23346 | 384 | 346 | 419 | 595 | 561 | 632 | 493 | 531 | 458 | 931 | 969 | 896 | 720 | 754 | 683 | 822 | 784 | 857 | 258 | 292 | 221 | 37 | 75 | 2 | 151 | 117 | 188 | 1021 | 987 | 1058 | 1242 | 1204 | 1277 | 1128 | 1162 | 1091 | 23346 |
| 23346 | 418 | 383 | 348 | 633 | 596 | 559 | 459 | 494 | 529 | 897 | 932 | 967 | 682 | 719 | 756 | 856 | 821 | 786 | 220 | 257 | 294 | 3 | 38 | 73 | 189 | 152 | 115 | 1059 | 1022 | 985 | 1276 | 1241 | 1206 | 1090 | 1127 | 1164 | 23346 |
| 23346 | 347 | 420 | 382 | 560 | 631 | 597 | 530 | 457 | 495 | 968 | 895 | 933 | 755 | 684 | 718 | 785 | 858 | 820 | 293 | 222 | 256 | 74 | 1 | 39 | 116 | 187 | 153 | 986 | 1057 | 1023 | 1205 | 1278 | 1240 | 1163 | 1092 | 1126 | 23346 |
| 23346 | 162 | 124 | 197 | 264 | 298 | 227 | 49 | 87 | 14 | 1117 | 1155 | 1082 | 1015 | 981 | 1052 | 1230 | 1192 | 1265 | 504 | 538 | 467 | 390 | 352 | 425 | 607 | 573 | 644 | 811 | 777 | 848 | 925 | 963 | 890 | 708 | 742 | 671 | 23346 |
| 23346 | 196 | 161 | 126 | 226 | 263 | 300 | 15 | 50 | 85 | 1083 | 1118 | 1153 | 1053 | 1016 | 979 | 1264 | 1229 | 1194 | 466 | 503 | 540 | 424 | 389 | 354 | 645 | 608 | 571 | 849 | 812 | 775 | 891 | 926 | 961 | 670 | 707 | 744 | 23346 |
| 23346 | 125 | 198 | 160 | 299 | 228 | 262 | 86 | 13 | 51 | 1154 | 1081 | 1119 | 980 | 1051 | 1017 | 1193 | 1266 | 1228 | 539 | 468 | 502 | 353 | 426 | 388 | 572 | 643 | 609 | 776 | 847 | 813 | 962 | 889 | 927 | 743 | 672 | 706 | 23346 |
| 23346 | 48 | 82 | 11 | 157 | 123 | 194 | 270 | 304 | 233 | 1231 | 1197 | 1268 | 1122 | 1156 | 1085 | 1009 | 975 | 1046 | 606 | 568 | 641 | 499 | 537 | 464 | 396 | 358 | 431 | 709 | 747 | 674 | 816 | 778 | 851 | 919 | 957 | 884 | 23346 |
| 23346 | 10 | 47 | 84 | 195 | 158 | 121 | 232 | 269 | 306 | 1269 | 1232 | 1195 | 1084 | 1121 | 1158 | 1047 | 1010 | 973 | 640 | 605 | 570 | 465 | 500 | 535 | 430 | 395 | 360 | 675 | 710 | 745 | 850 | 815 | 780 | 885 | 920 | 955 | 23346 |
| 23346 | 83 | 12 | 46 | 122 | 193 | 159 | 305 | 234 | 268 | 1196 | 1267 | 1233 | 1157 | 1086 | 1120 | 974 | 1045 | 1011 | 569 | 642 | 604 | 536 | 463 | 501 | 359 | 432 | 394 | 746 | 673 | 711 | 779 | 852 | 814 | 956 | 883 | 921 | 23346 |
| 23346 | 265 | 303 | 230 | 54 | 88 | 17 | 156 | 118 | 191 | 1014 | 976 | 1049 | 1225 | 1191 | 1262 | 1123 | 1161 | 1088 | 391 | 357 | 428 | 612 | 574 | 647 | 498 | 532 | 461 | 924 | 958 | 887 | 703 | 741 | 668 | 817 | 783 | 854 | 23346 |
| 23346 | 231 | 266 | 301 | 16 | 53 | 90 | 190 | 155 | 120 | 1048 | 1013 | 978 | 1263 | 1226 | 1189 | 1089 | 1124 | 1159 | 429 | 392 | 355 | 646 | 611 | 576 | 460 | 497 | 534 | 886 | 923 | 960 | 669 | 704 | 739 | 855 | 818 | 781 | 23346 |
| 23346 | 302 | 229 | 267 | 89 | 18 | 52 | 119 | 192 | 154 | 977 | 1050 | 1012 | 1190 | 1261 | 1227 | 1160 | 1087 | 1125 | 356 | 427 | 393 | 575 | 648 | 610 | 533 | 462 | 496 | 959 | 888 | 922 | 740 | 667 | 705 | 782 | 853 | 819 | 23346 |
| 23346 | 1152 | 1186 | 1115 | 1038 | 1000 | 1073 | 1255 | 1221 | 1292 | 163 | 129 | 200 | 277 | 315 | 242 | 60 | 94 | 23 | 810 | 772 | 845 | 912 | 946 | 875 | 697 | 735 | 662 | 469 | 507 | 434 | 367 | 333 | 404 | 582 | 544 | 617 | 23346 |
| 23346 | 1114 | 1151 | 1188 | 1072 | 1037 | 1002 | 1293 | 1256 | 1219 | 201 | 164 | 127 | 243 | 278 | 313 | 22 | 59 | 96 | 844 | 809 | 774 | 874 | 911 | 948 | 663 | 698 | 733 | 435 | 470 | 505 | 405 | 368 | 331 | 616 | 581 | 546 | 23346 |
| 23346 | 1187 | 1116 | 1150 | 1001 | 1074 | 1036 | 1220 | 1291 | 1257 | 128 | 199 | 165 | 314 | 241 | 279 | 95 | 24 | 58 | 773 | 846 | 808 | 947 | 876 | 910 | 734 | 661 | 699 | 506 | 433 | 471 | 332 | 403 | 369 | 545 | 618 | 580 | 23346 |
| 23346 | 1254 | 1216 | 1289 | 1147 | 1185 | 1112 | 1044 | 1006 | 1079 | 61 | 99 | 26 | 168 | 130 | 203 | 271 | 309 | 236 | 696 | 730 | 659 | 805 | 771 | 842 | 918 | 952 | 881 | 583 | 549 | 620 | 474 | 508 | 437 | 361 | 327 | 398 | 23346 |
| 23346 | 1288 | 1253 | 1218 | 1113 | 1148 | 1183 | 1078 | 1043 | 1008 | 27 | 62 | 97 | 202 | 167 | 132 | 237 | 272 | 307 | 658 | 695 | 732 | 843 | 806 | 769 | 880 | 917 | 954 | 621 | 584 | 547 | 436 | 473 | 510 | 399 | 362 | 325 | 23346 |
| 23346 | 1217 | 1290 | 1252 | 1184 | 1111 | 1149 | 1007 | 1080 | 1042 | 98 | 25 | 63 | 131 | 204 | 166 | 308 | 235 | 273 | 731 | 660 | 694 | 770 | 841 | 807 | 953 | 882 | 916 | 548 | 619 | 585 | 509 | 438 | 472 | 326 | 397 | 363 | 23346 |
| 23346 | 1039 | 1005 | 1076 | 1260 | 1222 | 1295 | 1146 | 1180 | 1109 | 276 | 310 | 239 | 55 | 93 | 20 | 169 | 135 | 206 | 913 | 951 | 878 | 702 | 736 | 665 | 804 | 766 | 839 | 366 | 328 | 401 | 577 | 543 | 614 | 475 | 513 | 440 | 23346 |
| 23346 | 1077 | 1040 | 1003 | 1294 | 1259 | 1224 | 1108 | 1145 | 1182 | 238 | 275 | 312 | 21 | 56 | 91 | 207 | 170 | 133 | 879 | 914 | 949 | 664 | 701 | 738 | 838 | 803 | 768 | 400 | 365 | 330 | 615 | 578 | 541 | 441 | 476 | 511 | 23346 |
| 23346 | 1004 | 1075 | 1041 | 1223 | 1296 | 1258 | 1181 | 1110 | 1144 | 311 | 240 | 274 | 92 | 19 | 57 | 134 | 205 | 171 | 950 | 877 | 915 | 737 | 666 | 700 | 767 | 840 | 802 | 329 | 402 | 364 | 542 | 613 | 579 | 512 | 439 | 477 | 23346 |
| 23346 | 793 | 759 | 830 | 907 | 945 | 872 | 690 | 724 | 653 | 486 | 520 | 449 | 372 | 334 | 407 | 589 | 555 | 626 | 1135 | 1173 | 1100 | 1033 | 999 | 1070 | 1248 | 1210 | 1283 | 180 | 142 | 215 | 282 | 316 | 245 | 67 | 105 | 32 | 23346 |
| 23346 | 831 | 794 | 757 | 873 | 908 | 943 | 652 | 689 | 726 | 448 | 485 | 522 | 406 | 371 | 336 | 627 | 590 | 553 | 1101 | 1136 | 1171 | 1071 | 1034 | 997 | 1282 | 1247 | 1212 | 214 | 179 | 144 | 244 | 281 | 318 | 33 | 68 | 103 | 23346 |
| 23346 | 758 | 829 | 795 | 944 | 871 | 909 | 725 | 654 | 688 | 521 | 450 | 484 | 335 | 408 | 370 | 554 | 625 | 591 | 1172 | 1099 | 1137 | 998 | 1069 | 1035 | 1211 | 1284 | 1246 | 143 | 216 | 178 | 317 | 246 | 280 | 104 | 31 | 69 | 23346 |
| 23346 | 691 | 729 | 656 | 798 | 760 | 833 | 901 | 939 | 866 | 588 | 550 | 623 | 481 | 519 | 446 | 378 | 340 | 413 | 1249 | 1215 | 1286 | 1140 | 1174 | 1103 | 1027 | 993 | 1064 | 66 | 100 | 29 | 175 | 141 | 212 | 288 | 322 | 251 | 23346 |
| 23346 | 657 | 692 | 727 | 832 | 797 | 762 | 867 | 902 | 937 | 622 | 587 | 552 | 447 | 482 | 517 | 412 | 377 | 342 | 1287 | 1250 | 1213 | 1102 | 1139 | 1176 | 1065 | 1028 | 991 | 28 | 65 | 102 | 213 | 176 | 139 | 250 | 287 | 324 | 23346 |
| 23346 | 728 | 655 | 693 | 761 | 834 | 796 | 938 | 865 | 903 | 551 | 624 | 586 | 518 | 445 | 483 | 341 | 414 | 376 | 1214 | 1285 | 1251 | 1175 | 1104 | 1138 | 992 | 1063 | 1029 | 101 | 30 | 64 | 140 | 211 | 177 | 323 | 252 | 286 | 23346 |
| 23346 | 906 | 940 | 869 | 685 | 723 | 650 | 799 | 765 | 836 | 373 | 339 | 410 | 594 | 556 | 629 | 480 | 514 | 443 | 1032 | 994 | 1067 | 1243 | 1209 | 1280 | 1141 | 1179 | 1106 | 283 | 321 | 248 | 72 | 106 | 35 | 174 | 136 | 209 | 23346 |
| 23346 | 868 | 905 | 942 | 651 | 686 | 721 | 837 | 800 | 763 | 411 | 374 | 337 | 628 | 593 | 558 | 442 | 479 | 516 | 1066 | 1031 | 996 | 1281 | 1244 | 1207 | 1107 | 1142 | 1177 | 249 | 284 | 319 | 34 | 71 | 108 | 208 | 173 | 138 | 23346 |
| 23346 | 941 | 870 | 904 | 722 | 649 | 687 | 764 | 835 | 801 | 338 | 409 | 375 | 557 | 630 | 592 | 515 | 444 | 478 | 995 | 1068 | 1030 | 1208 | 1279 | 1245 | 1178 | 1105 | 1143 | 320 | 247 | 285 | 107 | 36 | 70 | 137 | 210 | 172 | 23346 |
| | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |

In this case, each block of order 3 is a magic square with different magic sums, resulting in a **pan diagonal** magic of order 12 given in example below.

**Example 2.50.** *16 blocks of magic squares of order 3 constructed using Example 1.1, we get a* ***pan diagonal magic square*** *of order 12 given by*

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |
| | 1464 | 1158 | 1797 | 2481 | 2787 | 2148 | 438 | 780 | 123 | 3399 | 3057 | 3714 | 23346 |
| 23346 | 1806 | 1473 | 1140 | 2139 | 2472 | 2805 | 132 | 447 | 762 | 3705 | 3390 | 3075 | 23346 |
| 23346 | 1149 | 1788 | 1482 | 2796 | 2157 | 2463 | 771 | 114 | 456 | 3066 | 3723 | 3381 | 23346 |
| 23346 | 483 | 789 | 150 | 3354 | 3048 | 3687 | 1509 | 1167 | 1824 | 2436 | 2778 | 2121 | 23346 |
| 23346 | 141 | 474 | 807 | 3696 | 3363 | 3030 | 1815 | 1500 | 1185 | 2130 | 2445 | 2760 | 23346 |
| 23346 | 798 | 159 | 465 | 3039 | 3678 | 3372 | 1176 | 1833 | 1491 | 2769 | 2112 | 2454 | 23346 |
| 23346 | 3453 | 3111 | 3768 | 492 | 834 | 177 | 2427 | 2733 | 2094 | 1410 | 1104 | 1743 | 23346 |
| 23346 | 3759 | 3444 | 3129 | 186 | 501 | 816 | 2085 | 2418 | 2751 | 1752 | 1419 | 1086 | 23346 |
| 23346 | 3120 | 3777 | 3435 | 825 | 168 | 510 | 2742 | 2103 | 2409 | 1095 | 1734 | 1428 | 23346 |
| 23346 | 2382 | 2724 | 2067 | 1455 | 1113 | 1770 | 3408 | 3102 | 3741 | 537 | 843 | 204 | 23346 |
| 23346 | 2076 | 2391 | 2706 | 1761 | 1446 | 1131 | 3750 | 3417 | 3084 | 195 | 528 | 861 | 23346 |
| 23346 | 2715 | 2058 | 2400 | 1122 | 1779 | 1437 | 3093 | 3732 | 3426 | 852 | 213 | 519 | 23346 |
| | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |

The above magic square of order 36 is also formed by 16 blocks of magic square of order 9 resulting in a pan magic square of order 4:

**Example 2.51.** *Magic sums of* ***pan diagonal*** *magic squares of order 9 of Exemple 2.49 lead us to a* ***pan diagonal*** *magic of order 4 given by*

| | | | | | |
|---|---|---|---|---|---|
| | | 23346 | 23346 | 23346 | 23346 |
| | 4419 | 7416 | 1341 | 10170 | 23346 |
| 23346 | 1422 | 10089 | 4500 | 7335 | 23346 |
| 23346 | 10332 | 1503 | 7254 | 4257 | 23346 |
| 23346 | 7173 | 4338 | 10251 | 1584 | 23346 |
| | 23346 | 23346 | 23346 | 23346 | 23346 |

### 2.15.5 Fifth Approach: 36 Blocks of Order 6

**Example 2.52.** *A magic square of order 36 formed by 36 blocks of magic squares of order 6 constructed according to Example 1.4 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 23346 |
| 1 | 1225 | 1224 | 1153 | 72 | 216 | 2 | 1226 | 1223 | 1154 | 71 | 215 | 3 | 1227 | 1222 | 1155 | 70 | 214 | 4 | 1228 | 1221 | 1156 | 69 | 213 | 5 | 1229 | 1220 | 1157 | 68 | 212 | 6 | 1230 | 1219 | 1158 | 67 | 211 | 23346 |
| 1080 | 288 | 1008 | 289 | 361 | 865 | 1079 | 287 | 1007 | 290 | 362 | 866 | 1078 | 286 | 1006 | 291 | 363 | 867 | 1077 | 285 | 1005 | 292 | 364 | 868 | 1076 | 284 | 1004 | 293 | 365 | 869 | 1075 | 283 | 1003 | 294 | 366 | 870 | 23346 |
| 864 | 793 | 505 | 576 | 720 | 433 | 863 | 794 | 506 | 575 | 719 | 434 | 862 | 795 | 507 | 574 | 718 | 435 | 861 | 796 | 508 | 573 | 717 | 436 | 860 | 797 | 509 | 572 | 716 | 437 | 859 | 798 | 510 | 571 | 715 | 438 | 23346 |
| 648 | 504 | 721 | 792 | 577 | 649 | 647 | 503 | 722 | 791 | 578 | 650 | 646 | 502 | 723 | 790 | 579 | 651 | 645 | 501 | 724 | 789 | 580 | 652 | 644 | 500 | 725 | 788 | 581 | 653 | 643 | 499 | 726 | 787 | 582 | 654 | 23346 |
| 217 | 936 | 360 | 937 | 1009 | 432 | 218 | 935 | 359 | 938 | 1010 | 431 | 219 | 934 | 358 | 939 | 1011 | 430 | 220 | 933 | 357 | 940 | 1012 | 429 | 221 | 932 | 356 | 941 | 1013 | 428 | 222 | 931 | 355 | 942 | 1014 | 427 | 23346 |
| 1081 | 145 | 73 | 144 | 1152 | 1296 | 1082 | 146 | 74 | 143 | 1151 | 1295 | 1083 | 147 | 75 | 142 | 1150 | 1294 | 1084 | 148 | 76 | 141 | 1149 | 1293 | 1085 | 149 | 77 | 140 | 1148 | 1292 | 1086 | 150 | 78 | 139 | 1147 | 1291 | 23346 |
| 7 | 1231 | 1218 | 1159 | 66 | 210 | 8 | 1232 | 1217 | 1160 | 65 | 209 | 9 | 1233 | 1216 | 1161 | 64 | 208 | 10 | 1234 | 1215 | 1162 | 63 | 207 | 11 | 1235 | 1214 | 1163 | 62 | 206 | 12 | 1236 | 1213 | 1164 | 61 | 205 | 23346 |
| 1074 | 282 | 1002 | 295 | 367 | 871 | 1073 | 281 | 1001 | 296 | 368 | 872 | 1072 | 280 | 1000 | 297 | 369 | 873 | 1071 | 279 | 999 | 298 | 370 | 874 | 1070 | 278 | 998 | 299 | 371 | 875 | 1069 | 277 | 997 | 300 | 372 | 876 | 23346 |
| 858 | 799 | 511 | 570 | 714 | 439 | 857 | 800 | 512 | 569 | 713 | 440 | 856 | 801 | 513 | 568 | 712 | 441 | 855 | 802 | 514 | 567 | 711 | 442 | 854 | 803 | 515 | 566 | 710 | 443 | 853 | 804 | 516 | 565 | 709 | 444 | 23346 |
| 642 | 498 | 727 | 786 | 583 | 655 | 641 | 497 | 728 | 785 | 584 | 656 | 640 | 496 | 729 | 784 | 585 | 657 | 639 | 495 | 730 | 783 | 586 | 658 | 638 | 494 | 731 | 782 | 587 | 659 | 637 | 493 | 732 | 781 | 588 | 660 | 23346 |
| 223 | 930 | 354 | 943 | 1015 | 426 | 224 | 929 | 353 | 944 | 1016 | 425 | 225 | 928 | 352 | 945 | 1017 | 424 | 226 | 927 | 351 | 946 | 1018 | 423 | 227 | 926 | 350 | 947 | 1019 | 422 | 228 | 925 | 349 | 948 | 1020 | 421 | 23346 |
| 1087 | 151 | 79 | 138 | 1146 | 1290 | 1088 | 152 | 80 | 137 | 1145 | 1289 | 1089 | 153 | 81 | 136 | 1144 | 1288 | 1090 | 154 | 82 | 135 | 1143 | 1287 | 1091 | 155 | 83 | 134 | 1142 | 1286 | 1092 | 156 | 84 | 133 | 1141 | 1285 | 23346 |
| 13 | 1237 | 1212 | 1165 | 60 | 204 | 14 | 1238 | 1211 | 1166 | 59 | 203 | 15 | 1239 | 1210 | 1167 | 58 | 202 | 16 | 1240 | 1209 | 1168 | 57 | 201 | 17 | 1241 | 1208 | 1169 | 56 | 200 | 18 | 1242 | 1207 | 1170 | 55 | 199 | 23346 |
| 1068 | 276 | 996 | 301 | 373 | 877 | 1067 | 275 | 995 | 302 | 374 | 878 | 1066 | 274 | 994 | 303 | 375 | 879 | 1065 | 273 | 993 | 304 | 376 | 880 | 1064 | 272 | 992 | 305 | 377 | 881 | 1063 | 271 | 991 | 306 | 378 | 882 | 23346 |
| 852 | 805 | 517 | 564 | 708 | 445 | 851 | 806 | 518 | 563 | 707 | 446 | 850 | 807 | 519 | 562 | 706 | 447 | 849 | 808 | 520 | 561 | 705 | 448 | 848 | 809 | 521 | 560 | 704 | 449 | 847 | 810 | 522 | 559 | 703 | 450 | 23346 |
| 636 | 492 | 733 | 780 | 589 | 661 | 635 | 491 | 734 | 779 | 590 | 662 | 634 | 490 | 735 | 778 | 591 | 663 | 633 | 489 | 736 | 777 | 592 | 664 | 632 | 488 | 737 | 776 | 593 | 665 | 631 | 487 | 738 | 775 | 594 | 666 | 23346 |
| 229 | 924 | 348 | 949 | 1021 | 420 | 230 | 923 | 347 | 950 | 1022 | 419 | 231 | 922 | 346 | 951 | 1023 | 418 | 232 | 921 | 345 | 952 | 1024 | 417 | 233 | 920 | 344 | 953 | 1025 | 416 | 234 | 919 | 343 | 954 | 1026 | 415 | 23346 |
| 1093 | 157 | 85 | 132 | 1140 | 1284 | 1094 | 158 | 86 | 131 | 1139 | 1283 | 1095 | 159 | 87 | 130 | 1138 | 1282 | 1096 | 160 | 88 | 129 | 1137 | 1281 | 1097 | 161 | 89 | 128 | 1136 | 1280 | 1098 | 162 | 90 | 127 | 1135 | 1279 | 23346 |
| 19 | 1243 | 1206 | 1171 | 54 | 198 | 20 | 1244 | 1205 | 1172 | 53 | 197 | 21 | 1245 | 1204 | 1173 | 52 | 196 | 22 | 1246 | 1203 | 1174 | 51 | 195 | 23 | 1247 | 1202 | 1175 | 50 | 194 | 24 | 1248 | 1201 | 1176 | 49 | 193 | 23346 |
| 1062 | 270 | 990 | 307 | 379 | 883 | 1061 | 269 | 989 | 308 | 380 | 884 | 1060 | 268 | 988 | 309 | 381 | 885 | 1059 | 267 | 987 | 310 | 382 | 886 | 1058 | 266 | 986 | 311 | 383 | 887 | 1057 | 265 | 985 | 312 | 384 | 888 | 23346 |
| 846 | 811 | 523 | 558 | 702 | 451 | 845 | 812 | 524 | 557 | 701 | 452 | 844 | 813 | 525 | 556 | 700 | 453 | 843 | 814 | 526 | 555 | 699 | 454 | 842 | 815 | 527 | 554 | 698 | 455 | 841 | 816 | 528 | 553 | 697 | 456 | 23346 |
| 630 | 486 | 739 | 774 | 595 | 667 | 629 | 485 | 740 | 773 | 596 | 668 | 628 | 484 | 741 | 772 | 597 | 669 | 627 | 483 | 742 | 771 | 598 | 670 | 626 | 482 | 743 | 770 | 599 | 671 | 625 | 481 | 744 | 769 | 600 | 672 | 23346 |
| 235 | 918 | 342 | 955 | 1027 | 414 | 236 | 917 | 341 | 956 | 1028 | 413 | 237 | 916 | 340 | 957 | 1029 | 412 | 238 | 915 | 339 | 958 | 1030 | 411 | 239 | 914 | 338 | 959 | 1031 | 410 | 240 | 913 | 337 | 960 | 1032 | 409 | 23346 |
| 1099 | 163 | 91 | 126 | 1134 | 1278 | 1100 | 164 | 92 | 125 | 1133 | 1277 | 1101 | 165 | 93 | 124 | 1132 | 1276 | 1102 | 166 | 94 | 123 | 1131 | 1275 | 1103 | 167 | 95 | 122 | 1130 | 1274 | 1104 | 168 | 96 | 121 | 1129 | 1273 | 23346 |
| 25 | 1249 | 1200 | 1177 | 48 | 192 | 26 | 1250 | 1199 | 1178 | 47 | 191 | 27 | 1251 | 1198 | 1179 | 46 | 190 | 28 | 1252 | 1197 | 1180 | 45 | 189 | 29 | 1253 | 1196 | 1181 | 44 | 188 | 30 | 1254 | 1195 | 1182 | 43 | 187 | 23346 |
| 1056 | 264 | 984 | 313 | 385 | 889 | 1055 | 263 | 983 | 314 | 386 | 890 | 1054 | 262 | 982 | 315 | 387 | 891 | 1053 | 261 | 981 | 316 | 388 | 892 | 1052 | 260 | 980 | 317 | 389 | 893 | 1051 | 259 | 979 | 318 | 390 | 894 | 23346 |
| 840 | 817 | 529 | 552 | 696 | 457 | 839 | 818 | 530 | 551 | 695 | 458 | 838 | 819 | 531 | 550 | 694 | 459 | 837 | 820 | 532 | 549 | 693 | 460 | 836 | 821 | 533 | 548 | 692 | 461 | 835 | 822 | 534 | 547 | 691 | 462 | 23346 |
| 624 | 480 | 745 | 768 | 601 | 673 | 623 | 479 | 746 | 767 | 602 | 674 | 622 | 478 | 747 | 766 | 603 | 675 | 621 | 477 | 748 | 765 | 604 | 676 | 620 | 476 | 749 | 764 | 605 | 677 | 619 | 475 | 750 | 763 | 606 | 678 | 23346 |
| 241 | 912 | 336 | 961 | 1033 | 408 | 242 | 911 | 335 | 962 | 1034 | 407 | 243 | 910 | 334 | 963 | 1035 | 406 | 244 | 909 | 333 | 964 | 1036 | 405 | 245 | 908 | 332 | 965 | 1037 | 404 | 246 | 907 | 331 | 966 | 1038 | 403 | 23346 |
| 1105 | 169 | 97 | 120 | 1128 | 1272 | 1106 | 170 | 98 | 119 | 1127 | 1271 | 1107 | 171 | 99 | 118 | 1126 | 1270 | 1108 | 172 | 100 | 117 | 1125 | 1269 | 1109 | 173 | 101 | 116 | 1124 | 1268 | 1110 | 174 | 102 | 115 | 1123 | 1267 | 23346 |
| 31 | 1255 | 1194 | 1183 | 42 | 186 | 32 | 1256 | 1193 | 1184 | 41 | 185 | 33 | 1257 | 1192 | 1185 | 40 | 184 | 34 | 1258 | 1191 | 1186 | 39 | 183 | 35 | 1259 | 1190 | 1187 | 38 | 182 | 36 | 1260 | 1189 | 1188 | 37 | 181 | 23346 |
| 1050 | 258 | 978 | 319 | 391 | 895 | 1049 | 257 | 977 | 320 | 392 | 896 | 1048 | 256 | 976 | 321 | 393 | 897 | 1047 | 255 | 975 | 322 | 394 | 898 | 1046 | 254 | 974 | 323 | 395 | 899 | 1045 | 253 | 973 | 324 | 396 | 900 | 23346 |
| 834 | 823 | 535 | 546 | 690 | 463 | 833 | 824 | 536 | 545 | 689 | 464 | 832 | 825 | 537 | 544 | 688 | 465 | 831 | 826 | 538 | 543 | 687 | 466 | 830 | 827 | 539 | 542 | 686 | 467 | 829 | 828 | 540 | 541 | 685 | 468 | 23346 |
| 618 | 474 | 751 | 762 | 607 | 679 | 617 | 473 | 752 | 761 | 608 | 680 | 616 | 472 | 753 | 760 | 609 | 681 | 615 | 471 | 754 | 759 | 610 | 682 | 614 | 470 | 755 | 758 | 611 | 683 | 613 | 469 | 756 | 757 | 612 | 684 | 23346 |
| 247 | 906 | 330 | 967 | 1039 | 402 | 248 | 905 | 329 | 968 | 1040 | 401 | 249 | 904 | 328 | 969 | 1041 | 400 | 250 | 903 | 327 | 970 | 1042 | 399 | 251 | 902 | 326 | 971 | 1043 | 398 | 252 | 901 | 325 | 972 | 1044 | 397 | 23346 |
| 1111 | 175 | 103 | 114 | 1122 | 1266 | 1112 | 176 | 104 | 113 | 1121 | 1265 | 1113 | 177 | 105 | 112 | 1120 | 1264 | 1114 | 178 | 106 | 111 | 1119 | 1263 | 1115 | 179 | 107 | 110 | 1118 | 1262 | 1116 | 180 | 108 | 109 | 1117 | 1261 | 23346 |
| 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |

The 36 blocks of order 6 are magic squares of equal magic sums, $S_{6\times 6} := 3891$.

### 2.15.6 Sixth Approach: 16 Blocks of Bimagic Squares of Order 9

**Example 2.53.** *A magic square of order 36 formed by 16 blocks of* ***bimagic*** *squares of order 9 with different magic sums constructed according to Example 1.9 is given by*

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 23346 |
| 39 | 215 | 343 | 499 | 627 | 767 | 923 | 1051 | 1215 | 76 | 252 | 380 | 536 | 664 | 804 | 960 | 1088 | 1252 | 1 | 177 | 305 | 461 | 589 | 729 | 885 | 1013 | 1177 | 110 | 286 | 414 | 570 | 698 | 838 | 994 | 1122 | 1286 | 23346 |
| 491 | 619 | 783 | 903 | 1079 | 1207 | 67 | 195 | 335 | 528 | 656 | 820 | 940 | 1116 | 1244 | 104 | 232 | 372 | 453 | 581 | 745 | 865 | 1041 | 1169 | 29 | 157 | 297 | 562 | 690 | 854 | 974 | 1150 | 1278 | 138 | 266 | 406 | 23346 |
| 931 | 1059 | 1199 | 59 | 187 | 351 | 471 | 647 | 775 | 968 | 1096 | 1236 | 96 | 224 | 388 | 508 | 684 | 812 | 893 | 1021 | 1161 | 21 | 149 | 313 | 433 | 609 | 737 | 1002 | 1130 | 1270 | 130 | 258 | 422 | 542 | 718 | 846 | 23346 |
| 359 | 55 | 183 | 771 | 479 | 643 | 1195 | 927 | 1067 | 396 | 92 | 220 | 808 | 516 | 680 | 1232 | 964 | 1104 | 321 | 17 | 145 | 733 | 441 | 605 | 1157 | 889 | 1029 | 430 | 126 | 254 | 842 | 550 | 714 | 1266 | 998 | 1138 | 23346 |
| 763 | 495 | 635 | 1223 | 919 | 1047 | 339 | 47 | 211 | 800 | 532 | 672 | 1260 | 956 | 1084 | 376 | 84 | 248 | 725 | 457 | 597 | 1185 | 881 | 1009 | 301 | 9 | 173 | 834 | 566 | 706 | 1294 | 990 | 1118 | 410 | 118 | 282 | 23346 |
| 1203 | 911 | 1075 | 331 | 63 | 203 | 791 | 487 | 615 | 1240 | 948 | 1112 | 368 | 100 | 240 | 828 | 524 | 652 | 1165 | 873 | 1037 | 293 | 25 | 165 | 753 | 449 | 577 | 1274 | 982 | 1146 | 402 | 134 | 274 | 862 | 558 | 686 | 23346 |
| 199 | 327 | 71 | 623 | 787 | 483 | 1071 | 1211 | 907 | 236 | 364 | 108 | 660 | 824 | 520 | 1108 | 1248 | 944 | 161 | 289 | 33 | 585 | 749 | 445 | 1033 | 1173 | 869 | 270 | 398 | 142 | 694 | 858 | 554 | 1142 | 1282 | 978 | 23346 |
| 639 | 779 | 475 | 1063 | 1191 | 935 | 191 | 355 | 51 | 676 | 816 | 512 | 1100 | 1228 | 972 | 228 | 392 | 88 | 601 | 741 | 437 | 1025 | 1153 | 897 | 153 | 317 | 13 | 710 | 850 | 546 | 1134 | 1262 | 1006 | 262 | 426 | 122 | 23346 |
| 1055 | 1219 | 915 | 207 | 347 | 43 | 631 | 759 | 503 | 1092 | 1256 | 952 | 244 | 384 | 80 | 668 | 796 | 540 | 1017 | 1181 | 877 | 169 | 309 | 5 | 593 | 721 | 465 | 1126 | 1290 | 986 | 278 | 418 | 114 | 702 | 830 | 574 | 23346 |
| 2 | 178 | 306 | 462 | 590 | 730 | 886 | 1014 | 1178 | 109 | 285 | 413 | 569 | 697 | 837 | 993 | 1121 | 1285 | 40 | 216 | 344 | 500 | 628 | 768 | 924 | 1052 | 1216 | 75 | 251 | 379 | 535 | 663 | 803 | 959 | 1087 | 1251 | 23346 |
| 454 | 582 | 746 | 866 | 1042 | 1170 | 30 | 158 | 298 | 561 | 689 | 853 | 973 | 1149 | 1277 | 137 | 265 | 405 | 492 | 620 | 784 | 904 | 1080 | 1208 | 68 | 196 | 336 | 527 | 655 | 819 | 939 | 1115 | 1243 | 103 | 231 | 371 | 23346 |
| 894 | 1022 | 1162 | 22 | 150 | 314 | 434 | 610 | 738 | 1001 | 1129 | 1269 | 129 | 257 | 421 | 541 | 717 | 845 | 932 | 1060 | 1200 | 60 | 188 | 352 | 472 | 648 | 776 | 967 | 1095 | 1235 | 95 | 223 | 387 | 507 | 683 | 811 | 23346 |
| 322 | 18 | 146 | 734 | 442 | 606 | 1158 | 890 | 1030 | 429 | 125 | 253 | 841 | 549 | 713 | 1265 | 997 | 1137 | 360 | 56 | 184 | 772 | 480 | 644 | 1196 | 928 | 1068 | 395 | 91 | 219 | 807 | 515 | 679 | 1231 | 963 | 1103 | 23346 |
| 726 | 458 | 598 | 1186 | 882 | 1010 | 302 | 10 | 174 | 833 | 565 | 705 | 1293 | 989 | 1117 | 409 | 117 | 281 | 764 | 496 | 636 | 1224 | 920 | 1048 | 340 | 48 | 212 | 799 | 531 | 671 | 1259 | 955 | 1083 | 375 | 83 | 247 | 23346 |
| 1166 | 874 | 1038 | 294 | 26 | 166 | 754 | 450 | 578 | 1273 | 981 | 1145 | 401 | 133 | 273 | 861 | 557 | 685 | 1204 | 912 | 1076 | 332 | 64 | 204 | 792 | 488 | 616 | 1239 | 947 | 1111 | 367 | 99 | 239 | 827 | 523 | 651 | 23346 |
| 162 | 290 | 34 | 586 | 750 | 446 | 1034 | 1174 | 870 | 269 | 397 | 141 | 693 | 857 | 553 | 1141 | 1281 | 977 | 200 | 328 | 72 | 624 | 788 | 484 | 1072 | 1212 | 908 | 235 | 363 | 107 | 659 | 823 | 519 | 1107 | 1247 | 943 | 23346 |
| 602 | 742 | 438 | 1026 | 1154 | 898 | 154 | 318 | 14 | 709 | 849 | 545 | 1133 | 1261 | 1005 | 261 | 425 | 121 | 640 | 780 | 476 | 1064 | 1192 | 936 | 192 | 356 | 52 | 675 | 815 | 511 | 1099 | 1227 | 971 | 227 | 391 | 87 | 23346 |
| 1018 | 1182 | 878 | 170 | 310 | 6 | 594 | 722 | 466 | 1125 | 1289 | 985 | 277 | 417 | 113 | 701 | 829 | 573 | 1056 | 1220 | 916 | 208 | 348 | 44 | 632 | 760 | 504 | 1091 | 1255 | 951 | 243 | 383 | 79 | 667 | 795 | 539 | 23346 |
| 112 | 288 | 416 | 572 | 700 | 840 | 996 | 1124 | 1288 | 3 | 179 | 307 | 463 | 591 | 731 | 887 | 1015 | 1179 | 74 | 250 | 378 | 534 | 662 | 802 | 958 | 1086 | 1250 | 37 | 213 | 341 | 497 | 625 | 765 | 921 | 1049 | 1213 | 23346 |
| 564 | 692 | 856 | 976 | 1152 | 1280 | 140 | 268 | 408 | 455 | 583 | 747 | 867 | 1043 | 1171 | 31 | 159 | 299 | 526 | 654 | 818 | 938 | 1114 | 1242 | 102 | 230 | 370 | 489 | 617 | 781 | 901 | 1077 | 1205 | 65 | 193 | 333 | 23346 |
| 1004 | 1132 | 1272 | 132 | 260 | 424 | 544 | 720 | 848 | 895 | 1023 | 1163 | 23 | 151 | 315 | 435 | 611 | 739 | 966 | 1094 | 1234 | 94 | 222 | 386 | 506 | 682 | 810 | 929 | 1057 | 1197 | 57 | 185 | 349 | 469 | 645 | 773 | 23346 |
| 432 | 128 | 256 | 844 | 552 | 716 | 1268 | 1000 | 1140 | 323 | 19 | 147 | 735 | 443 | 607 | 1159 | 891 | 1031 | 394 | 90 | 218 | 806 | 514 | 678 | 1230 | 962 | 1102 | 357 | 53 | 181 | 769 | 477 | 641 | 1193 | 925 | 1065 | 23346 |
| 836 | 568 | 708 | 1296 | 992 | 1120 | 412 | 120 | 284 | 727 | 459 | 599 | 1187 | 883 | 1011 | 303 | 11 | 175 | 798 | 530 | 670 | 1258 | 954 | 1082 | 374 | 82 | 246 | 761 | 493 | 633 | 1221 | 917 | 1045 | 337 | 45 | 209 | 23346 |
| 1276 | 984 | 1148 | 404 | 136 | 276 | 864 | 560 | 688 | 1167 | 875 | 1039 | 295 | 27 | 167 | 755 | 451 | 579 | 1238 | 946 | 1110 | 366 | 98 | 238 | 826 | 522 | 650 | 1201 | 909 | 1073 | 329 | 61 | 201 | 789 | 485 | 613 | 23346 |
| 272 | 400 | 144 | 696 | 860 | 556 | 1144 | 1284 | 980 | 163 | 291 | 35 | 587 | 751 | 447 | 1035 | 1175 | 871 | 234 | 362 | 106 | 658 | 822 | 518 | 1106 | 1246 | 942 | 197 | 325 | 69 | 621 | 785 | 481 | 1069 | 1209 | 905 | 23346 |
| 712 | 852 | 548 | 1136 | 1264 | 1008 | 264 | 428 | 124 | 603 | 743 | 439 | 1027 | 1155 | 899 | 155 | 319 | 15 | 674 | 814 | 510 | 1098 | 1226 | 970 | 226 | 390 | 86 | 637 | 777 | 473 | 1061 | 1189 | 933 | 189 | 353 | 49 | 23346 |
| 1128 | 1292 | 988 | 280 | 420 | 116 | 704 | 832 | 576 | 1019 | 1183 | 879 | 171 | 311 | 7 | 595 | 723 | 467 | 1090 | 1254 | 950 | 242 | 382 | 78 | 666 | 794 | 538 | 1053 | 1217 | 913 | 205 | 345 | 41 | 629 | 757 | 501 | 23346 |
| 73 | 249 | 377 | 533 | 661 | 801 | 957 | 1085 | 1249 | 38 | 214 | 342 | 498 | 626 | 766 | 922 | 1050 | 1214 | 111 | 287 | 415 | 571 | 699 | 839 | 995 | 1123 | 1287 | 4 | 180 | 308 | 464 | 592 | 732 | 888 | 1016 | 1180 | 23346 |
| 525 | 653 | 817 | 937 | 1113 | 1241 | 101 | 229 | 369 | 490 | 618 | 782 | 902 | 1078 | 1206 | 66 | 194 | 334 | 563 | 691 | 855 | 975 | 1151 | 1279 | 139 | 267 | 407 | 456 | 584 | 748 | 868 | 1044 | 1172 | 32 | 160 | 300 | 23346 |
| 965 | 1093 | 1233 | 93 | 221 | 385 | 505 | 681 | 809 | 930 | 1058 | 1198 | 58 | 186 | 350 | 470 | 646 | 774 | 1003 | 1131 | 1271 | 131 | 259 | 423 | 543 | 719 | 847 | 896 | 1024 | 1164 | 24 | 152 | 316 | 436 | 612 | 740 | 23346 |
| 393 | 89 | 217 | 805 | 513 | 677 | 1229 | 961 | 1101 | 358 | 54 | 182 | 770 | 478 | 642 | 1194 | 926 | 1066 | 431 | 127 | 255 | 843 | 551 | 715 | 1267 | 999 | 1139 | 324 | 20 | 148 | 736 | 444 | 608 | 1160 | 892 | 1032 | 23346 |
| 797 | 529 | 669 | 1257 | 953 | 1081 | 373 | 81 | 245 | 762 | 494 | 634 | 1222 | 918 | 1046 | 338 | 46 | 210 | 835 | 567 | 707 | 1295 | 991 | 1119 | 411 | 119 | 283 | 728 | 460 | 600 | 1188 | 884 | 1012 | 304 | 12 | 176 | 23346 |
| 1237 | 945 | 1109 | 365 | 97 | 237 | 825 | 521 | 649 | 1202 | 910 | 1074 | 330 | 62 | 202 | 790 | 486 | 614 | 1275 | 983 | 1147 | 403 | 135 | 275 | 863 | 559 | 687 | 1168 | 876 | 1040 | 296 | 28 | 168 | 756 | 452 | 580 | 23346 |
| 233 | 361 | 105 | 657 | 821 | 517 | 1105 | 1245 | 941 | 198 | 326 | 70 | 622 | 786 | 482 | 1070 | 1210 | 906 | 271 | 399 | 143 | 695 | 859 | 555 | 1143 | 1283 | 979 | 164 | 292 | 36 | 588 | 752 | 448 | 1036 | 1176 | 872 | 23346 |
| 673 | 813 | 509 | 1097 | 1225 | 969 | 225 | 389 | 85 | 638 | 778 | 474 | 1062 | 1190 | 934 | 190 | 354 | 50 | 711 | 851 | 547 | 1135 | 1263 | 1007 | 263 | 427 | 123 | 604 | 744 | 440 | 1028 | 1156 | 900 | 156 | 320 | 16 | 23346 |
| 1089 | 1253 | 949 | 241 | 381 | 77 | 665 | 793 | 537 | 1054 | 1218 | 914 | 206 | 346 | 42 | 630 | 758 | 502 | 1127 | 1291 | 987 | 279 | 419 | 115 | 703 | 831 | 575 | 1020 | 1184 | 880 | 172 | 312 | 8 | 596 | 724 | 468 | 23346 |
| 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 | 23346 |

The 16 blocks of **bimagic** squares of order 9 brings a **pan diagonal** magic square of order 4 given in example below.

**Example 2.54.** *Magic sums of* ***pan diagonal*** *magic squares of order 9 of Exemple 2.49 lead us to a* ***pan diagonal*** *magic of order 4 given by*

| | | | | | |
|---|---|---|---|---|---|
| | | 23346 | 23346 | 23346 | 23346 |
| | 5679 | 6012 | 5337 | 6318 | 23346 |
| 23346 | 5346 | 6309 | 5688 | 6003 | 23346 |
| 23346 | 6336 | 5355 | 5994 | 5661 | 23346 |
| 23346 | 5985 | 5670 | 6327 | 5364 | 23346 |
| | 23346 | 23346 | 23346 | 23346 | 23346 |

Unfortunately, still we are unable to bring **bimagic** square of order 36, only the order 9 magic squares are **bimagic**.

## 3 Final Comments

This paper summarize some of the results done before by author on different ways are writing **block-wise** magic squares. Here, we have rewritten some of them. The construction details can be seen at [31, 32, 33, 34, 36] . This is done for the magic squares of orders 12 to 36, i.e. of orders 12, 15, 16, 18, 20, 21, 24, 25, 27, 28, 30, 32, 33, 35 and 36. In some cases, the magic squares are **pan diagonal** or **bimagic** or **semi-bimagic**.

We observed that in case of magic squares of type $4k$, it is always possible to write a pan diagonal magic square with equal magic sums of pan diagonal magic squares of order 4. This we have done the cases, of orders 8, 12, 16, 20, 24, 28, 32 and 36. Up to order 16 this type of construction is also developed by Candy [5].

In case of orders $6k$, it is also possible to write block-wise magic squares with sum magic squares of order 6. This we have done for the magic squares of orders 12, 18, 24, 30 and 36. In case of orders $3k$, we found two types of situations. One as blocks of equal sums semi-magic squares of order 3 (in rows and columns). This we have done for the magic squares of orders 12, 18, 24, 30 and 36. The second as blocks of magic squares of order 3 with different magic sums. This we have done for the magic squares of orders 15, 21, 27 and 33. Other block-wise situations for the orders 5, 7, 8, 9, 10, 11, 12 and 14 are also considered.

The bimagic squares of orders 8 and 9 given in examples 2.20 and 1.9 are the classical onea constructed in 1891 by G. Pfeffermann [13]. The block-wise **bimagic** squares of orders 16 and 25 is given by author in [22, 26]. The block-wise **bimagic** square of order 32 and **semi-bimagic** square of order 24 can be seen in Taneja [22]. A good collection of bimagic squares are given in a site by C. Boyer [8]. A mathematical study of bimagic squares of orders 16 and 25 is given in [6]. Some ideas of block-wise magic squares can be seen in [5, 15]. Programming calculations of block-wise magic squares up to order 20 are well explained in a site by Danielsson [11].

During past years the author worked with magic squares in different situations. These are given in details below:

### 3.1 Author's Contributions to Magic Squares

The item-wise author's work on magic squares is as follows:

(i) **Digital numbers** magic squares - [16, 17, 18, 19, 20, 21];

(ii) **Block-wise construction** of **bimagic squares** - [22];

(iii) Connections with **genetic tables** and **Shannon's entropy** - [23];

(iv) **Selfie** and **palindromic-type** magic squares - [24];

(v) **Intervally distributed** and **block-wise** magic squares - [25, 26, 27];

(vi) **Multi-digits** magic squares - [28];

(vii) **Perfect square sum** magic squares with **uniformity**, **minimum sum** and **Pythagorean triples**- [29, 30];

(viii) **Block-wise** equal and unequal sums magic squares of orders $3k$, $4k$ and $6k$ - [31, 32, 33, 36];

(ix) **Magic rectangles** in construction of **block-wise pan magic squares** - [34];

(x) **Magic crosses:** repeated and non repeated entries - [35];

(xi) Representations of **letters** and **numbers** With equal sums magic squares of orders 4 and 6 - [37, 38].

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