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# HERMITE – HADAMARD TYPE INEQUALITIES FOR FRACTIONAL INTEGRALS

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Abstract: In the present note, we have established an integral identity and some Hermite-Hadamard type integral inequalities for the fractional integrals.

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## 1. Introduction

The $f\colon I \subseteq R \to R$ be a convex function defined on the interval $I$ of real numbers and $a < b$. The following double inequality:

$$f\left(\frac{a+b}{2}\right) \le \frac{1}{b-a}\int_a^b f(x)\,dx \le \frac{f(a)+f(b)}{2}$$

is well known in the literature as Hermite – Hadamard's inequality.

Recently, many others [1–23] developed and discussed Hermite – Hadamard's inequality in terms of refinements, counterparts, generalizations and new Hermite - Hadamard's type inequalities.

In 2007, Varošanec [22] introduced a large class of non-negative functions, the so-called $h$ - convex functions. This class contains several well-known classes of functions such as non-negative convex functions (if $h(t) = t$) and $s$ - convex functions in the second sense (if $h(t) = t^s$). This class is defined in the following way: a non-negative function $f\colon I \to R$, $\emptyset \neq I \subseteq R$, is an interval, is called $h$ – convex if

$$f(tx + (1-t)y) \le h(t)\, f(x) + h(1-t) f(y)$$

holds for all $x, y \in I$ and $t \in [0,1]$, where $h\colon J \to R$ is a non-negative function, $h \not\equiv 0$ and $J$ is an interval, $(0,1) \subseteq J$.

In the following, we will give some necessary definitions and mathematical preliminaries of fractional calculus theory which are used further in this paper. For more details, one can consult [24, 25, 26].

Let $f \in L([a,b])$. The Riemann-Liouville integrals $\mathrm{I}_{a^+}^{\alpha} f$ and $\mathrm{I}_{b^-}^{\alpha} f$ of order $\alpha > 0$ with $a \ge 0$ are defined by

$$\mathrm{I}_{a^+}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)}\int_a^x (x-t)^{\alpha-1} f(t)dt \qquad (x > a),$$

and

$$\mathrm{I}_{b^-}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_x^b (t-x)^{\alpha-1} f(t) dt \qquad (x < b)$$

respectively. Here $\Gamma(\alpha)$ is the Gamma function and $\mathrm{I}_{a^+}^{0} f(x) = \mathrm{I}_{b^-}^{0} f(x) = f(x)$ .

For some recent results connected with fractional integral inequalities, see [5, 9, 19, 21].

The aim of this paper is to establish Hermite-Hadamard's type inequalities involving Riemann-Liouville fractional integral for functions whose derivatives are $h$ – convex using the identity is obtained for fractional integrals.

## 2. Main results

In order to prove our main theorems, we need the following lemma:

**Lemma 2.1.** Let $f: I \subset R \to R$ be differentiable on $I^\circ$ and $a, b \in I$, with $a < b$. If $f' \in L([a,b])$, then

$$f\left(\frac{a+b}{2}\right) - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b,-a)^{\alpha}} \left[\mathrm{I}_{\left(\frac{a+b}{2}\right)^-}^{\alpha} f(a) + \mathrm{I}_{\left(\frac{a+b}{2}\right)^+}^{\alpha} f(b)\right]$$

$$= \frac{b-a}{4} \int_0^1 \left[t^\alpha f'\left((1-t)a + t\frac{a+b}{2}\right) - (1-t)^\alpha f'\left((1-t)\frac{a+b}{2} + t\, b\right)\right] dt$$

and

$$\frac{f(a)+f(b)}{2} - \frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}} \left[\mathrm{I}_{a^+}^{\alpha} f\left(\frac{a+b}{2}\right) + \mathrm{I}_{b^-}^{\alpha} f\left(\frac{a+b}{2}\right)\right]$$

$$= \frac{b-a}{4} \int_0^1 \left[t^\alpha f'\left((1-t)\frac{a+b}{2} + tb\right) - (1-t)^\alpha f'\left((1-t)a + t\frac{a+b}{2}\right)\right] dt$$

Proof. Integrating by part and changing variables of integration yields

$$\int_0^1 \left[t^\alpha f'\left((1-t)a+t\frac{a+b}{2}\right)-(1-t)^\alpha f'\left((1-t)\frac{a+b}{2}+t\,b\right)\right]dt$$

$$=\frac{2}{b-a}\left[t^\alpha f\left((1-t)a+t\frac{a+b}{2}\right)\Big|_0^1-\alpha\int_0^1 t^{\alpha-1}f\left((1-t)a+t\frac{a+b}{2}\right)dt\right]$$

$$-\frac{2}{b-a}\left[(1-t)^\alpha f\left((1-t)\frac{a+b}{2}+t\,b\right)\Big|_0^1+\alpha\int_0^1(1-t)^{\alpha-1}f\left((1-t)\frac{a+b}{2}+t\,b\right)dt\right]$$

$$=\frac{4}{b-a}\,f\left(\frac{a+b}{2}\right)-\frac{2^{\alpha+1}\Gamma(\alpha+1)}{(b-\ a)^{\alpha+1}}\left[\mathrm{I}^\alpha_{\left(\frac{a+b}{2}\right)^-}f(a)+\mathrm{I}^\alpha_{\left(\frac{a+b}{2}\right)^+}f(b)\right]$$

and

$$\int_0^1 \left[t^\alpha f'\left((1-t)\frac{a+b}{2}+t\,b\right)-(1-t)^\alpha f'\left((1-t)a+t\frac{a+b}{2}\right)\right]dt$$

$$=\frac{2}{b-a}\left[t^\alpha f\left((1-t)\frac{a+b}{2}+t\,b\right)\Big|_0^1-\alpha\int_0^1 t^{\alpha-1}f\left((1-t)\frac{a+b}{2}+t\,b\right)dt\right]$$

$$-\frac{2}{b-a}\left[(1-t)^\alpha f\left((1-t)a+t\frac{a+b}{2}\right)\Big|_0^1+\alpha\int_0^1(1-t)^{\alpha-1}f\left((1-t)a+t\frac{a+b}{2}\right)dt\right]$$

$$=\frac{2}{b-a}[f(a)+f(b)]-\frac{2^{\alpha+1}\Gamma(\alpha+1)}{(b-a)^{\alpha+1}}\left[\mathrm{I}^\alpha_{a^+}f\left(\frac{a+b}{2}\right)+\mathrm{I}^\alpha_{b^-}f\left(\frac{a+b}{2}\right)\right].$$

This completes the proof. of Lemma 2.1.

Using the Lemma 2.1, we can obtain the following fractional integral inequalities.

**Theorem 2.1.** Let $f\colon\ I\subset R\to R$ be differentiable on $I^\circ$ and $a,b\in I$, with $a<b$, and $f'\in L([a,b])$. If $|f'|$ is $h-$convex on $[a,b]$, then

$$\left|f\left(\frac{a+b}{2}\right)-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha}\right|\left[\mathrm{I}^\alpha_{\left(\frac{a+b}{2}\right)^-}f(a)+\mathrm{I}^\alpha_{\left(\frac{a+b}{2}\right)^+}f(b)\right]$$

$$\leq \frac{b-a}{4}\left[2\cdot\left|f'\left(\frac{a+b}{2}\right)\right|\int_0^1 t^\alpha h(t)dt+(|f'(a)|+|f'(b)|)\int_0^1 t^\alpha h(1-t)dt\right]$$

and

$$\left|\frac{f(a)+f(b)}{2}-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha}\left[\mathrm{I}_{a^+}^\alpha f\left(\frac{a+b}{2}\right)+\mathrm{I}_{b^-}^\alpha f\left(\frac{a+b}{2}\right)\right]\right|$$

$$\leq \frac{b-a}{4}\left[2\cdot\left|f'\left(\frac{a+b}{2}\right)\right|\int_0^1 t^\alpha h(1-t)dt+(|f'(a)|+|f'(b)|)\int_0^1 t^\alpha h(t)dt\right].$$

Proof. By Lemma 2.1 and since $|f'|$ is $h$ – convex, then we have

$$\left|f\left(\frac{a+b}{2}\right)-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha}\left[\mathrm{I}_{\left(\frac{a+b}{2}\right)^-}^\alpha f(a)+\mathrm{I}_{\left(\frac{a+b}{2}\right)^+}^\alpha f(b)\right]\right|$$

$$\leq \frac{b-a}{4}\left[\int_0^1 t^\alpha\left|f'\left((1-t)a+t\frac{a+b}{2}\right)\right|dt+\int_0^1(1-t)^\alpha\left|f'\left((1-t)\frac{a+b}{2}+tb\right)\right|dt\right]$$

$$\leq \frac{b-a}{4}\left[\int_0^1 t^\alpha\left(h(1-t)|f'(a)|+h(t)\left|f'\left(\frac{a+b}{2}\right)\right|\right)dt\right.$$

$$\left.+\int_0^1(1-t)^\alpha\left(h(1-t)\left|f'\left(\frac{a+b}{2}\right)\right|+h(t)|f'(b)|\right)dt\right]$$

$$=\frac{b-a}{4}\left[2\cdot\left|f'\left(\frac{a+b}{2}\right)\right|\int_0^1 t^\alpha h(t)dt+(|f'(a)|+|f'(b)|)\int_0^1 t^\alpha h(1-t)dt\right]$$

and analogously

$$\left|\frac{f(a)+f(b)}{2}-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha}\left[\mathrm{I}_{a^+}^\alpha f\left(\frac{a+b}{2}\right)+\mathrm{I}_{b^-}^\alpha f\left(\frac{a+b}{2}\right)\right]\right|$$

$$\leq \frac{b-a}{4}\left[\int_0^1 t^\alpha \left|f'\left((1-t)\frac{a+b}{2}+tb\right)\right|dt+\int_0^1 (1-t)^\alpha \left|f'\left((1-t)a+t\frac{a+b}{2}\right)\right|dt\right]$$

$$\leq \frac{b-a}{4}\left[\int_0^1 t^\alpha \left(h(1-t)\left|f'\left(\frac{a+b}{2}\right)\right|+h(t)|f'(b)|\right)dt\right.$$

$$\left.+\int_0^1 (1-t)^\alpha \left(h(1-t)|f'(a)|+h(t)\left|f'\left(\frac{a+b}{2}\right)\right|\right)dt\right]$$

$$=\frac{b-a}{4}\left[2\cdot\left|f'\left(\frac{a+b}{2}\right)\right|\int_0^1 t^\alpha h(1-t)dt+(|f'(a)|+|f'(b)|)\int_0^1 t^\alpha h(t)dt\right].$$

This completes the required proof.

Corollary 1. In Theorem 1, if $|f'|$ is convex, then we get the following inequalities

$$\left|f\left(\frac{a+b}{2}\right)-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha}\left[\mathrm{I}^\alpha_{\left(\frac{a+b}{2}\right)^-}f(a)+\mathrm{I}^\alpha_{\left(\frac{a+b}{2}\right)^+}f(b)\right]\right|$$

$$\leq \frac{b-a}{4(\alpha+2)}\left[2\cdot\left|f'\left(\frac{a+b}{2}\right)\right|+\frac{|f'(a)|+|f'(b)|}{\alpha+1}\right]$$

and

$$\left|f\left(\frac{f(a)+f(b)}{2}\right)-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha}\left[I_{a^+}f\left(\frac{a+b}{2}\right)+I_{b^-}f\left(\frac{a+b}{2}\right)\right]\right|$$

$$\leq \frac{b-a}{4(\alpha+2)}\left[\frac{2}{\alpha+1}\left|f'\left(\frac{a+b}{2}\right)\right|+[|f'(a)|+|f'(b)|]\right].$$

Corollary 2. In Theorem 1, if $|f'|$ is $s$-convex, then we get the following inequalities

$$\left|f\left(\frac{a+b}{2}\right)-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha}\left[\mathrm{I}^\alpha_{\left(\frac{a+b}{2}\right)^-}f(a)+\mathrm{I}^\alpha_{\left(\frac{a+b}{2}\right)^+}f(b)\right]\right|$$

$$\leq \frac{b-a}{4}\left[\frac{2\left|f'\left(\frac{a+b}{2}\right)\right|}{\alpha+s+1}+\frac{\Gamma(\alpha+1)\Gamma(s+1)}{\Gamma(\alpha+s+2)}+(|f'(a)|+|f'(b)|)\right]$$

and

$$\left|\frac{f(a)+f(b)}{2}-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[I_{a^+}f\left(\frac{a+b}{2}\right)+I_{b^-}f\left(\frac{a+b}{2}\right)\right]\right|$$

$$\leq \frac{b-a}{4}\left[2\cdot\left|f'\left(\frac{a+b}{2}\right)\right|+\frac{\Gamma(\alpha+1)\Gamma(s+1)}{\Gamma(\alpha+s+2)}+\frac{|f'(a)|+|f'(b)|}{\alpha+s+1}\right]$$

**Theorem 2.2.** Let $f: I \subset R \to R$ be differentiable on $I^\circ$**,** $a, b \in I$, with $a < b$, and $f' \in L([a,b])$. If $|f'|^{q}$ is $h-$convex on $[a,b]$; $p, q > 1$; $\frac{1}{p}+\frac{1}{q}=1$, then following inequalities hold

$$\left|f\left(\frac{a+b}{2}\right)-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[\mathrm{I}^{\alpha}_{\left(\frac{a+b}{2}\right)^-}f(a)+\mathrm{I}^{\alpha}_{\left(\frac{a+b}{2}\right)^+}f(b)\right]\right|$$

$$\leq \frac{b-a}{4(\alpha p+1)^{\frac{1}{p}}}\left(\int_0^1 h(t)dt\right)^{\frac{1}{q}}\left[\left(|f'(a)|^q+\left|f'\left(\frac{a+b}{2}\right)\right|^q\right)^{\frac{1}{q}}+\left(|f'(b)|^q+\left|f'\left(\frac{a+b}{2}\right)\right|^q\right)^{\frac{1}{q}}\right]$$

and

$$\left|\frac{f(a)+f(b)}{2}-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[I^{\alpha}_{a^+}f\left(\frac{a+b}{2}\right)+I^{\alpha}_{b^-}f\left(\frac{a+b}{2}\right)\right]\right|$$

$$\leq \frac{b-a}{4(\alpha p+1)^{\frac{1}{p}}}\cdot\left(\int_0^1 h(t)dt\right)^{\frac{1}{q}}\left[\left(|f'(a)|^q+\left|f'\left(\frac{a+b}{2}\right)\right|^q\right)^{\frac{1}{q}}+\left(|f'(b)|^q+\left|f'\left(\frac{a+b}{2}\right)\right|^q\right)^{\frac{1}{q}}\right].$$

Proof. From Lemma 2.1 and using the Hőlder's integrals inequality, we have

$$\left|f\left(\frac{a+b}{2}\right)-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[\mathrm{I}^{\alpha}_{\left(\frac{a+b}{2}\right)^-}f(a)+\mathrm{I}^{\alpha}_{\left(\frac{a+b}{2}\right)^+}f(b)\right]\right|$$

$$\leq\frac{b-a}{4}\left[\left(\int_0^1 t^{\alpha p}\,dt\right)^{\frac{1}{p}}\cdot\left(\int_0^1\left|f'\left((1-t)a+t\frac{a+b}{2}\right)\right|^q dt\right)^{\frac{1}{q}}\right.$$

$$\left.+\left(\int_0^1(1-t)^{\alpha p}\,dt\right)^{\frac{1}{p}}\cdot\left(\int_0^1\left|f'\left((1-t)\frac{a+b}{2}+t\,b\right)\right|^q dt\right)^{\frac{1}{q}}\right]$$

$$\leq\frac{b-a}{4(\alpha p+1)^{\frac{1}{p}}}\cdot\left(\int_0^1 h(t)dt\right)^{\frac{1}{q}}\left[\left(|f'(a)|^q+\left|f'\left(\frac{a+b}{2}\right)\right|^q\right)^{\frac{1}{q}}+\left(|f'(b)|^q+\left|f'\left(\frac{a+b}{2}\right)\right|^q\right)^{\frac{1}{q}}\right].$$

In the analogous way, we can prove the second inequality.

**Theorem 2.3.** Let $f: I\subset R\to R$ be differentiable on $I^{\circ}$, $a,b\in I$, with $a<b$, and $f'\in L([a,b])$. If $|f'|^q$, $q\geq 1$, is $h$ - convex on $[a,b]$, then the following inequalities hold:

$$\left|f\left(\frac{a+b}{2}\right)-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[\mathrm{I}^{\alpha}_{\left(\frac{a+b}{2}\right)^-}f(a)+\mathrm{I}^{\alpha}_{\left(\frac{a+b}{2}\right)^+}f(b)\right]\right|$$

$$\leq\frac{b-a}{4}\left(\frac{1}{\alpha+1}\right)^{1-\frac{1}{q}}\left[\left(|f'(a)|^q\int_0^1 t^{\alpha}h(1-t)dt+\left|f'\left(\frac{a+b}{2}\right)\right|^q\int_0^1 t^{\alpha}h(t)dt\right)^{\frac{1}{q}}\right.$$

$$\left.+\left(|f'(b)|^q\int_0^1 t^{\alpha}h(1-t)dt+\left|f'\left(\frac{a+b}{2}\right)\right|^q\int_0^1 t^{\alpha}h(t)dt\right)^{\frac{1}{q}}\right]$$

and

$$\left|\frac{f(a)+f(b)}{2}-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^{\alpha}}\left[I^{\alpha}_{a^+}f\left(\frac{a+b}{2}\right)+I^{\alpha}_{b^-}f\left(\frac{a+b}{2}\right)\right]\right|$$

$$\le \frac{b-a}{4}\left(\frac{1}{\alpha+1}\right)^{1-\frac{1}{q}}\left[\left(|f'(a)|^q\int_0^1 t^\alpha h(t)dt+\left|f'\left(\frac{a+b}{2}\right)\right|^q\int_0^1 t^\alpha h(1-t)dt\right)^{\frac{1}{q}}\right.$$

$$\left.+\left(|f'(b)|^q\int_0^1 t^\alpha h(t)dt+\left|f'\left(\frac{a+b}{2}\right)\right|^q\int_0^1 t^\alpha h(1-t)dt\right)^{\frac{1}{q}}\right].$$

Proof. From Lemma 2.1 and using the well known power mean inequality, we have

$$\left|f\left(\frac{a+b}{2}\right)-\frac{2^{\alpha-1}\Gamma(\alpha+1)}{(b-a)^\alpha}\left[I^\alpha_{\left(\frac{a+b}{2}\right)^-}f(a)+I^\alpha_{\left(\frac{a+b}{2}\right)^+}f(b)\right]\right|$$

$$\le \frac{b-a}{4}\left[\int_0^1 t^\alpha\left|f'\left((1-t)a+t\frac{a+b}{2}\right)\right|dt+\int_0^1(1-t)^\alpha\left|f'\left((1-t)\frac{a+b}{2}+t\,b\right)\right|dt\right]$$

$$\le \frac{b-a}{4}\left(\int_0^1 t^\alpha dt\right)^{1-\frac{1}{q}}\left[\left(\int_0^1 t^\alpha\left|f'\left((1-t)a+t\frac{a+b}{2}\right)\right|^q dt\right)^{\frac{1}{q}}\right.$$

$$\left.+\left(\int_0^1(1-t)^\alpha\left|f'\left((1-t)\frac{a+b}{2}+t\,b\right)\right|^q dt\right)^{\frac{1}{q}}\right]$$

$$\le \frac{b-a}{4}\left(\frac{1}{\alpha+1}\right)^{1-\frac{1}{q}}\left[\left(|f'(a)|^q\int_0^1 t^\alpha h(1-t)dt+\left|f'\left(\frac{a+b}{2}\right)\right|^q\int_0^1 t^\alpha h(t)dt\right)^{\frac{1}{q}}\right.$$

$$\left.+\left(|f'(b)|^q\int_0^1 t^\alpha h(1-t)dt+\left|f'\left(\frac{a+b}{2}\right)\right|^q\int_0^1 t^\alpha h(t)dt\right)^{\frac{1}{q}}\right].$$

In analogous way we can prove the second inequality.

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