Crazy, Selfie, Fibonacci, Triangular, Amicable Types Representations of Numbers

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Inder J. Taneja[1]

SUMMARY

*The paper summarize author's work on numbers done during past years. It is commonly famous as **popular science mathematics** or **Pop-Sci-Math**. It contains the ideas as **crazy representations of numbers, selfie numbers, selfie fractions, equivalent fractions, amicable numbers, semi-selfie numbers, selfie expressions, embedded palprimes, primes patterns with fixed digits repetitions, magic square type palprimes, palindromic-type expressions**, etc. The **crazy representations** include representations of natural numbers in terms of 1 to 9 or 9 to 1 using basic operations; representations in terms of single digits and in terms of single letter "a", flexible power, etc. The idea of running expressions are also included. The **running expressions** are understood as representations of numbers from 1 to 9 or 9 to 1 or 9 to 0 separated by equalities. The **selfie numbers** are understood as numbers written in terms of its own digits with certain properties, such as, in **digit's order** or **reverse order of digits**, etc. These representations include applications of extra operations such as, **factorial, square-root, Fibonacci sequence values, triangular numbers, binomial coefficients, polygonal type numbers, concatenation property**, etc. The idea of **Selfie fractions** is also introduced. The **Selfie fractions** are understood as fraction represented by their own digits in numerator and denominators with basic operations. Also the study is made towards **equivalent fractions**. The **equivalent fractions** are those representing same fraction in different ways with out any operation. **Amicable numbers** are well known in the literature in terms of divisors. Here we worked on **amicable numbers** with multiplications and powers. **Semi-selfie numbers** are similar to **selfie numbers** having extra as a power. Still we worked on **selfie expressions**. The **selfie expressions** are those where we have same order in digits on both sides but with different operations. This we have done with use of **addition, power, factorial, Fibonacci sequence values, triangular numbers**, etc. **Embedded palindromic numbers** are studied in different situations. **Patterns in prime numbers** are established with **fixed digits repetitions**. Also the idea of **magic squares** is applied to bring **blocks of palindromic prime numbers** where rows, columns and principal diagonal are also palindromic prime numbers. Finally, **palindromic-type numbers** with addition are also studied.*

[1] *Formerly, Professor of Mathematics, Universidade Federal de Santa Catarina, Florianópolis, SC, Brazil (1978-2012). Also worked at Delhi University, India (1976-1978). **E-mail:** ijtaneja@gmail.com; **Web-sites:** http://inderjtaneja.com; http://ijtaneja.com; **Twitter:** @IJTANEJA.*

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Contents

1 Introduction

During last 5 years author worked on numbers. It includes the following main topics written in blocks:

(i) Crazy Representations of Numbers;
(ii) Selfie Numbers;
(iii) Selfie and Equivalent Fractions;
(iv) Amicable and Semi-Selfie Numbers;
(v) Selfie Expressions;
(vi) Embedded, Fixed Digits, Magic Square Type;
(vii) Palindromic-Type Expressions.

This type of study generally famous in the literature as famous as **Popular Science Mathematics**, or simply **Pop-Sci-Math**. Let's see below **block-wise** details.

The **first block** on **crazy representations of numbers** includes different ways of writing natural numbers. First as writing natural numbers in terms of digits 1 to 9 or 9 to 1 or 9 to 0. The second as writing natural numbers in terms of single digit. The third as writing natural numbers in terms of single letter, for example "a". It includes interesting patterns and power type numbers with single digit "a". The forth as writing natural numbers in terms of flexible powers, where bases and powers are of same digits with different permutations in powers. The idea is extended to running expressions also. The **running expressions** are understood as representations of numbers from 1 to 9 or 9 to 1 or 9 to 0 separated by equalities, single, double or more.

The **second block** block is on **selfie numbers**. By **selfie numbers** are understand as numbers represented by their own digits with certain properties, such as, representations of numbers in same order of digit or reverse order of digits. This work on **selfie numbers** brings numbers in different ways by use of basic operations, such as, **factorial, square-root, Fibonacci sequence, triangular numbers**, etc. The study also extend to **selfie numbers** with **binomial coefficients, polygonal type, concatenation type**, etc. Still the idea of **narcissistic numbers** and **narcissistic numbers with division** are extended to **flexible power** and positive and negative coefficients.

The **third block** brings **fraction type** numbers in two different situations. One when there are same digits in numerators and denominators on both sides of the fractions with basic operations, known by **selfie fractions**. The second type of fraction we studied are known as **equivalent fractions**. In this case the no operation is applied, just having different fractions with their equalities.

The **forth block** is little different. Here we worked with two different types of numbers calling as **amicable** and **semi-selfie** numbers. In case of amicable numbers again we considered two different situations, one with multiplication and another with potentiation. While the semi-selfie numbers are similar the selfie number, but are of powered type.

The **fifth block** is with expressions instead with numbers. We call it as **selfie expressions** as they have same order of digits on both sides of the expression but with different operations. Here we worked with operations, such as, **multiplication, factorial, power, Fibonacci sequence, triangular numbers**, etc.

The **sixth block** is different from other blocks. In this block we worked with three different type of prime numbers. The first one is **embedded palindromic prime numbers**. Here the palprimes are written in such a way that previous is in next, just like a tree. The second way is with different types of **prime patterns with repetitions of fixed digits**. Here the patterns are of pyramid type. The third type of work is with **palprime numbers** satisfying **magic square type** properties, i.e., they are palprimes in row, columns, and in principal diagonals.

The **seveth block**, i.e., the last block is with **palindromic-type expressions** separated by the operation of **addition**. Some interesting patterns are also given.

2 Crazy Representations of Natural Numbers

In this section natural numbers are represented in three different types.

2.1 Increasing and Decreasing

In 2014, author [1] wrote natural numbers in increasing and decreasing orders of 1 to 9 and 9 to 1. See examples below:

$$\begin{aligned}
\mathbf{100} &:= 1+2+3+4+5+6+7+8\times 9 &&= 9\times 8+7+6+5+4+3+2+1\\
\mathbf{101} &:= 1+2+34+5+6\times 7+8+9 &&= 9\times 8+7+6+5+4+3\times 2+1\\
\mathbf{102} &:= 12+3\times 4\times 5+6+7+8+9 &&= 9+8+7+6+5+4^3+2+1\\
\mathbf{103} &:= 1\times 2\times 34+5+6+7+8+9 &&= 9+8+7\times 6+5\times 4+3+21\\
\mathbf{104} &:= 1+23+4+5+6+7\times 8+9 &&= 9+8+7+65+4\times 3+2+1\\
\mathbf{105} &:= 1+2\times 3\times 4+56+7+8+9 &&= 9+8\times 7+6\times 5+4+3+2+1\\
\mathbf{106} &:= 12+3+4\times 5+6+7\times 8+9 &&= 9+8\times 7+6\times 5+4+3\times 2+1\\
\mathbf{107} &:= 1\times 23+4+56+7+8+9 &&= 9+8+76+5+4+3+2\times 1\\
\mathbf{108} &:= 1+2+3+4+5+6+78+9 &&= 9+8+76+5+4+3+2+1.
\end{aligned}$$

https://goo.gl/DSqYVs

See more examples,

$$\begin{aligned}
\mathbf{786} &:= 1+23+4+56+78\times 9 &&= 9+8\times 7+6\times 5\times 4\times 3\times 2+1\\
\mathbf{999} &:= 12\times 3\times (4+5)+(67+8)\times 9 &&= 9+8+7+654+321\\
\mathbf{2535} &:= 1+2345+(6+7+8)\times 9 &&= 9+87\times (6+5\times 4+3)+2+1\\
\mathbf{2607} &:= 123\times 4\times 5+6+(7+8)\times 9 &&= 987+6\times 54\times (3+2)\times 1\\
\mathbf{10483} &:= 1+23\times 456-7-8+9 &&= (9\times 87\times 6+543)\times 2+1\\
\mathbf{10549} &:= 1^2\times 34\times 5\times (6+7\times 8)+9 &&= 9+(87\times 6+5)\times 4\times (3+2)\times 1\\
\mathbf{10550} &:= 1^2+34\times 5\times (6+7\times 8)+9 &&= 9+(87\times 6+5)\times 4\times (3+2)+1\\
\mathbf{10553} &:= 1\times 23\times 456+7\times 8+9 &&= 9+8\times (7+6\times 5\times 43+21)\\
\mathbf{10554} &:= 1+23\times 456+7\times 8+9 &&= -9+(8+7)\times (6+5)\times 4^3+2+1\\
\mathbf{11807} &:= 1\times 234\times (5+6\times 7)+89 &&= -9+8+7\times (6+5)\times (4\times 3)^2\times 1.
\end{aligned}$$

https://goo.gl/DSqYVs

For more details refer author's complete work [1].

2.2 Comments and Discussions

This work has been considered as **"Improbable Research"**. For details see the link:

(*i*) *http://www.improbable.com/2013/02/12/lots-of-numbers-plain-and-almost-simple/ - [85];*

(*ii*) *http://www.improbable.com/2013/06/08/lots-more-numbers-deemed-crazy-sequential/ - [86].*

Also refer [107, 108] for more comments. We observe that the number 10958 is the only number among 0 to 11111 that is not possible to write in terms of 1 to 9 in the increasing case. We need to use extra operations such as **square-root** and/or **factorial** to write it. See below:

$$\mathbf{10958} := 1+2+3!!+(-4+5!+6-7)\times 89.$$

Recently, **Numberphile** produced two videos on **YouTube** using **concatenation** as an extra operation to bring representation of number 10958 in increasing order. See below both the links [109, 110]:

(*i*) **The 10,958 Problem - Numberphile;**

(*ii*) **A 10,958 Solution - Numberphile.**

Also this number has become a question of discussion at various web-sites. See the links below:

(*i*) *http://www.primepuzzles.net/puzzles/puzz_864.htm - [115];*

(*ii*) *https://www.futilitycloset.com/?s=10958 - [96];*

(*iii*) *https://puzzling.stackexchange.com/questions/51129/the-10-958-problem - [?];*

(*iv*) *https://puzzling.stackexchange.com/questions/47923/rendering-the-number-10-958-with-the-string-1-2-3-4-5-6-7-8-9;*

(*v*) *https://www.reddit.com/r/mathematics/comments/6hooq2/the_10958_problem/;*

(*vi*) *http://digg.com/video/10958-math-problem;*

(*vii*) *http://wssrmnn.net/index.php/2017/04/20/the-10958-problem/;*

(*viii*) *https://twitter.com/aap03102/status/854319054173220864;*

(*ix*) *https://inderjtaneja.com/2017/12/02/10958-problem-1-5-millions-views/.*

In the above sites, there are much more possibilities of writing the same number.

2.3 Extension to 20000 Numbers

The above subsection give the representations in terms of 1 to 9 and 9 to 1 numbers from 0 to 11111. If we want to extend up to 20000, we want to know how these numbers shall appears. We observed that 10958 is the only number that requires extra operations rathar than basic operation. In case of this extension, how many numbers shall require extra operations? After calculations, we found that there are 49 numbers in increasing order and 21 in decreasing orders. The can be written allowing **factorial** and/or **square-root**. See below these numbers:

2.3.1 Increasing Orders: 1 to 9

There are only 49 numbers that are not possible to write in terms of 1 to 9 just using basic operations. In this case, we need extra operations, such as, **factorial** and/or **square-root**. See below these numbers.

$\mathbf{11149} := 1^2 \times (-3! + 4^5) \times 6 + 7! - 8 + 9$
$\mathbf{11192} := 1 + 2 + 3! + 4^5 \times 6 + 7! + 8 - 9$
$\mathbf{11488} := (-1 \times 2 + 3!!) \times 4 \times 5!/(6 + 7 + 8 + 9)$
$\mathbf{11632} := 1 \times 2^{3!}/4 \times (5! \times 6 - 7 \times (8 - 9))$
$\mathbf{11642} := -1 - 2 + 3 + \sqrt{4} - 5 \times (-6! - 7 \times 8) \times \sqrt{9}$
$\mathbf{11852} := 1 - 2 - 3 + 456 \times 78/\sqrt{9}$
$\mathbf{11884} := (1 + 2) \times (3!!/\sqrt{4} \times (5 + 6) + 7) - 8 - 9$
$\mathbf{11948} := -1 - 2 + 3! \times 4^5 + (6! + 7) \times 8 - 9$
$\mathbf{12820} := (1 + 2) \times ((3!! - 4 - 5) \times 6 + 7) - 8 + 9$
$\mathbf{13964} := 123^{\sqrt{4}} + 5 + 6! - 7!/8 \times \sqrt{9}$
$\mathbf{14540} := -(1 + 2 + 3)! + \sqrt{4} + 5! - 6 + (7! + 8) \times \sqrt{9}$
$\mathbf{14611} := -1 \times 23 - 4 \times 5! + (6 + 7! - 8) \times \sqrt{9}$
$\mathbf{14612} := 1 \times 2 \times 3!! \times \sqrt{4} \times 5 + (6 + 7!/8)/\sqrt{9}$
$\mathbf{14648} := -(1 + 2) \times (3!! - 4) - 5 - 6 + 7^{8-\sqrt{9}}$
$\mathbf{14666} := 1 + (2 - 3! - 4) \times 5! + (6 + 7 - 8)^{(\sqrt{9})!}$
$\mathbf{14746} := (1 \times 2 + 3! \times 4!) \times (\sqrt{5^6} - 7 - 8 - 9)$
$\mathbf{14764} := (-1 + 234 - 5) \times 67 - 8^{\sqrt{9}}$
$\mathbf{14798} := (1 + 2) \times (3 + 4 - 5! + 6! \times 7) + 8 + 9$
$\mathbf{15971} := (-1 + 2) \times 3!! \times 4! - (5 + 6) \times 7 \times (8 + 9)$

$\mathbf{16004} := (1 - 2 \times 3!) \times (-4 + 5!) + 6! \times (7 + 8 + 9)$
$\mathbf{16034} := -1 \times 23 \times \sqrt{4} + 5! \times 67 \times (8 - (\sqrt{9})!)$
$\mathbf{16612} := (1 + 2 + 3!!) \times (\sqrt{4} \times 5 + 6 + 7) - 8 - 9$
$\mathbf{17312} := 1 + 2 + 3!! \times 4! + (5 + 6 + 7) + 8 + \sqrt{9}$
$\mathbf{17707} := 1 - 2 - 3 + (\sqrt{4^5} \times 6 + 7) \times 89$
$\mathbf{17726} := -1 + (23 - \sqrt{4} + 5! + 6! + 7! + 8) \times \sqrt{9}$
$\mathbf{17771} := 1^{23} \times 4 \times (5! - 6! + 7!) + 8 + \sqrt{9}$
$\mathbf{17773} := (1 + 2) \times (3!! + 4) + 5^6 - 7 - 8 - 9$
$\mathbf{17779} := \sqrt{-1 + 2 + 3} + 4 \times (5! - 6! + 7!) + 8 + 9$
$\mathbf{17780} := -1 - 2 - 3!! \times 4 + 5^6 + 7! - 8 + (\sqrt{9})!$
$\mathbf{17789} := 1 - 2 + (3!! \times 4 - 5 + 6 \times (7 + 8)) \times (\sqrt{9})!$
$\mathbf{17795} := 1 + (23 + 4! \times (5 + 6)) \times (7 \times 8 + (\sqrt{9})!)$
$\mathbf{17843} := -1 - 2 \times 3! + (4 + 5!) \times 6 \times (7 + 8 + 9)$
$\mathbf{17923} := (-1 \times 2 + 3!!) \times 4! - 5 + 6! - 7 - 8 - 9$
$\mathbf{17971} := 1 - (2 \times \sqrt{\sqrt{3^4}})! + 5 \times 6 \times 7 \times 89$
$\mathbf{17996} := (1^{23} + 4) \times 5 \times 6! + (7 - 8 - \sqrt{9})$

18014 $:= (1\times 2+3)\times(-\sqrt{4}+5\times 6!)+7+8+9$
18058 $:= -1\times 2+(3!!+\sqrt{4}-5!)\times(6+7+8+9)$
18266 $:= 1\times 2+(3!^{\sqrt{4}}+5+6!)\times(7+8+9)$
18536 $:= -1+(2+3!!)\times 4+5^6+7+8+9$
18812 $:= 1-2-3+\sqrt{4}\times 56\times 7\times 8\times\sqrt{9}$
18878 $:= (1+2)!\times 3!+\sqrt{4}+5!+6!\times 78/\sqrt{9}$
19084 $:= -1+(2+3!!)/\sqrt{4}\times 5+6!\times(7+8+9)$
19165 $:= (1+2^{3!})\times(4!+5)+6!\times(7+8+9)$
19166 $:= (-1+2+3!)\times(\sqrt{4}+(5!-6)\times(7+8+9))$
19861 $:= -1+2\times(3!^4-5)+6!\times(7+8+9)$
19867 $:= 1\times 2\times 3!^4-5+6!\times(7+8+9)$
19886 $:= (1+2)\times 3!!\times\sqrt{4}+5^6-7\times 8-\sqrt{9}$
19994 $:= 1\times 2+3!\times 4\times(56-7)\times(8+9)$
19996 $:= (-1+2+3)\times(-4!+(-5+6)\times 7!-8-9)$

2.3.2 Decreasing Orders: 9 to 1

There are only 21 numbers that are not possible to write in terms of 9 to 1 just using basic operations. In this case, we need extra operations, such as, **factorial** and/or **square-root**. See below these numbers.

14187 $:= \sqrt{9}\times(8\times(7+6!-5!)-4\times 32+1)$
14317 $:= \sqrt{9}-8-76+5!^{\sqrt{4}}-3+2-1$
14324 $:= \sqrt{9}+8-7+(6+5)^4-321$
17060 $:= \sqrt{9}+8!-7+(-6^5+4!)\times 3!/2\times 1$
17188 $:= -\sqrt{9}-8-76-5+4!\times(3\times 2\times 1)!$
17797 $:= (-9+8+7+6)\times(-5+(4!+3!!)\times 2)+1$
17806 $:= (98/7+6!-5!)\times(4!+3+2\times 1)$
17890 $:= ((\sqrt{9})!+87+6!)\times 5+4!^3+2-1$
18022 $:= \sqrt{9}+8+7+6!\times 5^{\sqrt{4}}+3+2-1$
18493 $:= 98+765\times 4!+3!^2-1$
18517 $:= (\sqrt{9})!\times 87+6!-5+4\times 3!!\times(2+1)!$
18860 $:= -\sqrt{9}+8\times 7\times 6\times 54+(3\times 2)!-1$
18995 $:= -\sqrt{9}-8+7!+6!/5+4!^3-2\times 1$
19042 $:= \sqrt{9}\times 8+(-7!+6!+5+4!^3)\times 2\times 1$
19078 $:= (-(\sqrt{9})!+8\times 7\times 6)\times 5!/\sqrt{4}-3!!-2\times 1$
19085 $:= ((\sqrt{9})!\times 8+7)\times(6!-5-4!+3)/2\times 1$
19102 $:= -9-8+7\times(-6+5!)\times 4!-32-1$
19157 $:= ((\sqrt{9})!\times(8+76)\times(-5+4!)+3)\times 2-1$
19301 $:= 9-8+7\times(6!-5)\times 4-3!!\times(2-1)$
19820 $:= \sqrt{9}-8+(-7+6!-5)\times(4!+3!-2)+1$
19931 $:= (9\times 8+7)\times(6+5!)\times\sqrt{4}+(3!-2)!-1$

The complete work of these numbers from 11112 to 20000 representing in terms of increasing as well as decreasing order of 1 to 9 or 9 to 1 shall be dealt elsewhere.

3 Flexible Power Representations

In the previous section, we represented natural numbers in terms of 1 to 9 and 9 to 1, where each digit appears once. There another way of representing these natural numbers using the idea of **flexible power**. Let's understand first this idea means.

Let's consider two numbers, 1 and 2. Using the idea of power and the operations of **addition** and

subtraction, we can write following 3 numbers in terms of 1 and 2, as

$$\mathbf{1} := -1^2 + 2^1$$
$$\mathbf{3} := 1^2 + 2^1$$
$$\mathbf{5} := 1^1 + 2^2$$

In this situation, we observe that the **bases** and the **exponents** are of same digits with different permutations. Permutations of exponent values helps in bringing more possibilities of different numbers. In case of repeated values, for example, $3 = 1^2 + 2^1 = -1^1 + 2^2$, only possibilities is considered. There is only one number having single digit, i.e., $1 = 1^1$. For simplicity, let us represent the above procedure as $(1,2)^{(1,2)}$, resulting in three possible values. The above procedure is with two digits. Instead having two digits, we can work with letters, such as,

$$(a,b)^{(a,b)}, (a,b,c)^{(a,b,c)}, \ldots, (a,b,c,d,e,f,g,h,i)^{(a,b,c,d,e,f,g,h,i)},$$

where $a,b,c,d,e,f,g,h,i \in \{1,2,3,4,5,6,7,8,9\}$, all distinct.

Below are two different ways of working with above procedure.

3.1 Unequal String Lengths

Since there are much more possibilities of writing same number in different ways. In this procedure we have chosen only that where we need less number of digits. Since, each representation is called as one string, that's why we called it as **unequal string length**. Also we have considered the possibilities without zero. See below some examples,

$$\mathbf{100} := 2^6 + 6^2$$
$$\mathbf{101} := 1^1 + 2^6 + 6^2$$
$$\mathbf{102} := -2^5 + 3^2 + 5^3$$
$$\mathbf{103} := 1^1 - 2^5 + 3^2 + 5^3$$
$$\mathbf{104} := -1^1 + 2^3 + 3^4 + 4^2$$
$$\mathbf{105} := 2^3 + 3^4 + 4^2$$
$$\mathbf{106} := 2^7 + 3^3 - 7^2$$
$$\mathbf{107} := -1^2 + 2^7 - 3^3 + 7^1$$
$$\mathbf{108} := 1^7 + 2^6 + 6^2 + 7^1$$
$$\mathbf{109} := 1^2 + 2^7 - 3^3 + 7^1$$

$$\mathbf{1751} := -1^3 + 2^6 + 3^1 + 4^5 + 5^4 + 6^2$$
$$\mathbf{1752} := 1^2 + 2^9 + 3^4 + 4^5 + 5^3 + 9^1$$
$$\mathbf{1753} := 2^8 + 3^6 - 4^2 + 6^4 - 8^3$$
$$\mathbf{1754} := -2^6 - 3^3 + 4^2 + 5^5 - 6^4$$
$$\mathbf{1755} := 2^7 + 3^3 + 4^5 + 5^4 - 7^2$$
$$\mathbf{1756} := -1^2 + 2^7 - 3^3 + 4^5 + 5^4 + 7^1$$
$$\mathbf{1757} := 1^3 - 2^4 + 3^6 + 4^5 + 5^2 - 6^1$$
$$\mathbf{1758} := 2^4 - 3^7 + 4^6 - 6^3 + 7^2$$
$$\mathbf{1759} := -1^3 - 2^6 - 3^2 + 4^1 + 5^5 - 6^4$$
$$\mathbf{1760} := -1^9 - 2^1 + 3^7 - 7^3 - 9^2.$$

https://goo.gl/64xJXH

See more examples,

$$\mathbf{638} := -1^5 - 2^1 - 4^2 + 5^4$$
$$\mathbf{666} := -2^5 + 3^2 + 4^3 + 5^4$$
$$\mathbf{786} := -1^4 + 3^6 + 4^3 - 6^1$$
$$\mathbf{1933} := -1^3 - 2^2 + 3^7 - 4^4 + 7^1$$
$$\mathbf{1934} := 2^9 + 3^6 - 6^2 + 9^3$$
$$\mathbf{3098} := -3^3 + 5^5$$
$$\mathbf{2280} := -1^1 - 2^6 + 4^5 + 5^2 + 6^4$$
$$\mathbf{6922} := -3^6 - 5^3 + 6^5$$
$$\mathbf{9711} := 1^3 + 2^4 + 3^8 + 4^2 + 5^5 - 8^1$$
$$\mathbf{9777} := 1^9 + 2^1 + 4^7 - 7^2 - 9^4$$
$$\mathbf{11110} := 1^1 + 2^2 + 3^9 - 5^6 + 6^5 - 9^3$$
$$\mathbf{11111} := -1^1 + 2^7 + 3^8 - 4^2 + 7^3 + 8^4.$$

https://goo.gl/64xJXH

Just to remember again, by unequal string length, we mean that each number is not represented by same number of digits. It uses minimum necessary digits for each representation. The powers and bases are with same digits with different permutations. By no means we can say that the above representation is unique. There are always alterative ways too. For complete work for the numbers from 0 to 11111 refer author's work [17].

3.2 Equal Lenght Strings

Based on procedure given in above subsection, still we can write natural numbers in a sequential way with uniform representations. Instead working with unequal strings as of previous section, here we shall work with equal string using the digits 0 to 9, i.e., using all the 10 digits, {0,1,2,3,4,5,6,7,8,9}. The results obtained are symmetric, i.e., writing in 0 to 9 or 9 to 0, the result is always same. See below some examples,

$$\mathbf{201} := 0^3 + 1^9 + 2^4 + 3^7 - 4^8 + 5^1 + 6^6 + 7^5 + 8^2 + 9^0$$
$$\mathbf{202} := 0^0 + 1^9 + 2^6 + 3^8 - 4^7 + 5^5 + 6^3 + 7^2 + 8^1 + 9^4$$
$$\mathbf{203} := 0^3 - 1^9 + 2^4 + 3^7 - 4^8 + 5^0 + 6^6 + 7^5 + 8^2 + 9^1$$
$$\mathbf{204} := 0^8 + 1^9 + 2^5 + 3^7 - 4^6 + 5^1 + 6^4 + 7^2 + 8^0 + 9^3$$
$$\mathbf{205} := 0^3 + 1^9 + 2^4 + 3^7 - 4^8 + 5^0 + 6^6 + 7^5 + 8^2 + 9^1$$
$$\mathbf{206} := 0^7 - 1^9 - 2^5 - 3^8 + 4^6 + 5^1 + 6^3 + 7^4 + 8^0 + 9^2$$
$$\mathbf{207} := 0^8 + 1^9 + 2^5 + 3^7 - 4^6 + 5^0 + 6^4 + 7^2 + 8^1 + 9^3$$
$$\mathbf{208} := 0^7 + 1^9 - 2^5 - 3^8 + 4^6 + 5^1 + 6^3 + 7^4 + 8^0 + 9^2$$
$$\mathbf{209} := 0^7 - 1^9 - 2^5 - 3^8 + 4^6 + 5^0 + 6^3 + 7^4 + 8^1 + 9^2$$
$$\mathbf{210} := 0^5 - 1^7 - 2^8 - 3^9 + 4^1 + 5^6 + 6^0 + 7^3 + 8^4 + 9^2.$$

https://goo.gl/N7Ld5z

Below are more examples,

11080 $:= 0^8+1^9+2^7+3^6+4^2+5^5+6^0+7^1+8^3+9^4$
11081 $:= 0^8-1^9+2^6+3^7+4^4+5^1+6^5+7^0+8^2+9^3$
11082 $:= 0^8+1^9+2^6+3^7+4^1+5^4+6^5+7^3+8^0+9^2$
11083 $:= 0^8+1^9+2^6+3^7+4^4+5^1+6^5+7^0+8^2+9^3$
11084 $:= 0^7+1^9+2^8+3^6+4^1+5^5+6^0+7^3+8^2+9^4$
11085 $:= 0^8+1^9+2^6+3^7+4^4+5^0+6^5+7^1+8^2+9^3$
11086 $:= 0^7+1^9+2^8+3^6+4^0+5^5+6^1+7^3+8^2+9^4$
11087 $:= 0^6+1^9-2^8+3^7+4^2+5^4+6^5+7^0+8^1+9^3$
11088 $:= 0^7+1^9+2^6-3^8+4^3+5^4+6^1+7^5+8^0+9^2$
11089 $:= 0^8-1^9-2^6+3^7+4^1+5^4+6^5+7^2+8^3+9^0$
11090 $:= 0^5+1^9+2^7-3^8+4^0+5^6+6^4+7^1+8^3+9^2$

https://goo.gl/N7Ld5z

For complete representations of numbers from 0 to 11111 refer auhor's work [34].

Analysing the procedures given in above Sections 2 and 3 , we observe that in the Section 2 , all the 9 digits from 1 to 9 or 9 to 1 are used to bring natural numbers, where each digit appears only once. In this case, the operations used are, **addition, subtraction, multiplication, division** and **potentiation** with extra operations as **factorial** and **square-root**. The Section 3 works with representations of natural numbers written in such a way that each digit appears twice, one in **bases** and second in **exponents** with different permutations. Subsection 3.1 choose the digits from 1 to 9, according to necessity, while Subsection 3.2 works with all the 10 digits, i.e., 0 to 9, along with positive and negative coefficients. Instead using all the 10 digits in 3.2, we can work with 9 digits $\{1,2,\dots,9\}$, but in this case not all the numbers are available.

4 Pyramidical and Multiple Type Representations

This section deals with pyramidical-type of representations of natural numbers in two different ways. One is based on the procedure used in Section 2 and second is based on procedure given in Section 3. This happens because, we can write same number in different ways.

4.1 Pyramidical Type Representations

4.1.1 Crazy Representations

Following the procedure of section 3 , we can write the natural numbers in pyramidical forms, for examples,

33 $:= 32+1\times 0!$
$:= 4\times 3+21\times 0!$
$:= 5+4+3\times(-2+10)$
$:= 6+5+43-21\times 0!$
$:= 76-5+4-32-10$
$:= 8+7+65-4\times 3!\times 2+1\times 0!$
$:= -9-8+7-6+54-3-2\times 1\times 0!$

48 $:= 3!\times(-2+10)$
$:= 4!+3+21\times 0!$
$:= -54\times 3+210$
$:= 6+5+4!+3!/2+10$
$:= 7\times(6+5)+4-32-1\times 0!$
$:= 8+7+6-5+4\times 3+2\times 10$
$:= -9+8\times 76-543+2-10$

https://goo.gl/MefiWn

$$\mathbf{434} := 432+1+0!$$
$$:= 54\times(3!+2)+1+0!$$
$$:= 6\times 54+(3+2)!-10$$
$$:= -7+654-3-210$$
$$:= 8\times(7+6)\times 5-4!-3\times 21+0!$$
$$:= (9+8+7+6+5-4)\times(-3!+2\times 10)$$

$$\mathbf{729} := 3^{(2+1)!\times 0!}$$
$$:= (4!+3)^2-1+0!$$
$$:= (5+4)^3\times(2-1)\times 0!$$
$$:= 6!+5+4+321\times 0$$
$$:= 765-4\times 3!-2-10$$
$$:= (8+76+5-4^3+2)^{(1+0!)}$$
$$:= (9+8+7+6-54-3)^2\times 1\times 0!$$

https://goo.gl/MefiWn

$$\mathbf{895} := -5+(4!+3!)^2\times 1\times 0!$$
$$:= 6\times(5+4\times 3!^2\times 1)+0!$$
$$:= 7\times(65+4^3)+2-10$$
$$:= -8-7+65\times(4\times 3+2)+1\times 0$$
$$:= 9+876+5-4+3^2\times 1\times 0!$$

$$\mathbf{947} := -5!+43+2^{10}$$
$$:= 6!+5+4\times 3+210$$
$$:= 7+((6+5)\times 4+3)\times 2\times 10$$
$$:= (8+7)\times 65-4!-3!+2+1\times 0$$
$$:= -9+8-76+5!+43\times 21+0!$$

https://goo.gl/MefiWn

For complete representations of natural numbers from 0 to 1000 refer author's work [33].

4.2 Pyramidical Type With Flexible Power

Following the procedure of Subsection 3.1 , we can write the natural numbers in pyramidical forms, for examples,

$$\mathbf{1} := 0^0$$
$$:= 0^1 + 1^0$$
$$:= 0^2 - 1^0 + 2^1$$
$$:= 0^0 + 1^3 - 2^2 + 3^1$$
$$:= 0^3 + 1^4 - 2^2 + 3^1 + 4^0$$
$$:= 0^2 + 1^4 - 2^5 + 3^3 + 4^1 + 5^0$$
$$:= 0^5 - 1^0 - 2^6 - 3^4 + 4^2 + 5^3 + 6^1$$
$$:= 0^2 + 1^7 + 2^5 - 3^6 + 4^3 + 5^4 + 6^0 + 7^1$$
$$:= 0^3 + 1^7 + 2^5 - 3^8 + 4^6 + 5^2 + 6^1 + 7^4 + 8^0$$
$$:= 0^5 - 1^8 + 2^9 + 3^7 - 4^6 + 5^4 + 6^2 + 7^0 + 8^1 + 9^3.$$

$$\mathbf{22} := 0^1 - 1^0 - 2^2 + 3^3$$
$$:= 0^2 + 1^3 + 2^4 + 3^0 + 4^1$$
$$:= 0^4 - 1^5 + 2^3 + 3^2 + 4^0 + 5^1$$
$$:= 0^2 + 1^6 + 2^5 - 3^4 + 4^3 + 5^1 + 60$$
$$:= 0^5 + 1^7 - 2^6 - 3^4 + 4^1 + 5^3 + 6^2 + 7^0$$
$$:= 0^1 + 1^4 + 2^8 + 3^5 - 4^7 + 5^6 + 6^3 + 7^0 + 8^2$$
$$:= 0^6 - 1^9 + 2^8 - 3^7 + 4^5 + 5^4 + 6^3 + 7^1 + 8^0 + 9^2.$$

https://goo.gl/K91g6W

$$\mathbf{666} := 0^1 - 1^3 + 2^5 + 3^2 + 4^0 + 5^4$$
$$:= 0^0 + 1^5 - 2^6 + 3^1 + 4^3 + 5^4 + 6^2$$
$$:= 0^5 + 1^7 - 2^6 + 3^1 + 4^3 + 5^4 + 6^2 + 7^0$$
$$:= 0^2 - 1^7 - 2^6 - 3^8 + 4^3 + 5^5 + 6^1 + 7^0 + 8^4$$
$$:= 0^7 + 1^9 - 2^5 - 3^8 + 4^6 + 5^2 + 6^1 + 7^4 + 8^0 + 9^3$$

$$\mathbf{1089} := 0^1 + 1^0 + 2^3 + 3^4 + 4^5 - 5^2$$
$$:= 0^4 - 1^6 + 2^1 + 3^3 + 4^5 + 5^0 + 6^2$$
$$:= 0^2 + 1^6 - 2^7 + 3^5 + 4^1 + 5^4 + 6^0 + 7^3$$
$$:= 0^0 - 1^7 + 2^4 - 3^8 + 4^6 + 5^5 + 6^1 + 7^3 + 8^2$$
$$:= 0^6 - 1^9 + 2^7 - 3^8 + 4^1 + 5^5 + 6^3 + 7^0 + 8^4 + 9^2.$$

$$\mathbf{1179} := 0^1 + 1^0 + 2^5 + 3^6 + 4^4 + 5^3 + 6^2$$
$$:= 0^2 + 1^6 + 2^4 - 3^7 + 4^0 + 5^5 + 6^3 + 7^1$$
$$:= 0^6 + 1^7 - 2^8 + 3^5 + 4^1 + 5^4 + 6^0 + 7^2 + 8^3$$
$$:= 0^6 + 1^9 - 2^8 - 3^7 + 4^5 + 5^3 + 6^1 + 7^4 + 8^2 + 9^0.$$

https://goo.gl/K91g6W

We observe that the digits appearing in **bases** and **exponents** are same in each number with different permutations. For complete representations of natural numbers from 0 to 1500 refer author's work [25].

4.3 Double Sequential Representations

This section deals with representations of natural numbers written in a sequential way of 3 to 8 digits ending in 0, such as {2,1,0}, {3,2,1,0},..., {9,8,7,6,5,4,32,1,0}. These representations are done combining both the processes given in Sections 2 and 3 . It is interesting to observe that the processes given in Section 2 uses operations such as, **addition, subtraction, multiplication, division, potentiation, square-root** and **factorial** with each digit appearing once. In case of process given in section 3 only addition and subtractions are used but each digit appears twice, once in base and another as a potentiation. Below are some examples,

$$\mathbf{1} := 2^1 - 1^0 + 0^2$$
$$:= 2 - 1 \times 0!.$$

$$\mathbf{2} := 2^0 + 1^2 + 0^1$$
$$:= 2 \times 1 \times 0!.$$

$$\mathbf{21} := 4^2 + 3^1 + 2^0 + 1^4 + 0^3$$
$$:= (4-3) \times 21 \times 0!.$$

$$\mathbf{116} := 5^2 + 4^0 + 3^4 + 2^3 + 1^5 + 0^1$$
$$:= 54 + 3 \times 21 - 0!.$$

$$\mathbf{227} := -7^5 + 6^1 + 5^4 + 4^7 + 3^2 + 2^3 + 1^6 + 0^0$$
$$:= (765 - 4!)/3 - 2 \times 10.$$

$$\mathbf{1406} := 6^4 + 5^1 + 4^3 + 3^2 + 2^5 - 1^6 + 0^0$$
$$:= 6 + 5! + 4 \times 32 \times 10.$$

$$\mathbf{1411} := 6^3 + 5^2 + 4^5 + 3^4 + 2^6 + 1^0 + 0^1$$
$$:= 6! - 5 - 4! + (3 \times 2 \times 1)! \times 0!.$$

$$\mathbf{1048} := -7^3 + 6^4 + 5^2 + 4^1 + 3^0 + 2^6 + 1^7 + 0^5$$
$$:= 7 - 6 + 5 \times 4 + 3 + 2^{10}.$$

$$\mathbf{2016} := 7^3 + 6^4 + 5^0 + 4^1 + 3^5 + 2^7 + 1^6 + 0^2$$
$$:= (7 + 65) \times (\sqrt{4} \times 3^2 + 10).$$

$$\mathbf{1087} := -8^2 + 7^1 + 6^0 + 5^3 + 4^4 + 3^6 + 2^5 + 1^8 + 0^7$$
$$:= -87 - 6 + (5 \times 4 \times 3! - 2) \times 10.$$

$$\mathbf{1417} := -9^0 + 8^3 - 7^4 + 6^1 + 5^2 + 4^5 + 3^7 + 2^6 + 1^9 + 0^8$$
$$:= 9 \times (8-7) + (6+5) \times 4 \times 32 \times 1 \times 0!.$$

(i) *https://goo.gl/YeYofd*; (ii) *https://goo.gl/Bb4V4e*; (iii) *https://goo.gl/nXVhvr*; (iv) *https://goo.gl/phHECc.*

For complete representations of numbers refer author's work [20, 24, 31, 32].

4.4 Triple Representations of Numbers

This section deals with the representations of natural numbers in three different ways. In each case the same digits are used. The first way is based on the representations given in Section 3 . The second and third are based on Section 2 in increasing and decreasing order of digits. In some cases, it is not possible to write all the numbers in three ways. For an idea see some examples below:

$$\mathbf{3} := 1^2+2^1 := 1+2 := 2+1$$

$$\mathbf{138} := -3^5+4^4+5^3 := -3!+4!+5! := 5!+4!-3!$$

$$\mathbf{32} := 1^1+2^2+3^3 := 32\times 1$$

$$\mathbf{425} := 1^6+2^4+3^5+4^1+5^3+6^2 := -1+23\times 4!-5!-6 := 6!+5\times(4-3\times 21)$$

https://goo.gl/sNcCNJ

$$\mathbf{905} := 2^6+3^2-4^7+5^4-6^3+7^5 := 2+(3\times 45-6)\times 7 := 7!/6+5+4!+3!^2$$

$$\mathbf{1840} := -4^5-5^8+6^7+7^6-8^4 := 4!\times(5+6)\times 7-8 := -8+7\times(6+5)\times 4!$$

$$\mathbf{2009} := 3^7-4^3-5^6-6^4+7^5 := -3!!+4!\times(5!-6)-7 := -7+(-6+5!)\times 4!-3!!$$

$$\mathbf{4278} := -3^3-4^6+5^4+6^5 := 3!\times(-\sqrt{4}-5+6!) := (6!-5-\sqrt{4})\times 3!$$

https://goo.gl/sNcCNJ

For complete detail refer refer to links [35]. The above work give the results only up to length 6. We can extend the results for higher length too. See examples below:

$$\mathbf{109} = 2^8-3^3-4^7-5^2+6^6+7^4-8^5 = 234+5\times 67+8 = 87+(6-5+43)/2.$$

$$\mathbf{512} = 1^7+2^5-3^8+4^6+5^2+6^1+7^4+8^3 = 1-23+456+78 = 8\times((7-6)^{54}+3\times 21).$$

$$\mathbf{944} = 3^9+4^8-5^7+6^6-7^5+8^4-9^3 = (-3+4!)\times 5+6!+7\times(8+9) = 987\times(6-5)+43.$$

$$\mathbf{288} = -2^3-3^8-4^9+5^4-6^7+7^5+8^2+9^6 = 2\times(34-5\times(67-89)) = (-9+87-6\times 5)\times 4\times 3/2.$$

https://goo.gl/sNcCNJ

$$\mathbf{1008} = 1^6 - 2^7 + 3^5 + 4^1 + 5^4 + 6^3 + 7^2 = (12-3) \times (45+67) = (-7+6+5) \times 4 \times 3 \times 21.$$

$$\mathbf{1172} = -2^9 - 3^2 + 4^8 - 5^7 - 6^6 + 7^4 - 8^3 + 9^5 = (2 \times 3)^4 + 5 + 6 - (7+8) \times 9 = (9+8) \times 76 - 5 \times 4 \times 3 \times 2.$$

https://goo.gl/sNcCNJ

5 Single Digits Representations

In previous Sections 2 and 3, all the nine digits are used to write natural numbers. Here the work is done in such a way that numbers are written in terms of each digit separately. See examples below:

$$\begin{aligned}
\mathbf{717} &:= (1+1)^{11} - 11^{(1+1+1)} \\
&:= 22^2 + 222 + 22/2 \\
&:= 3^{(3+3)} - 3 - 3 \times 3 \\
&:= 4 \times (4 \times 44 + 4) - 4 + 4/4 \\
&:= (55 \times (55+5+5) + 5 + 5)/5 \\
&:= (6 \times 6/(6+6))^6 - 6 - 6 \\
&:= 777 - 7 \times 7 - 77/7 \\
&:= 8 \times 88 + (88+8+8)/8 \\
&:= 9 \times 9 \times 9 - (99+9)/9.
\end{aligned}$$

$$\begin{aligned}
\mathbf{786} &:= ((1+1+1)^{(1+1+1)} + 1)^{(1+1)} + 1 + 1 \\
&:= (22+2+2+2)^2 + 2 \\
&:= 33 \times (3^3 - 3) - 3 - 3 \\
&:= 4 \times (4 \times (44+4) + 4) + (4+4)/4 \\
&:= 5 + (5^5 - 5/5)/(5 - 5/5) \\
&:= 66 \times (6+6) - 6 \\
&:= 777 + 7 + (7+7)/7 \\
&:= 8 \times (88+8) + 8 + (88-8)/8 \\
&:= 9 \times 99 - 99 - 9 + (9+9+9)/9
\end{aligned}$$

https://goo.gl/2L3mEk

$$\begin{aligned}
\mathbf{995} &:= (11-1)^{(1+1+1)} - (11-1)/(1+1) \\
&:= 22 + 2 \times (22^2 + 2) + 2/2 \\
&:= 3 \times 333 - 3 - 3/3 \\
&:= 4 \times (4^4 - 4 - 4) + 4 - 4/4 \\
&:= 5 \times (5+5) \times (5 \times 5 - 5) - 5 \\
&:= 666 + 6 \times 66 - 66 - 6/6 \\
&:= (7+7) \times (77-7) + 7 + 7 + 7/7 \\
&:= 888 + 88 + 8 + 88/8 \\
&:= 999 - (9+9+9+9)/9.
\end{aligned}$$

$$\begin{aligned}
\mathbf{1000} &:= (11-1)^{(1+1+1)} \\
&:= 2 \times (22^2 + 2^{(2+2)}) \\
&:= (3 \times 3 + 3/3)^3 \\
&:= 4 \times (4^4 - 4) - 4 - 4 \\
&:= 5 \times (5+5) \times (5 \times 5 - 5) \\
&:= ((66-6)/6)^{(6 \times 6/(6+6))} \\
&:= (7+7+7-7/7) \times (7 \times 7 + 7/7) \\
&:= 888 + 88 + 8 + 8 + 8 \\
&:= 999 + 9/9.
\end{aligned}$$

https://goo.gl/2L3mEk

$$\mathbf{1729} := (11+1)^{1+1+1}+1$$
$$:= (2/2+2)\times(22+2)^2+2/2$$
$$:= (3\times3+3)^3+3/3$$
$$:= 4\times(4\times44+4^4)+4/4$$
$$:= 55\times(5\times5-5)+(5^5-5)/5+5$$
$$:= 6\times6\times(6\times6+6+6)+6/6$$
$$:= 7\times7\times(7\times7-7-7)+7+7$$
$$:= 8\times(8\times(8+8)+88)+8/8$$
$$:= 9\times9\times9+999+9/9$$

$$\mathbf{2018} := (1+1)^{11}+(1+1+1)\times(1-11)$$
$$:= 2+2\times2\times(22^2-2+22)$$
$$:= 3+(33/3+3)^3-3^{3+3}$$
$$:= (4+4)/4+(4+4)\times(4^4-4)$$
$$:= 5+5^5-(5555+5)/5$$
$$:= 6+((6+6)/6)^{66/6}-6\times6$$
$$:= 7+7\times(7\times(7\times7-7)-7)+(7+7)/7$$
$$:= 8+(8888+88)/8+888$$
$$:= (9+9)/9\times(999+9/9+9)$$

https://goo.gl/2L3mEk

Values are calculated up to 1.000.000, but due to space, the work is written only from 0 to 1000. For details, refer to link [3].

Remark 5.1. *This work is also discussed in* ***YouTube*** *video by* ***Numberphile*** *as referred in Subsection 2.2. See again the link below:*

The 10,958 Problem - Numberphile *- [109].*

6 Single Letter Representations

In the Section 5, we wrote representations of numbers in terms of each digit. Instead representing in each digit separately, we can represent them in terms of single letter "a" . See examples below:

$$\mathbf{5} := \frac{aa-a}{a+a}$$
$$\mathbf{6} := \frac{aa+a}{a+a}$$
$$\mathbf{55} := \frac{aaa-a}{a+a}$$
$$\mathbf{56} := \frac{aaa+a}{a+a}$$
$$\mathbf{561} := \frac{aaaa+aa}{a+a}$$

$$\mathbf{666} := \frac{aaa\times(aa+a)}{(a+a)\times a}$$
$$\mathbf{786} := \frac{\left(\frac{(aa+a)\times aa}{a}-a\right)\times(aa+a)}{(a+a)\times a}$$
$$\mathbf{925} := \frac{aaaaa-aa}{aa+a}$$

where $a\in\{1,2,3,4,5,6,7,8,9\}$, and $aa=10\times a+a$, $aaa=10^2\times a+10\times a+a$, etc.

https://goo.gl/o74dcv

$$\mathbf{1089} := \frac{aaaa - aa - aa}{a}$$
$$\mathbf{1991} := \frac{\frac{aaaaaa}{aaa} \times (a+a) - aa}{a}$$
$$\mathbf{2018} := \frac{(aaaaa - aa - a) \times (a+a)}{aa \times a}$$
$$\mathbf{2020} := \frac{(aaaaa - a) \times (a+a)}{aa \times a}$$
$$\mathbf{2035} := \frac{(aaaa - a) \times aa}{(a+a+a) \times (a+a)}$$
$$\mathbf{4477} := \frac{aaa \times aa \times aa}{(a+a+a) \times a \times a}$$
$$\mathbf{4999} := \frac{aaaaa - aaaa - a - a}{(a+a)}$$
$$\mathbf{5000} := \frac{aaaaa - aaaa}{(a+a)}$$
$$\mathbf{122988} := \frac{(aaaa - a - a - a) \times aaa}{a \times a}.$$

where $a \in \{1,2,3,4,5,6,7,8,9\}$, and $aa = 10 \times a + a$, $aaa = 10^2 \times a + 10 \times a + a$, etc.

https://goo.gl/o74dcv

For more details refer author's complete works [6, 8, 83]. The first link [6] is up to 3000 numbers, while the second link [8] extend it to 5000 numbers. The third reference [83] give the numbers **fraction-type** representations.

6.1 Single Letter Power Representations

Above there are numbers written in terms of single letter "a". Using same idea, below are some examples, where the numbers are with exponential values written in terms of letter "a". Obviously, it is not possible to write all the numbers in terms of power, but only fews:

- **Power 2**

$$\mathbf{4} := 2^2 = \left(\frac{a+a}{a}\right)^{\frac{a+a}{a}}$$
$$\mathbf{9} := 3^2 = \left(\frac{a+a+a}{a}\right)^{\frac{a+a}{a}}$$
$$\mathbf{16} := 4^2 = \left(\frac{a+a+a+a}{a}\right)^{\frac{a+a}{a}}$$
$$\mathbf{25} := 5^2 = \left(\frac{aa-a}{a+a}\right)^{\frac{a+a}{a}}$$
$$\mathbf{36} := 6^2 = \left(\frac{aa+a}{a+a}\right)^{\frac{a+a}{a}}$$
$$\mathbf{49} := 7^2 = \left(\frac{aa-a-a-a-a}{a}\right)^{\frac{a+a}{a}}$$
$$\mathbf{64} := 7^2 = \left(\frac{aa-a-a-a}{a}\right)^{\frac{a+a}{a}}$$

where $a \in \{1,2,3,4,5,6,7,8,9\}$, and $aa = 10 \times a + a$, $aaa = 10^2 \times a + 10 \times a + a$, etc.

https://goo.gl/xYvcY5

- **Power 3**

$$\mathbf{8} := 2^3 = \left(\frac{a+a}{a}\right)^{\frac{a+a+a}{a}}$$

$$\mathbf{216} := 6^3 = \left(\frac{aa+a}{a+a}\right)^{\frac{a+a+a}{a}}$$

$$\mathbf{27} := 3^3 = \left(\frac{a+a+a}{a}\right)^{\frac{a+a+a}{a}}$$

$$\mathbf{343} := 7^3 = \left(\frac{aa-a-a-a-a}{a}\right)^{\frac{a+a+a}{a}}$$

$$\mathbf{64} := 4^3 = \left(\frac{a+a+a+a}{a}\right)^{\frac{a+a+a}{a}}$$

$$\mathbf{512} := 8^3 = \left(\frac{aa-a-a-a}{a}\right)^{\frac{a+a+a}{a}}$$

$$\mathbf{125} := 5^3 = \left(\frac{aa-a}{a+a}\right)^{\frac{a+a+a}{a}}$$

where $a \in \{1,2,3,4,5,6,7,8,9\}$, and $aa = 10 \times a + a$, $aaa = 10^2 \times a + 10 \times a + a$, etc.

https://goo.gl/xYvcY5

- **Power of 2**

$$\mathbf{4} := 2^2 = \left(\frac{a+a}{a}\right)^{\frac{a+a}{a}}$$

$$\mathbf{64} := 2^6 = \left(\frac{a+a}{a}\right)^{\frac{aa+a}{a+a}}$$

$$\mathbf{8} := 2^3 = \left(\frac{a+a}{a}\right)^{\frac{a+a+a}{a}}$$

$$\mathbf{128} := 2^7 = \left(\frac{a+a}{a}\right)^{\frac{aa-a-a-a-a}{a}}$$

$$\mathbf{16} := 2^4 = \left(\frac{a+a}{a}\right)^{\frac{a+a+a+a}{a}}$$

$$\mathbf{256} := 2^8 = \left(\frac{a+a}{a}\right)^{\frac{aa-a-a-a}{a}}$$

$$\mathbf{32} := 2^5 = \left(\frac{a+a}{a}\right)^{\frac{aa-a}{a+a}}$$

where $a \in \{1,2,3,4,5,6,7,8,9\}$, and $aa = 10 \times a + a$, $aaa = 10^2 \times a + 10 \times a + a$, etc.

https://goo.gl/xYvcY5

- **Power of 3**

$$\mathbf{9} := 3^2 = \left(\frac{a+a+a}{a}\right)^{\frac{(a+a}{a}}$$

$$\mathbf{27} := 3^3 = \left(\frac{a+a+a}{a}\right)^{\frac{(a+a+a}{a}}$$

$$\mathbf{81} := 3^4 = \left(\frac{a+a+a}{a}\right)^{\frac{a+a+a+a}{a}}$$

$$\mathbf{243} := 3^5 = \left(\frac{a+a+a}{a}\right)^{\frac{aa-a}{a+a}}$$

$$\mathbf{729} := 3^6 = \left(\frac{a+a+a}{a}\right)^{\frac{aa+a}{a+a}}$$

$$\mathbf{2187} := 3^7 = \left(\frac{a+a+a}{a}\right)^{\frac{aa-a-a-a-a}{a}}$$

$$\mathbf{6561} := 3^8 = \left(\frac{a+a+a}{a}\right)^{\frac{aa-a-a-a}{a}}$$

where $a \in \{1,2,3,4,5,6,7,8,9\}$, and $aa = 10 \times a + a$, $aaa = 10^2 \times a + 10 \times a + a$, etc.

https://goo.gl/xYvcY5

For full work, refer author's work [8].

6.2 Palindromic and Number Patterns

The idea of single letter representations of numbers given in Section 6 can be applied to **palindromic** and **number** type patterns. Throughout it is understood that

$$a \in \{1,2,3,4,5,6,7,8,9\},\ aa = 10 \times a + a,\ aaa = 10^2 \times a + 10 \times a + a, \text{ etc.}$$

See below some examples:

$$\mathbf{11} := \frac{a \times aa}{a \times a}$$

$$\mathbf{121} := \frac{aa \times aa}{a \times a}$$

$$\mathbf{12321} := \frac{aaa \times aaa}{a \times a}$$

$$\mathbf{1234321} := \frac{aaaa \times aaaa}{a \times a}$$

$$\mathbf{123454321} := \frac{aaaaa \times aaaaa}{a \times a}$$

$$\mathbf{12345654321} := \frac{aaaaaa \times aaaaaa}{a \times a}$$

$$\mathbf{1234567654321} := \frac{aaaaaaa \times aaaaaaa}{a \times a}$$

$$\mathbf{123456787654321} := \frac{aaaaaaaa \times aaaaaaaa}{a \times a}$$

$$\mathbf{12345678987654321} := \frac{aaaaaaaaa \times aaaaaaaaa}{a \times a}.$$

where $a \in \{1,2,3,4,5,6,7,8,9\}$, and $aa = 10 \times a + a$, $aaa = 10^2 \times a + 10 \times a + a$, etc.

https://goo.gl/xCP2Rv

$$\mathbf{1156 = 34^2} := \left(\frac{aa+aa+aa+a}{a}\right)^{\frac{a+a}{a}}$$

$$\mathbf{111556 = 334^2} := \left(\frac{aaa+aaa+aaa+a}{a}\right)^{\frac{a+a}{a}}$$

$$\mathbf{11115556 = 3334^2} := \left(\frac{aaaa+aaaa+aaaa+a}{a}\right)^{\frac{a+a}{a}}$$

$$\mathbf{1111155556 = 33334^2} := \left(\frac{aaaaa+aaaaa+aaaaa+a}{a}\right)^{\frac{a+a}{a}}$$

$$\mathbf{111111555556 = 333334^2} := \left(\frac{aaaaaa+aaaaaa+aaaaaa+a}{a}\right)^{\frac{a+a}{a}}$$

$$\mathbf{11111115555556 = 3333334^2} := \left(\frac{aaaaaaa+aaaaaaa+aaaaaaa+a}{a}\right)^{\frac{a+a}{a}}$$

$$\mathbf{1111111155555556 = 33333334^2} := \left(\frac{aaaaaaaa+aaaaaaaa+aaaaaaaa+a}{a}\right)^{\frac{a+a}{a}}.$$

where $a \in \{1,2,3,4,5,6,7,8,9\}$, and $aa = 10 \times a + a$, $aaa = 10^2 \times a + 10 \times a + a$, etc.

https://goo.gl/xCP2Rv

$$\mathbf{99 = 98 + 1} := \frac{aaa-aa-a}{a \times a}$$

$$\mathbf{999 = 987 + 12} := \frac{aaaa-aaa-a}{a \times a}$$

$$\mathbf{9999 = 9876 + 123} := \frac{aaaaa-aaaa-a}{a \times a}$$

$$\mathbf{99999 = 98765 + 1234} := \frac{aaaaaa-aaaaa-a}{a \times a}$$

$$\mathbf{999999 = 987654 + 12345} := \frac{aaaaaaa-aaaaaa-a}{a \times a}$$

$$\mathbf{9999999 = 9876543 + 123456} := \frac{aaaaaaaa-aaaaaaa-a}{a \times a}$$

$$\mathbf{99999999 = 98765432 + 1234567} := \frac{aaaaaaaaa-aaaaaaaa-a}{a \times a}$$

$$\mathbf{999999999 = 987654321 + 12345678} := \frac{aaaaaaaaaa-aaaaaaaaa-a}{a \times a}$$

$$\mathbf{9999999999 = 9876543210 + 123456789} := \frac{aaaaaaaaaaa-aaaaaaaaaa-a}{a \times a}.$$

where $a \in \{1,2,3,4,5,6,7,8,9\}$, and $aa = 10 \times a + a$, $aaa = 10^2 \times a + 10 \times a + a$, etc.

https://goo.gl/xCP2Rv

$$\mathbf{33 = 12 + 21} := \frac{(a+a+a)\times aa}{a\times a}$$
$$\mathbf{444 = 123 + 321} := \frac{(a+a+a+a)\times aaa}{a\times a}$$
$$\mathbf{5555 = 1234 + 4321} := \frac{(a+a+a+a+a)\times aaaa}{a\times a}$$
$$\mathbf{66666 = 12345 + 54321} := \frac{(a+a+a+a+a+a)\times aaaaa}{a\times a}$$
$$\mathbf{777777 = 123456 + 654321} := \frac{(aa-a-a-a-a)\times aaaaaa}{a\times a}$$
$$\mathbf{8888888 = 1234567 + 7654321} := \frac{(aa-a-a-a)\times aaaaaaa}{a\times a}$$
$$\mathbf{99999999 = 12345678 + 87654321} := \frac{(aa-a-a)\times aaaaaaaa}{a\times a}$$

where $a \in \{1,2,3,4,5,6,7,8,9\}$, and $aa = 10\times a + a$, $aaa = 10^2\times a + 10\times a + a$, etc.

https://goo.gl/xCP2Rv

$$\mathbf{111111111 = 12345679 \times 9 \times 1} := \frac{aaaaaaaaa\times a}{a\times a}$$
$$\mathbf{222222222 = 12345679 \times 9 \times 2} := \frac{aaaaaaaaa\times (a+a)}{a\times a}$$
$$\mathbf{333333333 = 12345679 \times 9 \times 3} := \frac{aaaaaaaaa\times (a+a+a)}{a\times a}$$
$$\mathbf{444444444 = 12345679 \times 9 \times 4} := \frac{aaaaaaaaa\times (a+a+a+a)}{a\times a}$$
$$\mathbf{555555555 = 12345679 \times 9 \times 5} := \frac{aaaaaaaaa\times (a+a+a+a+a)}{a\times a}$$
$$\mathbf{666666666 = 12345679 \times 9 \times 6} := \frac{aaaaaaaaa\times (a+a+a+a+a+a)}{a\times a}$$
$$\mathbf{777777777 = 12345679 \times 9 \times 7} := \frac{aaaaaaaaa\times (aa-a-a-a-a}{a\times a}$$
$$\mathbf{888888888 = 12345679 \times 9 \times 8} := \frac{aaaaaaaaa\times (aa-a-a-a}{a\times a}$$
$$\mathbf{999999999 = 12345679 \times 9 \times 9} := \frac{aaaaaaaaa\times (aa-a-a}{a\times a}.$$

The number 12345679 appearing above can be written as a division of $1/81$, i.e.

$$\frac{1}{81} = 0.012345679\,012345679\,012345679\,012345679\,.... = 0.\overline{012345679}$$

where $a \in \{1,2,3,4,5,6,7,8,9\}$, and $aa = 10\times a + a$, $aaa = 10^2\times a + 10\times a + a$, etc.

https://goo.gl/xCP2Rv

For full work, refer author's work [9, 10]. More studies on patterns in prime numbers without letter representations are given in Sections 21 and 22. Moreover, different types of patterns in many places are written connected with **selfie numbers** 9 to 14.

7 Running Expressions

It is well know that one can write, 12 = 3 × 4, 56 = 7 × 8. Here 9 remains alone. The aim of this work is to see how we can write expressions using 9 digits in a sequence in increasing and/or decreasing way. In the decreasing case, the possibility of 9 to 0 is also considered. The expressions are separated either by single or by double equality signs. This we have done using **factorial, square-root** and **Fibonacci sequence values**. For more details refer author's work [55, 56]. Below are some examples.

7.1 Running Expressions with Factorial and Square Root

7.1.1 Increasing Order

$$\mathbf{12} = 3+4+(5\times 6+7+8)/9$$
$$\mathbf{123} = 4+5+6\times 7+8\times 9$$
$$\mathbf{1234} = -5+6!\,+7+8^{\sqrt{9}}$$

$$12+3\times 4+5\times (6+7) = \mathbf{89}$$
$$1+23+45+6! = \mathbf{789}$$

https://goo.gl/rwqsyf
https://goo.gl/1jSyiR

7.1.2 Decreasing Order

$$98-7\times (6+5)\times (4-3) = \mathbf{21}$$
$$\sqrt{9}\times 87+6+54 = \mathbf{321}$$
$$9-8+7!-6\times 5! = \mathbf{4321}$$

$$9-8+7-6+5+4-3+2 = \mathbf{10}$$
$$9\times (8+7)+6+5+4^3 = \mathbf{210}$$
$$(9-87+6!)\times 5!/4! = \mathbf{3210}$$

https://goo.gl/rwqsyf
https://goo.gl/1jSyiR

$$\mathbf{98} = (7+6)\times 5+4\times 3+21$$
$$\mathbf{987} = 6!\ +5!\ +(4+3)\times 21$$

$$\mathbf{98} = 7+65+4+32-10$$
$$\mathbf{987} = 6!+54+3+210$$

https://goo.gl/rwqsyf
https://goo.gl/1jSyiR

7.1.3 Two or More Equalities

Above examples give running expressions in increasing or decreasing orders of 1 to 9 or 9 to 1 or 9 to 0 separated by single equality sign. But there are numbers, that can be separated by more equality signs, for example,

- **Increasing Order**

$$1\times 2\times 3! = (\sqrt{4+5})!+6 = 7+8-\sqrt{9}$$
$$1+2\times 3! = \sqrt{4}+5+6 = 78/(\sqrt{9})!$$
$$12+3! = \sqrt{4+5}\times 6 = 7+8+\sqrt{9}$$
$$1+23 = 4+5!/6 = 7+8+9$$
$$1\times (2^3)! = \sqrt{4+\sqrt{5\times 6!!}} = (7-8+9)!$$
$$(12-3)! = (\sqrt{4+5}+6)! = 7!\times 8\times 9$$

https://goo.gl/rwqsyf
https://goo.gl/1jSyiR

- **Decreasing Order**

$$\sqrt{9!/8!}\times 7! = 6!+5!^{\sqrt{4}} = 3!!\times 21$$
$$(9-8+7)! = (\sqrt{6!/5}-4)! = (3^2-1)!$$
$$(\sqrt{9!/8!})!\times 7 = 6\times(5+\sqrt{4}) = 32+10$$
$$(\sqrt{9})!!/(8+7) = -6+54 = 3!\times(-2+10)$$
$$9!/(8!/7) = 65-\sqrt{4} = 3\times 21$$
$$(9!-8!)/7! = \sqrt{6!\times 5}+4 = 32\times(1+0!)$$
$$(\sqrt{9})!!\times 8!/7! = 6!+(5+\sqrt{4})! = 3!!\times(-2+10)$$

https://goo.gl/rwqsyf
https://goo.gl/1jSyiR

7.1.4 Multiple Choices

- **Increasing Order**

$-(1+2)!!+(\sqrt{3^4})!+5!\times 6\times(-7+8)=\mathbf{9!}$
$(12-3)!+4-\sqrt{5\times 6!}+7\times 8=\mathbf{9!}$
$-1-(2\times 3)!+(4+5)!+6!-7+8=\mathbf{9!}$
$-1-(2^3)!+(4+5)!-6+7+8!=\mathbf{9!}$
$-1\times(2\times 3)!+(4+5)!+6!\times(-7+8)=\mathbf{9!}$
$1\times(2^3)!\times 4+5\times 6!\times 7\times 8=\mathbf{9!}$
$-1\times(2^3)!+(4+5)!+7!\times 8=\mathbf{9!}$
$-1\times(2+3)!+(4+5)!+(6+7-8)!=\mathbf{9!}$
$-1\times 2^3+(4+5)!+(-6+7)\times 8=\mathbf{9!}$
$-1\times 2-3+(4+5)!+6+7-8=\mathbf{9!}$
$-1^2\times 3+(4+5)!+\sqrt{-6+7+8}=\mathbf{9!}$
$-1+(2+3+4)!+(-5+6)^{78}=\mathbf{9!}$
$-12\times 3!+(4+5)!-6+78=\mathbf{9!}$
$1-2^3+(4+5)!+6-7+8=\mathbf{9!}$
$-12+(\sqrt{3^4})!+5+6-7+8=\mathbf{9!}$
$-1-2+(\sqrt{3^4})!+\sqrt{(5+67)/8}=\mathbf{9!}$
$-1-2-3+(4+5)!+6\times(-7+8)=\mathbf{9!}$
$(12-3)!-4!+(5-(-6+7)^8)!=\mathbf{9!}$
$(12-3)!-4-5-6+7+8=\mathbf{9!}$
$(12-3)!-\sqrt{4}-5+6-7+8=\mathbf{9!}$

https://goo.gl/rwqsyf
https://goo.gl/1jSyiR

- **Decreasing Order**

$(((\sqrt{9})!!+8)\times 7-6)/5+\sqrt{4}\times 3=\mathbf{2^{10}}$
$((\sqrt{9})!+8)\times(76-5)+4!+3!=\mathbf{2^{10}}$
$((\sqrt{9})!+8+7!+6)/5+4\times 3=\mathbf{2^{10}}$
$(-9\times 8+76)^5\times(4-3)=\mathbf{2^{10}}$
$(98/7-6)\times 5!+4^3=\mathbf{2^{10}}$
$(-98+7+6!-5!)\times\sqrt{4}+3!=\mathbf{2^{10}}$
$(\sqrt{9})!!+8\times(7\times 6-5)+4!/3=\mathbf{2^{10}}$
$-(\sqrt{9})!+(8\times 7+6)\times 5+(\sqrt{4}\times 3)!=\mathbf{2^{10}}$
$(\sqrt{9})!+8\times(7\times 6-5)+\sqrt{4}+3!!=\mathbf{2^{10}}$
$-(\sqrt{9})+8!/(7\times 6)-5+4!\times 3=\mathbf{2^{10}}$
$9\times(8\times(7+6)+5)+43=\mathbf{2^{10}}$
$98+7\times 6\times 5-4+3!!=\mathbf{2^{10}}$
$987+6\times 5+4+3=\mathbf{2^{10}}$
$987+6\times 5+4+3=\mathbf{2^{10}}$
$987+6+5^{\sqrt{4}}+3!=\mathbf{2^{10}}$
$-\sqrt{9}+8!/(7\times 6)-5+4!\times 3=\mathbf{2^{10}}$
$\sqrt{9}+8\times(7!/6!+5!)+\sqrt{4}+3=\mathbf{2^{10}}$
$-\sqrt{9}-8+7\times 6!/5+4!+3=\mathbf{2^{10}}$

https://goo.gl/rwqsyf
https://goo.gl/1jSyiR

For more details refer author's work [4, 55, 56]. The references [55, 56] are the revised version of [4].

7.2 Running Expressions with Fibonacci Sequence Numbers

Fibonacci sequence numbers are well known in literature [99, 102]. This sequence is defined as

$$F(0)=0,\quad F(1)=1,\quad F(n+1)=F(n)+F(n-1),\quad n\geq 1.$$

See below some running expressions using **Fibonacci sequence** values instead of factorial and/or square-root.

7.2.1 Increasing Order

$$\mathbf{12} = \boldsymbol{F}(3) \times \boldsymbol{F}(4) \times \boldsymbol{F}(5) + 6 - 7 - 8 - 9$$
$$\mathbf{123} = -4 \times 5 \times (6 - \boldsymbol{F}(7)) - 8 - 9$$
$$\mathbf{1234} = 5 \times \boldsymbol{F}(6) \times \boldsymbol{F}(7) + \boldsymbol{F}(8) \times \boldsymbol{F}(9)$$

$$1 + \boldsymbol{F}(2^3 + \boldsymbol{F}(4)) + (5 - 6)^7 = \mathbf{89}$$
$$1 \times 2 \times 3^4 \times 5 - \boldsymbol{F}(\boldsymbol{F}(6)) = \mathbf{789}$$
$$1 + 23 + \boldsymbol{F}(4 \times 5) = \mathbf{6789}.$$

https://goo.gl/ZF0JZ3

7.2.2 Decreasing Order

$$9 + (-\boldsymbol{F}(8)/7 + 6) \times 5 - \boldsymbol{F}(4)! + 3 = \mathbf{21}$$
$$-98 - \boldsymbol{F}(7) + \boldsymbol{F}(6) \times 54 = \mathbf{321}$$
$$(\boldsymbol{F}(9) \times \boldsymbol{F}(8) + 7) \times 6 - 5 = \mathbf{4321}$$

$$\mathbf{98} = (7 - 6) \times 5 + \boldsymbol{F}(4) \times (32 - 1)$$
$$\mathbf{987} = (6 - 5) \times \boldsymbol{F}(4 \times (3 + 2 - 1))$$

$$\mathbf{98} = -5 - 4 - 3 + 2 \times \boldsymbol{F}(10)$$
$$\mathbf{987} = (6 - 5)^4 \times \boldsymbol{F}(3 \times 2 + 10)$$
$$\mathbf{9876} = (\sqrt{5 + 4})! + \boldsymbol{F}(\boldsymbol{F}(3!) \times 2) \times 10$$

https://goo.gl/ZF0JZ3

7.2.3 Two or More Equalities

$$\boldsymbol{F}(-1+23)=\boldsymbol{F}(\boldsymbol{F}(\boldsymbol{F}(4))\times(5+6))=\boldsymbol{F}(7\times 8-\boldsymbol{F}(9))$$
$$\boldsymbol{F}(1\times 23)=\boldsymbol{F}(\boldsymbol{F}(4)\times 5+\boldsymbol{F}(6))=\boldsymbol{F}(-7+\boldsymbol{F}(8)+9)$$
$$\boldsymbol{F}(1+23)=\boldsymbol{F}(45-\boldsymbol{F}(\boldsymbol{F}(6)))=\boldsymbol{F}(7+8+9)$$

$$\boldsymbol{F}(-\boldsymbol{F}(9)+8\times 7)=\boldsymbol{F}(\boldsymbol{F}(\boldsymbol{F}(6))+5-4)=\boldsymbol{F}(\boldsymbol{F}(\boldsymbol{F}(3\times 2))+1)=\boldsymbol{F}(32-10)$$
$$\boldsymbol{F}(9+\boldsymbol{F}(8)-7)=\boldsymbol{F}(\boldsymbol{F}(6)+5\times\boldsymbol{F}(4))=\boldsymbol{F}(\boldsymbol{F}(3)+21)=\boldsymbol{F}(3+2\times 10)$$
$$\boldsymbol{F}(9+8+7)=\boldsymbol{F}(\boldsymbol{F}(6)\times(5-\boldsymbol{F}(\boldsymbol{F}(4))))=\boldsymbol{F}(3+21)$$

https://goo.gl/ZF0JZ3

For more details refer author's work [57]. Running expressions with **triangular numbers** shall be dealt elsewhere.

8 Narcissistic Type Numbers

An $n-$digit number that is the sum of the n^{th} powers of its digits is called an $n-$**narcissistic numbers**. Sometimes these numbers are also known by **Armstrong numbers**, **perfect digital invariants**, etc. Below are examples, of **narcissistic numbers** with 3 and 4 terms expressions.

$$\mathbf{153}:=1^3+5^3+3^3 \qquad \mathbf{1634}:=1^4+6^4+3^4+4^4$$
$$\mathbf{370}:=3^3+7^3+0^3 \qquad \mathbf{4151}:=4^5+1^5+5^5+1^5$$
$$\mathbf{371}:=3^3+7^3+1^3 \qquad \mathbf{8208}:=8^4+2^4+0^4+8^4$$
$$\mathbf{407}:=4^3+0^3+7^3. \qquad \mathbf{9472}:=9^4+4^4+7^4+2^4.$$

8.1 Flexible-Power-Narcissistic-Type Numbers

Above we have seen that the narcissistic numbers are with fixed power. Similar kind of study can be done if we replace the condition of fixed power by flexible power. Secondly, it is not necessary that we should also have positive constant. There may be numbers with positive and negative coefficients. Below are numbers with these situations.

8.1.1 Positive Coefficients

$$\mathbf{24} := 2^3 + 4^2$$
$$\mathbf{43} := 4^2 + 3^3$$
$$\mathbf{445} := 4^3 + 4^4 + 5^3$$
$$\mathbf{463} := 4^1 + 6^3 + 3^5$$
$$\mathbf{2283} := 2^4 + 2^4 + 8^2 + 3^7$$
$$\mathbf{2315} := 2^1 + 3^7 + 1^0 + 5^3$$
$$\mathbf{8977} := 8^1 + 9^4 + 7^1 + 7^4$$
$$\mathbf{8978} := 8^1 + 9^4 + 7^4 + 8^1$$
$$\mathbf{12634} := 1^1 + 2^5 + 6^5 + 3^6 + 4^6$$
$$\mathbf{13132} := 1^1 + 3^8 + 1^0 + 3^8 + 2^3$$
$$\mathbf{18347} := 1^1 + 8^3 + 3^1 + 4^5 + 7^5$$
$$\mathbf{18436} := 1^1 + 8^3 + 4^7 + 3^5 + 6^4$$
$$\mathbf{86345} := 8^4 + 6^0 + 3^3 + 4^6 + 5^7$$
$$\mathbf{86353} := 8^5 + 6^6 + 3^5 + 5^3 + 3^8$$
$$\mathbf{99758} := 9^3 + 9^0 + 7^5 + 5^7 + 8^4$$
$$\mathbf{99843} := 9^2 + 9^3 + 8^5 + 4^8 + 3^6$$

https://goo.gl/n8YkcQ

The above results are given with **positive coefficients**. These types of numbers, we can as **flexible power narcissistic numbers with positive coefficients**.

8.1.2 Positive and Negative Coefficients

There are many narcissistic numbers with flexibility in power and with positive and negative coefficients, for examples,

$$\mathbf{23} := -2^2 + 3^3$$
$$\mathbf{48} := -4^2 + 8^2$$
$$\mathbf{128} := -1^1 + 2^7 + 8^0$$
$$\mathbf{225} := -2^5 + 2^8 + 5^0$$
$$\mathbf{1526} := -1^1 - 5^2 + 2^8 + 6^4$$
$$\mathbf{1546} := -1^1 - 5^1 + 4^4 + 6^4$$
$$\mathbf{2137} := -2^8 + 1^0 - 3^2 + 7^4$$
$$\mathbf{6595} := -6^3 + 5^3 + 9^4 + 5^3$$
$$\mathbf{10176} := -1^1 - 0^0 + 1^0 + 7^4 + 6^5$$
$$\mathbf{10237} := -1^1 + 0^1 - 2^3 - 3^8 + 7^5$$
$$\mathbf{22904} := -2^3 - 2^5 + 9^4 - 0^0 + 4^7$$
$$\mathbf{85257} := -8^5 - 5^1 + 2^8 + 5^3 + 7^6$$
$$\mathbf{85259} := -8^2 + 5^3 + 2^9 + 5^7 + 9^4$$
$$\mathbf{99447} := -9^1 + 9^3 + 4^7 + 4^8 + 7^5$$
$$\mathbf{99843} := -9^3 + 9^2 + 8^5 + 4^8 + 3^7$$

https://goo.gl/n8YkcQ

The above results are given with positive and negative coefficients with flexible and powers. These types of numbers, we can as **flexible power narcissistic numbers with positive and negative coefficients**. For more details refer author's work [18, 81]. The reference [81] is the revised and enlarged version of the reference [18].

8.2 Fixed Power Narcissistic Numbers with Divisions

There are very few **narcissistic type numbers** with division and fixed power. See below these examples,

8.2.1 Positive Coefficients

$$\mathbf{37} := \frac{3^3+7^3}{3+7}$$
$$\mathbf{48} := \frac{4^3+8^3}{4+8}$$
$$\mathbf{241} := \frac{2^8+4^8+1^8}{2^4+4^4+1^1}$$
$$\mathbf{415} := \frac{4^5+1^5+5^5}{4+1+5}$$
$$\mathbf{2464} := \frac{2^5+4^5+6^5+4^5}{2^0+4^0+6^0+4^0}$$
$$\mathbf{4714} := \frac{4^5+7^5+1^5+4^5}{4^0+7^0+1^0+4^0}$$
$$\mathbf{5247} := \frac{5^5+2^5+4^5+7^5}{5^0+2^0+4^0+7^0}$$
$$\mathbf{8200} := \frac{8^5+2^5+0^5+0^5}{8^0+2^0+0^0+0^0}$$
$$\mathbf{15501} := \frac{1^9+5^9+5^9+0^9+1^9}{1^3+5^3+5^3+0^3+1^3}$$
$$\mathbf{142740} := \frac{1^7+4^7+2^7+7^7+4^7+0^7}{1^0+4^0+2^0+7^0+4^0+0^0}$$
$$\mathbf{231591} := \frac{2^7+3^7+1^7+5^7+9^7+1^7}{2+3+1+5+9+1}$$

https://goo.gl/oeebWZ

The above numbers are with positive coefficients. Allowing negative coefficients, still there are more numbers of similar kinds.

8.2.2 Positive and Negative Coefficients

$$\mathbf{13} := \frac{-1^3+3^3}{1^0+3^0}$$
$$\mathbf{21} := \frac{2^6-1^6}{2^1+1^1}$$
$$\mathbf{132} := \frac{-1^6+3^6+2^6}{1^1+3^1+2^1}$$
$$\mathbf{134} := \frac{-1^6+3^6+4^6}{-1^3-3^3+4^3}$$
$$\mathbf{763} := \frac{7^6-6^6+3^6}{7^2+6^2+3^2}$$
$$\mathbf{803} := \frac{8^4-0^4-3^4}{8^1-0^1-3^1}$$
$$\mathbf{6181} := \frac{-6^7-1^7+8^7-1^7}{-6^3-1^3+8^3-1^3}$$
$$\mathbf{8123} := \frac{8^5-1^5-2^5-3^5}{8^0+1^0+2^0+3^0}$$
$$\mathbf{26048} := \frac{2^5-6^5-0^5+4^5+8^5}{2^0+6^0+0^0-4^0-8^0}$$
$$\mathbf{26889} := \frac{2^6+6^6+8^6+8^6+9^6}{-2^2-6^2+8^2-8^2+9^2}$$
$$\mathbf{393425} := \frac{3^7+9^7-3^7+4^7-2^7-5^7}{3^1+9^1+3^1+4^1-2^1-5^1}$$
$$\mathbf{402589} := \frac{-4^6-0^6+2^6+5^6+8^6+9^6}{4^0+0^0+2^0+5^0-8^0-9^0}$$
$$\mathbf{1263641} := \frac{1^{16}+2^{16}+6^{16}+3^{16}-6^{16}+4^{16}-1^{16}}{1^6+2^6+6^6-3^6-6^6+4^6+1^6}$$
$$\mathbf{1457520} := \frac{-1^8+4^8+5^8+7^8-5^8-2^8-0^8}{-1^1+4^1+5^1-7^1+5^1-2^1-0^1}$$
$$\mathbf{9264867} := \frac{9^{10}-2^{10}-6^{10}+4^{10}-8^{10}-6^{10}+7^{10}}{9^2-2^2+6^2+4^2+8^2+6^2+7^2}$$
$$\mathbf{9926315} := \frac{9^7+9^7+2^7+6^7+3^7+1^7+5^7}{9^0+9^0+2^0+6^0-3^0-1^0-5^0}$$

https://goo.gl/oeebWZ

These are few examples. More detailed numbers of this kind can be seen in author's work [81].

8.3 Flexible Power Narcissistic Numbers with Divisions

Following the same idea of previous subsection 8.2 , here also we have written fractions with fixed and flexible powers along with positive-negative coefficients. See below some examples,

$$\mathbf{22} := \frac{2^1+2^6}{2^0+2^1} \qquad \mathbf{1177} := \frac{1^0+1^0+7^2+7^6}{1^2+1^2+7^2+7^2}$$

$$\mathbf{73} := \frac{7^2+3^5}{7^0+3^1} \qquad \mathbf{2089} := \frac{2^9+0^0+8^6+9^5}{2^3+0^0+8^2+9^2}$$

$$\mathbf{123} := \frac{1^0+2^3+3^6}{1^1+2^1+3^1} \qquad \mathbf{9933} := \frac{9^3+9^5+3^1+3^9}{9^0+9^0+3^1+3^1}$$

$$\mathbf{343} := \frac{3^6+4^4+3^8}{3^1+4^2+3^1} \qquad \mathbf{9985} := \frac{9^5+9^5+8^3+5^8}{9^1+9^1+8^1+5^2}$$

https://goo.gl/oeebWZ

The above numbers are with positive coefficients. Allowing negative coefficients, still there are more numbers of similar kind:

$$\mathbf{13} := \frac{-1^0+3^3}{1^0+3^0} \qquad \mathbf{1495} := \frac{-1^0+4^{10}-9^4+5^0}{-1^0+4^3+9^1+5^4}$$

$$\mathbf{91} := \frac{9^3-1^0}{9^1-1^0} \qquad \mathbf{8294} := \frac{8^0+2^5+9^5-4^5}{8^0+2^0+9^0+4^1}$$

$$\mathbf{266} := \frac{-2^{10}-6^1+6^4}{-2^0+6^0+6^0} \qquad \mathbf{9987} := \frac{9^1+9^6-8^6+7^3}{9^1+9^1+8^1+7^0}$$

$$\mathbf{269} := \frac{2^{10}-6^3-9^0}{2^0+6^0+9^0} \qquad \mathbf{9988} := \frac{9^1-9^4+8^4+8^6}{9^0+9^1+8^1+8^1}$$

https://goo.gl/oeebWZ

For more details refer author's work [18, 81, 82]. The references [81, 82] are the revised version of [18]. The reference [82] give the results with division.

8.4 Floor Function and Narcissistic Numbers with Divisions

In this section, we shall bring numbers in such a way that they becomes **narcissistic type numbers** with division in terms of **floor function**. Below are some examples,

$$\mathbf{21} := \left\lfloor \frac{2^6+1^6}{2+1} \right\rfloor \qquad \mathbf{115} := \left\lfloor \frac{1^5+1^5+5^5}{1^2+1^2+5^2} \right\rfloor$$

$$\mathbf{23} := \left\lfloor \frac{2^7+3^7}{2^4+3^4} \right\rfloor \qquad \mathbf{16737} := \left\lfloor \frac{1^{32}+6^{32}+7^{32}+3^{32}+7^{32}}{1^{27}+6^{27}+7^{27}+3^{27}+7^{27}} \right\rfloor$$

$$\mathbf{102} := \left\lfloor \frac{1^9+0^9+2^9}{1^2+0^2+2^2} \right\rfloor \qquad \mathbf{56494} := \left\lfloor \frac{5^{13}+6^{13}+4^{13}+9^{13}+4^{13}}{5^8+6^8+4^8+9^8+4^8} \right\rfloor$$

$$\mathbf{41527994} := \left\lfloor \frac{4^{18}+1^{18}+5^{18}+2^{18}+7^{18}+9^{18}+9^{18}+4^{18}}{4^{10}+1^{10}+5^{10}+2^{10}+7^{10}+9^{10}+9^{10}+4^{10}} \right\rfloor$$

$$\mathbf{51246026} := \left\lfloor \frac{5^{16}+1^{16}+2^{16}+4^{16}+6^{16}+0^{16}+2^{16}+6^{16}}{5^6+1^6+2^6+4^6+6^6+0^6+2^6+6^6} \right\rfloor$$

https://goo.gl/7qWeJf

For complete detail refer to author's work [19].

9 Selfie Numbers

Numbers represented by their own digits by use of certain operations are considered as **Selfie Number**. These numbers, we have divided in two categories. These two categories are again divided in two more, i.e., one in digit's order appearing in the numbers and their reverse, and the second is in increasing and decreasing order of digits. See below examples in each category:

- **Digit's Order**

$$\mathbf{936} := (\sqrt{9})!^3 + 6!$$

$$\mathbf{1296} := \sqrt{(1+2)!^9/6}$$

$$\mathbf{2896} := 2 \times (8 + (\sqrt{9})!! + 6!)$$

$$\mathbf{12969} := 1 \times 2 \times 9 \times 6! + 9$$

- **Reverse Order of Digits**

$$\mathbf{936} := 6! + (3!)^{\sqrt{9}}$$

$$\mathbf{1296} := 6^{(\sqrt{9}+2-1)}$$

$$\mathbf{2896} := (6! + (\sqrt{9})!! + 8) \times 2$$

$$\mathbf{20167} := 7 + (6+1+0!)!/2$$

https://goo.gl/iJXvi4

- **Increasing Order of Digits**

$$\mathbf{936} := 3!! + 6^{\sqrt{9})};$$
$$\mathbf{1296} := (1+2)! \times 6^{\sqrt{9}};$$
$$\mathbf{8397} := -3 - 7! + 8!/\sqrt{9};$$
$$\mathbf{241965} := (1 + (2 \times 4)! + 5) \times 6 + 9$$

- **Decreasing Order of Digits**

$$\mathbf{936} := (\sqrt{9})!! + 6^3$$
$$\mathbf{1296} := ((\sqrt{9})! \times 6)^2 \times 1$$
$$\mathbf{20148} := (8! - 4)/2 - 10$$
$$\mathbf{435609} := 9 + (6! - 5!/\sqrt{4})^{3-0!}$$

https://goo.gl/iJXvi4

Above we have given examples of **selfie numbers** in four different ways. This has been done using the basic operations along with **factorial** and **square-root**. See below more examples:

$$\mathbf{331779} := 3 + (31-7)^{\sqrt{7+9}} = \sqrt{9} + (7 \times 7 - 1)^3 \times 3$$
$$\mathbf{342995} := (3^4 - 2 - 9)^{\sqrt{9}} - 5 = -5 + (-9 + 9^2 - \sqrt{4})^3$$
$$\mathbf{759375} := (-7 + 59 - 37)^5 = (5 + 7 + 3)^{\sqrt{9}-5+7}.$$
$$\mathbf{759381} := 7 + (5 \times \sqrt{9})^{-3+8} - 1 = -1 + (8 \times 3 - 9)^5 + 7.$$

https://goo.gl/iJXvi4

See below following interesting numbers,

$$\mathbf{456} := 4 \times (5! - 6) = (-6 + 5!) \times 4$$
$$\mathbf{3456} := 3!! \times 4/5 \times 6 = 6!/5 \times 4 \times 3!!$$
$$\mathbf{34567} := (3 + 45) \times 6! + 7 = 7 + 6! \times (5 + 43)$$
$$\mathbf{345678} := (3! - \sqrt{4}) \times 5! \times 6! + 78.$$

https://goo.gl/iJXvi4

For full details, refer author's work [2, 5, 7, 13, 14, 15, 79]. The reference [79] is the revised version of some of the other references only in **digit's order**. See below some particular situations. Due to high quantity of numbers, the work is concentrated in only two ways, i.e., in order of digits and its reverse.

9.1 Addition, Subtraction and Factorial

Examples given above uses basic operation along with **factorial** and **square-root**. Madachy [106], page 167, 1966, gave few **factorial-type** examples. See below.

$$\mathbf{1} := 1!$$
$$\mathbf{2} := 2!$$
$$\mathbf{145} := 1! + 4! + 5!$$
$$\mathbf{40585} := 4! + 0! + 5! + 8! + 5!$$

Question arises, what else we can get using only the operations of **addition** and **subtraction** along with **factorial**? Below are some examples up to 6-digits:

$$\mathbf{145} := 1! + 4! + 5!$$
$$\mathbf{1463} := -1! + 4! + 6! + 3!!$$
$$\mathbf{10077} := -1! - 0! - 0! + 7! + 7!$$
$$\mathbf{40585} := 4! + 0! + 5! + 8! + 5!$$
$$\mathbf{80518} := 8! - 0! - 5! - 1! + 8!.$$
$$\mathbf{317489} := -3! - 1! - 7! - 4! - 8! + 9!$$
$$\mathbf{352797} := -3! + 5 - 2! - 7! + 9! - 7!$$
$$\mathbf{357592} := -3! - 5! - 7! - 5! + 9! - 2!$$
$$\mathbf{357941} := 3! + 5! - 7! + 9! - 4! - 1!$$
$$\mathbf{361469} := 3! - 6! - 1! + 4! - 6! + 9!$$
$$\mathbf{364292} := 3!! + 6! - 4! - 2! + 9! - 2!$$
$$\mathbf{397584} := -3!! + 9! - 7! + 5! + 8! + 4!$$
$$\mathbf{398173} := 3! + 9! + 8! + 1! - 7! + 3!$$
$$\mathbf{408937} := -4! + 0! + 8! + 9! + 3!! + 7!$$
$$\mathbf{715799} := -7! - 1! + 5! - 7! + 9! + 9!$$
$$\mathbf{720599} := -7! - 2! + 0! - 5! + 9! + 9!$$

https://goo.gl/AX54PJ

Still, we can have examples not necessarily having factorial on all the numbers. See below:

$$\mathbf{733} := 7 + 3!! + 3!$$
$$\mathbf{5177} := 5! + 17 + 7!$$
$$\mathbf{363239} := 36 + 323 + 9!$$
$$\mathbf{363269} := 363 + 26 + 9!$$
$$\mathbf{403199} := 40319 + 9!$$

https://goo.gl/AX54PJ

For more details refer to author's work [43, 79]. The reference [79] is the revised form of the reference [43] but only in **digit's order**.

9.2 Consecutive Symmetric Selfie Numbers

There are numbers those can be represented in symmetric and consecutive forms as block of 100 or blocks of 10. In some cases the numbers appears only in one way, i.e., either in **digit's order** or in **reverse order of digits** or in **both ways**. See below some examples:

9.2.1 Blocks of 100: Digit's Order

14400 $:= (1+4)!^{\sqrt{4}} + 00$
14401 $:= (1+4)!^{\sqrt{4}} + 01$
14402 $:= (1+4)!^{\sqrt{4}} + 02$
14403 $:= (1+4)!^{\sqrt{4}} + 03$
$\cdots \quad \cdots \quad \cdots$
14451 $:= (1+4)!^{\sqrt{4}} + 51$
14452 $:= (1+4)!^{\sqrt{4}} + 52$
14453 $:= (1+4)!^{\sqrt{4}} + 53$
14454 $:= (1+4)!^{\sqrt{4}} + 54$
$\cdots \quad \cdots \quad \cdots$
14496 $:= (1+4)!^{\sqrt{4}} + 96$
14497 $:= (1+4)!^{\sqrt{4}} + 97$
14498 $:= (1+4)!^{\sqrt{4}} + 98$
14499 $:= (1+4)!^{\sqrt{4}} + 99$

64800 $:= 6!^{\sqrt{4}}/8 + 00$
64801 $:= 6!^{\sqrt{4}}/8 + 01$
64802 $:= 6!^{\sqrt{4}}/8 + 02$
64803 $:= 6!^{\sqrt{4}}/8 + 03$
$\cdots \quad \cdots \quad \cdots$
64851 $:= 6!^{\sqrt{4}}/8 + 51$
64852 $:= 6!^{\sqrt{4}}/8 + 52$
64853 $:= 6!^{\sqrt{4}}/8 + 53$
64854 $:= 6!^{\sqrt{4}}/8 + 54$
$\cdots \quad \cdots \quad \cdots$
64896 $:= 6!^{\sqrt{4}}/8 + 96$
64897 $:= 6!^{\sqrt{4}}/8 + 97$
64898 $:= 6!^{\sqrt{4}}/8 + 98$
64899 $:= 6!^{\sqrt{4}}/8 + 99$

https://goo.gl/JDCa3i

$\mathbf{158400} := (-(1+5)!+8!)\times 4+00$
$\mathbf{158401} := (-(1+5)!+8!)\times 4+01$
$\mathbf{158402} := (-(1+5)!+8!)\times 4+02$
$\mathbf{158403} := (-(1+5)!+8!)\times 4+03$
... ...
$\mathbf{158451} := (-(1+5)!+8!)\times 4+51$
$\mathbf{158452} := (-(1+5)!+8!)\times 4+52$
$\mathbf{158453} := (-(1+5)!+8!)\times 4+53$
$\mathbf{158454} := (-(1+5)!+8!)\times 4+54$
... ...
$\mathbf{158496} := (-(1+5)!+8!)\times 4+96$
$\mathbf{158497} := (-(1+5)!+8!)\times 4+97$
$\mathbf{158498} := (-(1+5)!+8!)\times 4+98$
$\mathbf{158499} := (-(1+5)!+8!)\times 4+99$

$\mathbf{363600} := (3^{6/3})!+6!+00$
$\mathbf{363601} := (3^{6/3})!+6!+01$
$\mathbf{363602} := (3^{6/3})!+6!+02$
$\mathbf{363603} := (3^{6/3})!+6!+03$
... ...
$\mathbf{363651} := (3^{6/3})!+6!+51$
$\mathbf{363652} := (3^{6/3})!+6!+52$
$\mathbf{363653} := (3^{6/3})!+6!+53$
$\mathbf{363654} := (3^{6/3})!+6!+54$
... ...
$\mathbf{363696} := (3^{6/3})!+6!+96$
$\mathbf{363697} := (3^{6/3})!+6!+97$
$\mathbf{363698} := (3^{6/3})!+6!+98$
$\mathbf{363699} := (3^{6/3})!+6!+99$

https://goo.gl/JDCa3i

9.2.2 Blocks of 10: Digit's Order

$\mathbf{64980} := (6!+\sqrt{4})\times(\sqrt{9})!!/8+0$
$\mathbf{64981} := (6!+\sqrt{4})\times(\sqrt{9})!!/8+1$
$\mathbf{64982} := (6!+\sqrt{4})\times(\sqrt{9})!!/8+2$
$\mathbf{64983} := (6!+\sqrt{4})\times(\sqrt{9})!!/8+3$
$\mathbf{64984} := (6!+\sqrt{4})\times(\sqrt{9})!!/8+4$
$\mathbf{64985} := (6!+\sqrt{4})\times(\sqrt{9})!!/8+5$
$\mathbf{64986} := (6!+\sqrt{4})\times(\sqrt{9})!!/8+6$
$\mathbf{64987} := (6!+\sqrt{4})\times(\sqrt{9})!!/8+7$
$\mathbf{64988} := (6!+\sqrt{4})\times(\sqrt{9})!!/8+8$
$\mathbf{64989} := (6!+\sqrt{4})\times(\sqrt{9})!!/8+9$

$\mathbf{83520} := 8!+3\times 5!^2+0$
$\mathbf{83521} := 8!+3\times 5!^2+1$
$\mathbf{83522} := 8!+3\times 5!^2+2$
$\mathbf{83523} := 8!+3\times 5!^2+3$
$\mathbf{83524} := 8!+3\times 5!^2+4$
$\mathbf{83525} := 8!+3\times 5!^2+5$
$\mathbf{83526} := 8!+3\times 5!^2+6$
$\mathbf{83527} := 8!+3\times 5!^2+7$
$\mathbf{83528} := 8!+3\times 5!^2+8$
$\mathbf{83529} := 8!+3\times 5!^2+9$

https://goo.gl/ynYHGX

9.2.3 Blocks of 10: Reverse order of Digits

43560 $:= 0+(6!+5!\times 3!!)/\sqrt{4}$
43561 $:= 1+(6!+5!\times 3!!)/\sqrt{4}$
43562 $:= 2+(6!+5!\times 3!!)/\sqrt{4}$
43563 $:= 3+(6!+5!\times 3!!)/\sqrt{4}$
43564 $:= 4+(6!+5!\times 3!!)/\sqrt{4}$
43565 $:= 5+(6!+5!\times 3!!)/\sqrt{4}$
43566 $:= 6+(6!+5!\times 3!!)/\sqrt{4}$
43567 $:= 7+(6!+5!\times 3!!)/\sqrt{4}$
43568 $:= 8+(6!+5!\times 3!!)/\sqrt{4}$
43569 $:= 9+(6!+5!\times 3!!)/\sqrt{4}$

53880 $:= 0+8!+8!/3+5!$
53881 $:= 1+8!+8!/3+5!$
53882 $:= 2+8!+8!/3+5!$
53883 $:= 3+8!+8!/3+5!$
53884 $:= 4+8!+8!/3+5!$
53885 $:= 5+8!+8!/3+5!$
53886 $:= 6+8!+8!/3+5!$
53887 $:= 7+8!+8!/3+5!$
53888 $:= 8+8!+8!/3+5!$
53889 $:= 9+8!+8!/3+5!$

https://goo.gl/ynYHGX

9.2.4 Blocks of 10: Both Ways

466560 $:= (4+6)\times 6^5\times 6+0=0+6^5\times 6\times (6+4)$
466561 $:= (4+6)\times 6^5\times 6+1=1+6^5\times 6\times (6+4)$
466562 $:= (4+6)\times 6^5\times 6+2=2+6^5\times 6\times (6+4)$
466563 $:= (4+6)\times 6^5\times 6+3=3+6^5\times 6\times (6+4)$
466564 $:= (4+6)\times 6^5\times 6+4=4+6^5\times 6\times (6+4)$
466565 $:= (4+6)\times 6^5\times 6+5=5+6^5\times 6\times (6+4)$
466566 $:= (4+6)\times 6^5\times 6+6=6+6^5\times 6\times (6+4)$
466567 $:= (4+6)\times 6^5\times 6+7=7+6^5\times 6\times (6+4)$
466568 $:= (4+6)\times 6^5\times 6+8=8+6^5\times 6\times (6+4)$
466569 $:= (4+6)\times 6^5\times 6+9=9+6^5\times 6\times (6+4)$

64840 $:= 6!^{\sqrt{4}}/8+40=0+4^8+4!-6!$
64841 $:= 6!^{\sqrt{4}}/8+41=1+4^8+4!-6!$
64842 $:= 6!^{\sqrt{4}}/8+42=2+4^8+4!-6!$
64843 $:= 6!^{\sqrt{4}}/8+43=3+4^8+4!-6!$
64844 $:= 6!^{\sqrt{4}}/8+44=4+4^8+4!-6!$
64845 $:= 6!^{\sqrt{4}}/8+45=5+4^8+4!-6!$
64846 $:= 6!^{\sqrt{4}}/8+46=6+4^8+4!-6!$
64847 $:= 6!^{\sqrt{4}}/8+47=7+4^8+4!-6!$
64848 $:= 6!^{\sqrt{4}}/8+48=8+4^8+4!-6!$
64849 $:= 6!^{\sqrt{4}}/8+49=9+4^8+4!-6!$

https://goo.gl/ynYHGX

$$\mathbf{518400} := (5+1)!^{8/4} + 00 = 00 + (4!/8)!! \times (1+5)!$$
$$\mathbf{518411} := (5+1)!^{8/4} + 11 = 11 + (4!/8)!! \times (1+5)!$$
$$\mathbf{518422} := (5+1)!^{8/4} + 22 = 22 + (4!/8)!! \times (1+5)!$$
$$\mathbf{518433} := (5+1)!^{8/4} + 33 = 33 + (4!/8)!! \times (1+5)!$$
$$\mathbf{518444} := (5+1)!^{8/4} + 44 = 44 + (4!/8)!! \times (1+5)!$$
$$\mathbf{518455} := (5+1)!^{8/4} + 55 = 55 + (4!/8)!! \times (1+5)!$$
$$\mathbf{518466} := (5+1)!^{8/4} + 66 = 66 + (4!/8)!! \times (1+5)!$$
$$\mathbf{518477} := (5+1)!^{8/4} + 77 = 77 + (4!/8)!! \times (1+5)!$$
$$\mathbf{518488} := (5+1)!^{8/4} + 88 = 88 + (4!/8)!! \times (1+5)!$$
$$\mathbf{518499} := (5+1)!^{8/4} + 99 = 99 + (4!/8)!! \times (1+5)!$$

$$\mathbf{363390} := 3! + 6! - 3!^3 + 9! + 0 = 0 + 9! + (3 \times 3)!/6! + 3!$$
$$\mathbf{363391} := 3! + 6! - 3!^3 + 9! + 1 = 1 + 9! + (3 \times 3)!/6! + 3!$$
$$\mathbf{363392} := 3! + 6! - 3!^3 + 9! + 2 = 2 + 9! + (3 \times 3)!/6! + 3!$$
$$\mathbf{363393} := 3! + 6! - 3!^3 + 9! + 3 = 3 + 9! + (3 \times 3)!/6! + 3!$$
$$\mathbf{363394} := 3! + 6! - 3!^3 + 9! + 4 = 4 + 9! + (3 \times 3)!/6! + 3!$$
$$\mathbf{363395} := 3! + 6! - 3!^3 + 9! + 5 = 5 + 9! + (3 \times 3)!/6! + 3!$$
$$\mathbf{363396} := 3! + 6! - 3!^3 + 9! + 6 = 6 + 9! + (3 \times 3)!/6! + 3!$$
$$\mathbf{363397} := 3! + 6! - 3!^3 + 9! + 7 = 7 + 9! + (3 \times 3)!/6! + 3!$$
$$\mathbf{363398} := 3! + 6! - 3!^3 + 9! + 8 = 8 + 9! + (3 \times 3)!/6! + 3!$$
$$\mathbf{363399} := 3! + 6! - 3!^3 + 9! + 9 = 9 + 9! + (3 \times 3)!/6! + 3!$$

https://goo.gl/ynYHGX

The numbers 518400, 518411, 518422, etc. are not consecutive but the representation is symmetric. For full details, refer author's work [14, 15, 44, 79]. The reference [79] is the revised version of some of above works, but is only in **digit's order**.

9.3 Unified Representations

From examples above, we observe that there are numbers such as 936, 1296, etc., those can be written in all the four ways. These types of numbers we call as **unified selfie numbers**. More clearly,

Unified Selfie number := Order of digits
:= Reverse order of digits
:= Increasing order of digits
:= Decreasing order of digits.

Below are examples of **unified selfie numbers** written in all the four ways,

$$\begin{aligned}\mathbf{729} &= (\sqrt{7+2})!! + 9\\ &= 9 + (\sqrt{2+7})!!\\ &= (2+7)^{\sqrt{9}}\\ &= 9^{\sqrt{7+2}}.\end{aligned}$$

$$\begin{aligned}\mathbf{97632} &= -(\sqrt{9})!! + 7! + 6^{3!} \times 2\\ &= 2 \times 3!^6 + 7! - (\sqrt{9})!!\\ &= 2 \times 3!^6 + 7! - (\sqrt{9})!!\\ &= -(\sqrt{9})!! + 7! + 6^{3!} \times 2.\end{aligned}$$

https://goo.gl/VVMmvA

$$\mathbf{114688} = (11\times\sqrt{4}+6)\times\sqrt{8^8} = (8+8\times 6)\times\sqrt{4^{11}} = (11\times\sqrt{4}+6)\times\sqrt{8^8} = (8+8\times 6)\times\sqrt{4^{11}}.$$

$$\mathbf{139968} = (13\times 9-9)\times\sqrt{6^8} = 8\times 6^{\sqrt{9}}\times 9^{3-1} = 1\times\sqrt{36^8}/(\sqrt{9}+9) = (-\sqrt{9}+9)^8/(6\times(3-1)).$$

https://goo.gl/VVMmvA

$$\mathbf{326627} = (3+2+6^{\sqrt{6^2}})\times 7 = \sqrt{7^2}\times(6^6+2+3) = (\sqrt{2+23}+6^6)\times 7 = 7\times(6^6+\sqrt{3+22}).$$

$$\mathbf{531439} = -5+3+(-1+4)^{3+9} = (9\times 3)^4\times 1+3-5 = 1-3+3^{\sqrt{4+5}+9} = 9^{5+4-3}-\sqrt{3+1}.$$

https://goo.gl/VVMmvA

For full details, refer author's work [11, 13, 14, 15].

9.4 Patterns in Selfie Numbers

Numbers extended by same properties multiplying by zero, we call as **patterns in numbers**. Few examples of this kind are studied long back in 1966 by Madachy [106], page 174-175. See below:

$$3^4\times 425 := 34425 \qquad 31^2\times 325 := 312325$$
$$3^4\times 4250 := 344250 \qquad 31^2\times 3250 := 3123250$$
$$3^4\times 42500 := 3442500 \qquad 31^2\times 32500 := 31232500$$

For simplicity, let us write these numbers as **patterns in selfie numbers**. Subsections below give some examples.

9.4.1 Digit's Order

$$\mathbf{1285} := (1+2^8)\times 5$$
$$\mathbf{12850} := (1+2^8)\times 50$$
$$\mathbf{128500} := (1+2^8)\times 500$$

$$\mathbf{15585} := 1\times(5^5-8)\times 5$$
$$\mathbf{155850} := 1\times(5^5-8)\times 50$$
$$\mathbf{1558500} := 1\times(5^5-8)\times 500$$

$$\mathbf{8192} := 8^{1+\sqrt{9}}\times 2$$
$$\mathbf{81920} := 8^{1+\sqrt{9}}\times 20$$
$$\mathbf{819200} := 8^{1+\sqrt{9}}\times 200$$

$$\mathbf{9435} := \sqrt{29^4}\times 35$$
$$\mathbf{294350} := \sqrt{29^4}\times 350$$
$$\mathbf{2943500} := \sqrt{29^4}\times 3500$$

https://goo.gl/ru1AUo

9.4.2 Decreasing Order of Digits

$$\mathbf{1827} := 87\times 21$$
$$\mathbf{18270} := 87\times 210$$
$$\mathbf{182700} := 87\times 2100$$

$$\mathbf{19683} := \sqrt{9^8}\times(6-3)\times 1$$
$$\mathbf{196830} := \sqrt{9^8}\times(6-3)\times 10$$
$$\mathbf{1968300} := \sqrt{9^8}\times(6-3)\times 100$$

$$\mathbf{2916} := (9\times 6)^2\times 1$$
$$\mathbf{29160} := (9\times 6)^2\times 10$$
$$\mathbf{291600} := (9\times 6)^2\times 100$$

$$\mathbf{95544} := ((\sqrt{9}+9)^5+54)\times 4$$
$$\mathbf{9955440} := ((\sqrt{9}+9)^5+54)\times 40$$
$$\mathbf{99554400} := ((\sqrt{9}+9)^5+54)\times 400$$

https://goo.gl/ru1AUo

For the complete work of this section on **selfie numbers**, refer author's work [2, 5, 7, 11, 12, 13, 14, 15, 43, 44, 79]. The Sections 11, 12, 13 and 14 brings more work on **selfie numbers** with some extra functions. The following section give **selfie numbers** but with different way. It is an extension of **narcissistic type selfie numbers**.

10 Flexible Power Selfie Numbers

From the numbers given in Section 3 we observe that there are numbers, 23, 1239, 1364, 1654, 1837, 2137, 2173, 2537, 3125, 3275, 3529 and 4316 uses the same digits in bases and powers, such as,

$$\mathbf{23} := -2^2+3^3$$
$$\mathbf{1239} := 1^2+2^9-3^1+9^3$$
$$\mathbf{1364} := 1^6+3^1+6^4+4^3$$
$$\mathbf{1654} := -1^6+6^1+5^4+4^5$$
$$\mathbf{1837} := 1^8-8^1+3^7-7^3$$
$$\mathbf{2137} := -2^1+1^3+3^7-7^2$$
$$\mathbf{2173} := -2^3+1^2-7^1+3^7$$
$$\mathbf{2537} := 2^5-5^2+3^7+7^3$$
$$\mathbf{3125} := -3^2+1^1+2^3+5^5$$
$$\mathbf{3275} := -3^3+2^7+7^2+5^5$$
$$\mathbf{3529} := -3^3+5^5+2^9-9^2$$
$$\mathbf{4316} := 4^6+3^1+1^4+6^3$$

https://goo.gl/oXT6Wf

These representations, we call as **flexible power selfie numbers**. On the other side, they can also be considered as particular cases of **flexible power narcissistic numbers** given in Section 8. Motivated by this idea author wrote a work on **flexible power selfie numbers** with different digits [16, 21, 22, 23]. See below some examples,

$$\mathbf{1} := 1^1$$
$$\mathbf{23} := -2^2+3^3$$
$$\mathbf{1654} := -1^6+6^1+5^4+4^5$$
$$\mathbf{3435} := 3^3+4^4+3^3+5^5$$
$$\mathbf{4355} := 4^5+3^4+5^3+5^5$$
$$\mathbf{39339} := -3^3+9^3+3^9+3^9-9^3$$
$$\mathbf{46350} := -4^3+6^6-3^5+5^0+0^4$$
$$\mathbf{46360} := 4^0+6^6-3^4-6^3+0^6$$
$$\mathbf{397612} := 3^2+9^1+7^6+6^7+1^9+2^3$$
$$\mathbf{423858} := 4^3+2^8+3^4+8^2+5^8+8^5$$
$$\mathbf{637395} := 6^5+3^3+7^3+3^9+9^6+5^7$$
$$\mathbf{758014} := 7^7+5^1+8^0+0^5+1^4-4^8$$
$$\mathbf{778530} := 7^7+7^3+8^5-5^7+3^0+0^8$$
$$\mathbf{804637} := 8^0+0^4-4^8+6^6-3^3+7^7$$

https://goo.gl/ZWepNN
https://goo.gl/mxiewN

$$\mathbf{15647982} := 1^5-5^9+6^2+4^4+7^7-9^1+8^8+2^6$$
$$\mathbf{17946238} := 1^6+7^8+9^4+4^2+6^9+2^3+3^1+8^7$$
$$\mathbf{57396108} := -5^6+7^9+3^5+9^3+6^7+1^1+0^0+8^8$$
$$\mathbf{134287690} := 1^2+3^8+4^7+2^4+8^9+7^3+6^6+9^0+0^1$$
$$\mathbf{387945261} := 3^3+8^2+7^6+9^9+4^7+5^8+2^4+6^1+1^5$$
$$\mathbf{392876054} := 3^0+9^9-2^2-8^5+7^8-6^7+0^3-5^4+4^6$$
$$\mathbf{392876540} := -3^0+9^9-2^4-8^5+7^8-6^7-5^3+4^6+0^2$$

https://goo.gl/ZWepNN
https://goo.gl/mxiewN

We observe that the bases and powers are of same digits with different permutations.

The work is divided in two categories, i.e., **with zero** or **without zero**. Also it goes maximum up to 9 digits. For details refer author's work [16, 21, 22, 23]. The references [21, 22, 23] include the work appearing in [16].

11 Fibonacci and Triangular Type Selfie Numbers

11.1 Fibonacci Sequence and Selfie Numbers

Below are examples of **selfie numbers** by use of **Fibonacci sequence values**. This we have done in different situations, such as using $F(\cdot)$ and $F(F(\cdot))$ in separate works. For more details refer author's work [36, 37, 38, 80]. The definition of **Fibonacci sequence** is already given in Subection 7.2. For simplicity, let's write again

$$F(0)=0, \quad F(1)=1, \quad F(n+1)=F(n)+F(n-1), \quad n\geq 1.$$

See below some examples:

11.1.1 Digit's Order

$$\mathbf{34} := F(3\times F(4))$$
$$\mathbf{233} := F(F(-2+3\times 3))$$
$$\mathbf{630} := F(F(6))\times 30$$
$$\mathbf{1178} := F(11)\times F(7)+F(8)$$
$$\mathbf{2079} := (-2+F(F(07)))\times 9$$
$$\mathbf{4128} := (F(4!)+(F((1+2)!))!)/F(8)$$
$$\mathbf{4147} := -F(F(F(4)!)+1)+F(F(4)!+F(7))$$
$$\mathbf{4160} := -F(F(F(4)!))+F(-1+F(F(6))-0!)$$
$$\mathbf{4864} := F(F(4))^8\times (F(F(6))-F(F(4)))$$
$$\mathbf{6780} := F(6)+7+F(F(8)-0!)$$
$$\mathbf{6930} := 6\times (F(9)^{F(3)}-0!)$$
$$\mathbf{6944} := F(6)+F(9)^{F(F(4))}\times F(4)!$$
$$\mathbf{8759} := -F(9-5)^7+F(F(8))$$
$$\mathbf{8849} := -9\times F(F(F(F(F(4)))-8))+F(F(8))$$
$$\mathbf{9239} := F(9)^2\times F(3!)-9$$
$$\mathbf{9349} := -F(F(9)/F(F(4)))+F(F(F(-3+9)))$$

(i) *https://goo.gl/ETctFz*;
(ii) *https://goo.gl/3f3zub*;
(iii) *https://goo.gl/MxAjXh*;
(iv) *https://goo.gl/bdqHD6*;

27450 $:= F(2+F(7))\times 45+0$
27451 $:= F(2+F(7))\times 45+1$
27452 $:= F(2+F(7))\times 45+2$
27453 $:= F(2+F(7))\times 45+3$
27454 $:= F(2+F(7))\times 45+4$
27455 $:= F(2+F(7))\times 45+5$
27456 $:= F(2+F(7))\times 45+6$
27457 $:= F(2+F(7))\times 45+7$
27458 $:= F(2+F(7))\times 45+8$
27459 $:= F(2+F(7))\times 45+9$

86920 $:= 8\times(F(F(F(6)))-9^2)+0$
86921 $:= 8\times(F(F(F(6)))-9^2)+1$
86922 $:= 8\times(F(F(F(6)))-9^2)+2$
86923 $:= 8\times(F(F(F(6)))-9^2)+3$
86924 $:= 8\times(F(F(F(6)))-9^2)+4$
86925 $:= 8\times(F(F(F(6)))-9^2)+5$
86926 $:= 8\times(F(F(F(6)))-9^2)+6$
86927 $:= 8\times(F(F(F(6)))-9^2)+7$
86928 $:= 8\times(F(F(F(6)))-9^2)+8$
86929 $:= 8\times(F(F(F(6)))-9^2)+9$

(i) *https://goo.gl/ETctFz*;
(ii) *https://goo.gl/3f3zub*;
(iii) *https://goo.gl/MxAjXh*;
(iv) *https://goo.gl/bdqHD6*;

11.1.2 Reverse Order of Digits

36 $:= 6^{F(3)}$
143 $:= F(3\times 4)-1$
231 $:= F(13)-2$
377 $:= F(-7+7\times 3)$
986 $:= (F(6)+F(8))\times F(9)$
1596 $:= F(F(6)+9)-F(F(F(5-1)))$
2592 $:= F(2\times 9)+F(5+F(2))$
3728 $:= 8\times 2\times F(7+3!)$

3736 $:= -6!+F(3)^{F(7)}/F(3)$
4317 $:= (7-1)!\times 3!-F(4)$
4344 $:= (4+(F(4)\times F(3))!)\times F(4)!$
5417 $:= 7!+F(-1+F(4)\times 5)$
5439 $:= -F(9)+F(F(F(3!)))/(-F(4)+5)$
8427 $:= (-7!+2)/F(F(4))+F(F(8))$
8447 $:= -7!/F(F(4))+F(F(F(4)!))+F(F(8))$
9756 $:= F(F(F(6)))-5\times 7\times F(9)$

(i) *https://goo.gl/ETctFz*;
(ii) *https://goo.gl/3f3zub*;
(iii) *https://goo.gl/MxAjXh*;
(iv) *https://goo.gl/bdqHD6*;

$$\mathbf{43740} := 0 + F(4)^7 \times (F(F(3!)) - F(F(F(4))))$$
$$\mathbf{43741} := 1 + F(4)^7 \times (F(F(3!)) - F(F(F(4))))$$
$$\mathbf{43742} := 2 + F(4)^7 \times (F(F(3!)) - F(F(F(4))))$$
$$\mathbf{43743} := 3 + F(4)^7 \times (F(F(3!)) - F(F(F(4))))$$
$$\mathbf{43744} := 4 + F(4)^7 \times (F(F(3!)) - F(F(F(4))))$$
$$\mathbf{43745} := 5 + F(4)^7 \times (F(F(3!)) - F(F(F(4))))$$
$$\mathbf{43746} := 6 + F(4)^7 \times (F(F(3!)) - F(F(F(4))))$$
$$\mathbf{43747} := 7 + F(4)^7 \times (F(F(3!)) - F(F(F(4))))$$
$$\mathbf{43748} := 8 + F(4)^7 \times (F(F(3!)) - F(F(F(4))))$$
$$\mathbf{43749} := 9 + F(4)^7 \times (F(F(3!)) - F(F(F(4))))$$

$$\mathbf{74830} := 0 + (-F(3)^8 + F(F(F(F(4)!)))) \times 7$$
$$\mathbf{74831} := 1 + (-F(3)^8 + F(F(F(F(4)!)))) \times 7$$
$$\mathbf{74832} := 2 + (-F(3)^8 + F(F(F(F(4)!)))) \times 7$$
$$\mathbf{74833} := 3 + (-F(3)^8 + F(F(F(F(4)!)))) \times 7$$
$$\mathbf{74834} := 4 + (-F(3)^8 + F(F(F(F(4)!)))) \times 7$$
$$\mathbf{74835} := 5 + (-F(3)^8 + F(F(F(F(4)!)))) \times 7$$
$$\mathbf{74836} := 6 + (-F(3)^8 + F(F(F(F(4)!)))) \times 7$$
$$\mathbf{74837} := 7 + (-F(3)^8 + F(F(F(F(4)!)))) \times 7$$
$$\mathbf{74838} := 8 + (-F(3)^8 + F(F(F(F(4)!)))) \times 7$$
$$\mathbf{74839} := 9 + (-F(3)^8 + F(F(F(F(4)!)))) \times 7$$

(i) *https://goo.gl/ETctFz*;
(ii) *https://goo.gl/3f3zub*;
(iii) *https://goo.gl/MxAjXh*;
(iv) *https://goo.gl/bdqHD6*;

11.1.3 Both Ways

$$\mathbf{143} := -1 + F(4 \times 3) = F(3 \times 4) - 1$$
$$\mathbf{986} := F(9) \times (F(8) + F(6)) = (F(6) + F(8)) \times F(9)$$
$$\mathbf{1178} := F(11) \times F(7) + F(8) = F(8) + F(7) \times F(11)$$
$$\mathbf{2585} := F(2) + F(5 + 8 + 5) = F(5 + 8 + 5) + F(2)$$
$$\mathbf{12819} := 1 + F(2 \times (8 - 1)) \times F(9) = F(9) \times F((-1 + 8) \times 2) + 1$$
$$\mathbf{24297} := F(2 \times 4) \times F(2 + 9) \times F(7) = F(7) \times F(9 + 2) \times F(4 \times 2)$$
$$\mathbf{39394} := -3 + 93 + F(9)^{F(4)} = (-4 + F(9)) \times 3 + F(9)^3$$
$$\mathbf{74997} := -7 \times 4 + F(9 + 9 + 7) = F(7 + 9 + 9) - 4 \times 7$$
$$\mathbf{87937} := -8 + F(7) \times F(9 \times 3 - 7) = F(7) \times F(3 \times 9 - 7) - 8$$
$$\mathbf{98703} := 9 \times (F(8) + F(7 \times 03)) = (F(3 \times 07) + F(8)) \times 9$$

(i) *https://goo.gl/ETctFz*;
(ii) *https://goo.gl/3f3zub*;
(iii) *https://goo.gl/MxAjXh*;
(iv) *https://goo.gl/bdqHD6*;

$$\mathbf{3840} := (F(3!))!/F(8) \times F(F(4)) + 0 = 0 + F(F(4)) \times 8!/F(F(3!))$$
$$\mathbf{3841} := (F(3!))!/F(8) \times F(F(4)) + 1 = 1 + F(F(4)) \times 8!/F(F(3!))$$
$$\mathbf{3842} := (F(3!))!/F(8) \times F(F(4)) + 2 = 2 + F(F(4)) \times 8!/F(F(3!))$$
$$\mathbf{3843} := (F(3!))!/F(8) \times F(F(4)) + 3 = 3 + F(F(4)) \times 8!/F(F(3!))$$
$$\mathbf{3844} := (F(3!))!/F(8) \times F(F(4)) + 4 = 4 + F(F(4)) \times 8!/F(F(3!))$$
$$\mathbf{3845} := (F(3!))!/F(8) \times F(F(4)) + 5 = 5 + F(F(4)) \times 8!/F(F(3!))$$
$$\mathbf{3846} := (F(3!))!/F(8) \times F(F(4)) + 6 = 6 + F(F(4)) \times 8!/F(F(3!))$$
$$\mathbf{3847} := (F(3!))!/F(8) \times F(F(4)) + 7 = 7 + F(F(4)) \times 8!/F(F(3!))$$
$$\mathbf{3848} := (F(3!))!/F(8) \times F(F(4)) + 8 = 8 + F(F(4)) \times 8!/F(F(3!))$$
$$\mathbf{3849} := (F(3!))!/F(8) \times F(F(4)) + 9 = 9 + F(F(4)) \times 8!/F(F(3!))$$

(i) *https://goo.gl/ETctFz*; (iii) *https://goo.gl/MxAjXh*;
(ii) *https://goo.gl/3f3zub*; (iv) *https://goo.gl/bdqHD6*;

$$\mathbf{54670} := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 0 = 0 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5$$
$$\mathbf{54671} := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 1 = 1 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5$$
$$\mathbf{54672} := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 2 = 2 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5$$
$$\mathbf{54673} := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 3 = 3 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5$$
$$\mathbf{54674} := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 4 = 4 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5$$
$$\mathbf{54675} := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 5 = 5 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5$$
$$\mathbf{54676} := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 6 = 6 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5$$
$$\mathbf{54677} := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 7 = 7 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5$$
$$\mathbf{54678} := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 8 = 8 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5$$
$$\mathbf{54679} := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 9 = 9 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5$$

(i) *https://goo.gl/ETctFz*; (iii) *https://goo.gl/MxAjXh*;
(ii) *https://goo.gl/3f3zub*; (iv) *https://goo.gl/bdqHD6*;

$$\mathbf{823540} := (8-F(2))^{F(3)+5} - F(4) + 0 = 0 - F(4) + (5+F(3))^{-F(2)+8}$$
$$\mathbf{823541} := (8-F(2))^{F(3)+5} - F(4) + 1 = 1 - F(4) + (5+F(3))^{-F(2)+8}$$
$$\mathbf{823542} := (8-F(2))^{F(3)+5} - F(4) + 2 = 2 - F(4) + (5+F(3))^{-F(2)+8}$$
$$\mathbf{823543} := (8-F(2))^{F(3)+5} - F(4) + 3 = 3 - F(4) + (5+F(3))^{-F(2)+8}$$
$$\mathbf{823544} := (8-F(2))^{F(3)+5} - F(4) + 4 = 4 - F(4) + (5+F(3))^{-F(2)+8}$$
$$\mathbf{823545} := (8-F(2))^{F(3)+5} - F(4) + 5 = 5 - F(4) + (5+F(3))^{-F(2)+8}$$
$$\mathbf{823546} := (8-F(2))^{F(3)+5} - F(4) + 6 = 6 - F(4) + (5+F(3))^{-F(2)+8}$$
$$\mathbf{823547} := (8-F(2))^{F(3)+5} - F(4) + 7 = 7 - F(4) + (5+F(3))^{-F(2)+8}$$
$$\mathbf{823548} := (8-F(2))^{F(3)+5} - F(4) + 8 = 8 - F(4) + (5+F(3))^{-F(2)+8}$$
$$\mathbf{823549} := (8-F(2))^{F(3)+5} - F(4) + 9 = 9 - F(4) + (5+F(3))^{-F(2)+8}$$

(i) *https://goo.gl/ETctFz*;
(ii) *https://goo.gl/3f3zub*;
(iii) *https://goo.gl/MxAjXh*;
(iv) *https://goo.gl/bdqHD6*;

11.1.4 Patterns with Fibonacci Sequence Values

Below are representations of numbers those can be extended just multiplying by 10. These numbers we call, number patterns with Fibonacci numbers sequence values:

$$\mathbf{1365} := 13 \times F(F(6)) \times 5$$
$$\mathbf{13650} := 13 \times F(F(6)) \times 50$$
$$\mathbf{136500} := 13 \times F(F(6)) \times 500$$

$$\mathbf{3528} := F(3+5)^2 \times 8$$
$$\mathbf{35280} := F(3+5)^2 \times 80$$
$$\mathbf{352800} := F(3+5)^2 \times 800$$

$$\mathbf{1687} := (F(F(1+6))+8) \times 7$$
$$\mathbf{16870} := (F(F(1+6))+8) \times 70$$
$$\mathbf{168700} := (F(F(1+6))+8) \times 700$$

$$\mathbf{3635} := (3^6 - F(3)) \times 5$$
$$\mathbf{36350} := (3^6 - F(3)) \times 50$$
$$\mathbf{363500} := (3^6 - F(3)) \times 500$$

(i) *https://goo.gl/ETctFz*;
(ii) *https://goo.gl/3f3zub*;
(iii) *https://goo.gl/MxAjXh*;
(iv) *https://goo.gl/bdqHD6*;

For more details refer author's work [36, 37, 38, 80]. The reference [80] is the combined version of the references [36, 37, 38] but only in digit's order.

11.2 Triangular Numbers

Triangular numbers are very much famous in the literature of mathematics. The general formula to write these numbers is given by

$$T(n) = 1+2+3+\ldots = \frac{n(n+1)}{2} = C(n+1,2)$$

The examples given in above subsections are with **factorial**, **square-root**, **Fibonacci sequence** numbers, etc. Still, one can have similar kind of results using **Triangular numbers**. For more details see author's work [62]. See below some examples:

11.2.1 Digit's Order

$$\begin{aligned}
\mathbf{1069} &:= T(10) - T(6) + T(T(9)) \\
\mathbf{1081} &:= T(1 + T(08 + 1)) \\
\mathbf{2887} &:= T(T(T(T(2)))) + T(T(8) + T(8)) + T(7) \\
\mathbf{4965} &:= T(-4 + 9) + T(-T(6) + T(T(5))) \\
\mathbf{4999} &:= 49 + T(99) \\
\mathbf{99545} &:= T(9) + T(9) \times T(T(T(5) - 4)) + 5 \\
\mathbf{99546} &:= T(9) + T(9) \times T(T(T(5) - 4)) + 6.
\end{aligned}$$

https://goo.gl/8cNpaq

$$\begin{aligned}
\mathbf{190} &:= T(19) + 0 & \mathbf{1090} &:= T(10) + T(T(9)) + 0 \\
\mathbf{191} &:= T(19) + 1 & \mathbf{1091} &:= T(10) + T(T(9)) + 1 \\
\mathbf{192} &:= T(19) + 2 & \mathbf{1092} &:= T(10) + T(T(9)) + 2 \\
\mathbf{193} &:= T(19) + 3 & \mathbf{1093} &:= T(10) + T(T(9)) + 3 \\
\mathbf{194} &:= T(19) + 4 & \mathbf{1094} &:= T(10) + T(T(9)) + 4 \\
\mathbf{195} &:= T(19) + 5 & \mathbf{1095} &:= T(10) + T(T(9)) + 5 \\
\mathbf{196} &:= T(19) + 6 & \mathbf{1096} &:= T(10) + T(T(9)) + 6 \\
\mathbf{197} &:= T(19) + 7 & \mathbf{1097} &:= T(10) + T(T(9)) + 7 \\
\mathbf{198} &:= T(19) + 8 & \mathbf{1098} &:= T(10) + T(T(9)) + 8 \\
\mathbf{199} &:= T(19) + 9 & \mathbf{1099} &:= T(10) + T(T(9)) + 9.
\end{aligned}$$

https://goo.gl/8cNpaq

11.2.2 Reverse Order of Digits

$$\mathbf{874} := T(T(T(4))) - T(T(7)+8)$$
$$\mathbf{0105} := 50 + T(10)$$
$$\mathbf{1155} := -T(T(5)) + T(51-1)$$
$$\mathbf{1224} := T(T(T(4)) - T(T(2))) - 2 + 1$$
$$\mathbf{2418} := T(81) - T(42)$$
$$\mathbf{99632} := 2 + (3 + T(T(6) + T(9))) \times T(9)$$
$$\mathbf{99633} := 3 + (3 + T(T(6) + T(9))) \times T(9).$$

https://goo.gl/8cNpaq

$$\mathbf{0210} := 0 + T(1 \times 20)$$
$$\mathbf{0211} := 1 + T(1 \times 20)$$
$$\mathbf{0212} := 2 + T(1 \times 20)$$
$$\mathbf{0213} := 3 + T(1 \times 20)$$
$$\mathbf{0214} := 4 + T(1 \times 20)$$
$$\mathbf{0215} := 5 + T(1 \times 20)$$
$$\mathbf{0216} := 6 + T(1 \times 20)$$
$$\mathbf{0217} := 7 + T(1 \times 20)$$
$$\mathbf{0218} := 8 + T(1 \times 20)$$
$$\mathbf{0219} := 9 + T(1 \times 20)$$

$$\mathbf{2080} := 0 + T(8^{02})$$
$$\mathbf{2081} := 1 + T(8^{02})$$
$$\mathbf{2082} := 2 + T(8^{02})$$
$$\mathbf{2083} := 3 + T(8^{02})$$
$$\mathbf{2084} := 4 + T(8^{02})$$
$$\mathbf{2085} := 5 + T(8^{02})$$
$$\mathbf{2086} := 6 + T(8^{02})$$
$$\mathbf{2087} := 7 + T(8^{02})$$
$$\mathbf{2088} := 8 + T(8^{02})$$
$$\mathbf{2089} := 9 + T(8^{02}).$$

$$\mathbf{4190} := 0 + T(91) + 4$$
$$\mathbf{4191} := 1 + T(91) + 4$$
$$\mathbf{4192} := 2 + T(91) + 4$$
$$\mathbf{4193} := 3 + T(91) + 4$$
$$\mathbf{4194} := 4 + T(91) + 4$$
$$\mathbf{4195} := 5 + T(91) + 4$$
$$\mathbf{4196} := 6 + T(91) + 4$$
$$\mathbf{4197} := 7 + T(91) + 4$$
$$\mathbf{4198} := 8 + T(91) + 4$$
$$\mathbf{4199} := 9 + T(91) + 4.$$

https://goo.gl/8cNpaq

11.2.3 Both Ways

$$\mathbf{1932} := (1+T(9))\times T(T(3))\times 2 = 2\times T(T(3))\times (T(9)+1)$$
$$\mathbf{1937} := -1+T(T(9))+T(T(3)\times 7) = T(7\times T(3))+T(T(9))-1$$
$$\mathbf{1938} := T(T(1\times 9))+T(T(3)+T(8)) = T(T(8)+T(3))+T(T(9\times 1))$$
$$\mathbf{2183} := -T(T(T(2))+1)+T(T(8+3)) = T(T(3+8))-T(1+T(T(2)))$$
$$\mathbf{3518} := 3+5\times T(1+T(8)) = T(T(8)+1)\times 5+3$$
$$\mathbf{6472} := -6+T(4)+T(7)\times T(T(T(T(2)))) = T(T(T(T(2))))\times T(7)+T(4)-6$$
$$\mathbf{9981} := 9\times T(T(9))+T(T(8\times 1)) = T(T(1\times 8))+T(T(9))\times 9$$
$$\mathbf{9985} := T(T(9)/9)\times T(T(8))-5 = T(5)\times T(T(8))-T(9)/9$$

https://goo.gl/8cNpaq

$$\mathbf{2210} := T(T(T(T(T(T(2)))))/T(T(T(2)))))-1+0 = 0-1+T(T(T(T(T(T(2)))))/T(T(T(2)))))$$
$$\mathbf{2211} := T(T(T(T(T(T(2)))))/T(T(T(2)))))-1+1 = 1-1+T(T(T(T(T(T(2)))))/T(T(T(2)))))$$
$$\mathbf{2212} := T(T(T(T(T(T(2)))))/T(T(T(2)))))-1+2 = 2-1+T(T(T(T(T(T(2)))))/T(T(T(2)))))$$
$$\mathbf{2213} := T(T(T(T(T(T(2)))))/T(T(T(2)))))-1+3 = 3-1+T(T(T(T(T(T(2)))))/T(T(T(2)))))$$
$$\mathbf{2214} := T(T(T(T(T(T(2)))))/T(T(T(2)))))-1+4 = 4-1+T(T(T(T(T(T(2)))))/T(T(T(2)))))$$
$$\mathbf{2215} := T(T(T(T(T(T(2)))))/T(T(T(2)))))-1+5 = 5-1+T(T(T(T(T(T(2)))))/T(T(T(2)))))$$
$$\mathbf{2216} := T(T(T(T(T(T(2)))))/T(T(T(2)))))-1+6 = 6-1+T(T(T(T(T(T(2)))))/T(T(T(2)))))$$
$$\mathbf{2217} := T(T(T(T(T(T(2)))))/T(T(T(2)))))-1+7 = 7-1+T(T(T(T(T(T(2)))))/T(T(T(2)))))$$
$$\mathbf{2218} := T(T(T(T(T(T(2)))))/T(T(T(2)))))-1+8 = 8-1+T(T(T(T(T(T(2)))))/T(T(T(2)))))$$
$$\mathbf{2219} := T(T(T(T(T(T(2)))))/T(T(T(2)))))-1+9 = 9-1+T(T(T(T(T(T(2)))))/T(T(T(2))))).$$

https://goo.gl/8cNpaq

$$\mathbf{8460} := T(8)\times(4+T(T(6)))+0 = 0+(T(T(6))+4)\times T(8)$$
$$\mathbf{8461} := T(8)\times(4+T(T(6)))+1 = 1+(T(T(6))+4)\times T(8)$$
$$\mathbf{8462} := T(8)\times(4+T(T(6)))+2 = 2+(T(T(6))+4)\times T(8)$$
$$\mathbf{8463} := T(8)\times(4+T(T(6)))+3 = 3+(T(T(6))+4)\times T(8)$$
$$\mathbf{8464} := T(8)\times(4+T(T(6)))+4 = 4+(T(T(6))+4)\times T(8)$$
$$\mathbf{8465} := T(8)\times(4+T(T(6)))+5 = 5+(T(T(6))+4)\times T(8)$$
$$\mathbf{8466} := T(8)\times(4+T(T(6)))+6 = 6+(T(T(6))+4)\times T(8)$$
$$\mathbf{8467} := T(8)\times(4+T(T(6)))+7 = 7+(T(T(6))+4)\times T(8)$$
$$\mathbf{8468} := T(8)\times(4+T(T(6)))+8 = 8+(T(T(6))+4)\times T(8)$$
$$\mathbf{8469} := T(8)\times(4+T(T(6)))+9 = 9+(T(T(6))+4)\times T(8).$$

https://goo.gl/8cNpaq

11.2.4 Patterns With Triangular Numbers

$$\mathbf{21} := T(T(T(2)))\times 1$$
$$\mathbf{210} := T(T(T(2)))\times 10$$
$$\mathbf{2100} := T(T(T(2)))\times 100$$

$$\mathbf{2688} := 2\times T(6)\times 8\times 8$$
$$\mathbf{26880} := 2\times T(6)\times 8\times 80$$
$$\mathbf{268800} := 2\times T(6)\times 8\times 800$$

$$\mathbf{245} := (-T(T(2))+T(T(4)))\times 5$$
$$\mathbf{2450} := (-T(T(2))+T(T(4)))\times 50$$
$$\mathbf{24500} := (-T(T(2))+T(T(4)))\times 500$$

$$\mathbf{9936} := T(T(T(9))/T(9))\times 36$$
$$\mathbf{99360} := T(T(T(9))/T(9))\times 360$$
$$\mathbf{993600} := T(T(T(9))/T(9))\times 3600$$

https://goo.gl/8cNpaq

For more details see author's work [62]. Due to high quantity of numbers, we worked only up to 4 digits, i.e., from 1 to 9999.

11.3 Simultaneous Representations: Fibonacci and Triangular Numbers

In this subsection, we shall bring numbers those can be written with Fibonacci sequence values and triangular numbers simultaneously. For more details see author's work [63]. Below are some examples.

11.3.1 Digit's Order

$$\mathbf{1446} := (-1+F(4))\times(F(4)+6!) = (1+4!\times T(4))\times 6$$
$$\mathbf{1448} := -1+F(4!)/(4\times 8) = -1+T(T(T(4)))-T(T(4))-T(8)$$
$$\mathbf{1456} := F(1+F(4)!)\times(5!-F(6)) = (1+T(T(4)))\times(5+T(6))$$
$$\mathbf{7874} := (F(F(7))+F(8))\times(7+4!) = 7!-T(T(8))+7!-T(T(T(4)))$$
$$\mathbf{7920} := F(F(7))\times F(9)-2+0 = -7!+(9!/T(T(T(2))+0!))$$
$$\mathbf{7942} := F(F(7))\times F(9)+F(F(F(4)!))-F(2) = (T(T(7))-T(9))\times(4!-2)$$
$$\mathbf{8085} := F(8)+08!/5 = (T(8)-0!)\times T(T(8)-T(5))$$
$$\mathbf{8317} := 8!/3!+F(17) = T(8)\times T(T(T(3)))+1^7$$
$$\mathbf{8856} := (F(8+8)+5!)\times F(6) = T(8)\times(T(8)+5)\times 6$$
$$\mathbf{8972} := F(F(8))-F(9+7)\times 2 = 8\times T(T(9))-T(7)+(T(T(2)))!$$
$$\mathbf{9243} := -9\times 2+F(F(F(4)!))^3 = 9\times(2^{T(4)}+3)$$
$$\mathbf{9244} := F(9)^2\times F(F(4)!)-4 = (9-T(2))\times T(T(T(4)))+4$$

https://goo.gl/qEPB1V

$$\mathbf{4350} := F(4)!\times(3!!+5)+0 = T(T(\sqrt{4}))\times(T(3)!+5)+0$$
$$\mathbf{4351} := F(4)!\times(3!!+5)+1 = T(T(\sqrt{4}))\times(T(3)!+5)+1$$
$$\mathbf{4352} := F(4)!\times(3!!+5)+2 = T(T(\sqrt{4}))\times(T(3)!+5)+2$$
$$\mathbf{4353} := F(4)!\times(3!!+5)+3 = T(T(\sqrt{4}))\times(T(3)!+5)+3$$
$$\mathbf{4354} := F(4)!\times(3!!+5)+4 = T(T(\sqrt{4}))\times(T(3)!+5)+4$$
$$\mathbf{4355} := F(4)!\times(3!!+5)+5 = T(T(\sqrt{4}))\times(T(3)!+5)+5$$
$$\mathbf{4356} := F(4)!\times(3!!+5)+6 = T(T(\sqrt{4}))\times(T(3)!+5)+6$$
$$\mathbf{4357} := F(4)!\times(3!!+5)+7 = T(T(\sqrt{4}))\times(T(3)!+5)+7$$
$$\mathbf{4358} := F(4)!\times(3!!+5)+8 = T(T(\sqrt{4}))\times(T(3)!+5)+8$$
$$\mathbf{4359} := F(4)!\times(3!!+5)+9 = T(T(\sqrt{4}))\times(T(3)!+5)+9.$$

https://goo.gl/qEPB1V

$$\mathbf{5490} := F(5\times F(4))\times 9+0 = (5!+\sqrt{4})\times T(9)+0$$
$$\mathbf{5491} := F(5\times F(4))\times 9+1 = (5!+\sqrt{4})\times T(9)+1$$
$$\mathbf{5492} := F(5\times F(4))\times 9+2 = (5!+\sqrt{4})\times T(9)+2$$
$$\mathbf{5493} := F(5\times F(4))\times 9+3 = (5!+\sqrt{4})\times T(9)+3$$
$$\mathbf{5494} := F(5\times F(4))\times 9+4 = (5!+\sqrt{4})\times T(9)+4$$
$$\mathbf{5495} := F(5\times F(4))\times 9+5 = (5!+\sqrt{4})\times T(9)+5$$
$$\mathbf{5496} := F(5\times F(4))\times 9+6 = (5!+\sqrt{4})\times T(9)+6$$
$$\mathbf{5497} := F(5\times F(4))\times 9+7 = (5!+\sqrt{4})\times T(9)+7$$
$$\mathbf{5498} := F(5\times F(4))\times 9+8 = (5!+\sqrt{4})\times T(9)+8$$
$$\mathbf{5499} := F(5\times F(4))\times 9+9 = (5!+\sqrt{4})\times T(9)+9$$

https://goo.gl/qEPB1V

11.3.2 Reverse Order of Digits

$$\mathbf{0169} := F(9)\times(6-1)-0! = (T(T(9))-T(6))/T(T(1+0!))$$
$$\mathbf{0176} := F(6)\times(F(7+1)+0!) = -T(T(6))+T(T(7))\times 1+0!$$
$$\mathbf{0234} := F(4+3^2)+0! = 4\times T(3)+T(20)$$
$$\mathbf{0244} := F(4)^{F(4)+2}+0! = 4!+T(4)+T(20)$$
$$\mathbf{3024} := (F(4)^2)!/(-0!+3!)! = 4!\times T(T(2))\times T(T(03))$$
$$\mathbf{3045} := (5!+4!+0!)\times F(F(3!)) = T(5+4!)\times(0!+T(3))$$
$$\mathbf{3165} := -5\times 6!+F(-1+F(F(3!))) = T(5)\times T(T(6))-T((1+3)!)$$
$$\mathbf{3276} := F(F(6))\times(F(7)\times 2)\times 3! = T(6+7)\times T(2^3)$$
$$\mathbf{3297} := -7+F(9\times 2)+3!! = (T(7+9)+T(T(T(2))))\times T(T(3))$$
$$\mathbf{3303} := 3!!-0!+F(3\times 3!) = T((3+0!)!)+T(T(T(T(3))))/3)$$
$$\mathbf{3304} := F(4!-03!)+3!! = T(4!)+0!+T(T(T(T(3))))/3)$$
$$\mathbf{3325} := 5\times(-F(2+F(3!))+3!!) = 5\times(-T(T(-2+T(3)))+T(3)!).$$

https://goo.gl/qEPB1V

$\mathbf{3840} := 0+\sqrt{4}\times 8!/F(F(3!)) = 0+\sqrt{4}\times 8!/T(T(3))$

$\mathbf{3841} := 1+\sqrt{4}\times 8!/F(F(3!)) = 1+\sqrt{4}\times 8!/T(T(3))$

$\mathbf{3842} := 2+\sqrt{4}\times 8!/F(F(3!)) = 2+\sqrt{4}\times 8!/T(T(3))$

$\mathbf{3843} := 3+\sqrt{4}\times 8!/F(F(3!)) = 3+\sqrt{4}\times 8!/T(T(3))$

$\mathbf{3844} := 4+\sqrt{4}\times 8!/F(F(3!)) = 4+\sqrt{4}\times 8!/T(T(3))$

$\mathbf{3845} := 5+\sqrt{4}\times 8!/F(F(3!)) = 5+\sqrt{4}\times 8!/T(T(3))$

$\mathbf{3846} := 6+\sqrt{4}\times 8!/F(F(3!)) = 6+\sqrt{4}\times 8!/T(T(3))$

$\mathbf{3847} := 7+\sqrt{4}\times 8!/F(F(3!)) = 7+\sqrt{4}\times 8!/T(T(3))$

$\mathbf{3848} := 8+\sqrt{4}\times 8!/F(F(3!)) = 8+\sqrt{4}\times 8!/T(T(3))$

$\mathbf{3849} := 9+\sqrt{4}\times 8!/F(F(3!)) = 9+\sqrt{4}\times 8!/T(T(3))$

https://goo.gl/qEPB1V

$\mathbf{5490} := 0+9\times F(F(4)\times 5) = 0+T(9)\times(\sqrt{4}+5!)$

$\mathbf{5491} := 1+9\times F(F(4)\times 5) = 1+T(9)\times(\sqrt{4}+5!)$

$\mathbf{5492} := 2+9\times F(F(4)\times 5) = 2+T(9)\times(\sqrt{4}+5!)$

$\mathbf{5493} := 3+9\times F(F(4)\times 5) = 3+T(9)\times(\sqrt{4}+5!)$

$\mathbf{5494} := 4+9\times F(F(4)\times 5) = 4+T(9)\times(\sqrt{4}+5!)$

$\mathbf{5495} := 5+9\times F(F(4)\times 5) = 5+T(9)\times(\sqrt{4}+5!)$

$\mathbf{5496} := 6+9\times F(F(4)\times 5) = 6+T(9)\times(\sqrt{4}+5!)$

$\mathbf{5497} := 7+9\times F(F(4)\times 5) = 7+T(9)\times(\sqrt{4}+5!)$

$\mathbf{5498} := 8+9\times F(F(4)\times 5) = 8+T(9)\times(\sqrt{4}+5!)$

$\mathbf{5499} := 9+9\times F(F(4)\times 5) = 9+T(9)\times(\sqrt{4}+5!)$

https://goo.gl/qEPB1V

11.3.3 Patterns

$$\mathbf{48} := F(4)! \times 8 \quad = T(T(\sqrt{4})) \times 8$$
$$\mathbf{480} := F(4)! \times 80 \quad = T(T(\sqrt{4})) \times 80$$
$$\mathbf{4800} := F(4)! \times 800 = T(T(\sqrt{4})) \times 800$$

$$\mathbf{1365} := 13 \times F(F(6)) \times 5 \quad = 13 \times T(6) \times 5$$
$$\mathbf{13650} := 13 \times F(F(6)) \times 50 \quad = 13 \times T(6) \times 50$$
$$\mathbf{136500} := 13 \times F(F(6)) \times 500 = 13 \times T(6) \times 500$$

https://goo.gl/qEPB1V

$$\mathbf{3325} := (3!! - F(F(3!) + 2)) \times 5 \quad = (T(3)! - T(T(T(3) - 2))) \times 5$$
$$\mathbf{33250} := (3!! - F(F(3!) + 2)) \times 50 \quad = (T(3)! - T(T(T(3) - 2))) \times 50$$
$$\mathbf{332500} := (3!! - F(F(3!) + 2)) \times 500 = (T(3)! - T(T(T(3) - 2))) \times 500$$

$$\mathbf{9425} := F((\sqrt{9})! + F(F(4)!)) \times 25 \quad = (-T(\sqrt{9}) + T(T(T(4)) + T(T(2)))) \times 5$$
$$\mathbf{94250} := F((\sqrt{9})! + F(F(4)!)) \times 250 \quad = (-T(\sqrt{9}) + T(T(T(4)) + T(T(2)))) \times 50$$
$$\mathbf{942500} := F((\sqrt{9})! + F(F(4)!)) \times 2500 = (-T(\sqrt{9}) + T(T(T(4)) + T(T(2)))) \times 500$$

https://goo.gl/qEPB1V

For more details see author's work [63].

12 Binomial Coefficients and Selfie Numbers

In the previous sections, we worked with **selfie numbers** by use of functions, such as, **factorial, square-root, Fibonacci** and **Triangular numbers**. This section deals with **selfie numbers** by use of **binomial coefficients**. The results are in digit's order, reverse order of digits. The work is limited only up to five digits numbers. Higher orders shall be dealt elsewhere. The **Binomial Coefficients** are defined as

$$C(m,r) = \frac{m!}{r! \times (m-r)!}, \; m \geq r \geq 0, \; m, r \in \boldsymbol{N}.$$

In above subsections, we gave examples of selfie numbers with **Fibonacci sequence**, **Triangular numbers**, etc. Still, one can have similar kind results using **binomial coefficients**. For more details refer author's complete work [54]. See below some examples written in **both ways**, **digit's order** and **reverse order of digits**.

12.1 Digit's Order

3125 $:= (C(3,1)+2)^5$

3495 $:= C(3!,\sqrt{4})^{\sqrt{9}}+5!$

3591 $:= 3!!\times 5-C(9,1)$

3597 $:= -3+5\times(\sqrt{C(9,7)})!$

3978 $:= C(3!\times\sqrt{9},7)/8$

12650 $:= C(-1+26,\ 5-0!)$

12870 $:= C(1\times 2\times 8,\ 7+0!)$

14950 $:= C(-1+4!+\sqrt{9},\ 5-0!)$

18564 $:= C(18,\ (5-6+4)!)$

19448 $:= C(19-\sqrt{4},\ \sqrt{4}+8)$

26334 $:= C(2+C(6,\ 3),\ 3+\sqrt{4})$

43758 $:= C(4!-3!,\ 7-5+8)$

53130 $:= C(5^{3-1},\ 3!-0!)$

66564 $:= (6!-C(6+5,6))^{\sqrt{4}}$

69498 $:= 6\times 9\times C(4+9,8)$

69557 $:= -C(6\times\sqrt{9},5)+5^7$

73998 $:= 7!\times 3!+C(9+9,8)$

98283 $:= \sqrt{9}+C(C(8,2),8-3)$

https://goo.gl/NQrvWO

25920 $:= (-2+5)!!\times C(9,2)+0$

25921 $:= (-2+5)!!\times C(9,2)+1$

25922 $:= (-2+5)!!\times C(9,2)+2$

25923 $:= (-2+5)!!\times C(9,2)+3$

25924 $:= (-2+5)!!\times C(9,2)+4$

25925 $:= (-2+5)!!\times C(9,2)+5$

25926 $:= (-2+5)!!\times C(9,2)+6$

25927 $:= (-2+5)!!\times C(9,2)+7$

25928 $:= (-2+5)!!\times C(9,2)+8$

25929 $:= (-2+5)!!\times C(9,2)+9$

95760 $:= (C(9,5)+7)\times 6!+0$

95761 $:= (C(9,5)+7)\times 6!+1$

95762 $:= (C(9,5)+7)\times 6!+2$

95763 $:= (C(9,5)+7)\times 6!+3$

95764 $:= (C(9,5)+7)\times 6!+4$

95765 $:= (C(9,5)+7)\times 6!+5$

95766 $:= (C(9,5)+7)\times 6!+6$

95767 $:= (C(9,5)+7)\times 6!+7$

95768 $:= (C(9,5)+7)\times 6!+8$

95769 $:= (C(9,5)+7)\times 6!+9$

https://goo.gl/NQrvWO

12.2 Reverse Order of Digits

28 $:= C(8,\ 2)$

792 $:= C(2\times(\sqrt{9})!,\ 7)$

924 $:= C(4!/2,\ (\sqrt{9})!)$

2024 $:= C(4!,\ 2+(0\times 2)!)$

3125 $:= 5^{C(2,1)+3}$

3456 $:= 6!/5\times C(4,3)!$

3464 $:= \sqrt{4}\times 6!+C(4!,3)$

3654 $:= C(4!+5,6-3)$

4296 $:= 6!\times\sqrt{C(9,2)}-4!$

4845 $:= C(5\times 4,8-4)$

4944 $:= -4!\times 4+(9-\sqrt{4})!$

00378 $:= C(C(8,\ \sqrt{7-3}),\ 0!+0!)$

00792 $:= C(2\times(\sqrt{9})!,\ 7-0!-0!)$

00924 $:= C(4!/2,\ \sqrt{9}\times(0!+0!))$

20349 $:= C(-\sqrt{9}+4!,3+02)$

20474 $:= C(4\times 7,4)-(0\times 2)!$

31824 $:= C(4!+2-8,1+3!)$

32928 $:= C(8,2)^{\sqrt{9}}/2\times 3$

97489 $:= -(\sqrt{9})!!\times C(8,\sqrt{4})+7^{(\sqrt{9})!}$

98448 $:= 8\times C(4!,4)+8!/\sqrt{9}$

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23760 $:= 0+6!\times(C(7,3)-2)$

23761 $:= 1+6!\times(C(7,3)-2)$

23762 $:= 2+6!\times(C(7,3)-2)$

23763 $:= 3+6!\times(C(7,3)-2)$

23764 $:= 4+6!\times(C(7,3)-2)$

23765 $:= 5+6!\times(C(7,3)-2)$

23766 $:= 6+6!\times(C(7,3)-2)$

23767 $:= 7+6!\times(C(7,3)-2)$

23768 $:= 8+6!\times(C(7,3)-2)$

23769 $:= 9+6!\times(C(7,3)-2)$

98280 $:= 0+C(C(8,2),8-\sqrt{9})$

98281 $:= 1+C(C(8,2),8-\sqrt{9})$

98282 $:= 2+C(C(8,2),8-\sqrt{9})$

98283 $:= 3+C(C(8,2),8-\sqrt{9})$

98284 $:= 4+C(C(8,2),8-\sqrt{9})$

98285 $:= 5+C(C(8,2),8-\sqrt{9})$

98286 $:= 6+C(C(8,2),8-\sqrt{9})$

98287 $:= 7+C(C(8,2),8-\sqrt{9})$

98288 $:= 8+C(C(8,2),8-\sqrt{9})$

98289 $:= 9+C(C(8,2),8-\sqrt{9})$

https://goo.gl/NQrvWO

12.3 Both Ways

$$\begin{aligned}
\mathbf{3599} &:= 3!! \times 5 - C(9,9) &&= -C(9,9) + 5 \times 3!! \\
\mathbf{3723} &:= 3!! + C(7 \times 2, 3!) &&= 3!! + C(2 \times 7, 3!) \\
\mathbf{6435} &:= C(C(6,4), 3+5) &&= C(5 \times 3, \sqrt{4}+6) \\
\mathbf{7993} &:= -7 + C((\sqrt{9})!, \sqrt{9})^3 &&= C(3!, \sqrt{9})^{\sqrt{9}} - 7 \\
\mathbf{10624} &:= -1 - 0! + C((6-2)!, 4) &&= C(4!, -2+6) - 0! - 1 \\
\mathbf{10626} &:= C((10-6)!, -2+6) &&= C((6-2)!, 6-0!-1) \\
\mathbf{15504} &:= C(15+5,\ 0!+4) &&= C(4 \times 05,\ 5 \times 1) \\
\mathbf{42504} &:= C(4!,\ \sqrt{2 \times 50/4}) &&= C(4!,\ -05+24) \\
\mathbf{54264} &:= C(5+4^2,\ C(6,\ 4)) &&= C(4!-6/2,\ (\sqrt{4+5})!) \\
\mathbf{63748} &:= 6 \times C((-3+7)!, 4) - 8 &&= -8 + C(4!, 7-3) \times 6 \\
\mathbf{64468} &:= 6! + C(4!, 4) \times 6 - 8 &&= -8 + 6! + C(4!, 4) \times 6 \\
\mathbf{74319} &:= 7 \times (C(4!, 3+1) - 9) &&= (-9 + C((1+3)!, 4)) \times 7 \\
\mathbf{74376} &:= 7 \times C(4!, -3+7) - 6 &&= -6 + C((7-3)!, 4) \times 7
\end{aligned}$$

https://goo.gl/NQrvWO

For more details refer author's work [54].

13 Polygonal Type Selfie Numbers

In this section we shall work with polygonal type selfie numbers. We work with only two types. One is known as **s-gonal** numbers and another as **centered polygonal numbers**. In both the cases, we worked **selfie numbers** in **digit's order**, **reverse order of digits** and **both ways**. For more details on polygonal type numbers refer online sites [88, 125, 126, 101].

13.1 S-gonal numbers

The general formula for **s-sides of a polygon** (**s-gonal**) is given by

$$\boldsymbol{P_s}(n) := \frac{n(n-1)(s-2)}{2} + n, \quad s > 2.$$

See below some particular cases:

Triangle (3-gonal): $P_3(n) = \dfrac{n(n+1)}{2}$

Sequence values: $1, 3, 6, 10, 15, \ldots.$

Square (4-gonal): $\boldsymbol{P_4}(n) = n^2$

Sequence values: $1, 4, 9, 16, 25, \ldots.$

$$\textbf{Pentagonal (5-gonal):} \quad P_5(n) = \frac{n(3n-1)}{2}$$
$$\textbf{Sequence values:} \quad 1, 5, 12, 22, \ldots.$$

$$\textbf{Hexagonal (6-gonal):} \quad P_6(n) = n(2n-1)$$
$$\textbf{Sequence values:} \quad 1, 6, 15, 28, \ldots.$$

From now onwards we shall use the notation $\boldsymbol{P(n,s)}$ for **s-gonal numbers**, i.e.,

$$P(n,s) := P_s(n), s \geq 3.$$

From mathematical point of view, we can calculate values of $P(n,s)$ for $s \leq 2$, but from practical point of view, **s-gonal numbers** are considered for $s \geq 3$.

Three subsections below give examples of **s-gonal selfie numbers** in three different ways. One with digits order and its reverse both ways, second in digit's order and third in reverse order of digits

This subsection brings some examples of selfie numbers using **S-gonal numbers**. This section brings examples in **digit's order** and in **reverse order of digits**. For more details refer author's complete work [61].

13.1.1 Digit's Order

$$\mathbf{357} := P(3!,5) \times 7$$
$$\mathbf{384} := 3! \times P(8,4)$$
$$\mathbf{4992} := P(4!,9+9+2)$$
$$\mathbf{1653} := P(-1+6\times 5,3!)$$
$$\mathbf{1944} := P(1\times 9,4) \times 4!$$
$$\mathbf{7744} := (P(7,7)-4!)^{\sqrt{4}}$$
$$\mathbf{7896} := 7 \times P(8\times\sqrt{9},6)$$
$$\mathbf{17760} := P(1+7,7) \times (6-0!)!$$
$$\mathbf{18355} := P(1+8,3!) \times 5! - 5$$
$$\mathbf{30982} := 3! + P(-0!+9,8)^2$$
$$\mathbf{33327} := (P(3!,3!)+3)^2 \times 7$$
$$\mathbf{65485} := -P(6,5) + \sqrt{4} \times 8^5$$
$$\mathbf{65943} := P(6,5) \times ((\sqrt{9})!^4 - 3)$$
$$\mathbf{67977} := (6+7) \times (P(9,7)+7!)$$
$$\mathbf{72495} := -P(7+2,4) + 9!/5$$
$$\mathbf{83544} := \sqrt{P(8,3)} \times (5! - \sqrt{4})^{\sqrt{4}}$$
$$\mathbf{98535} := \sqrt{9^8} \times P(5,3) + 5!$$
$$\mathbf{99543} := \sqrt{9^9} \times 5 + P(4!,3!).$$

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32760 $:= 3!!/2 \times P(7,6) + 0$
32761 $:= 3!!/2 \times P(7,6) + 1$
32762 $:= 3!!/2 \times P(7,6) + 2$
32763 $:= 3!!/2 \times P(7,6) + 3$
32764 $:= 3!!/2 \times P(7,6) + 4$
32765 $:= 3!!/2 \times P(7,6) + 5$
32766 $:= 3!!/2 \times P(7,6) + 6$
32767 $:= 3!!/2 \times P(7,6) + 7$
32768 $:= 3!!/2 \times P(7,6) + 8$
32769 $:= 3!!/2 \times P(7,6) + 9$

86640 $:= P(8,6) \times (6! + \sqrt{4}) + 0$
86641 $:= P(8,6) \times (6! + \sqrt{4}) + 1$
86642 $:= P(8,6) \times (6! + \sqrt{4}) + 2$
86643 $:= P(8,6) \times (6! + \sqrt{4}) + 3$
86644 $:= P(8,6) \times (6! + \sqrt{4}) + 4$
86645 $:= P(8,6) \times (6! + \sqrt{4}) + 5$
86646 $:= P(8,6) \times (6! + \sqrt{4}) + 6$
86647 $:= P(8,6) \times (6! + \sqrt{4}) + 7$
86648 $:= P(8,6) \times (6! + \sqrt{4}) + 8$
86649 $:= P(8,6) \times (6! + \sqrt{4}) + 9$

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13.1.2 Reverse Order of Digits

189 $:= P(9,8-1)$
325 $:= P(5^2,3)$
0148 $:= P(8,(4-1)!+0!)$
0179 $:= P(9,7) - 10$
0288 $:= 8 \times P(8,2+0!)$
0377 $:= P(7+7,3!) - 0!$
0564 $:= P(4!,6)/\sqrt{5-0!}$
0637 $:= P(7,3!) \times (6+0!)$
8967 $:= 7 \times P(P(6,\sqrt{9}),8)$
9504 $:= 4! \times P(\sqrt{0!+5!},9)$
9744 $:= 4! \times P(4 \times 7,\sqrt{9})$
16448 $:= P(8,4) + 4^{6+1}$
17537 $:= P(7+3!,5) \times 71$

22338 $:= (8! + P(3!,3!)^2)/2$
23409 $:= P(9,(0 \times 4+3)!)^2$
24575 $:= 5 \times 7! - P(5,4)^2$
36726 $:= P(6,-2+7) \times 6! + 3!$
37449 $:= 9 \times (P(4!,4!-7) - 3)$
49281 $:= 1 \times 8! + P(29,4!)$
49548 $:= -8! - P(4!,5) + 9!/4$
50424 $:= 4! \times P(-2+4!,\sqrt{0!+5!})$
52895 $:= (5+P(9,8))^2 - 5$
53995 $:= (5! - P(9,\sqrt{9})) \times 3!! - 5$
97364 $:= 46^3 + P(7,\sqrt{9})$
98464 $:= (4+6!) \times P(4!-8,\sqrt{9})$

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00840 $:= 0+P(4!,8)/(0!+0!)$
00841 $:= 1+P(4!,8)/(0!+0!)$
00842 $:= 2+P(4!,8)/(0!+0!)$
00843 $:= 3+P(4!,8)/(0!+0!)$
00844 $:= 4+P(4!,8)/(0!+0!)$
00845 $:= 5+P(4!,8)/(0!+0!)$
00846 $:= 6+P(4!,8)/(0!+0!)$
00847 $:= 7+P(4!,8)/(0!+0!)$
00848 $:= 8+P(4!,8)/(0!+0!)$
00849 $:= 9+P(4!,8)/(0!+0!)$

73440 $:= 0+4!\times P(4!,3!+7)$
73441 $:= 1+4!\times P(4!,3!+7)$
73442 $:= 2+4!\times P(4!,3!+7)$
73443 $:= 3+4!\times P(4!,3!+7)$
73444 $:= 4+4!\times P(4!,3!+7)$
73445 $:= 5+4!\times P(4!,3!+7)$
73446 $:= 6+4!\times P(4!,3!+7)$
73447 $:= 7+4!\times P(4!,3!+7)$
73448 $:= 8+4!\times P(4!,3!+7)$
73449 $:= 9+4!\times P(4!,3!+7)$

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13.1.3 Both Ways

66 $:= P(6,6) = P(6,6)$
396 $:= P(3!,(\sqrt{9})!)\times 6 = P(6,(\sqrt{9})!)\times 3!$
699 $:= 6!-P((\sqrt{9})!,\sqrt{9}) = -P((\sqrt{9})!,\sqrt{9})+6!$
1949 $:= -1-(\sqrt{9})!+P(4!,9) = -(\sqrt{9})!+P(4!,9)-1$
4164 $:= P(4!,-1-6+4!) = P(4!,-6-1+4!)$
7497 $:= 7\times P(4!-\sqrt{9},7) = 7\times P(-\sqrt{9}+4!,7)$
8344 $:= P(8\times 3+4,4!) = P(4!+4,3\times 8)$
9927 $:= 7!\times 2-P(9,(\sqrt{9})!) = -P(9,(\sqrt{9})!)+2\times 7!$

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$$
\begin{aligned}
\mathbf{36936} &:= P(3\times 6,\sqrt{9})\times\sqrt{3!^6} &&= P(6\times 3,\sqrt{9})\times 6^3\\
\mathbf{37435} &:= 3!!\times(P(7,4)+3)-5 &&= -5+3!!\times(4!+P(7,3))\\
\mathbf{38888} &:= -(3+P(8,8))\times 8+8! &&= 8!-8\times(P(8,8)+3)\\
\mathbf{44976} &:= P(4!,4!)+9\times(7!-6!) &&= (-6!+7!)\times 9+P(4!,4!)\\
\mathbf{77949} &:= P(7,7)\times((\sqrt{9})!!-4!)-\sqrt{9} &&= -\sqrt{9}+(-4!+(\sqrt{9})!!)\times P(7,7)\\
\mathbf{80788} &:= P(8,07)+8!+8! &&= 8!+P(8,7)+(08)!\\
\mathbf{93488} &:= (\sqrt{9})!^{3!}\times\sqrt{4}+P(8,8) &&= P(8,8)+\sqrt{4}\times 3!^{(\sqrt{9})!}\\
\mathbf{96624} &:= P((\sqrt{9})!,6)\times(6!\times 2+4!) &&= (4!+2\times 6!)\times P(6,(\sqrt{9})!)
\end{aligned}
$$

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For more details refer author's complete work [61].

13.2 Centered Polygonal Numbers

The centered polygonal numbers are the extensions of s-gonal numbers. The general formula for **centered polygonal numbers** is given by

$$K_t(n) := \frac{t\,n(n-1)}{2}+1,\quad t>2.$$

See below some particular cases:

Centered trianglar numbers: $K_3(n) := \frac{3\,n(n-1)}{2}+1$

Sequence values: $1, 4, 10, 19, 31, ...$

Centered square numbers: $K_4(n) := \frac{4\,n(n-1)}{2}+1$

Sequence values: $1, 5, 13, 25, 41, ...$

Centered pentagonal numbers: $K_5(n) := \frac{5\,n(n-1)}{2}+1$

Sequence values: $1, 6, 16, 31, 51, ...$

Centered hexagonal numbers: $K_6(n) := \frac{6\,n(n-1)}{2}+1$

Sequence values: $1, 7, 19, 37, 61, ...$

From now onwards we shall use the notation $K(n,t)$ for **centered polygonal numbers**, i.e.,

$$K(n,t) := K_t(n),\ t\geq 3.$$

From mathematical point of view, we can calculate values of $K(n,t)$ for $t \leq 2$, but from practical point of view, **centered polygonal numbers** are considered for $t \geq 3$.

Below are some examples of selfie numbers with **centered polygonal numbers**. These are in **digit's order** and in**reverse order of digits**. For more details refer author's complete work [61].

13.2.1 Digit's Order

$\mathbf{197} := K(-1+9,7)$
$\mathbf{829} := K((8/2)!,\sqrt{9})$
$\mathbf{973} := K(9,7)+3!!$
$\mathbf{1083} := K(10,8)\times 3$
$\mathbf{1489} := K(1\times 4!,8)-(\sqrt{9})!!$
$\mathbf{1519} := K(-1+(5-1)!,(\sqrt{9})!)$
$\mathbf{2269} := K(2+26,(\sqrt{9})!)$
$\mathbf{2735} := K(2\times 7,3!)\times 5$
$\mathbf{2888} := K(2+8,8)\times 8$
$\mathbf{3640} := K(3!,6)\times 40$
$\mathbf{5887} := K(5!/8,8)\times 7$
$\mathbf{8475} := K(8,4)\times 75$
$\mathbf{9459} := K(-\sqrt{9}+4!,5)\times 9$
$\mathbf{10269} := K(10\times 2,6)\times 9$
$\mathbf{14939} := -1+(K(4!,(\sqrt{9})!)+3)\times 9$
$\mathbf{14959} := (-1+K(4!,(\sqrt{9})!)+5)\times 9$
$\mathbf{15144} := K(15,(-1+4)!)\times 4!$
$\mathbf{15399} := K(1\times 5!/3!,9)\times 9$
$\mathbf{79984} := K(7,\sqrt{9})-(\sqrt{9})!!+8!\times\sqrt{4}$
$\mathbf{80768} := 8!+0!+K(7,6)+8!$
$\mathbf{94590} := K(-\sqrt{9}+4!,5)\times 90$
$\mathbf{95749} := -(\sqrt{9})!-5+7!\times K(4,\sqrt{9})$
$\mathbf{99991} := -9+(K(\sqrt{9},\sqrt{9})^{(\sqrt{9})!-1})$
$\mathbf{99994} := -(\sqrt{9})!+K(\sqrt{9},\sqrt{9})^{9-4}$

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$\mathbf{81360} := K(8,1+3)\times 6!+0$
$\mathbf{81361} := K(8,1+3)\times 6!+1$
$\mathbf{81362} := K(8,1+3)\times 6!+2$
$\mathbf{81363} := K(8,1+3)\times 6!+3$
$\mathbf{81364} := K(8,1+3)\times 6!+4$
$\mathbf{81365} := K(8,1+3)\times 6!+5$
$\mathbf{81366} := K(8,1+3)\times 6!+6$
$\mathbf{81367} := K(8,1+3)\times 6!+7$
$\mathbf{81368} := K(8,1+3)\times 6!+8$
$\mathbf{81369} := K(8,1+3)\times 6!+9$

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13.2.2 Reverse Order of Digits

$\mathbf{127} := K(7,(2+1)!)$

$\mathbf{364} := 4\times K(6,3!)$

$\mathbf{637} := 7\times K(3!,6)$

$\mathbf{0109} := K(9,0!+1+0!)$

$\mathbf{0198} := K(8,(\sqrt{9})!+1)+0!$

$\mathbf{0357} := 7\times K(5,3!-0!)$

$\mathbf{0735} := K(5\times 3,7)-0!$

$\mathbf{0961} := K(16,9-0!)$

$\mathbf{0985} := 5\times K(8,(\sqrt{9})!+0!)$

$\mathbf{2924} := K(4!+2,9)-2$

$\mathbf{3249} := 9\times K(4^2,3)$

$\mathbf{4566} := 6\times (6!+K(5,4))$

$\mathbf{7372} := K(27,3\times 7)$

$\mathbf{8433} := K(3^3,4!)+8$

$\mathbf{9384} := 4!+\sqrt{K(8,3!)}\times(\sqrt{9})!!$

$\mathbf{9919} := 91\times K(9,\sqrt{9})$

$\mathbf{9942} := (K(24,(\sqrt{9})!)\times(\sqrt{9})!)$

$\mathbf{00938} := K(\sqrt{K(8,3!)},(\sqrt{9})!)\times(0!+0!)$

$\mathbf{01051} := K(15,010)$

$\mathbf{59938} := K(8,3!)+(\sqrt{9})!!+9^5$

$\mathbf{62424} := 4!\times K(2+4!,2+6)$

$\mathbf{63973} := K(3!+7,9)\times K(3!,6)$

$\mathbf{77882} := 2\times(8!-K(8,7)\times 7)$

$\mathbf{84289} := ((\sqrt{9})!!+8!)\times 2+K(4!,8)$

$\mathbf{86435} := 5!\times 3!!+\sqrt{K(4!-6,8)}$

$\mathbf{91437} := K(7,3!)\times(4-1)!!-\sqrt{9}$

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$\mathbf{33840} := 0+\sqrt{K(4!,8)}\times(3+3)!$

$\mathbf{33841} := 1+\sqrt{K(4!,8)}\times(3+3)!$

$\mathbf{33842} := 2+\sqrt{K(4!,8)}\times(3+3)!$

$\mathbf{33843} := 3+\sqrt{K(4!,8)}\times(3+3)!$

$\mathbf{33844} := 4+\sqrt{K(4!,8)}\times(3+3)!$

$\mathbf{33845} := 5+\sqrt{K(4!,8)}\times(3+3)!$

$\mathbf{33846} := 6+\sqrt{K(4!,8)}\times(3+3)!$

$\mathbf{33847} := 7+\sqrt{K(4!,8)}\times(3+3)!$

$\mathbf{33848} := 8+\sqrt{K(4!,8)}\times(3+3)!$

$\mathbf{33849} := 9+\sqrt{K(4!,8)}\times(3+3)!$

$\mathbf{59760} := 0-6!\times(-7-K((\sqrt{9})!,5))$

$\mathbf{59761} := 1-6!\times(-7-K((\sqrt{9})!,5))$

$\mathbf{59762} := 2-6!\times(-7-K((\sqrt{9})!,5))$

$\mathbf{59763} := 3-6!\times(-7-K((\sqrt{9})!,5))$

$\mathbf{59764} := 4-6!\times(-7-K((\sqrt{9})!,5))$

$\mathbf{59765} := 5-6!\times(-7-K((\sqrt{9})!,5))$

$\mathbf{59766} := 6-6!\times(-7-K((\sqrt{9})!,5))$

$\mathbf{59767} := 7-6!\times(-7-K((\sqrt{9})!,5))$

$\mathbf{59768} := 8-6!\times(-7-K((\sqrt{9})!,5))$

$\mathbf{59769} := 9-6!\times(-7-K((\sqrt{9})!,5))$

https://goo.gl/73o86h

13.2.3 Both Ways

$$\mathbf{1199} := 11 \times K(9,\sqrt{9}) = K(9,\sqrt{9}) \times 11$$
$$\mathbf{1464} := 1 \times 4! \times K(6,4) = 4! \times K(6,4) \times 1$$
$$\mathbf{3786} := 3! \times K(7+8,6) = 6 \times K(8+7,3!)$$
$$\mathbf{4444} := K(4!,4) \times 4 + 4! = K(4!,4) \times 4 + 4!$$
$$\mathbf{4799} := -4! + 7! - K(9,(\sqrt{9})!) = -K(9,(\sqrt{9})!) + 7! - 4!$$
$$\mathbf{9994} := -(\sqrt{9})! + K(\sqrt{9},\sqrt{9})^4 = (\sqrt{4} + K(4!,(\sqrt{9})!)) \times (\sqrt{9})!$$
$$\mathbf{12435} := (1+2) \times K(4!,3) \times 5 = 5 \times 3 \times K(4!,2+1)$$
$$\mathbf{13689} := K((-1+3!)!/6,8) \times 9 = 9 \times \sqrt{K(8,6)^{3-1}}$$

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$$\mathbf{14402} := K(1+4!,4!) \times 02 = 2 \times K(0!+4!,4!) \times 1$$
$$\mathbf{15599} := -1 + 5! \times (5! + K(\sqrt{9},\sqrt{9})) = (K(\sqrt{9},\sqrt{9}) + 5!) \times 5! - 1$$
$$\mathbf{69646} := (K(6,(\sqrt{9})!) + 6) \times (-\sqrt{4} + 6!) = (6! - \sqrt{4}) \times (K(6,(\sqrt{9})!) + 6)$$
$$\mathbf{78444} := (7 + 8! - K(4!,4)) \times \sqrt{4} = -\sqrt{4} \times ((K(4!,4) - 8!) - 7)$$
$$\mathbf{80788} := K(8-0!,7) + 8! + 8! = 8! + 8! + K(7,-0!+8)$$
$$\mathbf{91694} := K((\sqrt{9})! + 1,6) \times ((\sqrt{9})!! + \sqrt{4}) = (\sqrt{4} + (\sqrt{9})!!) \times K(6+1,(\sqrt{9})!)$$
$$\mathbf{92744} := -(\sqrt{9})! + 2 \times 7 \times K(4!,4!) = K(4!,4!) \times 7 \times 2 - (\sqrt{9})!$$
$$\mathbf{93332} := (K(\sqrt{9},3) + 3!^{3!}) \times 2 = 2 \times (3!^{3!} + K(3,\sqrt{9}))$$

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$$\mathbf{33120} := 3!! \times K(3!, 1+2) + 0 = 0 + (2+1)!! \times K(3!, 3)$$
$$\mathbf{33121} := 3!! \times K(3!, 1+2) + 1 = 1 + (2+1)!! \times K(3!, 3)$$
$$\mathbf{33122} := 3!! \times K(3!, 1+2) + 2 = 2 + (2+1)!! \times K(3!, 3)$$
$$\mathbf{33123} := 3!! \times K(3!, 1+2) + 3 = 3 + (2+1)!! \times K(3!, 3)$$
$$\mathbf{33124} := 3!! \times K(3!, 1+2) + 4 = 4 + (2+1)!! \times K(3!, 3)$$
$$\mathbf{33125} := 3!! \times K(3!, 1+2) + 5 = 5 + (2+1)!! \times K(3!, 3)$$
$$\mathbf{33126} := 3!! \times K(3!, 1+2) + 6 = 6 + (2+1)!! \times K(3!, 3)$$
$$\mathbf{33127} := 3!! \times K(3!, 1+2) + 7 = 7 + (2+1)!! \times K(3!, 3)$$
$$\mathbf{33128} := 3!! \times K(3!, 1+2) + 8 = 8 + (2+1)!! \times K(3!, 3)$$
$$\mathbf{33129} := 3!! \times K(3!, 1+2) + 9 = 9 + (2+1)!! \times K(3!, 3)$$

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$$\mathbf{99360} := K((\sqrt{9})!, \sqrt{9}) \times 3 \times 6! + 0 = 0 + 6! \times K(3!, \sqrt{9}) \times \sqrt{9}$$
$$\mathbf{99361} := K((\sqrt{9})!, \sqrt{9}) \times 3 \times 6! + 1 = 1 + 6! \times K(3!, \sqrt{9}) \times \sqrt{9}$$
$$\mathbf{99362} := K((\sqrt{9})!, \sqrt{9}) \times 3 \times 6! + 2 = 2 + 6! \times K(3!, \sqrt{9}) \times \sqrt{9}$$
$$\mathbf{99363} := K((\sqrt{9})!, \sqrt{9}) \times 3 \times 6! + 3 = 3 + 6! \times K(3!, \sqrt{9}) \times \sqrt{9}$$
$$\mathbf{99364} := K((\sqrt{9})!, \sqrt{9}) \times 3 \times 6! + 4 = 4 + 6! \times K(3!, \sqrt{9}) \times \sqrt{9}$$
$$\mathbf{99365} := K((\sqrt{9})!, \sqrt{9}) \times 3 \times 6! + 5 = 5 + 6! \times K(3!, \sqrt{9}) \times \sqrt{9}$$
$$\mathbf{99366} := K((\sqrt{9})!, \sqrt{9}) \times 3 \times 6! + 6 = 6 + 6! \times K(3!, \sqrt{9}) \times \sqrt{9}$$
$$\mathbf{99367} := K((\sqrt{9})!, \sqrt{9}) \times 3 \times 6! + 7 = 7 + 6! \times K(3!, \sqrt{9}) \times \sqrt{9}$$
$$\mathbf{99368} := K((\sqrt{9})!, \sqrt{9}) \times 3 \times 6! + 8 = 8 + 6! \times K(3!, \sqrt{9}) \times \sqrt{9}$$
$$\mathbf{99369} := K((\sqrt{9})!, \sqrt{9}) \times 3 \times 6! + 9 = 9 + 6! \times K(3!, \sqrt{9}) \times \sqrt{9}$$

https://goo.gl/73o86h

For more details refer author's complete work [61].

14 Concatenation Type Selfie Numbers

Let's define **concatenation function** between two numbers as

$$\langle a \parallel b \rangle := 10 \times a + b; \; a \in \mathbf{N}, \; b \in \{0, 1, 2, ..., 9\}.$$

Based on this definition, below are some **concatenation type selfie numbers**. For more details refer [78].

14.1 Digit's Order

$$\mathbf{492} := \left\langle 4! \parallel (\sqrt{9})! \right\rangle \times 2$$
$$\mathbf{793} := \left\langle 7 \parallel \sqrt{9} \right\rangle + 3!!$$
$$\mathbf{796} := \left\langle 7 \parallel (\sqrt{9})! \right\rangle + 6!$$
$$\mathbf{1446} := \left(-1 + \left\langle 4! \parallel \sqrt{4} \right\rangle\right) \times 6$$
$$\mathbf{1476} := (-1 + \langle 4! \parallel 7 \rangle) \times 6$$
$$\mathbf{2904} := \left\langle 2 \times (\sqrt{9})! \parallel 0! \right\rangle \times 4!$$
$$\mathbf{15129} := \langle (-1+5) \parallel 1 \rangle^2 \times 9$$
$$\mathbf{30172} := 3! \times (-\langle 0! \parallel 1 \rangle + 7!) - 2$$
$$\mathbf{33124} := \langle 3 \times 3 \parallel 1 \rangle^2 \times 4$$
$$\mathbf{55080} := 5! \times \langle 5 \parallel 0! \rangle \times (8 + 0!)$$
$$\mathbf{58035} := 5! + \langle 8 \parallel 0! \rangle \times (3!! - 5)$$

https://goo.gl/VgLeWb

$$\mathbf{37440} := 3!! \times \left\langle \left(7 - \sqrt{4}\right) \parallel \sqrt{4} \right\rangle + 0$$
$$\mathbf{37441} := 3!! \times \left\langle \left(7 - \sqrt{4}\right) \parallel \sqrt{4} \right\rangle + 1$$
$$\mathbf{37442} := 3!! \times \left\langle \left(7 - \sqrt{4}\right) \parallel \sqrt{4} \right\rangle + 2$$
$$\mathbf{37443} := 3!! \times \left\langle \left(7 - \sqrt{4}\right) \parallel \sqrt{4} \right\rangle + 3$$
$$\mathbf{37444} := 3!! \times \left\langle \left(7 - \sqrt{4}\right) \parallel \sqrt{4} \right\rangle + 4$$
$$\mathbf{37445} := 3!! \times \left\langle \left(7 - \sqrt{4}\right) \parallel \sqrt{4} \right\rangle + 5$$
$$\mathbf{37446} := 3!! \times \left\langle \left(7 - \sqrt{4}\right) \parallel \sqrt{4} \right\rangle + 6$$
$$\mathbf{37447} := 3!! \times \left\langle \left(7 - \sqrt{4}\right) \parallel \sqrt{4} \right\rangle + 7$$
$$\mathbf{37448} := 3!! \times \left\langle \left(7 - \sqrt{4}\right) \parallel \sqrt{4} \right\rangle + 8$$
$$\mathbf{37449} := 3!! \times \left\langle \left(7 - \sqrt{4}\right) \parallel \sqrt{4} \right\rangle + 9$$
$$\mathbf{44640} := (4!/4)! \times \left\langle 6 \parallel \sqrt{4} \right\rangle + 0$$
$$\mathbf{44641} := (4!/4)! \times \left\langle 6 \parallel \sqrt{4} \right\rangle + 1$$
$$\mathbf{44642} := (4!/4)! \times \left\langle 6 \parallel \sqrt{4} \right\rangle + 2$$
$$\mathbf{44643} := (4!/4)! \times \left\langle 6 \parallel \sqrt{4} \right\rangle + 3$$
$$\mathbf{44644} := (4!/4)! \times \left\langle 6 \parallel \sqrt{4} \right\rangle + 4$$
$$\mathbf{44645} := (4!/4)! \times \left\langle 6 \parallel \sqrt{4} \right\rangle + 5$$
$$\mathbf{44646} := (4!/4)! \times \left\langle 6 \parallel \sqrt{4} \right\rangle + 6$$
$$\mathbf{44647} := (4!/4)! \times \left\langle 6 \parallel \sqrt{4} \right\rangle + 7$$
$$\mathbf{44648} := (4!/4)! \times \left\langle 6 \parallel \sqrt{4} \right\rangle + 8$$
$$\mathbf{44649} := (4!/4)! \times \left\langle 6 \parallel \sqrt{4} \right\rangle + 9$$

https://goo.gl/VgLeWb

14.2 Reverse Order of Digits

$\mathbf{264} := \langle 4! \parallel (6-2)! \rangle$

$\mathbf{396} := \left\langle 6 \parallel (\sqrt{9})! \right\rangle \times 3!$

$\mathbf{0105} := 5 \times \langle (0!+1) \parallel 0! \rangle$

$\mathbf{0109} := \left((\sqrt{9})! - 0!\right)! - \langle 1 \parallel 0! \rangle$

$\mathbf{1255} := 5 \times \langle 5^2 \parallel 1 \rangle$

$\mathbf{1288} := 8 \times \langle 8 \times 2 \parallel 1 \rangle$

$\mathbf{1359} := 9 \times \langle 5 \times 3 \parallel 1 \rangle$

$\mathbf{1449} := 9 \times \langle 4 \times 4 \parallel 1 \rangle$

$\mathbf{4979} := \sqrt{9} + 7! - \left\langle (\sqrt{9})! \parallel 4 \right\rangle$

$\mathbf{03864} := 4! \times \langle 6 \times 8/3 \parallel 0! \rangle$

$\mathbf{15477} := 77 \times \langle 4 \times 5 \parallel 1 \rangle$

$\mathbf{20048} := 8!/\sqrt{4} - \langle \langle 0! \parallel 0! \rangle \parallel 2 \rangle$

$\mathbf{24964} := (4 + \langle (6+9) \parallel 4 \rangle)^2$

$\mathbf{26896} := (\langle 6+9 \parallel 8 \rangle + 6)^2$

$\mathbf{39304} := (\langle 4 \parallel 03 \rangle - 9)^3$

$\mathbf{40108} := 8! - \left\langle \langle (0!+1) \parallel 0! \rangle \parallel \sqrt{4} \right\rangle$

$\mathbf{47424} := 4 \times 2 \times \langle 4! \parallel 7 \rangle \times 4!$

$\mathbf{47448} := 8 \times 4! \times \langle 4! \parallel 7 \rangle + 4!$

https://goo.gl/VgLeWb

$\mathbf{84960} := 0 + 6! \times \left\langle \left(9 + \sqrt{4}\right) \parallel 8 \right\rangle$

$\mathbf{84961} := 1 + 6! \times \left\langle \left(9 + \sqrt{4}\right) \parallel 8 \right\rangle$

$\mathbf{84962} := 2 + 6! \times \left\langle \left(9 + \sqrt{4}\right) \parallel 8 \right\rangle$

$\mathbf{84963} := 3 + 6! \times \left\langle \left(9 + \sqrt{4}\right) \parallel 8 \right\rangle$

$\mathbf{84964} := 4 + 6! \times \left\langle \left(9 + \sqrt{4}\right) \parallel 8 \right\rangle$

$\mathbf{84965} := 5 + 6! \times \left\langle \left(9 + \sqrt{4}\right) \parallel 8 \right\rangle$

$\mathbf{84966} := 6 + 6! \times \left\langle \left(9 + \sqrt{4}\right) \parallel 8 \right\rangle$

$\mathbf{84967} := 7 + 6! \times \left\langle \left(9 + \sqrt{4}\right) \parallel 8 \right\rangle$

$\mathbf{84968} := 8 + 6! \times \left\langle \left(9 + \sqrt{4}\right) \parallel 8 \right\rangle$

$\mathbf{84969} := 9 + 6! \times \left\langle \left(9 + \sqrt{4}\right) \parallel 8 \right\rangle$

$\mathbf{45360} := 0 + 6! \times \left\langle 3! \parallel \sqrt{5+4} \right\rangle$

$\mathbf{45361} := 1 + 6! \times \left\langle 3! \parallel \sqrt{5+4} \right\rangle$

$\mathbf{45362} := 2 + 6! \times \left\langle 3! \parallel \sqrt{5+4} \right\rangle$

$\mathbf{45363} := 3 + 6! \times \left\langle 3! \parallel \sqrt{5+4} \right\rangle$

$\mathbf{45364} := 4 + 6! \times \left\langle 3! \parallel \sqrt{5+4} \right\rangle$

$\mathbf{45365} := 5 + 6! \times \left\langle 3! \parallel \sqrt{5+4} \right\rangle$

$\mathbf{45366} := 6 + 6! \times \left\langle 3! \parallel \sqrt{5+4} \right\rangle$

$\mathbf{45367} := 7 + 6! \times \left\langle 3! \parallel \sqrt{5+4} \right\rangle$

$\mathbf{45368} := 8 + 6! \times \left\langle 3! \parallel \sqrt{5+4} \right\rangle$

$\mathbf{45369} := 9 + 6! \times \left\langle 3! \parallel \sqrt{5+4} \right\rangle$

https://goo.gl/VgLeWb

14.3 Both Ways

$$\mathbf{19844} := \left\langle 4! \parallel \sqrt{4} \right\rangle \times \left(\left\langle 8 \parallel \sqrt{9} \right) - 1 \right\rangle = \left(1 + \sqrt{\sqrt{9^8}} \right) \times \left\langle 4! \parallel \sqrt{4} \right\rangle$$

$$\mathbf{20147} := 7! \times 4 - \langle 1 \parallel 0! \rangle - 2 = -2 - \langle 0! \parallel 1 \rangle + 4 \times 7!$$

$$\mathbf{23593} := \left\langle 3 \parallel \sqrt{9} \right\rangle \times (-5 + 3!!) - 2 = -2 + (3!! - 5) \times \left\langle \sqrt{9} \parallel 3 \right\rangle$$

$$\mathbf{24964} := (4 + \langle (6+9) \parallel 4 \rangle)^2 = (2 + \langle (4! - 9) \parallel 6 \rangle)^{\sqrt{4}}$$

$$\mathbf{29789} := \left\langle \sqrt{9} \parallel (8-7) \right\rangle^{\sqrt{9}} - 2 = -2 + \left\langle \sqrt{9} \parallel (-7+8) \right\rangle^{\sqrt{9}}$$

$$\mathbf{29793} := \left\langle 3 \parallel \left(-\left(\sqrt{9} \right)! + 7 \right) \right\rangle^{\sqrt{9}} + 2 = 2 + \left\langle \sqrt{9} \parallel \left(7 - \left(\sqrt{9} \right)! \right) \right\rangle^3$$

$$\mathbf{30172} := -2 + (7! - \langle 1 \parallel 0! \rangle) \times 3! = 3! \times (-\langle 0! \parallel 1 \rangle + 7!) - 2$$

$$\mathbf{30174} := \sqrt{4} \times (7! - \langle 1 \parallel 0! \rangle) \times 3 = -3 \times (\langle 0! \parallel 1 \rangle - 7!) \times \sqrt{4}$$

$$\mathbf{30282} := \left\langle \sqrt{2 \times 8} \parallel 2 \right\rangle \times (0! + 3!!) = (3!! + 0!) \times \left\langle \sqrt{2 \times 8} \parallel 2 \right\rangle$$

$$\mathbf{30606} := ((6 + 0!)! + \langle 6 \parallel 0! \rangle) \times 3! = ((3! + 0!)! + \langle 6 \parallel 0! \rangle) \times 6$$

https://goo.gl/VgLeWb

$$\mathbf{30960} := \left\langle (3+0!) \parallel \sqrt{9} \right\rangle \times 6! + 0 = 0 + 6! \times \left\langle \left(\sqrt{9} + 0! \right) \parallel 3 \right\rangle$$

$$\mathbf{30961} := \left\langle (3+0!) \parallel \sqrt{9} \right\rangle \times 6! + 1 = 1 + 6! \times \left\langle \left(\sqrt{9} + 0! \right) \parallel 3 \right\rangle$$

$$\mathbf{30962} := \left\langle (3+0!) \parallel \sqrt{9} \right\rangle \times 6! + 2 = 2 + 6! \times \left\langle \left(\sqrt{9} + 0! \right) \parallel 3 \right\rangle$$

$$\mathbf{30963} := \left\langle (3+0!) \parallel \sqrt{9} \right\rangle \times 6! + 3 = 3 + 6! \times \left\langle \left(\sqrt{9} + 0! \right) \parallel 3 \right\rangle$$

$$\mathbf{30964} := \left\langle (3+0!) \parallel \sqrt{9} \right\rangle \times 6! + 4 = 4 + 6! \times \left\langle \left(\sqrt{9} + 0! \right) \parallel 3 \right\rangle$$

$$\mathbf{30965} := \left\langle (3+0!) \parallel \sqrt{9} \right\rangle \times 6! + 5 = 5 + 6! \times \left\langle \left(\sqrt{9} + 0! \right) \parallel 3 \right\rangle$$

$$\mathbf{30966} := \left\langle (3+0!) \parallel \sqrt{9} \right\rangle \times 6! + 6 = 6 + 6! \times \left\langle \left(\sqrt{9} + 0! \right) \parallel 3 \right\rangle$$

$$\mathbf{30967} := \left\langle (3+0!) \parallel \sqrt{9} \right\rangle \times 6! + 7 = 7 + 6! \times \left\langle \left(\sqrt{9} + 0! \right) \parallel 3 \right\rangle$$

$$\mathbf{30968} := \left\langle (3+0!) \parallel \sqrt{9} \right\rangle \times 6! + 8 = 8 + 6! \times \left\langle \left(\sqrt{9} + 0! \right) \parallel 3 \right\rangle$$

$$\mathbf{30969} := \left\langle (3+0!) \parallel \sqrt{9} \right\rangle \times 6! + 9 = 9 + 6! \times \left\langle \left(\sqrt{9} + 0! \right) \parallel 3 \right\rangle$$

https://goo.gl/VgLeWb

14.4 Pattern in Concatenation-Type Selfie Numbers

$$\mathbf{305} := \langle 3! \parallel 0! \rangle \times 5$$
$$\mathbf{3050} := \langle 3! \parallel 0! \rangle \times 50$$
$$\mathbf{30500} := \langle 3! \parallel 0! \rangle \times 500$$

$$\mathbf{10635} := (-\langle 1 \parallel 0! \rangle + 6!) \times 3 \times 5$$
$$\mathbf{106350} := (-\langle 1 \parallel 0! \rangle + 6!) \times 3 \times 50$$
$$\mathbf{1063500} := (-\langle 1 \parallel 0! \rangle + 6!) \times 3 \times 500$$

$$\mathbf{1968} := \left\langle \left(1+\sqrt{9}\right)! \parallel 6 \right\rangle \times 8$$
$$\mathbf{19680} := \left\langle \left(1+\sqrt{9}\right)! \parallel 6 \right\rangle \times 80$$
$$\mathbf{196800} := \left\langle \left(1+\sqrt{9}\right)! \parallel 6 \right\rangle \times 800$$

$$\mathbf{69408} := 6 \times \left(\sqrt{9}\right)! \times \langle 4! \parallel 0! \rangle \times 8$$
$$\mathbf{694080} := 6 \times \left(\sqrt{9}\right)! \times \langle 4! \parallel 0! \rangle \times 80$$
$$\mathbf{6940800} := 6 \times \left(\sqrt{9}\right)! \times \langle 4! \parallel 0! \rangle \times 800$$

https://goo.gl/VgLeWb

For complete details refer author's work [78].

15 Semi-Selfie Numbers

Before defining the idea of **Semi-Selfie Numbers**, let us consider following historical example (Madachy [106], p.167-170, and Heinz [95]):

$$\begin{aligned}
81 &:= (8+1)^2\\
512 &:= (5+1+2)^3\\
4913 &:= (4+9+1+3)^3\\
17576 &:= (1+7+5+7+6)^3\\
234256 &:= (2+3+4+2+5+6)^4\\
1679616 &:= (1+6+7+9+6+1+6)^4\\
17210368 &:= (1+7+2+1+0+3+6+8)^5\\
205962976 &:= (2+0+5+9+6+2+9+7+6)^5\\
8303765625 &:= (8+3+0+3+7+6+5+6+2+5)^6\\
24794911296 &:= (2+4+7+9+4+9+1+1+2+9+6)^6\\
271818611107 &:= (2+7+1+8+1+8+6+1+1+1+0+7)^7\\
6722988818432 &:= (6+7+2+2+9+8+8+8+1+8+4+3+2)^7\\
72301961339136 &:= (7+2+3+0+1+9+6+1+3+3+9+1+3+6)^8\\
248155780267521 &:= (2+4+8+1+5+5+7+8+0+2+6+7+5+2+1)^8.
\end{aligned}$$

The above representation is in such a way that the both sides we have same digits except the power. Let us call these numbers as **semi-selfie numbers**. Moreover, above numbers are separated by a **single digit** and are with **positive coefficient**.

The aim of this work is extend above type of numbers for **flexible powers** and with **positive and negative coefficients**. Also, the idea is to extend for any **more digits** instead of **single digit**. Some of the above numbers can also be written as

$$\begin{aligned}
\mathbf{512} &:= (5+1+2)^3 = (5-1-2)^9\\
\mathbf{234256} &:= (2+3+4+2+5+6)^4 = (-234+256)^4\\
\mathbf{1679616} &:= (1+6+7+9+6+1+6)^4 = (1+679+616)^2\\
\mathbf{17210368} &:= (1+7+2+1+0+3+6+8)^5 = (1+72-1-036-8)^5\\
\mathbf{205962976} &:= (2+0+5+9+6+2+9+7+6)^5 = (2-05+96+29-76)^5\\
\mathbf{8303765625} &:= (8+3+0+3+7+6+5+6+2+5)^6 = (83+037+6-56-25)^6\\
\mathbf{24794911296} &:= (2+4+7+9+4+9+1+1+2+9+6)^6 = (247+9+4+91-1-296)^6.
\end{aligned}$$

https://goo.gl/r83KpP;
https://goo.gl/yf7W1q.

For more details refer author's complete work [59, 66, 72]. Below are some examples.

15.1 Single Digit Semi-Selfie Numbers

$$\begin{aligned}
\mathbf{64} &:= (6-4)^6 & \mathbf{1594323} &:= (1+5-9+4-3+2+3)^{13}\\
\mathbf{196} &:= (-1+9+6)^2 & \mathbf{7962624} &:= (7+9-6+2+6+2+4)^5\\
\mathbf{243} &:= (2+4-3)^5 & \mathbf{24137569} &:= (2+4-1+3+7+5+6-9)^6\\
\mathbf{2048} &:= (-2+0-4+8)^{11} & \mathbf{35831808} &:= (3+5+8-3-1+8+0-8)^7\\
\mathbf{8192} &:= (8+1-9+2)^{13} & \mathbf{282475249} &:= (2+8+2+4+7-5+2-4-9)^{10}\\
\mathbf{15625} &:= (1+5+6-2-5)^6 & \mathbf{387420489} &:= (3+8+7+4+2+0+4+8-9)^6\\
\mathbf{19683} &:= (1-9+6+8-3)^9 & \mathbf{429981696} &:= (4+2+9+9+8+1-6-9-6)^8\\
\mathbf{371293} &:= (3+7-1-2+9-3)^5 & \mathbf{594823321} &:= (5+9+4+8+2+3-3+2-1)^6\\
\mathbf{390625} &:= (3-9+0-6+2+5)^8 & \mathbf{2357947691} &:= (2+3+5+7+9-4-7+6-9-1)^9\\
\mathbf{823543} &:= (8-2+3+5-4-3)^7 & \mathbf{3518743761} &:= (3+5+1+8+7+4-3+7+6+1)^6
\end{aligned}$$

https://goo.gl/r83KpP;
https://goo.gl/yf7W1q.

15.2 Equal Digits Semi-Selfie Numbers

$$\mathbf{2025} := (20+25)^2$$
$$\mathbf{3025} := (30+25)^2$$
$$\mathbf{9801} := (98+01)^2$$
$$\mathbf{20151121} := (20+15+11+21)^4$$
$$\mathbf{1222830961} := (12+22+83+09+61)^4$$
$$\mathbf{1536953616} := (15+36+95+36+16)^4$$
$$\mathbf{1568239201} := (15+68+23+92+01)^4$$
$$\mathbf{2897022976} := (28+97+02+29+76)^4$$
$$\mathbf{3486784401} := (34+86+78+44+01)^4$$
$$\mathbf{4097152081} := (40+97+15+20+81)^4$$
$$\mathbf{6690585616} := (66+90+58+56+16)^4$$
$$\mathbf{494209} := (494+209)^2$$
$$\mathbf{998001} := (998+001)^2$$
$$\mathbf{24502500} := (2450+2500)^2$$
$$\mathbf{25502500} := (2550+2500)^2$$
$$\mathbf{52881984} := (5288+1984)^2$$
$$\mathbf{60481729} := (6048+1729)^2$$
$$\mathbf{99980001} := (9998+0001)^2$$
$$\mathbf{6049417284} := (60494+17284)^2$$
$$\mathbf{6832014336} := (68320+14336)^2$$
$$\mathbf{9048004641} := (90480+04641)^2$$
$$\mathbf{9999800001} := (99998+00001)^2.$$

https://goo.gl/r83KpP;
https://goo.gl/yf7W1q.

$$\mathbf{195112} := (19+51-12)^3$$
$$\mathbf{456533} := (45+65-33)^3$$
$$\mathbf{20511149} := (20-51+11+49)^5$$
$$\mathbf{24137569} := (24-13+75-69)^6$$
$$\mathbf{81450625} := (81+45-06-25)^4$$
$$\mathbf{1838265625} := (18-38-26+56+25)^6$$
$$\mathbf{9509900499} := (95+09+90+04-99)^5$$
$$:= (95-09-90+04+99)^5$$
$$\mathbf{132496} := (-132+496)^2$$
$$\mathbf{234256} := (-234+256)^4$$
$$\mathbf{456533} := (-456+533)^3$$
$$\mathbf{474552} := (-474+552)^3$$
$$\mathbf{105413504} := (105+413-504)^7$$
$$\mathbf{188132517} := (188-132+517)^3$$
$$\mathbf{258474853} := (258-474+853)^3$$
$$\mathbf{312900721} := (312-900+721)^4$$
$$\mathbf{360944128} := (-360+944+128)^3$$
$$\mathbf{365525875} := (365-525+875)^3$$
$$\mathbf{384240583} := (384-240+583)^3$$
$$\mathbf{429981696} := (429-981+696)^4$$
$$\mathbf{547981281} := (-547+981-281)^4$$
$$\mathbf{605495736} := (605-495+736)^3$$
$$\mathbf{786330467} := (786-330+467)^3$$
$$\mathbf{69426531} := (6942-6531)^3$$

https://goo.gl/r83KpP;
https://goo.gl/yf7W1q.

15.3 Multiple Digits Semi-Selfie Numbers

3581577000 $:= (3+5+815+7+700+0)^3$
3722098081 $:= (3+72+2+09+80+81)^4$
3862125316 $:= (3+8+62125+3+1+6)^2$
3899547224 $:= (389+954+7+224)^3$
4034866688 $:= (4+034+866+688)^3$
4097152081 $:= (4+09+7+152+081)^4$
$:= (40+9+71+52+081)^4$
$:= (40+97+15+20+81)^4$

11830695361 $:= (118306-9536-1)^2$
13413413376 $:= (-1341+341+3376)^3$
16620420608 $:= (-1662+04206+08)^3$
17777155561 $:= (-1+77771+55561)^2$
17777955556 $:= (-1+77779+55556)^2$
18454135716 $:= (184-54+135716)^2$
30900024072 $:= (3090-0024+072)^3$
33061785241 $:= (3306+178524-1)^2$

https://goo.gl/r83KpP;
https://goo.gl/yf7W1q.

15.4 Same Number With Different Representations

1296 $:= (1+29+6)^2 = (1+2+9-6)^4$
19683 $:= (1+9+6+8+3)^3 = (-1+9+6-8-3)^9$
234256 $:= (2+3+4+2+5+6)^4 = (-234+256)^4$
551368 $:= (55+13+6+8)^3 = (55-1+36-8)^3$
970299 $:= (9+70+2+9+9)^3 = (9+70+29-9)^3$
7311616 $:= (7+31+1+6+1+6)^4 = (73-11+6-16)^4$
8503056 $:= (8+5+030+5+6)^4 = (85-030+5-6)^4$

https://goo.gl/r83KpP;
https://goo.gl/yf7W1q.

$$\mathbf{11316496} := (1+1+31+6+4+9+6)^4 = (11-3+1-6+49+6)^4$$
$$\mathbf{13845841} := (1+38+4+5+8+4+1)^4 = (13+8-45+84+1)^4$$
$$\mathbf{973242271} := (973+2+4+2+2+7+1)^3 = (973+2+42-27+1)^3$$
$$\mathbf{992436543} := (9+9+24+3+6+5+4+3)^5 = (99-24+36-5-43)^5$$
$$\mathbf{1003875856} := (1+0038+75+8+56)^4 = (100+38-7+58-5-6)^4$$
$$\mathbf{1027243729} := (10+27+243+729)^3 = (1027+2+43-72+9)^3$$
$$\mathbf{29906468864} := (2990+6+4+6+8+86+4)^3 = (2990-6-4+68-8+64)^3$$
$$\mathbf{29964315016} := (2996+43+1+50+16)^3 = (-29-9+6+4+3150-16)^3$$

https://goo.gl/r83KpP;
https://goo.gl/yf7W1q.

$$\mathbf{1296} := (1+2+9-6)^4$$
$$:= (1+29+6)^2$$
$$\mathbf{15625} := (1+5+6-2-5)^6$$
$$:= (1+5-6+25)^3$$
$$\mathbf{32768} := (-3-2-7+6+8)^{15}$$
$$:= (3-2-7+6+8)^5$$
$$:= (3+27-6+8)^3$$
$$\mathbf{390625} := (-3+9+0+6-2-5)^8$$
$$:= (3-9+0+6+25)^4$$

$$\mathbf{262144} := (2-6-2+14-4)^9$$
$$:= (2-6+2+14-4)^6$$
$$\mathbf{1771561} := (1+77-1-5-61)^6$$
$$:= (177-1-56+1)^3$$
$$:= (-1+771+561)^2$$
$$\mathbf{2097152} := (20-9-7+1-5+2)^{21}$$
$$:= (20-9-7+1+5-2)^7$$
$$:= (-20+97-1+52)^3$$

https://goo.gl/r83KpP;
https://goo.gl/yf7W1q.

$$\mathbf{60466176} := (60-46+6-1-7-6)^{10}$$
$$:= (6+04+6+6+1+7+6)^{5}$$
$$\mathbf{67108864} := (67-1+08-8-64)^{26}$$
$$:= (67+1+08-8-64)^{13}$$
$$\mathbf{387420489} := (3+8+7+4+20-48+9)^{18}$$
$$:= (3+8-7+4-20+4+8+9)^{9}$$
$$:= (3+8-7+4-20+48-9)^{6}$$
$$\mathbf{429981696} := (4+2+99+8+1-6-96)^{8}$$
$$:= (4+2+99+8+16+9+6)^{4}$$
$$\mathbf{6975757441} := (69+75-75-7-44-1)^{8}$$
$$:= (6+9+7+5-757+441)^{4}$$
$$\mathbf{8589934592} := (8+58-99-3+45-9+2)^{33}$$
$$:= (8+58-99+3+45-9+2)^{11}$$
$$\mathbf{30840979456} := (308-409+7+94+56)^{6}$$
$$:= (-30+8+4097-945+6)^{3}$$
$$\mathbf{31381059609} := (31-38-1+05+9+6-09)^{12}$$
$$:= (31+3-81+059+6-09)^{11}.$$

https://goo.gl/r83KpP; https://goo.gl/yf7W1q.

For more details refer author's complete work [59, 66, 72]. The references [66, 72] are the revised versions of the reference [59]

15.5 Patterns in Semi-Selfie Numbers

$$\mathbf{3025} := (30+25)^{2}$$
$$\mathbf{98903025} := (9890+30+25)^{2}$$
$$\mathbf{9989003025} := (99890+030+25)^{2}$$
$$\mathbf{999890003025} := (999890+0030+25)^{2}$$

$$\mathbf{912025} := (910+20+25)^{2}$$
$$\mathbf{99120025} := (9910+20+025)^{2}$$
$$\mathbf{9991200025} := (99910+20+0025)^{2}$$
$$\mathbf{999912000025} := (999910+20+00025)^{2}$$

$$\mathbf{11881} := (118-8-1)^{2}$$
$$\mathbf{1018081} := (1018-08-1)^{2}$$
$$\mathbf{100180081} := (10018-008-1)^{2}$$
$$\mathbf{10001800081} := (100018-0008-1)^{2}$$

$$\mathbf{978121} := (978+12-1)^{2}$$
$$\mathbf{99780121} := (9978+012-1)^{2}$$
$$\mathbf{9997800121} := (99978+0012-1)^{2}$$
$$\mathbf{999978000121} := (999978+00012-1)^{2}$$

https://goo.gl/r83KpP

$$\mathbf{245025} := (2-4+502-5)^2$$
$$\mathbf{2499500025} := (2-4+9-9+50002-5)^2$$
$$\mathbf{24999950000025} := (2-4+99-99+5000002-5)^2$$
$$\mathbf{249999995000000025} := (2-4+999-999+500000002-5)^2$$

$$\mathbf{991026973} := (991+02-69+73)^3$$
$$\mathbf{999910002699973} := (99991+0002-69+9-9+73)^3$$
$$\mathbf{999999100000269999973} := (9999991+000002-69+99-99+73)^3$$
$$\mathbf{999999991000000026999999973} := (999999991+00000002-69+999-999+73)^3$$

https://goo.gl/r83KpP

$$\mathbf{6398880049} := (-63-9+8+8+80049)^2$$
$$\mathbf{63999888000049} := (-63+9-9-9+8+8+8000049)^2$$
$$\mathbf{639999988800000049} := (-63+99-99-9+8+8+800000049)^2$$
$$\mathbf{6399999998880000000049} := (-63+999-999-9+8+8+80000000049)^2$$

https://goo.gl/r83KpP

For more details refer author's complete work [69].

16 Amicable Numbers

In the history, there are numbers known by **"Amicable numbers"** (see Madachy [106], p. 155). There are many different ways of expressing these numbers. Most famous among them is with operation of addition, such as 220 and 284. In this case the summing the divisors of one we get another number. See below:

Divisors of **284** : **1**, **2**, **4**, **71** and **142**

Sum : **1+2+4+71+142** := **220**

Divisors of **220** : **1**, **2**, **4**, **5**, **10**, **11**, **20**, **22**, **44**, **55** and **110**

Sum : **1+2+4+5+10+11+20+22+44+55+110** := **284**.

More studies on this type of numbers can be seen in [106, 87, 124, 127].

The other type of **amicable numbers in pairs** (ref. Madachy [106], p. 165-167) are in terms of squares of each others, for examples,

$$\mathbf{3869} := \mathbf{62}^2+\mathbf{05}^2 \leftrightarrows \mathbf{6205} := \mathbf{38}^2+\mathbf{69}^2$$
$$\mathbf{5965} := \mathbf{77}^2+\mathbf{06}^2 \leftrightarrows \mathbf{7706} := \mathbf{59}^2+\mathbf{65}^2.$$

Instead of squares of each others, it may happen with same numbers too, for example,

$$\mathbf{1233} := \mathbf{12}^2 + \mathbf{33}^2$$
$$\mathbf{990100} := \mathbf{990}^2 + \mathbf{100}^2.$$

This type of situation let's call as **self-amicable** numbers. It is not necessary that it happens only with addition, we may have results with subtraction, such as,

$$\mathbf{48} := -\mathbf{4}^2 + \mathbf{8}^2$$
$$\mathbf{3468} := -\mathbf{34}^2 + \mathbf{68}^2$$
$$\mathbf{416768} := -\mathbf{416}^2 + \mathbf{768}^2.$$

These numbers are alos **self-amicable** numbers. For more details refer [84]. Below some examples on these type of numbers in different situations.

16.1 Product-Type Amicable Numbers

16.1.1 In Pairs

- **Positive Coefficients**

$$\mathbf{15} := \mathbf{3} \times 5 + \mathbf{0} \times 5 \leftrightarrows \mathbf{30} := \mathbf{1} \times 5 + \mathbf{5} \times 5$$
$$\mathbf{104} := \mathbf{1} \times 8 + \mathbf{12} \times 8 \leftrightarrows \mathbf{112} := \mathbf{10} \times 8 + \mathbf{4} \times 8$$
$$\mathbf{160} := \mathbf{12} \times 8 + \mathbf{8} \times 8 \leftrightarrows \mathbf{128} := \mathbf{16} \times 8 + \mathbf{0} \times 8$$
$$\mathbf{165} := \mathbf{33} \times 5 + \mathbf{0} \times 5 \leftrightarrows \mathbf{330} := \mathbf{1} \times 5 + \mathbf{65} \times 5$$
$$\mathbf{176} := \mathbf{6} \times 8 + \mathbf{16} \times 8 \leftrightarrows \mathbf{616} := \mathbf{1} \times 8 + \mathbf{76} \times 8$$
$$\mathbf{1650} := \mathbf{325} \times 5 + \mathbf{5} \times 5 \leftrightarrows \mathbf{3255} := \mathbf{1} \times 5 + \mathbf{650} \times 5$$
$$\mathbf{11840} := \mathbf{1472} \times 8 + \mathbf{8} \times 8 \leftrightarrows \mathbf{14728} := \mathbf{1} \times 8 + \mathbf{1840} \times 8$$
$$\mathbf{16665} := \mathbf{3333} \times 5 + \mathbf{0} \times 5 \leftrightarrows \mathbf{33330} := \mathbf{1} \times 5 + \mathbf{6665} \times 5$$
$$\mathbf{166665} := \mathbf{3} \times 5 + \mathbf{33330} \times 5 \leftrightarrows \mathbf{333330} := \mathbf{1} \times 5 + \mathbf{66665} \times 5$$
$$\mathbf{237024} := \mathbf{29620} \times 8 + \mathbf{8} \times 8 \leftrightarrows \mathbf{296208} := \mathbf{2} \times 8 + \mathbf{37024} \times 8$$
$$\mathbf{6730152} := \mathbf{5} \times 8 + \mathbf{841264} \times 8 \leftrightarrows \mathbf{5841264} := \mathbf{6} \times 8 + \mathbf{730152} \times 8$$
$$\mathbf{6984120} := \mathbf{7} \times 8 + \mathbf{873008} \times 8 \leftrightarrows \mathbf{7873008} := \mathbf{6} \times 8 + \mathbf{984120} \times 8$$

https://goo.gl/dBpWt8

- **Positive and Negative Coefficients**

$$63 := 8 \times 9 - 1 \times 9 \quad \leftrightarrows \quad 81 := 6 \times 9 + 3 \times 9$$

$$126 := 16 \times 9 - 2 \times 9 \quad \leftrightarrows \quad 162 := 12 \times 9 + 6 \times 9$$
$$792 := 89 \times 9 - 1 \times 9 \quad \leftrightarrows \quad 891 := 7 \times 9 + 92 \times 9$$

$$1267 := 187 \times 7 - 6 \times 7 \quad \leftrightarrows \quad 1876 := 1 \times 7 + 267 \times 7$$
$$4563 := 510 \times 9 - 3 \times 9 \quad \leftrightarrows \quad 5103 := 4 \times 9 + 563 \times 9$$
$$13860 := 2316 \times 6 - 6 \times 6 \leftrightarrows 23166 := 1 \times 6 + 3860 \times 6$$

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16.1.2 Self-Amicable

$$64 := 8 \times 8 + 0 \times 8$$
$$72 := 7 \times 8 + 2 \times 8$$
$$126 := 12 \times 7 + 6 \times 7$$
$$729 := 72 \times 9 + 9 \times 9$$
$$891 := 8 \times 9 + 91 \times 9$$
$$1332 := 1 \times 4 + 332 \times 4$$
$$1998 := 1 \times 2 + 998 \times 2$$
$$13332 := 1 \times 4 + 3332 \times 4$$

$$79992 := 7 \times 8 + 9992 \times 8$$
$$89991 := 8 \times 9 + 9991 \times 9$$
$$199998 := 1 \times 2 + 99998 \times 2$$
$$1142856 := 1 \times 8 + 142856 \times 8$$
$$4666662 := 4 \times 7 + 666662 \times 7$$
$$6999993 := 6 \times 7 + 999993 \times 7$$
$$7999992 := 7 \times 8 + 999992 \times 8$$
$$8999991 := 8 \times 9 + 999991 \times 9$$

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16.2 Power-Type Amicable Numbers

16.2.1 In Pairs

- **Positive Coefficients**

$$3869 := 62^2 + 5^2 \leftrightarrows 6205 := 38^2 + 69^2$$
$$5965 := 77^2 + 6^2 \leftrightarrows 7706 := 59^2 + 65^2$$
$$43354 := 127^2 + 165^2 \leftrightarrows 127165 := 43^2 + 354^2$$
$$137461 := 231^2 + 290^2 \leftrightarrows 231290 := 137^2 + 461^2$$
$$1261485 := 222^2 + 1101^2 \leftrightarrows 2221101 := 126^2 + 1485^2$$
$$1528804 := 298^2 + 1200^2 \leftrightarrows 2981200 := 1528^2 + 804^2$$
$$7414650 := 2217^2 + 1581^2 \leftrightarrows 22171581 := 741^2 + 4650^2$$

https://goo.gl/dBpWt8

- **Positive and Negative Coefficients**

$$16 := -3^2 + 5^2 \leftrightarrows 35 := -1^2 + 6^2$$
$$369 := 12^2 + 15^2 \leftrightarrows 1215 := 36^2 - 9^2$$
$$1155 := -31^2 + 46^2 \leftrightarrows 3146 := 11^2 + 55^2$$
$$2205 := 42^2 + 21^2 \leftrightarrows 42021 := -2^2 + 205^2$$
$$88836 := 706^2 - 640^2 \leftrightarrows 706640 := 88^2 + 836^2$$
$$134784 := -596^2 + 700^2 \leftrightarrows 596700 := -134^2 + 784^2$$
$$275808 := 577^2 - 239^2 \leftrightarrows 577239 := -275^2 + 808^2$$
$$384912 := 684^2 - 288^2 \leftrightarrows 684288 := -384^2 + 912^2$$
$$970299 := 1030^2 - 301^2 \leftrightarrows 1030301 := 970^2 + 299^2$$
$$6445779 := 3298^2 - 2105^2 \leftrightarrows 32982105 := -644^2 + 5779^2$$
$$8556048 := 732^2 + 02832^2 \leftrightarrows 73202832 := 8556^2 - 48^2.$$

https://goo.gl/dBpWt8

16.3 Self-Amicable

- **Positive Coefficients**

$$100 := 10^2 + 0^2$$
$$101 := 10^2 + 1^2$$
$$407 := 4^3 + 7^3$$
$$1000 := 10^3 + 00^3$$
$$1001 := 10^3 + 01^3$$
$$1233 := 12^2 + 33^2$$
$$8833 := 88^2 + 33^2$$
$$10000 := 100^2 + 00^2$$
$$10000 := 10^4 + 000^4$$
$$10001 := 100^2 + 01^2$$
$$10001 := 10^4 + 001^4$$
$$10100 := 10^2 + 100^2$$
$$340067 := 34^3 + 0067^3$$
$$990100 := 990^2 + 100^2$$
$$1000000 := 1000^2 + 000^2$$
$$5882353 := 588^2 + 2353^2$$

https://goo.gl/dBpWt8

- **Positive and Negative Coefficients**

$$48 := -4^2 + 8^2$$
$$147 := 14^2 - 7^2$$
$$3468 := -34^2 + 68^2$$
$$10101 := -10^2 + 101^2$$
$$13467 := 134^2 - 67^2$$
$$16128 := -16^2 + 128^2$$
$$34188 := -34^2 + 188^2$$
$$140400 := -140^2 + 400^2$$
$$190476 := -190^2 + 476^2$$
$$216513 := -216^2 + 513^2$$
$$300625 := -300^2 + 625^2$$
$$334668 := -334^2 + 668^2$$
$$416768 := -416^2 + 768^2$$
$$484848 := -484^2 + 848^2$$
$$530901 := -530^2 + 901^2$$
$$1010100 := 1010^2 - 100^2$$
$$1016127 := 1016^2 - 127^2$$
$$1034187 := 1034^2 - 187^2$$
$$1140399 := 1140^2 - 399^2$$
$$1190475 := 1190^2 - 475^2$$
$$1216512 := 1216^2 - 512^2$$
$$1300624 := 1300^2 - 624^2$$
$$1334667 := 1334^2 - 667^2$$
$$1416767 := 1416^2 - 767^2$$
$$1484847 := 1484^2 - 847^2$$
$$1530900 := 1530^2 - 900^2$$

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16.4 Patterns in Amicable Numbers

The idea of patterns in amicable numbers is not known in the literature. See below some examples **patterns** in **pairs of amicable numbers** and **self-amicable numbers**:

16.4.1 In Pairs

$$\begin{aligned}
165 &:= 33\times5+0\times5 = 3\times5+30\times5 \leftrightarrows 330 := 1\times5+65\times5\\
1665 &:= 333\times5+0\times5 = 3\times5+330\times5 \leftrightarrows 3330 := 1\times5+665\times5\\
16665 &:= 3333\times5+0\times5 = 3\times5+3330\times5 \leftrightarrows 33330 := 1\times5+6665\times5\\
166665 &:= 33333\times5+0\times5 = 3\times5+33330\times5 \leftrightarrows 333330 := 1\times5+66665\times5\\
1666665 &:= 333333\times5+0\times5 = 3\times5+333330\times5 \leftrightarrows 3333330 := 1\times5+666665\times5
\end{aligned}$$

https://goo.gl/dBpWt8

$$\begin{aligned}
352 &:= 44\times8+0\times8 = 4\times8+40\times8 \leftrightarrows 440 := 3\times8+52\times8\\
3552 &:= 444\times8+0\times8 = 4\times8+440\times8 \leftrightarrows 4440 := 3\times8+552\times8\\
35552 &:= 4444\times8+0\times8 = 4\times8+4440\times8 \leftrightarrows 44440 := 3\times8+5552\times8\\
355552 &:= 44444\times8+0\times8 = 4\times8+44440\times8 \leftrightarrows 444440 := 3\times8+55552\times8\\
3555552 &:= 444444\times8+0\times8 = 4\times8+444440\times8 \leftrightarrows 4444440 := 3\times8+555552\times8
\end{aligned}$$

https://goo.gl/dBpWt8

$$\begin{aligned}
176 &:= 6\times8+16\times8 \leftrightarrows 616 := 1\times8+76\times8\\
1776 &:= 6\times8+216\times8 \leftrightarrows 6216 := 1\times8+776\times8\\
17776 &:= 6\times8+2216\times8 \leftrightarrows 62216 := 1\times8+7776\times8\\
177776 &:= 6\times8+22216\times8 \leftrightarrows 622216 := 1\times8+77776\times8\\
1777776 &:= 6\times8+222216\times8 \leftrightarrows 6222216 := 1\times8+777776\times8
\end{aligned}$$

https://goo.gl/dBpWt8

$$\begin{aligned}
264 &:= 5\times8+28\times8 \leftrightarrows 528 := 2\times8+64\times8\\
2664 &:= 5\times8+328\times8 \leftrightarrows 5328 := 2\times8+664\times8\\
26664 &:= 5\times8+3328\times8 \leftrightarrows 53328 := 2\times8+6664\times8\\
266664 &:= 5\times8+33328\times8 \leftrightarrows 533328 := 2\times8+66664\times8\\
2666664 &:= 5\times8+333328\times8 \leftrightarrows 5333328 := 2\times8+666664\times8
\end{aligned}$$

https://goo.gl/dBpWt8

$$\begin{aligned}
\mathbf{1650} &:= \mathbf{325}\times 5+\mathbf{5}\times 5 \leftrightarrows \mathbf{3255} := \mathbf{1}\times 5+\mathbf{650}\times 5\\
\mathbf{16650} &:= \mathbf{3325}\times 5+\mathbf{5}\times 5 \leftrightarrows \mathbf{33255} := \mathbf{1}\times 5+\mathbf{6650}\times 5\\
\mathbf{166650} &:= \mathbf{33325}\times 5+\mathbf{5}\times 5 \leftrightarrows \mathbf{333255} := \mathbf{1}\times 5+\mathbf{66650}\times 5\\
\mathbf{1666650} &:= \mathbf{333325}\times 5+\mathbf{5}\times 5 \leftrightarrows \mathbf{3333255} := \mathbf{1}\times 5+\mathbf{666650}\times 5
\end{aligned}$$

https://goo.gl/dBpWt8

$$\begin{aligned}
\mathbf{3544} &:= \mathbf{437}\times 8+\mathbf{6}\times 8 \leftrightarrows \mathbf{4376} := \mathbf{3}\times 8+\mathbf{544}\times 8\\
\mathbf{35544} &:= \mathbf{4437}\times 8+\mathbf{6}\times 8 \leftrightarrows \mathbf{44376} := \mathbf{3}\times 8+\mathbf{5544}\times 8\\
\mathbf{355544} &:= \mathbf{44437}\times 8+\mathbf{6}\times 8 \leftrightarrows \mathbf{444376} := \mathbf{3}\times 8+\mathbf{55544}\times 8\\
\mathbf{3555544} &:= \mathbf{444437}\times 8+\mathbf{6}\times 8 \leftrightarrows \mathbf{4444376} := \mathbf{3}\times 8+\mathbf{555544}\times 8
\end{aligned}$$

https://goo.gl/dBpWt8

16.4.2 Self-Amicable

$\mathbf{132} := \mathbf{1}\times 4+\mathbf{32}\times 4$	$\mathbf{264} := \mathbf{2}\times 4+\mathbf{64}\times 4$	$\mathbf{396} := \mathbf{3}\times 4+\mathbf{96}\times 4$
$\mathbf{1332} := \mathbf{1}\times 4+\mathbf{332}\times 4$	$\mathbf{2664} := \mathbf{2}\times 4+\mathbf{664}\times 4$	$\mathbf{3996} := \mathbf{3}\times 4+\mathbf{996}\times 4$
$\mathbf{13332} := \mathbf{1}\times 4+\mathbf{3332}\times 4$	$\mathbf{26664} := \mathbf{2}\times 4+\mathbf{6664}\times 4$	$\mathbf{39996} := \mathbf{3}\times 4+\mathbf{9996}\times 4$
$\mathbf{133332} := \mathbf{1}\times 4+\mathbf{33332}\times 4$	$\mathbf{266664} := \mathbf{2}\times 4+\mathbf{66664}\times 4$	$\mathbf{399996} := \mathbf{3}\times 4+\mathbf{99996}\times 4$

https://goo.gl/dBpWt8

$\mathbf{462} := \mathbf{4}\times 7+\mathbf{62}\times 7$	$\mathbf{792} := \mathbf{7}\times 8+\mathbf{92}\times 8$	$\mathbf{891} := \mathbf{8}\times 9+\mathbf{91}\times 9$
$\mathbf{4662} := \mathbf{4}\times 7+\mathbf{662}\times 7$	$\mathbf{7992} := \mathbf{7}\times 8+\mathbf{992}\times 8$	$\mathbf{8991} := \mathbf{8}\times 9+\mathbf{991}\times 9$
$\mathbf{46662} := \mathbf{4}\times 7+\mathbf{6662}\times 7$	$\mathbf{79992} := \mathbf{7}\times 8+\mathbf{9992}\times 8$	$\mathbf{89991} := \mathbf{8}\times 9+\mathbf{9991}\times 9$
$\mathbf{466662} := \mathbf{4}\times 7+\mathbf{66662}\times 7$	$\mathbf{799992} := \mathbf{7}\times 8+\mathbf{99992}\times 8$	$\mathbf{899991} := \mathbf{8}\times 9+\mathbf{99991}\times 9$

https://goo.gl/dBpWt8

For complete details refer author's work [84].

17 Selfie Fractions

A **addable fraction** is a proper fraction where addition signs can be inserted into numerator and denominator, and the resulting fraction is equal to the original. The same is true for operations also, such as with **addition, multiplication, potentiation,** etc. For more details refer author's complete work [26, 27, 28]. Below are some examples:

- **Addable**

$$\frac{\mathbf{96}}{\mathbf{352}} := \frac{9+6}{3+52}, \quad \frac{\mathbf{182}}{\mathbf{6734}} := \frac{18+2}{6+734}, \text{ etc.}$$

- **Subtractable**

$$\frac{\mathbf{204}}{\mathbf{357}} := \frac{20-4}{35-7}, \quad \frac{\mathbf{726}}{\mathbf{1089}} := \frac{72-6}{108-9}, \text{ etc.}$$

- **Dottable**

$$\frac{\mathbf{13}}{\mathbf{624}} := \frac{1\times 3}{6\times 24}, \quad \frac{\mathbf{416}}{\mathbf{728}} := \frac{4\times 16}{7\times 2\times 8}, \text{ etc.}$$

(i) *https://goo.gl/8atQMY*;
(ii) *https://goo.gl/qidrGQ*;
(iii) *https://goo.gl/8zFbq7*.

- **Dottable with Potentiation**

$$\frac{95}{342} := \frac{9\times 5}{3^4\times 2}, \quad \frac{\mathbf{728}}{\mathbf{1456}} := \frac{7^2\times 8}{14\times 56}, \text{ etc.}$$

- **Mixed: All Operations**

$$\frac{\mathbf{4980}}{\mathbf{5312}} := \frac{4-9+80}{5\times(3+1)^2}, \quad \frac{\mathbf{3249}}{\mathbf{5168}} := \frac{(3+2^4)\times 9}{(5-1)\times 68}, \text{ etc.}$$

(i) *https://goo.gl/8atQMY*;
(ii) *https://goo.gl/qidrGQ*;
(iii) *https://goo.gl/8zFbq7*.

Observing above examples, the numerator and denominator follows the same order of digits in both sides of each fraction separated by operations. These type of fractions, we call *Selfie fractions*. There are two situations. One when digits appearing in each fraction are distinct, and second, when there are repetitions of digits. Studies on non-repeated digits are summarized in [26, 27, 28].

17.1 Equivalent Selfie Fractions

Above we have given *selfie fractions* with single value in each case. There are many fractions, that can be written in more than one way, for example,

- **Addable**

$$\mathbf{\frac{1453}{2906}} := \frac{1+453}{2+906} = \frac{145+3}{290+6} = \frac{1+45+3}{2+90+6}.$$

- **Subtractable**

$$\mathbf{\frac{932}{1864}} := \frac{9-32}{18-64} = \frac{93-2}{186-4}.$$

- **Dottable and Addable**

$$\mathbf{\frac{1680}{59472}} := \frac{1\times 6\times 80}{59\times 4\times 72} = \frac{1+6+8+0}{59+472}.$$

- **Dottable, Addable and Subtractable**

$$\mathbf{\frac{302}{8154}} := \frac{30\times 2}{81\times 5\times 4} = \frac{3+02}{81+54} = \frac{3-02}{81-54}.$$

(i) *https://goo.gl/yf7W1q*
(ii) *https://goo.gl/Gyj51q*

- **Symmetric Addable and Subtractable**

$$\mathbf{\frac{645}{1290}} := \frac{6-45}{12-90} = \frac{6+45}{12+90}.$$

- **Dottable and Addable together**

$$\mathbf{\frac{284}{639}} := \frac{2\times 8+4}{6+39} = \frac{28+4}{6\times (3+9)}.$$

- **Mixed - All Operations**

$$\mathbf{\frac{73842}{90516}} := \frac{7-3\times (8-4^2)}{9\times 05-1-6} = \frac{7\times (3+8)+4^2}{90+(5-1)\times 6} = \frac{738+4+2}{905+1+6}.$$

(i) *https://goo.gl/yf7W1q*
(ii) *https://goo.gl/Gyj51q*

Equivalent expression given in equation (8), let us classify it as *symmetric equivalent fraction*. In this case we just change plus with minus and vice-versa. There are many fractions *double symmetric equivalent fraction* too. In this paper, we shall work with ***equivalent fractions*** given in equations (6)-(10). Below are few examples of equivalent fractions with all operations:

$$\mathbf{\frac{15}{240}} := \frac{1^5}{2^{4-0}} = \frac{1\times 5}{2\times 40}$$
$$\mathbf{\frac{42}{1785}} := \frac{4^2)}{17\times 8\times 5} = \frac{4-2)}{((1^7)\times 85}$$
$$\mathbf{\frac{95}{3420}} := \frac{9-5)}{(3\times 4)^{2-0}} = \frac{9\times 5}{3^4\times 20}$$
$$\mathbf{\frac{163}{7824}} := \frac{1^{63}}{7\times 8-2\times 4} = \frac{16-3}{78\times 2\times 4}$$

$$\mathbf{\frac{189}{4725}} := \frac{1^{89}}{4-7\times(2-5)} = \frac{1-8+9}{47-2+5} = \frac{18-9}{(47-2)\times 5}$$
$$\mathbf{\frac{206}{3914}} := \frac{2-0\times 6}{39-1^4} = \frac{2+06}{(39-1)\times 4}$$
$$\mathbf{\frac{384}{9216}} := \frac{3\times(8-4)}{9\times 2\times 16} = \frac{3\times 8-4}{(9^2-1)\times 6} = \frac{3^{8-4}}{9\times 216}$$
$$\mathbf{\frac{624}{1950}} := \frac{(6-2)\times 4}{(1^9\times 50} = \frac{6\times 24)}{1\times 9\times 50}$$

(i) *https://goo.gl/yf7W1q*
(ii) *https://goo.gl/Gyj51q*

$$\mathbf{\frac{142}{69580}} := \frac{1^{42}}{6\times 95-80} = \frac{1\times 4\times 2}{(6\times 9-5)\times 80}$$
$$\mathbf{\frac{149}{36207}} := \frac{1^{49}}{3^{6\times 2}-07} = \frac{1-4+9}{3^6\times(2-0\times 7)}$$
$$\mathbf{\frac{172}{63984}} := \frac{1^7\times 2}{6\times(39-8)\times 4} = \frac{1\times 7-2}{6^3\times 9-84}$$
$$\mathbf{\frac{173}{29064}} := \frac{1^{73}}{(-2+9)\times 06\times 4} = \frac{1\times 7-3}{2\times(90-6)\times 4}$$
$$\mathbf{\frac{1975}{6320}} := \frac{1^{97}\times 5}{6\times 3-2-0} = \frac{(19-7)\times 5}{6\times(32-0)}$$
$$\mathbf{\frac{2457}{8190}} := \frac{(2-4+5)^7)}{81\times 90} = \frac{2\times 4\times 57}{8\times 190}$$

$$\mathbf{\frac{4256}{83790}} := \frac{4\times(2\times 5-6)}{(8-3)\times 7\times(9-0)} = \frac{4\times 2^5\times 6}{8\times 3\times 7\times 90}$$
$$\mathbf{\frac{5608}{79213}} := \frac{(-5+6)\times 08}{7\times 9\times 2-13} = \frac{56-0\times 8}{792-1^3}$$
$$\mathbf{\frac{7690}{13842}} := \frac{7-6+9-0}{1\times 3\times(8-4+2)} = \frac{(7-6)\times 90}{(1\times 3^{8-4}\times 2}$$
$$\mathbf{\frac{9246}{37185}} := \frac{92-46}{37\times 1^8\times 5} = \frac{9\times 2\times 46}{37\times 18\times 5}$$
$$\mathbf{\frac{21480}{73569}} := \frac{(2-1+4)\times(8-0)}{73-5+69} = \frac{2-1^4\times 80}{7-3+5\times 6\times 9}$$
$$\mathbf{\frac{67284}{90513}} := \frac{6\times 7\times 2\times(8-4)}{90\times 5-1+3} = \frac{6\times 7\times 2^{8-4}}{905-1^3}$$

(i) *https://goo.gl/yf7W1q*
(ii) *https://goo.gl/Gyj51q*

$$\mathbf{\frac{30927}{61854}} = \frac{3+0\times 927}{6^{1854}} = \frac{(3+09)\times 2+7}{6+(1+8+5)\times 4} = \frac{30\times(9+2\times 7)}{(61+8)\times 5\times 4} = \frac{30\times(9+27)}{6\times 18\times 5\times 4} = \frac{3\times(0\times 9+27)}{6\times(18+5+4)}$$

$$= \frac{3+092+7}{(6+(1+8)\times 5)\times 4} = \frac{(3+09\times 2)\times 7}{6\times((1+8)\times 5+4)} = \frac{3\times 0927}{618\times(5+4)} = \frac{(30+92)\times 7}{61\times(8+5\times 4)} = \frac{3\times 09^{2+7}}{6\times(1+8)^{5+4}}$$

$$= \frac{3\times(0\times 9+2)+7}{6+1^8\times 5\times 4} = \frac{3+09+27}{6+1\times 8\times(5+4)} = \frac{30+9+2+7}{6+1+85+4} = \frac{3^{0\times 9+2}\times 7}{6\times(1^8+5\times 4)} = \frac{3+09\times(2+7)}{6\times 1\times(8+5\times 4)}$$

$$= \frac{3\times(09+27)}{(6+18)\times(5+4)} = \frac{30+9\times 2\times 7}{6\times 1\times(8+5)\times 4} = \frac{3\times 09\times(2+7)}{6\times(1+8)\times(5+4)} = \frac{3\times 09\times 2\times 7}{(6+1\times 8)\times 54} = \frac{(30+9\times 2)\times 7}{618+54}.$$

$$= \frac{3+0\times 9+2\times 7}{6+1\times 8+5\times 4} = \frac{3\times(09+2)+7}{(6+1+8+5)\times 4} = \frac{3\times(09+2+7)}{6\times(1+8+5+4)} = \frac{3+0\times 9+2+7}{6+1+8+5+4} = \frac{(30+9)\times 2+7}{6+(1+8\times 5)\times 4}$$

$$= \frac{30+9^2+7}{(6\times(1+8)+5)\times 4} = \frac{(3+09)\times 2\times 7}{6\times(1+8+5)\times 4} = \frac{3+09\times 27}{6+(1+8)\times 54} = \frac{3+0927}{6+1854}$$

$$= \frac{3+0\times 9+27}{6+1^8\times 54} = \frac{3\times 09+2\times 7}{6\times 1\times(8+5)+4} = \frac{30+9\times 2+7}{(6+1)\times 8+54} = \frac{3\times(09\times 2+7)}{61+85+4} = \frac{(3+09+2)\times 7}{(6+1)\times(8+5\times 4)}$$

$$= \frac{3+09+2^7}{(6+1\times 8)\times 5\times 4} = \frac{30\times(9+2+7)}{6\times(1+8)\times 5\times 4} = \frac{3\times(0\times 9+2+7)}{6\times 1^8\times(5+4)} = \frac{3\times 0\times 9+2\times 7}{6+1^{85}\times 4}$$

(i) *https://goo.gl/yf7W1q*
(ii) *https://goo.gl/Gyj51q*

The work on **equivalent selfie fractions** is divided in different papers [29, 30].

18 Equivalent Fractions

This section deals with *equivalent fractions* without use of any operations, just with different digits. Below are some examples to understand the idea of *equivalent fractions* for different digits:

- $\frac{3}{24}=\frac{4}{32}$, • $\frac{4}{356}=\frac{6}{534}$, • $\frac{27}{3456}=\frac{42}{5376}$,
- $\frac{315}{4620}=\frac{342}{5016}$, • $\frac{123}{58764}=\frac{164}{78352}$,
- $\frac{357}{26418}=\frac{618}{45732}=\frac{714}{52836}=\frac{732}{54168}=\frac{738}{54612}$, etc.

https://goo.gl/6AjHbm

Some studies in this direction can be seen in [103, 124, 128]. Work of these authors is concentrated on equivalent fractions for the digits 1 to 9, calling *pandigital fractions*. While, we worked for all digits, i.e., for 3 to 10 digits. For example, in case of 3-digits, we tried to found equivalent fractions 3 by 3, i.e., [0, 1, 2], [1, 2, 3], [2, 3, 4], [3, 4, 5], [4, 5, 6], [5, 6, 7], [6, 7, 8] and [7, 8, 9]. In this case, we found only one result, i.e., $\frac{3}{24}=\frac{4}{32}$. The same procedure is done for 4 by 4, 5 by 5, etc. Below are highest possible equivalent fractions for 6 digits onwards. For more details refer to author's complete work [39, 40, 41]. Below are few examples.

18.1 6-Digits Higher Equivalent Fractions: 3 Expressions

In this case we have five possibilities, i.e., 0 to 5, 1 to 6, 2 to 7, 3 to 8, and 4 to 9. The number we got is for 2 to 7 and 4 to 9. Both with 3 expressions:

- $\frac{267}{534}=\frac{273}{546}=\frac{327}{654}$
- $\frac{497}{568}=\frac{749}{856}=\frac{854}{976}$

https://goo.gl/6AjHbm

18.2 7-Digits Higher Equivalent Fractions: 5 Expressions

In this case we have four possibilities, i.e., 0 to 6, 1 to 7, 2 to 8, and 3 to 9. The only higher number we got is in case of 3 to 9. See below:

- $\frac{697}{3485}=\frac{769}{3845}=\frac{937}{4685}=\frac{967}{4835}=\frac{973}{4865}.$

https://goo.gl/6AjHbm

18.3 8-Digits Higher Equivalent Fractions: 12 Expressions

In this case we have three possibilities, i.e., 0 to 7, 1 to 8, and 2 to 9. The only higher number we got is in case of 1 to 8. See below:

- $\frac{1728}{3456}=\frac{1764}{3528}=\frac{1782}{3564}=\frac{1827}{3654}=\frac{2178}{4356}=\frac{2358}{4716}=\frac{2718}{5436}=\frac{2817}{5634}=\frac{3564}{7128}=\frac{3582}{7164}=\frac{4176}{8352}=\frac{4356}{8712}.$

https://goo.gl/6AjHbm

18.4 9-Digits Higher Equivalent Fractions: 46 Expressions

In this case we have two possibilities, i.e., 0 to 8 and 1 to 9. The only higher number we got is in case of 1 to 9. See below:

$$\bullet\ \frac{3187}{25496} = \frac{4589}{36712} = \frac{4591}{36728} = \frac{4689}{37512} = \frac{4691}{37528} = \frac{4769}{38152} = \frac{5237}{41896} = \frac{5371}{42968} = \frac{5789}{46312} = \frac{5791}{46328} = \frac{5839}{46712}$$
$$= \frac{5892}{47136} = \frac{5916}{47328} = \frac{5921}{47368} = \frac{6479}{51832} = \frac{6741}{53928} = \frac{6789}{54312} = \frac{6791}{54328} = \frac{6839}{54712} = \frac{7123}{56984} = \frac{7312}{58496}$$
$$= \frac{7364}{58912} = \frac{7416}{59328} = \frac{7421}{59368} = \frac{7894}{63152} = \frac{7941}{63528} = \frac{8174}{65392} = \frac{8179}{65432} = \frac{8394}{67152} = \frac{8419}{67352} = \frac{8439}{67512}$$
$$= \frac{8932}{71456} = \frac{8942}{71536} = \frac{8953}{71624} = \frac{8954}{71632} = \frac{9156}{73248} = \frac{9158}{73264} = \frac{9182}{73456} = \frac{9316}{74528} = \frac{9321}{74568} = \frac{9352}{74816}$$
$$= \frac{9416}{75328} = \frac{9421}{75368} = \frac{9523}{76184} = \frac{9531}{76248} = \frac{9541}{76328}$$

https://goo.gl/AYaGZb

18.5 10-Digits Higher Equivalent Fractions: 7 Expressions

In this case we have only one possibility, i.e., 0 to 9. See below the higher equivalent fraction with 78 expressions:

$$\bullet\ \frac{16748}{20935} = \frac{18476}{23095} = \frac{18940}{23675} = \frac{18964}{23705} = \frac{19076}{23845} = \frac{19708}{24635} = \frac{24716}{30895} = \frac{24876}{31095} = \frac{25496}{31870} = \frac{28940}{36175} = \frac{29180}{36475} = \frac{29684}{37105}$$
$$= \frac{32716}{40895} = \frac{32876}{41095} = \frac{36712}{45890} = \frac{36728}{45910} = \frac{37512}{46890} = \frac{37528}{46910} = \frac{38092}{47615} = \frac{38152}{47690} = \frac{39012}{48765} = \frac{41896}{52370} = \frac{42968}{53710}$$
$$= \frac{46312}{57890} = \frac{46328}{57910} = \frac{46712}{58390} = \frac{47136}{58920} = \frac{47328}{59160} = \frac{47368}{59210} = \frac{48732}{60915} = \frac{49380}{61725} = \frac{49708}{62135} = \frac{51832}{64790} = \frac{53928}{67410}$$
$$= \frac{54312}{67890} = \frac{54328}{67910} = \frac{54712}{68390} = \frac{56984}{71230} = \frac{58496}{73120} = \frac{58912}{73640} = \frac{59328}{74160} = \frac{59368}{74210} = \frac{63124}{78905} = \frac{63140}{78925} = \frac{63152}{78940}$$
$$= \frac{63284}{79105} = \frac{63528}{79410} = \frac{64732}{80915} = \frac{65392}{81740} = \frac{65432}{81790} = \frac{67124}{83905} = \frac{67140}{83925} = \frac{67152}{83940} = \frac{67352}{84190} = \frac{67512}{84390} = \frac{71236}{89045}$$
$$= \frac{71364}{89205} = \frac{71456}{89320} = \frac{71460}{89325} = \frac{71536}{89420} = \frac{71624}{89530} = \frac{71632}{89540} = \frac{72836}{91045} = \frac{73248}{91560} = \frac{73264}{91580} = \frac{73284}{91605} = \frac{73456}{91820}$$
$$= \frac{73460}{91825} = \frac{73684}{92105} = \frac{74108}{92635} = \frac{74528}{93160} = \frac{74568}{93210} = \frac{74816}{93520} = \frac{75328}{94160} = \frac{75368}{94210} = \frac{76184}{95230} = \frac{76248}{95310} = \frac{76328}{95410}.$$

https://goo.gl/gJYujy

For complete details refer to author's work [39, 40, 41]. The first work [39] is from 3 to 8 digits. The second work [40] is for 9 digits and the third work [41] is for 10 digits. All the three papers are for different digits. The repetition of digits give much more equivalent fractions. This shall be dealt elsewhere.

19 Selfie Expressions: Multiplicative, Power and Factorial

Selfie expressions are very much similar to **selfie numbers**. Selfie numbers are represented by its own digits by use of some operations, while **selfie expressions** are the expressions where both sides have same digits, not necessarily with same operations on both sides, i.e., **same digits equality expressions**. Below are different ways of expressing **selfie expressions** with same digits on both sides:

- **Multiplicative Equalities**

$$abcd... \times efgh... = cbad... \times gfhe..\ \forall a,b,c,d,e,... \in N_+ \tag{1}$$

- **Power and Addition**

$$a^b + c^d + ... = ab + cd + ...,\ \forall a,b,c,d,.. \in N. \tag{2}$$

- **Factorial and Power**

$$a! \times b! + (c! + d!) \times e! + ... = a^a + b^b - c^c \times (d^d - e^e) + ...,\ \forall a,b,c,d,e,... \in N_+, \text{ etc.} \tag{3}$$

$$a! \times b! + (c! + d!) \times e! + ... = a^c + (b^d - c^a) \times d^e - e^b + ...,\ \forall a,b,c,d,e,... \in N_+, \text{ etc.} \tag{4}$$

We observe that the (4) is different from the (3) in right side of the expression. In case (3), the power of digits is same as of bases. In case of (4), it is not necessary that the power is same as of digits, but is a permutation of same digits as of bases. See below more general way.

$$(a!, b!, c!, ...) = (a^a, b^b, c^c, ..)$$
$$(a!, b!, c!, ...) = (a, b, c, ..)^{(a,b,c,...)}.$$

The first expression is simplified form of (3) and the second expression is similar to (4). Let's explain one by one by examples, the four **selfie expressions** given in (1), (2) , (3) and (4).

19.1 Multiplicative-Type Selfie Equalities

This subsection brings results based on the expression (1). By **multiplicative selfie equalities**, we understand that there are equalities, where each side is separated by operation of multiplications having same digits on both sides, not necessarily in same order. There are many ways of writing these kind of numbers explained in following subsections.

19.1.1 First Type

In this case, we have multiplicative equalities with equal number of digits on both sides and also in each multiplicative factor. The operation of multiplications is with number and its reverse forming a palindromic-type expression. For example, Based on idea of expressions are written in such a way that numbers formed by same digits multiplied by its reverse are equal to another group of multiplicative factors with same digits but of different numbers. See below some examples:

⋄ $37468 \times 86473 = 47386 \times 68374$
⋄ $37596 \times 69573 = 39756 \times 65793$
⋄ $39648 \times 84693 = 48396 \times 69384$
⋄ $45495 \times 59454 = 49545 \times 54594$
⋄ $46069 \times 96064 = 64096 \times 69046.$

⋄ $120024 \times 420021 = 210042 \times 240012$
⋄ $102204 \times 402201 = 201402 \times 204102$
⋄ $130026 \times 620031 = 260013 \times 310062$
⋄ $120036 \times 630021 = 210063 \times 360012$
⋄ $102306 \times 603201 = 201603 \times 306102.$

https://goo.gl/DUWQ4o

19.1.2 Second Type

The second case is similar to first one, having the same number of digits in each multiplicative factor but not forming a palindromic-type expression. For example,

$\diamond\, 2017 \times 3404 = 1702 \times 4034$
$\diamond\, 2017 \times 6808 = 1702 \times 8068$
$\diamond\, 1729 \times 3584 = 1792 \times 3458$
$\diamond\, 1729 \times 3854 = 1927 \times 3458.$
$\diamond\, 1729 \times 4358 = 2179 \times 3458$
$\diamond\, 1729 \times 4732 = 2197 \times 3724$
$\diamond\, 1729 \times 5438 = 2719 \times 3458$
$\diamond\, 1729 \times 5781 = 1927 \times 5187.$

https://goo.gl/DUWQ4o

19.1.3 Third Type

The third case is similar to second one, but there is no rule with order of digits. Only thing is that on both sides of the equality sign, there are same digits. There are many numbers, but we have written only those with more than one equality sign. See below examples,

$\diamond\, 162 \times 8064 = 216 \times 6048 = 648 \times 2016$
$\diamond\, 162 \times 8073 = 207 \times 6318 = 702 \times 1863$
$\diamond\, 17 \times 35945 = 35 \times 17459 = 395 \times 1547$
$\diamond\, 176 \times 7469 = 194 \times 6776 = 776 \times 1694$
$\diamond\, 18 \times 39879 = 189 \times 3798 = 378 \times 1899$
$\diamond\, 18 \times 41553 = 54 \times 13851 = 513 \times 1458.$

https://goo.gl/DUWQ4o

$\diamond\, 1782 \times 43956 = 2178 \times 35964 = 3564 \times 21978 = 4356 \times 17982$
$\diamond\, 18 \times 2830464 = 486 \times 104832 = 1404 \times 36288 = 3024 \times 16848$
$\diamond\, 18 \times 5204736 = 162 \times 578304 = 3456 \times 27108 = 4518 \times 20736$
$\diamond\, 198 \times 179982 = 297 \times 119988 = 1188 \times 29997 = 1782 \times 19998$
$\diamond\, 198 \times 339966 = 396 \times 169983 = 1683 \times 39996 = 3366 \times 19998$
$\diamond\, 2 \times 12089121 = 11 \times 2198022 = 222 \times 108911 = 1221 \times 19802.$

https://goo.gl/DUWQ4o

Due to large quantity of numbers, we worked only with double or higher equality signs. These expressions with single equality are famous as **vamp numbers**. For more details see author's complete work [59]:

19.2 Power and Addition

Following the idea of expression (2) the author wrote the numbers **2017** [46] and **1729** [47] as:

$$\begin{aligned}
\mathbf{2017} &:= 4^4+41^2+77^0+79^1 &&= 44+412+770+791\\
&:= 1^4+44^2+77^0+79^1 &&= 14+442+770+791\\
&:= 2^4+2^8+4^2+12^3+180^0 &&= 24+28+42+123+1800\\
&:= 1^1+3^6+5^4+5^4+6^2+180^0 &&= 11+36+54+54+62+1800
\end{aligned}$$

$$\begin{aligned}
\mathbf{1729} &:= 2^7+40^2+130^0 &&= 27+402+1300\\
&:= 2^6+40^2+64^1+66^0 &&= 26+402+641+660\\
&:= 1^6+41^2+46^1+84^0 &&= 16+412+461+840.
\end{aligned}$$

https://goo.gl/As0eJA
https://goo.gl/tGA0ea

Below are more examples,

$$\begin{aligned}
\mathbf{81} &:= 2^3+2^6+3^2 &&= 23+26+32\\
\mathbf{99} &:= 2^3+3^3+4^3 &&= 23+33+43\\
\mathbf{121} &:= 2^3+2^6+7^2 &&= 23+26+72\\
\mathbf{170} &:= 2^6+5^2+9^2 &&= 26+52+92\\
\mathbf{246} &:= 2^2+11^2+11^2 &&= 22+112+112.
\end{aligned}$$

$$\begin{aligned}
\mathbf{246} &:= 5^2+5^2+14^2 &&= 52+52+142\\
\mathbf{266} &:= 4^2+9^2+13^2 &&= 42+92+132\\
\mathbf{286} &:= 6^2+9^2+13^2 &&= 62+92+132\\
\mathbf{306} &:= 8^2+11^2+11^2 &&= 82+112+112\\
&:= 9^2+9^2+12^2 &&= 92+92+122.
\end{aligned}$$

https://goo.gl/As0eJA
https://goo.gl/tGA0ea

In the above examples, the equality expressions are formed by three terms on both sides, while the numbers 2017 and 1729 are with **different terms expressions**. More detailed study can be seen at author's work [46, 47]. In these works, instead of using only positive sign, both positive and negative signs are used.

19.3 Factorial and Power

Recently, author [53, 54] worked on results arising due to (3) and (4). This we have done in three different ways. One without any repetition of digits. The second we have done with repetition of digits. Third with permutable powers. Both sides of the equality are with the operations: **addition**, **subtraction**, and **multiplication** along with composite relation. See below some examples in each case:

19.3.1 Different Digits

$$\begin{aligned}
\mathbf{144} &:= (2!-1!)\times 3!\times 4! &&= -2^2\times(1^1+3^3)+4^4\\
\mathbf{147} &:= 1!+2!+3!\times 4! &&= -1^1-2^2\times 3^3+4^4\\
\mathbf{148} &:= (1!+4!)\times 3!-2! &&= 1^1\times 4^4-3^3\times 2^2\\
\mathbf{152} &:= 2!+3!\times(1!+4!) &&= 2^2\times(-3^3+1^1)+4^4\\
\mathbf{286} &:= (-1!+3!\times 4!)\times 2! &&= -1^1+3^3+4^4+2^2\\
\mathbf{287} &:= -1!+2!\times 3!\times 4! &&= 1^1\times 2^2+3^3+4^4\\
\mathbf{288} &:= 1!\times 2!\times 3!\times 4! &&= 1^1+2^2+3^3+4^4.
\end{aligned}$$

https://goo.gl/XenV9x
https://goo.gl/AvVRxE

19.3.2 Repetition of Digits

$$\begin{aligned}
\mathbf{108} &:= 2!\times(3!+4!+4!) &&= 2^2\times 3^3+4^4-4^4\\
&:= 3!\times(3!+3!\times 2!) &&= (3^3+3^3-3^3)\times 2^2\\
&:= -5!+2!\times(5!-3!) &&= (5^5+2^2-5^5)\times 3^3\\
&:= (-3!+5!)\times 2!-5! &&= 3^3\times(5^5+2^2-5^5)\\
&:= (2!\times 3!+3!)\times 3!\times 1! &&= (2^2+3^3-3^3)\times 3^3\times 1^1\\
&:= (1!\times 1!+2!)\times 3!\times 3! &&= (-1^1-1^1+2^2)\times(3^3+3^3)\\
&:= (1!\times 3!+3!+3!)\times 3! &&= 1^1\times 3^3+3^3+3^3+3^3\\
&:= (4!+3!\times 1!+4!)\times 2! &&= (4^4+3^3\times 1^1-4^4)\times 2^2\\
&:= (-3!+5!\times 1!)\times 2!-5! &&= (5^5\times 1^1+3^3-5^5)\times 2^2.
\end{aligned}$$

$$\begin{aligned}
\mathbf{1008} &:= ((4!-2!)\times 4!-4!)\times 2! &&= (4^4-2^2-4^4+4^4)\times 2^2\\
&:= (2!+2!+4!)\times 3!\times 3! &&= 2^2\times(-2^2+4^4)-3^3+3^3\\
&:= (2!-1!+3!)\times 3!\times 4! &&= -2^2+(1^1+3^3)\times 3^3+4^4\\
&:= 2!\times(2!\times(5!+5!)+4!) &&= 2^2\times(-2^2-5^5+5^5+4^4).
\end{aligned}$$

https://goo.gl/XenV9x
https://goo.gl/AvVRxE

19.3.3 Permutable Powers

In the above two subsections powers on left side are the same as of bases, below are examples, where powers permutations of bases:

$$\begin{aligned}
\mathbf{3648} &:= 1!\times 6!+(2!+5!)\times 4! = (1^5+6^2)\times 2^6+5^1\times 4^4\\
&:= 1!\times 6!+(5!+2!)\times 4! = (1^4\times 6^2+5^1)\times 2^6+4^5\\
\mathbf{3649} &:= 1!+4!\times(2!+5!)+6! = 1^4+4^5+2^6\times(5^1+6^2)\\
\mathbf{3690} &:= (1!+2!+5!)\times(3!+4!) = (1^2+2^1)\times(5^3+3^4+4^5)\\
\mathbf{3744} &:= (1!\times 3!+5!)\times 4!+6! = (1^6\times 3^5+5^3+4^4)\times 6^1\\
\mathbf{3745} &:= 1!+(3!+5!)\times 4!+6! = 1^6+(3^5+5^3+4^4)\times 6^1\\
\mathbf{3840} &:= (1!\times 4!+2!+3!)\times 5! = (1^4+4^3)\times(2^1+3^2)+5^5\\
& = 1^4\times 4^3\times(2^5+3^1+5^2)\\
&:= 1!\times 5!\times(4!+2!)+6! = (1^5+5^1)\times 4^4+2^6\times 6^2
\end{aligned}$$

$$\begin{aligned}
\mathbf{4320} &:= (2!-1!)\times 3!\times 6! = (-2^3-1^2+3^6)\times 6^1\\
& = 2^1\times(1^6+3^2)\times 6^3\\
&:= (2!-1!)\times 7!-6! = (2^7-1^6-7^1)\times 6^2\\
\mathbf{4326} &:= 3!\times(2!-1!+6!) = (3^6-2^3)\times 1^2\times 6^1\\
\mathbf{4332} &:= 1!\times 3!\times(2!+6!) = (1^2+3^6-2^3)\times 6^1\\
\mathbf{5050} &:= 2!\times(3!-1!)+7! = (2^7-3^3)\times(1^1+7^2)\\
\mathbf{5058} &:= (2!+1!)\times 3!+7! = 2^1\times(-1^2+3^7+7^3).
\end{aligned}$$

https://goo.gl/XenV9x
https://goo.gl/AvVRxE

For more details refer author's work [58, 70].

20 Selfie Expressions: Factorial, Fibonacci and Triangular Numbers

This section extend the work given in Section 19 specially subsection 19.3 using **Factorial, Fibonacci** and **Triangular** type functions instead of power. In order to write these expression, the digit order remains the same independent of operations.

20.1 Factorial-Fibonacci-Triangular-Type Selfie Expressions

- **Two-Terms Expressions**

$$\mathbf{2} := 1!\times 2! = F(1)+F(2) = -T(1)+T(2)$$

- **Three-Terms Expressions**

$$\begin{aligned}
\mathbf{3} &:= -1!-2!+3! = F(1)\times F(2)+F(3) = -T(1)\times T(2)+T(3)\\
\mathbf{4} &:= -1!\times 2!+3! = F(1)+F(2)+F(3) = T(1)-T(2)+T(3)
\end{aligned}$$

https://goo.gl/Bvbrir

- **Four-Terms Expressions**

$$\begin{aligned}
\mathbf{0} &:= (1!-3!)\times 4!+5! &&= F(1)\times F(3)+F(4)-F(5) &&= T(1)-T(3)-T(4)+T(5)\\
&:= 4!\times(1!-3!)+5! &&= F(4)\times F(1)+F(3)-F(5) &&= T(4)-T(1)+T(3)-T(5)\\
\mathbf{1} &:= 1!+3!\times 5!-6! &&= -F(1)\times F(3)-F(5)+F(6) &&= T(1)+T(3)+T(5)-T(6)\\
&:= -6!+3!\times 5!+1! &&= F(6)-F(3)-F(5)\times F(1) &&= -T(6)+T(3)+T(5)+T(1)\\
\mathbf{6} &:= -(1!+2!)\times 3!+4! &&= F(1)\times F(2)+F(3)+F(4) &&= -T(1)+T(2)-T(3)+T(4)\\
\mathbf{12} &:= -1!\times 2!\times 3!+4! &&= (F(1)+F(2)+F(3))\times F(4) &&= -T(1)-T(2)+T(3)+T(4)\\
&:= 3!\times(-2!+4!)-5! &&= -F(3)-F(2)+F(4)\times F(5) &&= T(3)\times(-T(2)-T(4)+T(5))\\
&:= 3!\times(2!+5!)-6! &&= -F(3)+F(2)+F(5)+F(6) &&= T(3)\times T(2)+T(5)-T(6)\\
\mathbf{18} &:= 3!\times(-1!+4!)-5! &&= F(3)+F(1)+F(4)\times F(5) &&= -T(3)-T(1)+T(4)+T(5)\\
\mathbf{22} &:= -2!+4!\times 3!-5! &&= F(2)+F(4)\times(F(3)+F(5)) &&= T(2)+T(4)-T(3)+T(5)\\
\mathbf{24} &:= 3!\times 5!+4!-6! &&= F(3)\times(F(5)+F(4))+F(6) &&= T(3)\times(T(5)+T(4)-T(6))\\
&:= -1!\times 5!+4!\times 3! &&= (-F(1)+F(5))\times F(4)\times F(3) &&= (-T(1)+T(5)-T(4))\times T(3)\\
\mathbf{25} &:= 1!+3!\times 4!-5! &&= (F(1)\times F(3)+F(4))\times F(5) &&= (T(1)-T(3))\times(T(4)-T(5))\\
\mathbf{30} &:= (1!+4!)\times 3!-5! &&= (F(1)+F(4)+F(3))\times F(5) &&= -T(1)+T(4)+T(3)+T(5)\\
\mathbf{120} &:= 5!\times(1!+3!)-6! &&= F(5)\times(F(1)+F(3))\times F(6) &&= T(5)-(T(1)-T(3))\times T(6)
\end{aligned}$$

https://goo.gl/Bvbrir

- **Five-Terms Expressions**

$$\begin{aligned}
\mathbf{48} &:= (1!-3!+2!)\times 4!+5! &&= (F(1)+F(3))\times(F(2)+F(4)\times F(5)) &&= T(1)\times T(3)\times(T(2)-T(4)+T(5))\\
&:= 2!\times 4!+3!\times 5!-6! &&= (F(2)+F(4))\times F(3)\times F(5)+F(6) &&= (-T(2)+T(4))\times T(3)-T(5)+T(6)\\
&:= (3!+1!)\times(4!-5!)+6! &&= (F(3)+F(1))\times(F(4)+F(5)+F(6)) &&= T(3)\times(-T(1)+T(4))+T(5)-T(6)\\
\mathbf{60} &:= -1!\times 2!\times(3!+4!)+5! &&= (F(1)+F(2)+F(3))\times F(4)\times F(5) &&= (T(1)-T(2))\times T(3)\times(T(4)-T(5))\\
\mathbf{96} &:= (1!+3!)\times 5!-4!-6! &&= ((F(1)+F(3))\times F(5)-F(4))\times F(6) &&= T(1)\times T(3)\times(-T(5)+T(4)+T(6))\\
\mathbf{120} &:= (-1!+2!-3!)\times 5!+6! &&= (F(1)\times F(2)+F(3))\times F(5)\times F(6) &&= (T(1)+T(2))\times(-T(3)+T(5)+T(6))\\
&:= (2!+3!)\times 7!+5!-8! &&= F(2)\times F(3)+F(7)+F(5)\times F(8) &&= T(2)+T(3)\times T(7)-T(5)-T(8)\\
\mathbf{132} &:= -3!\times(2!+5!-4!)+6! &&= F(3)\times(F(2)+F(5))\times(F(4)+F(6)) &&= (T(3)-T(2)-T(5))\times(T(4)-T(6))
\end{aligned}$$

https://goo.gl/Bvbrir

- **Five-Terms Expressions**

$$\begin{aligned}
\mathbf{144} &:= 1! \times 3! \times (4!+5!)-6! = (F(1)+F(3)+F(4)\times F(5))\times F(6) = (-T(1)\times T(3)+T(4))\times(T(5)+T(6))\\
&:= (2!-3!)\times(4!+5!)+6! = (F(2)+F(3)+F(4)\times F(5))\times F(6) = (T(2)+T(3))\times(T(4)-T(5)+T(6))\\
&:= 3!\times(4!+6!+5!)-7! = (-F(3)+F(4))\times F(6)\times(F(5)+F(7)) = T(3)\times(-T(4)+T(6)-T(5)+T(7))\\
\mathbf{264} &:= (2!+3!)\times 5!+4!-6! = (F(2)+F(3)\times F(5))\times F(4)\times F(6) = (T(2)+T(3)+T(5))\times(-T(4)+T(6))\\
\mathbf{576} &:= -3!\times(4!+6!)\times 1!+7! = F(3)\times F(4)\times F(6)\times(-F(1)+F(7)) = T(3)+T(4)+(T(6)-T(1))\times T(7)\\
\mathbf{600} &:= -5!-3!\times 6!\times 1!+7! = F(5)\times(F(3)+F(6))\times(-F(1)+F(7)) = -T(5)+T(3)+T(6)\times(T(1)+T(7))\\
&:= -5!+6!\times(2!+3!)-7! = F(5)\times F(6)\times F(2)\times(F(3)+F(7)) = T(5)\times(T(6)-T(2)-T(3)+T(7))\\
\mathbf{864} &:= 4!+5!+7!-3!\times 6! = F(4)\times(F(5)+F(7))\times F(3)\times F(6) = T(4)\times T(5)+(T(7)+T(3))\times T(6)\\
\mathbf{960} &:= -3!\times 6!+5!\times 2!+7! = F(3)\times F(6)\times F(5)\times(-F(2)+F(7)) = -T(3)+T(6)\times(T(5)+T(2)+T(7))\\
\mathbf{1560} &:= -(3!-1!)\times 6!+5!+7! = (F(3)+F(1))\times F(6)\times F(5)\times F(7) = T(3)\times(T(1)-T(6))\times(T(5)-T(7))
\end{aligned}$$

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20.2 Factorial-Fibonacci-Type Selfie Expressions

- **Two-Terms Expressions**

$$\mathbf{1} := -1!+2! = F(1)\times F(2)$$

- **Three-Terms Expressions**

$$\begin{aligned}
\mathbf{2} &:= 2!+3!\times 5!-6! = F(2)\times F(3)\times F(5)-F(6)\\
\mathbf{6} &:= (1!+5!)\times 3!-6! = F(1)-F(5)+F(3)+F(6)
\end{aligned}$$

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- **Four-Terms Expressions**

$$\begin{aligned}
\mathbf{10} &:= -(1!+3!)\times 2!+4! = F(1)+(F(3)+F(2))\times F(4)\\
&:= 4!-2!\times(1!+3!) \quad = (F(4)+F(2)+F(1))\times F(3)\\
\mathbf{24} &:= (-3!+2!)\times 4!+5! = (F(3)+F(2))\times(F(4)+F(5))\\
\mathbf{26} &:= 2!+3!\times 4!-5! \quad = F(2)+(F(3)+F(4))\times F(5)\\
\mathbf{36} &:= -5!+(2!+4!)\times 3! = (F(5)+F(2))\times F(4)\times F(3)
\end{aligned}$$

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- **Five-Terms Expressions**

$$\begin{aligned}
\mathbf{120} &:= (1!+3!)\times 6!+5!-7! \quad = (F(1)-F(3)\times F(6))\times(F(5)-F(7))\\
&:= (3!+2!-1!)\times 5!-6! \quad = (F(3)+F(2))\times F(1)\times F(5)\times F(6)\\
&:= 5!+8!-(3!+2!)\times 7! \quad = (F(5)\times F(8)+F(3))\times F(2)+F(7)\\
&:= -7!\times(2!+3!)+8!+5! = F(7)\times F(2)+F(3)+F(8)\times F(5)\\
\mathbf{146} &:= 3!\times(4!+5!)+2!-6! \quad = F(3)+F(4)\times(F(5)+F(2))\times F(6)\\
\mathbf{216} &:= -4!+5!\times(3!+2!)-6! = F(4)\times(F(5)\times F(3)-F(2))\times F(6)\\
\mathbf{720} &:= (2!\times 5!+6!)\times 3!-7! \quad = (F(2)+F(5))\times F(6)\times(F(3)+F(7))\\
&:= -(3!+4!)\times 5!+7!-6! = (F(3)+F(4))\times(F(5)+F(7))\times F(6)\\
&:= -2!\times 6!-5!\times 4!+7! \quad = (F(2)+F(6))\times F(5)\times(F(4)+F(7))
\end{aligned}$$

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- **Five-Terms Expressions**

$$\begin{aligned}
\mathbf{744} &:= (4!-6!)\times 3!+7!-5! \quad = F(4)\times F(6)\times(F(3)\times F(7)+F(5))\\
\mathbf{816} &:= -4!+5!-3!\times 6!+7! \quad = (F(4)+F(5))\times(-F(3)+F(6)\times F(7))\\
\mathbf{864} &:= -3!\times 6!+4!+5!+7! \quad = F(3)\times F(6)\times F(4)\times(F(5)+F(7))\\
\mathbf{960} &:= 5!\times 2!+7!-3!\times 6! \quad = F(5)\times(-F(2)+F(7))\times F(3)\times F(6)\\
\mathbf{1440} &:= 4!\times 5!\times 2!-7!+6! \quad = F(4)\times F(5)\times(-F(2)+F(7))\times F(6)\\
&:= 1!\times 7!-4!\times 5!-6! \quad = (-F(1)+F(7))\times F(4)\times F(5)\times F(6)\\
\mathbf{1680} &:= -5!\times(4!-2!)+7!-6! = F(5)\times F(4)\times(F(2)+F(7))\times F(6)\\
\mathbf{9240} &:= 8!-5!-6!-7!\times 3! \quad = F(8)\times F(5)\times F(6)\times(F(7)-F(3))
\end{aligned}$$

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20.3 Factorial-Triangular-Type Selfie Expressions

20.3.1 Positive Coefficients

$\mathbf{6} := 3! = T(3)$

$\mathbf{7} := 1! + 3! = T(1) + T(3)$

$\mathbf{9} := 1! + 2! + 3! = T(1) \times T(2) + T(3)$

$\mathbf{18} := (1! + 2!) \times 3! = T(1) \times T(2) \times T(3)$

$\mathbf{36} := 3! \times 2! + 4! = T(3) + T(2) \times T(4)$

$\mathbf{150} := 3! + 4! + 5! = T(3) \times (T(4) + T(5))$

$\mathbf{37} := 1! + 3! \times 2! + 4! = T(1) + T(3) + T(2) \times T(4)$

$\mathbf{78} := (1! + 2!) \times 4! + 3! = (T(1) \times T(2) + T(4)) \times T(3)$

$\mathbf{90} := (1! + 2!) \times (3! + 4!) = (T(1) \times T(2) + T(3)) \times T(4)$

$\mathbf{150} := 1! \times 3! + 4! + 5! = T(1) \times T(3) \times (T(4) + T(5))$

$\mathbf{151} := 1! + 3! + 4! + 5! = T(1) + T(3) \times (T(4) + T(5))$

$\mathbf{168} := 2! \times 1! \times 4! + 5! = T(2) + (T(1) + T(4)) \times T(5)$

$\mathbf{300} := 2! \times (3! + 5! + 4!) = T(2) \times (T(3) \times T(5) + T(4))$

$\mathbf{960} := 1! \times 6! + 2! \times 5! = (T(1) + T(6) \times T(2)) \times T(5)$

$\mathbf{2160} := 2! \times 6! + 3! \times 5! = (T(2) + T(6)) \times T(3) \times T(5)$

$\mathbf{1008} := (2! \times 3!) \times 4! + 6! = T(2) \times (T(3) + T(4)) \times T(6)$

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$\mathbf{174} := 3! \times 1! + 2! \times 4! + 5! = T(3) \times (T(1) + T(2)) + T(4) \times T(5)$

$\mathbf{301} := 1! + 2! \times (4! + 3!) + 5! = T(1) + T(2) \times (T(4) + T(3) \times T(5))$

$\mathbf{456} := (1! + 3!) \times 2! \times 4! + 5! = T(1) \times T(3) + (T(2) \times T(4)) \times T(5)$

$\mathbf{757} := 1! + 3! \times 2! + 4! + 6! = T(1) + (T(3) + T(2) \times T(4)) \times T(6)$

$\mathbf{810} := (1! + 2!) \times (4! + 3!) + 6! = T(1) \times T(2) \times T(4) \times (T(3) + T(6))$

$\mathbf{972} := (3! \times 1! + 5!) \times 2! + 6! = T(3) + (T(1) + T(5) \times T(2)) \times T(6)$

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$\mathbf{1728} := 3! \times (4! \times 2! + 5!) + 6! = (T(3) + T(4)) \times T(2) \times (T(5) + T(6))$

$\mathbf{1968} := 2! \times (5! + 6! + 4! \times 3!) = (T(2) + T(5) \times T(6) + T(4)) \times T(3)$

$\mathbf{2160} := 1! \times 3! \times 5! \times 2! + 6! = T(1) \times T(3) \times T(5) \times (T(2) + T(6))$

$\mathbf{4320} := 1! \times 5! \times (4! + 3!) + 6! = (T(1) + T(5)) \times T(4) \times (T(3) + T(6))$

$\mathbf{7560} := (1! + 2! + 3!) \times (5! + 6!) = (T(1) + T(2)) \times T(3) \times T(5) \times T(6)$

$\mathbf{12240} := (7! + 3! \times 5!) \times 2! + 6! = (T(7) + T(3)) \times T(5) \times (T(2) + T(6))$

$\mathbf{25200} := (1! + 3!) \times 4! \times 5! + 7! = T(1) \times T(3) \times T(4) \times T(5) \times T(7)$

$\mathbf{30240} := 4! \times (5! + 6!) + 2! \times 7! = T(4) \times (T(5) + T(6)) \times T(2) \times T(7)$

$\mathbf{725760} := 8! \times 3! + 4! \times 7! + 9! = (T(8) \times (T(3) + T(4)) \times T(7)) \times T(9)$

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20.3.2 Positive-Negative Coefficients

- **Single-Term Expressions**

$$\mathbf{6} := 3! = T(3)$$

- **Two-Terms Expressions**

$$\begin{aligned}
\mathbf{5} &:= -1!+3! = -T(1)+T(3)\\
\mathbf{6} &:= 1!\times 3! \quad = T(1)\times T(3)\\
\mathbf{7} &:= 1!+3! \quad = T(1)+T(3)
\end{aligned}$$

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- **Three-Terms Expressions**

$$\begin{aligned}
\mathbf{8} &:= 1!\times 2!+3! &&= -T(1)+T(2)+T(3)\\
\mathbf{9} &:= 1!+2!+3! &&= T(1)\times T(2)+T(3)\\
\mathbf{10} &:= (-1!+3!)\times 2! &&= T(1)+T(3)+T(2)\\
\mathbf{12} &:= 1!\times 2!\times 3! &&= (-T(1)+T(2))\times T(3)\\
&:= -2!\times 3!+4! &&= T(2)\times(-T(3)+T(4))\\
\mathbf{17} &:= -1!-3!+4! &&= T(1)+T(3)+T(4)\\
\mathbf{18} &:= (1!+2!)\times 3! &&= T(1)\times T(2)\times T(3)\\
\mathbf{27} &:= 1!+4!+2! &&= (-T(1)+T(4))\times T(2)
\end{aligned}$$

$$\begin{aligned}
\mathbf{28} &:= -2!+3!+4! &&= T(2)\times T(3)+T(4)\\
\mathbf{36} &:= 3!\times 2!+4! &&= T(3)+T(2)\times T(4)\\
\mathbf{42} &:= 2!\times 4!-3! &&= (-T(2)+T(4))\times T(3)\\
\mathbf{90} &:= -3!+5!-4! &&= (-T(3)+T(5))\times T(4)\\
\mathbf{108} &:= -3!\times 2!+5! &&= T(3)\times(T(2)+T(5))\\
\mathbf{150} &:= 3!+4!+5! &&= T(3)\times(T(4)+T(5))
\end{aligned}$$

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- **Four-Terms Expressions**

$$\begin{aligned}
\mathbf{11} &:= -1! - 2! \times 3! + 4! &&= -T(1) + T(2) \times (-T(3) + T(4))\\
\mathbf{28} &:= -1! \times 2! + 3! + 4! &&= T(1) \times T(2) \times T(3) + T(4)\\
\mathbf{34} &:= 2! \times (-1! - 3! + 4!) &&= (T(2) + T(1)) \times T(3) + T(4)\\
\mathbf{48} &:= -(2! + 1!) \times 4! + 5! &&= T(2) \times (T(1) + T(4)) + T(5)\\
\mathbf{53} &:= -1! + 3! + 4! \times 2! &&= (-T(1) + T(3)) \times T(4) + T(2)\\
\mathbf{56} &:= 2! \times (1! + 4!) + 3! &&= -T(2) - T(1) + T(4) \times T(3)\\
\mathbf{72} &:= -2! \times 1! \times 4! + 5! &&= T(2) \times (-T(1) + T(4) + T(5))\\
\mathbf{100} &:= (3! - 2!) \times (1! + 4!) &&= (T(3) + T(2) + T(1)) \times T(4)\\
\mathbf{144} &:= 3! \times (4! + 5!) - 6! &&= (-T(3) + T(4)) \times (T(5) + T(6))\\
\mathbf{168} &:= 2! \times 3! \times 4! - 5! &&= T(2) \times T(3) + T(4) \times T(5)
\end{aligned}$$

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- **Four-Terms Expressions**

$$\begin{aligned}
\mathbf{240} &:= (-1! + 3!) \times 4! + 5! &&= (T(1) \times T(3) + T(4)) \times T(5)\\
\mathbf{468} &:= -(3! + 5!) \times 2! + 6! &&= T(3) \times (T(5) + T(2) \times T(6))\\
\mathbf{624} &:= (-3! + 2!) \times 4! + 6! &&= -T(3) + (T(2) \times T(4)) \times T(6)\\
\mathbf{630} &:= 3! + 4! - 5! + 6! &&= T(3) \times (-T(4) + T(5)) \times T(6)\\
\mathbf{750} &:= 3! \times (1! + 5!) + 4! &&= (T(3) - T(1)) \times T(5) \times T(4)\\
\mathbf{960} &:= 1! \times 6! + 2! \times 5! &&= (T(1) + T(6) \times T(2)) \times T(5)\\
\mathbf{1008} &:= (2! \times 3!) \times 4! + 6! &&= T(2) \times (T(3) + T(4)) \times T(6)\\
\mathbf{2160} &:= 2! \times 6! + 3! \times 5! &&= (T(2) + T(6)) \times T(3) \times T(5)\\
\mathbf{3612} &:= 2! \times (3! - 6!) + 7! &&= (T(2) + T(3) \times T(6)) \times T(7)\\
\mathbf{5040} &:= -(1! + 3!) \times 7! + 8! &&= (-T(1) + T(3)) \times T(7) \times T(8)
\end{aligned}$$

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- **Five-Terms Expressions**

$$
\begin{aligned}
\mathbf{21} &:= 3! \times 4! - 1! - 2! - 5! &&= -T(3) + T(4) - T(1) + T(2) + T(5)\\
\mathbf{36} &:= 2! \times (3! \times (-1! + 4!) - 5!) &&= (T(2) + T(3)) \times (-T(1) - T(4) + T(5))\\
\mathbf{37} &:= 1! + (4! + 2!) \times 3! - 5! &&= T(1) \times T(4) - T(2) \times (T(3) - T(5))\\
\mathbf{42} &:= (1! + 4! + 2!) \times 3! - 5! &&= (T(1) + T(4)) \times T(2) - T(3) + T(5)\\
\mathbf{46} &:= 2! \times (3! \times 4! - 1! - 5!) &&= -(T(2) - T(3)) \times T(4) + T(1) + T(5)\\
\mathbf{62} &:= (1! + 3!) \times (2! + 4!) - 5! &&= -T(1) + T(3) \times (T(2) + T(4)) - T(5)\\
\mathbf{71} &:= -1! + (2! + 3!) \times 4! - 5! &&= -T(1) - T(2) + T(3) \times T(4) + T(5)\\
\mathbf{82} &:= -(1! + 3!) \times 2! - 4! + 5! &&= T(1) + T(3) + T(2) \times (T(4) + T(5))
\end{aligned}
$$

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- **Five-Terms Expressions**

$$
\begin{aligned}
\mathbf{135} &:= -1! - 2! - 3! + 4! + 5! &&= T(1) \times T(2) \times (T(3) \times T(4) - T(5))\\
\mathbf{141} &:= 1! + 2! - 3! + 4! + 5! &&= -T(1) \times T(2) - T(3) + T(4) \times T(5)\\
\mathbf{164} &:= (1! + 4!) \times 2! - 3! + 5! &&= -T(1) + T(4) \times T(2) \times T(3) - T(5)\\
\mathbf{178} &:= 2! \times (-3! - 1! - 4! + 5!) &&= T(2) + (T(3) + T(1)) \times (T(4) + T(5))\\
\mathbf{180} &:= (1! \times 3! + 4!) \times 2! + 5! &&= (-T(1) \times T(3) + T(4)) \times T(2) \times T(5)\\
\mathbf{1009} &:= 1! + 2! \times 3! \times 4! + 6! &&= T(1) + T(2) \times (T(3) + T(4)) \times T(6)\\
\mathbf{1154} &:= (1! + 6! - 4! - 5!) \times 2! &&= -T(1) + T(6) \times (T(4) + T(5) \times T(2))\\
\mathbf{1170} &:= -3! - 4! + 2! \times (6! - 5!) &&= (((T(3) \times T(4)) - T(2)) + T(6)) \times T(5)
\end{aligned}
$$

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- **Five-Terms Expressions**

$$\begin{aligned}
\mathbf{10260} &:= 2! \times (-3!+5!-4!+7!) &&= T(2)\times T(3)\times T(5)\times (T(4)+T(7))\\
\mathbf{11400} &:= -5!+6!\times(-1!+4!)-7! &&= T(5)\times (T(6)-T(1))\times (T(4)+T(7))\\
\mathbf{18144} &:= -(6!-3!)\times 4!-7!+8! &&= T(6)\times (T(3)-T(4)+T(7))\times T(8)\\
\mathbf{36288} &:= 3!\times(4!\times 2!-6!)+8! &&= (T(3)+T(4))\times T(2)\times T(6)\times T(8)\\
\mathbf{37440} &:= (3!-4!)\times(6!-7!)-8! &&= (T(3)\times T(4))\times (T(6)\times T(7)+T(8))\\
\mathbf{82800} &:= 4!\times(5!+7!)-6!-8! &&= T(4)\times T(5)\times (T(7)\times T(6)-T(8))\\
\mathbf{89880} &:= -5!-6!+(-3!+4!)\times 7! &&= (T(5)\times T(6)+T(3))\times T(4)\times T(7)
\end{aligned}$$

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- **Five-Terms Expressions**

$$\begin{aligned}
\mathbf{108864} &:= (4!-5!)\times(3!-6!)+8! &&= (T(4)\times T(5)-T(3))\times T(6)\times T(8)\\
\mathbf{116640} &:= 3!\times(5!-8!)-7!+9! &&= T(3)\times T(5)\times (T(8)+T(7)\times T(9))\\
\mathbf{120960} &:= (-3!+6!)\times 5!-7!+8! &&= T(3)\times T(6)\times T(5)\times (T(7)+T(8))\\
\mathbf{233280} &:= 2!\times(-(6!+8!)\times 3!+9!) &&= (T(2)+T(6))\times T(8)\times T(3)\times T(9)\\
\mathbf{339984} &:= (3!+6!)\times 4!+9!-8! &&= (-T(3)+T(6)\times T(4)\times T(9))\times T(8)\\
\mathbf{443520} &:= 4!\times 7!-8!\times 1!+9! &&= T(4)\times T(7)\times T(8)\times (-T(1)+T(9))\\
\mathbf{453600} &:= (2!+5!-4!)\times 7!-8! &&= T(2)\times T(5)\times T(4)\times T(7)\times T(8)\\
\mathbf{725760} &:= 7!\times 4!+3!\times 8!+9! &&= T(7)\times (T(4)+T(3))\times T(8)\times T(9)\\
\mathbf{816480} &:= 3!\times(2!\times 8!-7!)+9! &&= T(3)\times T(2)\times T(8)\times T(7)\times T(9)\\
\mathbf{1360800} &:= (4!-2!)\times(7!+8!)+9! &&= T(4)\times T(2)\times T(7)\times T(8)\times T(9)\\
\mathbf{4082400} &:= (3!+5!)\times(8!-7!)-9! &&= T(3)\times T(5)\times T(8)\times T(7)\times T(9)
\end{aligned}$$

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For more details see the author's work [71].

20.4 Fibonacci and Triangular Type Selfie Expressions

20.4.1 Positive Coefficients

- **Two-Terms Expressions**

$$\mathbf{42} := F(3) \times F(8) = T(3) + T(8)$$

- **Three-Terms Expressions**

$$\begin{aligned}
\mathbf{43} &:= F(1) + F(3) \times F(8) &&= T(1) + T(3) + T(8)\\
\mathbf{66} &:= F(4) \times (F(2) + F(8)) &&= T(4) \times T(2) + T(8)\\
\mathbf{105} &:= F(2) + F(7) \times F(6) &&= T(2) \times T(7) + T(6)
\end{aligned}$$

https://goo.gl/gcCnbp

- **Four-Terms Expressions**

$$\begin{aligned}
\mathbf{31} &:= F(1) + F(3) \times F(4) \times F(5) &&= T(1) \times T(3) + T(4) + T(5)\\
\mathbf{46} &:= (F(1) + F(2)) \times (F(3) + F(8)) &&= T(1) + T(2) + T(3) + T(8)\\
\mathbf{52} &:= (F(1) + F(2) + F(3)) \times F(7) &&= (T(1) + T(2)) \times T(3) + T(7)\\
\mathbf{96} &:= F(3) \times F(6) \times (F(2) + F(5)) &&= (T(3) + T(6)) \times T(2) + T(5)\\
\mathbf{114} &:= F(2) + F(6) + F(5) \times F(8) &&= T(2) \times T(6) + T(5) + T(8)\\
\mathbf{448} &:= F(4) \times F(3) + F(7) \times F(9) &&= T(4) + T(3) \times (T(7) + T(9))\\
\mathbf{615} &:= F(4) + (F(7) + F(5)) \times F(9) &&= (T(4) + T(7)) \times T(5) + T(9)\\
\mathbf{2145} &:= (F(2) + F(8) \times F(9)) \times F(4) &&= (T(2) + T(8)) \times (T(9) + T(4))\\
\mathbf{2205} &:= F(4) \times F(8) \times (F(2) + F(9)) &&= (T(4) + T(8) + T(2)) \times T(9)\\
\mathbf{2346} &:= F(4) \times (F(8) + F(3)) \times F(9) &&= (T(4) + T(8)) \times (T(3) + T(9))\\
\mathbf{2352} &:= (F(2) + F(7)) \times F(6) \times F(8) &&= T(2) \times (T(7) + T(6) \times T(8))
\end{aligned}$$

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- **Five-Terms Expressions**

$$
\begin{aligned}
\mathbf{34} &:= F(1)+(F(2)+F(5)\times F(3))\times F(4) &&= T(1)\times T(2)+T(5)+T(3)+T(4)\\
\mathbf{48} &:= (F(1)\times F(2)+F(3))\times (F(4)+F(7)) &&= T(1)+T(2)+T(3)+T(4)+T(7)\\
\mathbf{93} &:= F(2)\times F(4)\times (F(6)+F(3)+F(8)) &&= T(2)\times T(4)+T(6)+T(3)+T(8)\\
\mathbf{95} &:= (F(1)+F(2))\times (F(4)+F(9))+F(8) &&= T(1)+T(2)+T(4)+T(9)+T(8)\\
\mathbf{946} &:= (F(1)+F(8))\times (F(4)+F(5)\times F(6)) &&= (T(1)+T(8))\times (T(4)+T(5))+T(6)\\
\mathbf{955} &:= F(3)\times F(9)\times (F(7)+F(2))+F(4) &&= T(3)+(T(9)+T(7))\times (T(2)+T(4))
\end{aligned}
$$

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- **Five-Terms Expressions**

$$
\begin{aligned}
\mathbf{3150} &:= (F(3)+F(7))\times F(5)\times (F(6)+F(9)) &&= (T(3)+T(7)+T(5)+T(6))\times T(9)\\
\mathbf{3192} &:= F(8)\times F(6)\times (F(2)+F(5)+F(7)) &&= (T(8)+T(6)\times T(2)+T(5))\times T(7)\\
\mathbf{9384} &:= (F(3)+F(7)\times F(8)+F(1))\times F(9) &&= (T(3)\times T(7)+T(8))\times (T(1)+T(9))\\
\mathbf{9792} &:= ((F(2)+F(8))\times F(7)+F(3))\times F(9) &&= T(2)\times (T(8)+T(7))\times (T(3)+T(9))\\
\mathbf{10584} &:= (F(3)+F(9))\times F(8)\times (F(2)+F(7)) &&= (T(3)\times T(9)+T(8)\times T(2))\times T(7)\\
\mathbf{10710} &:= (F(1)+F(7)\times F(6))\times F(4)\times F(9) &&= (T(1)\times T(7)+T(6)\times T(4))\times T(9)\\
\mathbf{22848} &:= F(3)\times F(8)\times F(9)\times (F(4)+F(7)) &&= (T(3)+(T(8)+T(9))\times T(4))\times T(7)\\
\mathbf{53550} &:= F(5)\times F(9)\times (F(7)+F(3))\times F(8) &&= (T(5)+T(9)\times T(7))\times (T(3)+T(8))
\end{aligned}
$$

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20.4.2 Positive and Negative Coefficients

- **Two-Terms Expressions**

$$
\begin{aligned}
\mathbf{3} &:= F(2)+F(3) &&= -T(2)+T(3)\\
\mathbf{8} &:= -F(7)+F(8) &&= -T(7)+T(8)\\
\mathbf{11} &:= F(4)+F(6) &&= -T(4)+T(6)
\end{aligned}
$$

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- **Three-Terms Expressions**

$$\mathbf{4} := -F(1)+F(3)+F(4) = -T(1)\times T(3)+T(4)$$
$$:= -F(2)-F(6)+F(7) = -T(2)-T(6)+T(7)$$
$$\mathbf{10} := F(1)\times F(3)\times F(5) = T(1)-T(3)+T(5)$$
$$:= -F(1)+F(4)+F(6) = -T(1)-T(4)+T(6)$$
$$:= -F(2)-F(3)+F(7) = -T(2)\times T(3)+T(7)$$
$$\mathbf{21} := F(2)\times F(6)+F(7) = T(2)\times(-T(6)+T(7))$$
$$\mathbf{26} := (F(1)+F(2))\times F(7) = T(1)-T(2)+T(7)$$
$$\mathbf{27} := F(2)-F(6)+F(9) = T(2)-T(6)+T(9)$$
$$\mathbf{39} := -F(2)+F(5)\times F(6) = T(2)+T(5)+T(6)$$
$$\mathbf{41} := -F(1)+F(3)\times F(8) = -T(1)+T(3)+T(8)$$
$$\mathbf{42} := F(3)\times(F(6)+F(7)) = T(3)\times(-T(6)+T(7))$$
$$\mathbf{48} := (F(5)+F(2))\times F(6) = -T(5)+T(2)\times T(6)$$
$$\mathbf{60} := F(3)\times F(9)-F(6) = -T(3)+T(9)+T(6)$$
$$\mathbf{91} := (-F(2)+F(6))\times F(7) = T(2)\times T(6)+T(7)$$
$$\mathbf{105} := (F(2)+F(9))\times F(4) = T(2)\times(T(9)-T(4))$$
$$:= F(5)\times(F(6)+F(7)) = T(5)\times(-T(6)+T(7))$$
$$\mathbf{180} := (F(3)+F(9))\times F(5) = T(3)\times(T(9)-T(5))$$

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- **Four-Terms Expressions**

$$\mathbf{12} := -F(1)-F(4)-F(5)+F(8) = T(1)-T(4)-T(5)+T(8)$$
$$:= F(1)-F(3)+F(5)+F(6) = T(1)\times T(3)-T(5)+T(6)$$
$$:= -(F(2)+F(3))\times F(4)+F(8) = T(2)\times(T(3)+T(4))-T(8)$$
$$:= (F(2)+F(4))\times F(5)-F(6) = T(2)\times(T(4)+T(5)-T(6))$$
$$:= -F(2)-F(6)-F(7)+F(9) = T(2)\times(T(6)+T(7)-T(9))$$
$$:= F(2)-F(3)+F(5)+F(6) = T(2)\times T(3)+T(5)-T(6)$$
$$:= F(2)-F(3)-F(6)+F(8) = T(2)-T(3)-T(6)+T(8)$$
$$:= F(2)-F(3)-F(8)+F(9) = -T(2)+T(3)-T(8)+T(9)$$
$$:= F(3)\times F(2)-F(4)+F(7) = T(3)\times(T(2)\times T(4)-T(7))$$
$$\mathbf{152} := (-F(2)+F(9))\times F(5)-F(7) = T(2)\times(T(9)+T(5))-T(7)$$
$$\mathbf{153} := -F(3)-F(7)+F(6)\times F(8) = T(3)\times T(7)+T(6)-T(8)$$
$$:= F(2)-F(6)\times(F(3)-F(8)) = -T(2)\times T(6)+T(3)\times T(8)$$

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- **Four-Terms Expressions**

$$\begin{aligned}
\mathbf{1680} &:= F(6)\times(-F(4)+F(7))\times F(8) = T(6)\times T(4)\times(-T(7)+T(8))\\
\mathbf{1911} &:= F(7)\times(F(6)-F(2))\times F(8) = (T(7)+T(6))\times(T(2)+T(8))\\
\mathbf{2184} &:= F(2)\times F(7)\times F(6)\times F(8) = T(2)\times(-T(7)+T(6)\times T(8))\\
\mathbf{2205} &:= F(5)\times F(8)\times(-F(7)+F(9)) = (-T(5)+T(8)-T(7))\times T(9)\\
\mathbf{2520} &:= (F(3)+F(7))\times F(6)\times F(8) = T(3)\times T(7)\times(-T(6)+T(8))\\
\mathbf{4368} &:= F(3)\times F(7)\times F(6)\times F(8) = T(3)\times(-T(7)+T(6)\times T(8))\\
\mathbf{6552} &:= F(4)\times F(6)\times F(7)\times F(8) = (T(4)\times T(6)-T(7))\times T(8)\\
\mathbf{10920} &:= F(5)\times F(7)\times F(6)\times F(8) = T(5)\times(-T(7)+T(6)\times T(8))
\end{aligned}$$

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- **Five-Terms Expressions**

$$\begin{aligned}
\mathbf{7} &:= -F(3)-F(1)-F(2)+F(4)+F(6) = -T(3)-T(1)+T(2)-T(4)+T(6)\\
\mathbf{26} &:= F(4)\times F(2)+F(7)+F(3)\times F(5) = T(4)-T(2)+T(7)+T(3)-T(5)\\
\mathbf{35} &:= F(3)\times(-F(4)+F(9)-F(7))-F(2) = -T(3)-T(4)+(T(9)-T(7))\times T(2)\\
\mathbf{95} &:= -F(1)-F(9)+F(5)\times F(3)\times F(7) = T(1)+T(9)+T(5)+T(3)+T(7)\\
\mathbf{180} &:= F(4)\times(F(2)+F(3))\times(F(8)-F(1)) = -T(4)\times T(2)+T(3)\times(T(8)-T(1))\\
\mathbf{181} &:= (F(4)-F(9))\times(F(3)-F(6))-F(5) = T(4)-T(9)+T(3)\times(T(6)+T(5))\\
\mathbf{335} &:= -F(1)\times F(2)+F(8)\times(F(4)+F(7)) = T(1)\times T(2)+T(8)\times T(4)-T(7)\\
\mathbf{340} &:= (-F(5)+F(7)-F(2)+F(4))\times F(9) = -T(5)+(T(7)+T(2))\times T(4)+T(9)
\end{aligned}$$

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- **Five-Terms Expressions**

$$\mathbf{484} := -F(1)+(F(9)+F(4)\times F(8))\times F(5) = -T(1)-T(9)-T(4)+T(8)\times T(5)$$
$$\mathbf{715} := (-F(5)+F(7)\times F(4))\times F(8)+F(1) = T(5)-T(7)\times (T(4)-T(8)+T(1))$$
$$\mathbf{985} := -F(4)+F(2)+(F(9)+F(7))\times F(8) = (T(4)+T(2))\times (T(9)+T(7))+T(8)$$
$$\mathbf{994} := -F(5)+F(7)+F(9)\times (F(6)+F(8)) = -T(5)+T(7)+T(9)\times T(6)+T(8)$$
$$\mathbf{999} := F(5)\times (-F(3)+F(6))\times F(9)-F(8) = (T(5)\times T(3)+T(6))\times (T(9)-T(8))$$
$$\mathbf{1722} := F(3)\times F(8)\times (F(2)+F(6)\times F(5)) = -T(3)+T(8)\times (T(2)\times T(6)-T(5))$$
$$\mathbf{1728} := (F(2)+F(5))\times (F(3)+F(9))\times F(6) = (-T(2)+T(5))\times T(3)\times (T(9)-T(6))$$
$$\mathbf{9999} := F(4)+(F(7)+F(1))\times F(8)\times F(9) = (T(4)\times T(7)-T(1))\times T(8)-T(9)$$

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20.5 Interesting Results: Fibonacci and Triangular

20.5.1 Multiplication With Fibonacci Sequence Values

Here we have **selfie expressions** where only multiplication operation is used with Fibonacci sequence values.

Multiplication With Fibonacci Terms

$$\mathbf{42} := F(3)\times F(8) = T(3)+T(8)$$

$$\mathbf{2} := F(1)\times F(2)\times F(3) = -T(1)-T(2)+T(3)$$
$$\mathbf{10} := F(1)\times F(3)\times F(5) = T(1)-T(3)+T(5)$$
$$\mathbf{15} := F(2)\times F(4)\times F(5) = T(2)\times T(4)-T(5)$$
$$\mathbf{16} := F(1)\times F(3)\times F(6) = T(1)-T(3)+T(6)$$
$$\mathbf{30} := F(3)\times F(4)\times F(5) = T(3)\times (-T(4)+T(5))$$
$$\mathbf{39} := F(1)\times F(4)\times F(7) = T(1)+T(4)+T(7)$$

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Multiplication With Fibonacci Terms

$$\mathbf{15} := F(1) \times F(2) \times F(4) \times F(5) = T(1) \times T(2) \times T(4) - T(5)$$
$$\mathbf{26} := F(1) \times F(2) \times F(3) \times F(7) = T(1) + T(2) - T(3) + T(7)$$
$$\mathbf{30} := F(2) \times F(3) \times F(5) \times F(4) = (T(2) \times T(3) - T(5)) \times T(4)$$
$$\mathbf{40} := F(1) \times F(2) \times F(5) \times F(6) = T(1) + T(2) + T(5) + T(6)$$
$$\mathbf{102} := F(2) \times F(1) \times F(4) \times F(9) = T(2) \times (-T(1) - T(4) + T(9))$$
$$\mathbf{120} := F(2) \times F(4) \times F(6) \times F(5) = (-T(2) - T(4) + T(6)) \times T(5)$$
$$\mathbf{168} := F(2) \times F(1) \times F(6) \times F(8) = T(2) \times (-T(1) + T(6) + T(8))$$
$$\mathbf{315} := F(2) \times F(5) \times F(4) \times F(8) = -T(2) \times T(5) + T(4) \times T(8)$$

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Multiplication With Fibonacci Terms

$$\mathbf{336} := F(3) \times F(1) \times F(6) \times F(8) = T(3) \times (-T(1) + T(6) + T(8))$$
$$\mathbf{504} := F(2) \times F(4) \times F(6) \times F(8) = (T(2) - T(4) + T(6)) \times T(8)$$
$$\mathbf{510} := F(5) \times F(1) \times F(4) \times F(9) = T(5) + (T(1) + T(4)) \times T(9)$$
$$\mathbf{840} := F(5) \times F(1) \times F(6) \times F(8) = T(5) \times (-T(1) + T(6) + T(8))$$
$$\mathbf{2184} := F(2) \times F(7) \times F(6) \times F(8) = T(2) \times (-T(7) + T(6) \times T(8))$$
$$\mathbf{4368} := F(3) \times F(7) \times F(6) \times F(8) = T(3) \times (-T(7) + T(6) \times T(8))$$
$$\mathbf{6552} := F(4) \times F(6) \times F(7) \times F(8) = (T(4) \times T(6) - T(7)) \times T(8)$$
$$\mathbf{10920} := F(5) \times F(7) \times F(6) \times F(8) = T(5) \times (-T(7) + T(6) \times T(8))$$

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20.5.2 Addition With Fibonacci Sequence Values

Addition with Fibonacci Terms

$$\mathbf{3} := F(2)+F(3) \qquad = -T(2)+T(3)$$
$$\mathbf{11} := F(4)+F(6) \qquad = -T(4)+T(6)$$

$$\mathbf{8} := F(1)+F(3)+F(5) = -T(1)-T(3)+T(5)$$
$$\mathbf{12} := F(1)+F(4)+F(6) = T(1)-T(4)+T(6)$$
$$\mathbf{25} := F(1)+F(4)+F(8) = -T(1)-T(4)+T(8)$$
$$\mathbf{38} := F(2)+F(4)+F(9) = T(2)-T(4)+T(9)$$

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Addition with Fibonacci Terms

$$\mathbf{7} := F(1)+F(2)+F(3)+F(4) = T(1)\times T(2)-T(3)+T(4)$$
$$\mathbf{9} := F(1)+F(2)+F(3)+F(5) = (T(1)+T(2))\times T(3)-T(5)$$
$$\mathbf{11} := F(1)+F(3)+F(4)+F(5) = T(1)\times T(3)-T(4)+T(5)$$
$$\mathbf{13} := F(1)+F(2)+F(4)+F(6) = -T(1)+T(2)-T(4)+T(6)$$
$$\mathbf{14} := F(2)+F(3)+F(4)+F(6) = -T(2)+T(3)-T(4)+T(6)$$
$$\mathbf{28} := F(3)+F(5)+F(6)+F(7) = -T(3)-T(5)+T(6)+T(7)$$
$$\mathbf{30} := F(1)+F(4)+F(5)+F(8) = -T(1)+T(4)-T(5)+T(8)$$
$$\mathbf{40} := F(1)+F(3)+F(4)+F(9) = -T(1)+T(3)-T(4)+T(9)$$
$$\mathbf{42} := F(2)+F(3)+F(5)+F(9) = -T(2)\times T(3)+T(5)+T(9)$$
$$\mathbf{43} := F(1)+F(6)+F(7)+F(8) = -T(1)\times T(6)+T(7)+T(8)$$
$$\mathbf{44} := F(3)+F(4)+F(5)+F(9) = -T(3)-T(4)+T(5)+T(9)$$
$$\mathbf{48} := F(2)+F(5)+F(6)+F(9) = -T(2)-T(5)+T(6)+T(9)$$

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https://goo.gl/gcCnbp

Here we have **selfie expressions** where only the operation of addition is used with Fibonacci sequence values.

20.5.3 Multiplication with Fibonacci and Triangular Numbers

We have few examples when all the terms with Fibonacci or triangular numbers are with multiplication sign.

$$\mathbf{1} := -1! + 2! = F(1) \times F(2)$$
$$\mathbf{1560} := (1! - 4!) \times 5! - 6! + 7! = F(1) \times F(4) \times F(5) \times F(6) \times F(7)$$

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https://goo.gl/gcCnbp

Multiplication With Triangular Terms

$$\begin{aligned}
\mathbf{6} &:= 1! \times 3! &&= T(1) \times T(3)\\
\mathbf{18} &:= (1! + 2!) \times 3! &&= T(1) \times T(2) \times T(3)\\
\mathbf{2700} &:= (1! + 4!) \times (5! - 2! \times 3!) &&= T(1) \times T(4) \times T(5) \times T(2) \times T(3)\\
\mathbf{25200} &:= (1! + 3!) \times 4! \times 5! + 7! &&= T(1) \times T(3) \times T(4) \times T(5) \times T(7)\\
\mathbf{181440} &:= (3! - 2! + 4!) \times 7! + 8! &&= T(3) \times T(2) \times T(4) \times T(7) \times T(8)\\
\mathbf{453600} &:= (2! + 5! - 4!) \times 7! - 8! &&= T(2) \times T(5) \times T(4) \times T(7) \times T(8)\\
\mathbf{816480} &:= 3! \times (2! \times 8! - 7!) + 9! &&= T(3) \times T(2) \times T(8) \times T(7) \times T(9)\\
\mathbf{1360800} &:= (4! - 2!) \times (7! + 8!) + 9! &&= T(4) \times T(2) \times T(7) \times T(8) \times T(9)\\
\mathbf{4082400} &:= (3! + 5!) \times (8! - 7!) - 9! &&= T(3) \times T(5) \times T(8) \times T(7) \times T(9)
\end{aligned}$$

https://goo.gl/LzsxWH
https://goo.gl/gcCnbp

20.5.4 Power-Factorial-Triangular Numbers

There are only three values, where **factorial**, **power** and **triangular numbers** are equal with same digit's order.

$$\begin{aligned}
\mathbf{1} &:= 1! = 1^1 &&= T(1)\\
\mathbf{3} &:= 1! + 2! = -1^1 + 2^2 &&= T(1) \times T(2)\\
\mathbf{2760} &:= (-1! + 5! + 2! - 3!) \times 4! = -1^1 + 5^5 - 2^2 \times 3^3 - 4^4 &&= (T(1) + T(5) \times T(2)) \times T(3) \times T(4)
\end{aligned}$$

https://goo.gl/LzsxWH
https://goo.gl/gcCnbp

In some cases, there are multiple ways of writing same number. Due to space, we have written only one way. For more details refer author's work [75, 76].

21 Embedded Palprime Patterns

Palindromic prime numbers are well known in the literature. In some particular situations, they are prime number too. For example, 11, 131, 191, 10301, etc. Some times they are called as, **palprime numbers**, instead, **palindromic prime numbers**. In this section, our aim is work with palprime numbers in such a way that previous one is in the next one, forming as pyramid. More more details refer author's complete work [74, 77]. See below some examples,

► 101
31013
3310133
933101339
1593310133951
13159331013395131
171315933101339513171
1617131593310133951317161
96161713159331013395131716169
36396161713159331013395131716169363

https://goo.gl/4mX1j3

This type of pyramidical embedded palprime patterns are well known in literature. Recently, author [74, 77] worked with embedded palprimes for 3 and 5 digits giving few examples in each cases. See below:

► 101
31013
3310133
933101339
1093310133901
1111093310133901111.

► 101
31013
3310133
933101339
1093310133901
1611093310133901161.

► 101
31013
3310133
933101339
1093310133901
1701093310133901071.

► 353
33533
1335331
17133533171
7171335331717
1027171335331717201.

► 353
33533
1335331
17133533171
7171335331717
1137171335331717311.

► 353
33533
1335331
17133533171
7171335331717
1147171335331717411.

https://goo.gl/4mX1j3

► 10501	► 10501	► 10501
111050111	301050103	741050147
1911105011191	3830105010383	7074105014707
17191110501119171	938301050103839	970741050147079
1171911105011191711.	19383010501038391.	99707410501470799.
► 12721	► 12721	► 12721
9127219	9127219	9127219
35912721953	35912721953	35912721953
3359127219533	3359127219533	3359127219533
133591272195331.	18433591272195334 81.	31335912721953313.

https://goo.gl/4mX1j3

More details can be seen at [74, 77]. Examples of embedded palprimes starting with 7 digits are understudy. Above examples are in the general case, still we can write embedded trees with specific digits with some extra properties.

21.1 Special Type of Embedded Palprimes

21.1.1 Fixed Digits

Below are examples of some fixed digits embedded palprimes:

9271729	9271729
77927172977	92927172929
917792717297719	1929271729291
7791779271729771977.	1191929271729291911.
771020177	71021012017
120771020177021	117102101201711
1212077102017702121	7117102101201711 7
12121207710201770212121.	1271171021012017117 21.

https://goo.gl/4mX1j3

71317	71317
71171317117	71171317117
17171171317117171	33711713171173 3
3171711713171171713	7733711713171173377
1113171711713171171713111	777337117131711733777
71917	71917
77771917777	77771917777
17177771917777171	9117777191777711 9
7717177771917777171 77	99911777719177771 1999
1771717777191777717 1771	19999117777191777711 99991

https://goo.gl/4mX1j3

For example, the first row is just with digits 1, 7, 2 and 9. Second row with 2, 0, 1 and 7. Third row digits 1, 3 and 7. Finally, the last row with digits 1, 7 and 9.

21.1.2 Complimentary Embedded Palprimes

Below are examples of complimentary embedded palprimes. By complimentary embedded we understand that we can inter change digits in pair.

16661	19991
1191166611911	1161199911611
1111111191166611911111111	1111111161199911611111111

https://goo.gl/4mX1j3

131	191
71317	71917
77771317777	77771917777
11111777771317777711111	11111777771917777711111
1111111111111777771317777711111111111111	1111111111111777771917777711111111111111

https://goo.gl/4mX1j3

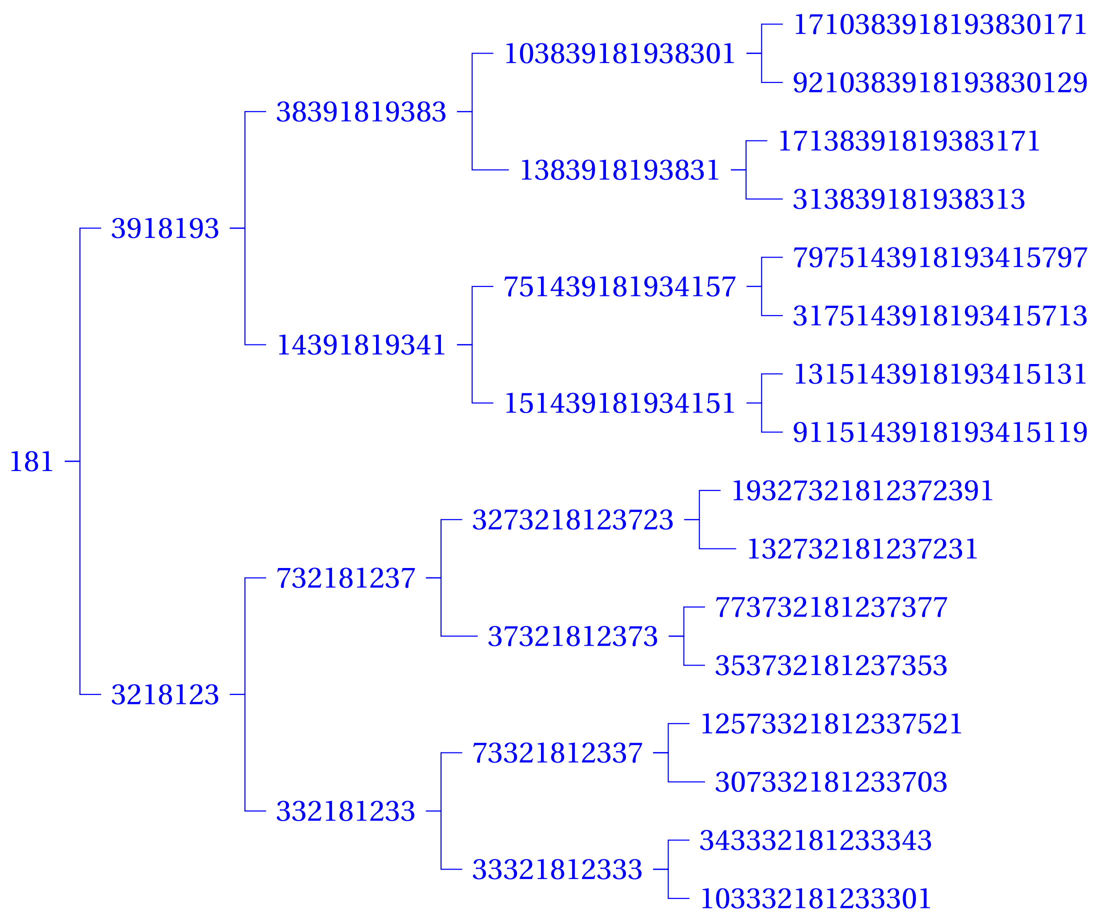
131	191
71317	71917
77771317777	77771917777
1177777777131777777711	1177777777191777777711
1111111111111177777777131777777771111111111111 1	11111111111111777777771917777777711111111111111

https://goo.gl/4mX1j3

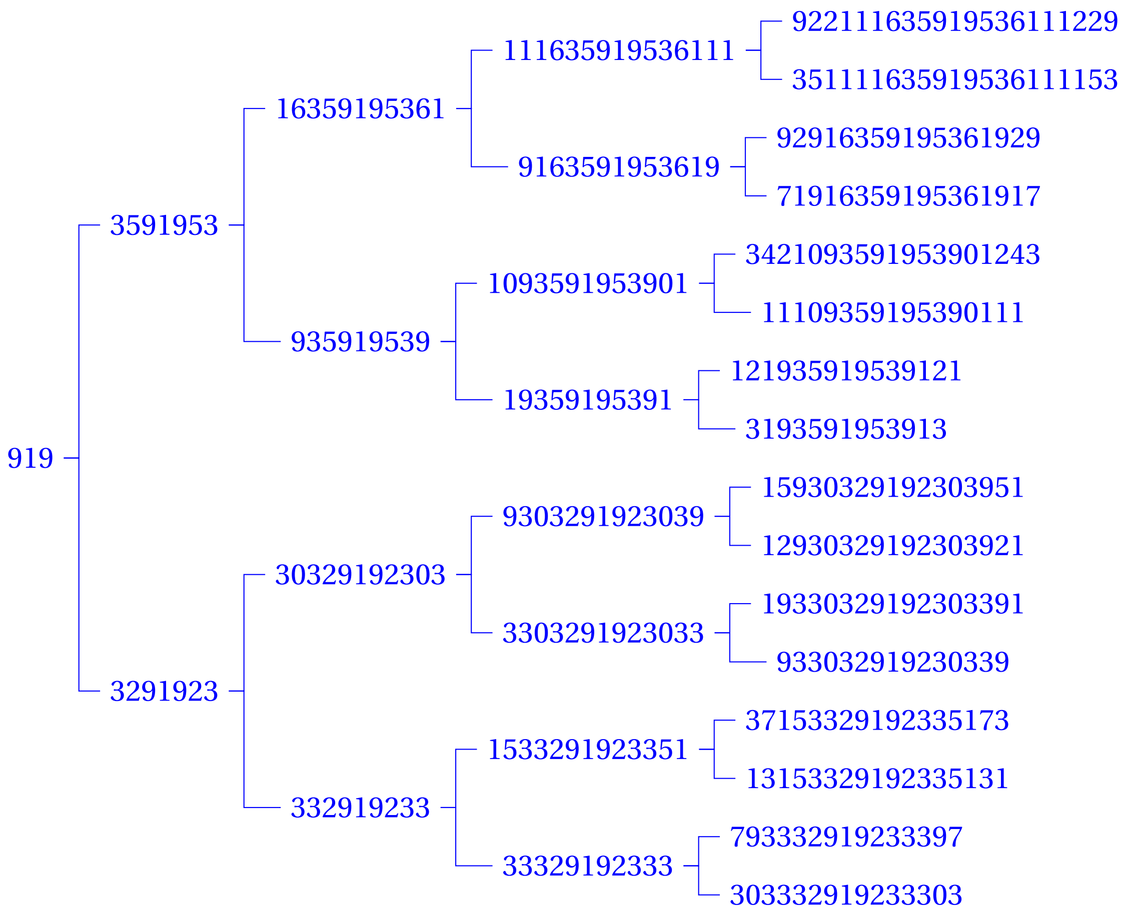
In the first example 6 and 9 are changable. In the second and third examples, 3 and 9 are changables. Due this we call these types of embedded palprimes as **complimentary or paired embedded palprimes**. Detailed study of this kind of results shall be dealt elsewhere.

21.1.3 Embedded Palprimes Trees

Above we have 3 different examples in each case, i.e., for the palprimes, 101, 353, 10501, 12721, etc., but there are much more cases. Recently, author considered 15 palprimes of 3-digits and wrote embedded tree two-by-two up to fifth order. See below some examples:



https://goo.gl/uDE4vx



https://goo.gl/uDE4vx

More more details refer author's complete work [74, 77].

22 Fixed Digits Repetitions Prime Patterns

First let us understand what we mean by **"patterns in prime numbers"**. It is understood as repetition of same digits in **limited way** resulting numbers are continues as prime numbers. See some examples,

- 71
  701
  7001
  70001
  700001.

- 11
  1 1614 1
  1 1614 1614 1
  1 1614 1614 1614 1
  1 1614 1614 1614 1614 1.

Let us analyse the above two prime patterns. In the first case digit 0 repeats 4 times. In the second case the digit 1614 repeats also 4 times. In both cases, the next number in the string is not a prime number. See below:

$$700001 = 197 \times 35533;$$
$$116141614161416141416141 = 82549 \times 14069415033666809;$$

Based on this we can say that both the patterns are **"patterns of prime numbers"** of length 5. For more study of **"patterns in prime number"** see author's work [50, 64, 65, 67, 68]. Below are examples of fixed length multiple choice fixed digits prime patterns.

22.1 Multiple Choice Prime Patterns: Length 10

Below is are examples of prime patterns of lenght 10 written in two different forms.

► **56 29933**
56 219 **29933**
56 219 219 **29933**
56 219 219 219 **29933**
56 219 219 219 219 **29933**
56 219 219 219 219 219 **29933**
56 219 219 219 219 219 219 **29933**
56 219 219 219 219 219 219 219 **29933**
56 219 219 219 219 219 219 219 219 **29933**
56 219 219 219 219 219 219 219 219 219 **29933**.

► **562 9933**
562 192 **9933**
562 192 192 **9933**
562 192 192 192 **9933**
562 192 192 192 192 **9933**
562 192 192 192 192 192 **9933**
562 192 192 192 192 192 192 **9933**
562 192 192 192 192 192 192 192 **9933**
562 192 192 192 192 192 192 192 192 **9933**
562 192 192 192 192 192 192 192 192 192 **9933**.

https://goo.gl/PquvOe
https://goo.gl/rPyzjr

Actually, these are two prime patterns are same, but the difference is in distributions. We have few of this type of examples. Below are more two examples of single choice:

► **7 54993**
7 678 **54993**
7 678 678 **54993**
7 678 678 678 **54993**
7 678 678 678 678 **54993**
7 678 678 678 678 678 **54993**
7 678 678 678 678 678 678 **54993**
7 678 678 678 678 678 678 678 **54993**
7 678 678 678 678 678 678 678 678 **54993**
7 678 678 678 678 678 678 678 678 678 **54993**.

► **1 407727**
1 105 **407727**
1 105 105 **407727**
1 105 105 105 **407727**
1 105 105 105 105 **407727**
1 105 105 105 105 105 **407727**
1 105 105 105 105 105 105 **407727**
1 105 105 105 105 105 105 105 **407727**
1 105 105 105 105 105 105 105 105 **407727**
1 105 105 105 105 105 105 105 105 105 **407727**.

https://goo.gl/PquvOe
https://goo.gl/rPyzjr

Above we have given pattern of length 10. Still, we don't have this kind of pattern of length 11 or higher [116].

22.2 Multiple Choice Prime Patterns: Length 9

In this case we have only two possibilities with triple choice patterns with prime numbers of length 9. Triple choice means that there are the patterns of prime numbers where each one starts with same prime number.

► **50 487611**
50 489 **487611**
50 489 489 **487611**
50 489 489 489 **487611**
50 489 489 489 489 **487611**
50 489 489 489 489 489 **487611**
50 489 489 489 489 489 489 **487611**
50 489 489 489 489 489 489 489 **487611**
50 489 489 489 489 489 489 489 489 **487611**.

► **504 87611**
504 894 **87611**
504 894 894 **87611**
504 894 894 894 **87611**
504 894 894 894 894 **87611**
504 894 894 894 894 894 **87611**
504 894 894 894 894 894 894 **87611**
504 894 894 894 894 894 894 894 **87611**
504 894 894 894 894 894 894 894 894 **87611**.

► **5048 7611**
5048 948 **7611**
5048 948 948 **7611**
5048 948 948 948 **7611**
5048 948 948 948 948 **7611**
5048 948 948 948 948 948 **7611**
5048 948 948 948 948 948 948 **7611**
5048 948 948 948 948 948 948 948 **7611**
5048 948 948 948 948 948 948 948 948 **7611**

https://goo.gl/PquvOe
https://goo.gl/rPyzjr

Here also we have very few examples of this kind.

22.3 Multiple Choice Prime Patterns: Length 8

This subsection brings multiple patterns of 8 digits with triple choice. See below example:

► **3 80377**
3 8130 **80377**
3 8130 8130 **80377**
3 8130 8130 8130 **80377**
3 8130 8130 8130 8130 **80377**
3 8130 8130 8130 8130 8130 **80377**
3 8130 8130 8130 8130 8130 8130 **80377**
3 8130 8130 8130 8130 8130 8130 8130 **80377**.

► **38 0377**
38 1308 **0377**
38 1308 1308 **0377**
38 1308 1308 1308 **0377**
38 1308 1308 1308 1308 **0377**
38 1308 1308 1308 1308 1308 **0377**
38 1308 1308 1308 1308 1308 1308 **0377**
38 1308 1308 1308 1308 1308 1308 1308 **0377**.

► **38 0377**
38 992607 **0377**
38 992607 992607 **0377**
38 992607 992607 992607 **0377**
38 992607 992607 992607 992607 **0377**
38 992607 992607 992607 992607 992607 **0377**
38 992607 992607 992607 992607 992607 992607 **0377**
38 992607 992607 992607 992607 992607 992607 992607 **0377**.

https://goo.gl/PquvOe
https://goo.gl/rPyzjr

22.4 Multiple Choice Prime Patterns: Length 7

In this case we have maximum 5 patterns staring with same prime number of length 7. This we calculated using repetition of numbers from 1 to 6 digits. See below a example:

► **155663**
1551 **155663**
1551 1551 **155663**
1551 1551 1551 **155663**
1551 1551 1551 1551 **155663**
1551 1551 1551 1551 1551 **155663**
1551 1551 1551 1551 1551 1551 **155663**.

► **155663**
15 5115 **5663**
15 5115 5115 **5663**
15 5115 5115 5115 **5663**
15 5115 5115 5115 5115 **5663**
15 5115 5115 5115 5115 5115 **5663**
15 5115 5115 5115 5115 5115 5115 **5663**.

► **155663**
1 5511 **55663**
1 5511 5511 **55663**
1 5511 5511 5511 **55663**
1 5511 5511 5511 5511 **55663**
1 5511 5511 5511 5511 5511 **55663**
1 5511 5511 5511 5511 5511 5511 **55663**.

► **155663**
155 1155 **663**
155 1155 1155 **663**
155 1155 1155 1155 **663**
155 1155 1155 1155 1155 **663**
155 1155 1155 1155 1155 1155 **663**
155 1155 1155 1155 1155 1155 1155 **663**.

► **155663**
20127 **155663**
20127 20127 **155663**
20127 20127 20127 **155663**
20127 20127 20127 20127 **155663**
20127 20127 20127 20127 20127 **155663**
20127 20127 20127 20127 20127 20127 **155663**.

https://goo.gl/PquvOe
https://goo.gl/rPyzjr

22.5 Multiple Choice Prime Patterns: Length 6

In this case we have maximum 14 patterns staring with same prime number of length 6. This we calculated using repetition of numbers from 1 to 6 digits. See below a example:

► **5009**
112743 **5009**
112743 112743 **5009**
112743 112743 112743 **5009**
112743 112743 112743 112743 **5009**
112743 112743 112743 112743 112743 **5009**.

► **5009**
5 899811 **009**
5 899811 899811 **009**
5 899811 899811 899811 **009**
5 899811 899811 899811 899811 **009**
5 899811 899811 899811 899811 899811 **009**.

► **5009**
500 37350 **9**
500 37350 37350 **9**
500 37350 37350 37350 **9**
500 37350 37350 37350 37350 **9**
500 37350 37350 37350 37350 37350 **9**.

► **5009**
597807 **5009**
597807 597807 **5009**
597807 597807 597807 **5009**
597807 597807 597807 597807 **5009**
597807 597807 597807 597807 597807 **5009**.

► **5009**
5 978075 **009**
5 978075 978075 **009**
5 978075 978075 978075 **009**
5 978075 978075 978075 978075 **009**
5 978075 978075 978075 978075 978075 **009**.

► **5009**
500 901746 **9**
500 901746 901746 **9**
500 901746 901746 901746 **9**
500 901746 901746 901746 901746 **9**
500 901746 901746 901746 901746 901746 **9**.

► **5009**
5 39 **009**
5 39 39 **009**
5 39 39 39 **009**
5 39 39 39 39 **009**
5 39 39 39 39 39 **009**.

► **5009**
50 786942 **09**
50 786942 786942 **09**
50 786942 786942 786942 **09**
50 786942 786942 786942 786942 **09**
50 786942 786942 786942 786942 786942 **09**.

► **5009**
500 948129 **9**
500 948129 948129 **9**
500 948129 948129 948129 **9**
500 948129 948129 948129 948129 **9**
500 948129 948129 948129 948129 948129 **9**.

► **5009**
5 833742 **009**
5 833742 833742 **009**
5 833742 833742 833742 **009**
5 833742 833742 833742 833742 **009**
5 833742 833742 833742 833742 833742 **009**.

► **5009**
500 30966 **9**
500 30966 30966 **9**
500 30966 30966 30966 **9**
500 30966 30966 30966 30966 **9**
500 30966 30966 30966 30966 30966 **9**.

► **5009**
5009 74907
5009 74907 74907
5009 74907 74907 74907
5009 74907 74907 74907 74907
5009 74907 74907 74907 74907 74907.

► **5009**
5009 481299
5009 481299 481299
5009 481299 481299 481299
5009 481299 481299 481299 481299
5009 481299 481299 481299 481299 481299.

► **5009**
5009 728277
5009 728277 728277
5009 728277 728277 728277
5009 728277 728277 728277 728277
5009 728277 728277 728277 728277 728277.

https://goo.gl/PquvOe
https://goo.gl/oW9EB6
https://goo.gl/WbgsJE

22.6 More Examples

Below are some examples of prime patterns.

- **Prime Patterns With 71117 to 71917**

► 11 **71117**

11 555 **71117**

11 555 555 **71117**

11 555 555 555 **71117**

11 555 555 555 555 **71117**

11 555 555 555 555 555 **71117**

► **1 71217 3**

759 1 **71217 3**

759 759 1 **71217 3**

759 759 759 1 **71217 3**

759 759 759 759 1 **71217 3**

759 759 759 759 759 1 **71217 3**

► **15 71417**

15 51 **71417**

15 51 51 **71417**

15 51 51 51 **71417**

15 51 51 51 51 **71417**

15 51 51 51 51 51 **71417**

► **71317**

7131 606912 **7**

7131 606912 606912 **7**

7131 606912 606912 606912 **7**

7131 606912 606912 606912 606912 **7**

7131 606912 606912 606912 606912 606912 **7**

► **4 71517 7**

4 71517 909 **7**

4 71517 909 909 **7**

4 71517 909 909 909 **7**

4 71517 909 909 909 909 **7**

4 71517 909 909 909 909 909 **7**

► **71617 73**

71617 207 **73**

71617 207 207 **73**

71617 207 207 207 **73**

71617 207 207 207 207 **73**

71617 207 207 207 207 207 **73**

► **5 6 71717**

5 51 **6 71717**

5 51 51 **6 71717**

5 51 51 51 **6 71717**

5 51 51 51 51 **6 71717**

5 51 51 51 51 51 **6 71717**

► **3471817**

168 **34 71817**

168 168 **34 71817**

168 168 168 **34 71817**

168 168 168 168 **34 71817**

168 168 168 168 168 **34 71817**

► **60 71917**

60 555 **71917**

60 555 555 **71917**

60 555 555 555 **71917**

60 555 555 555 555 **71917**

60 555 555 555 555 555 **71917**

https://goo.gl/PquvOe
https://goo.gl/oW9EB6
https://goo.gl/WbgsJE

For more details refer author's complete work [50, 64, 65, 67, 68].

23 Magic Square Type Palprimes

We studied "palprime distributions of orders 5 × 5, 7 × 7 and 9 × 9 in such a way that they have magic-square-type properties, i.e., rows, columns, principal diagonals are all palprimes along with extended rows also palprimes. Some particular cases, such as symmetric in rows and columns, embedded, extended columns, etc. are also studied. For example, see below examples, of embedded and extended rows and columns palprimes. For complete details refer author's work [51, 52, 53]. For the previous works refer [118, 117, 122].

23.1 Palprimes of Order 5 × 5

1. 7 1 9 1 7
   7 4 0 4 7
   9 3 7 3 9
   7 4 0 4 7
   7 1 9 1 7

2. 9 3 7 3 9
   7 7 9 7 7
   3 2 3 2 3
   7 7 9 7 7
   9 3 7 3 9

Extended rows and columns palprimes:

1. 7191774047937397404771917
   7797714341907091434177977

2. 9373977977323237797793739
   9737937273793973727397379

https://goo.gl/Vv1v3G

Above two examples are palprime distributions of order 5 × 5 in such a way that rows and columns both are palprimes. For details refer [59].

23.2 Palprimes of Order 7 × 7

Below some examples of embedded or pyramid type property of palprime distributions of order 7 × 7:

1117111
3586853
1883881
7930397
1883881
3586853
1117111

1117111
7693967
7452547
7014107
7452547
7693967
1117111

https://goo.gl/Vv1v3G

- **Embedded property**

7930397

188388179303971883881

3586853188388179303971883881358685 3

111711135868531883881793039718838813586853111 7111

7014107

745254770141077452547

769396774525477014107745254776939 67

1117111769396774525477014107745254776939671117111

https://goo.gl/Vv1v3G

Above two examples are of embedded or pyramid type property of palprime distributions. These are also **palprimes**. There are very few examples of this type of distributions.

Below are three magic square type palprimes, i.e, each row columns and principal diagonals are palindromic primes with embedded property. Each member in embedded pattern is also a palprime. Also there is symmetry in respective rows and columns.

9173719	9711179	3337333
1909091	7136317	3245423
7079707	1374731	3487843
3998993	1643461	7576757
7079707	1374731	3487843
1909091	7136317	3245423
9173719	9711179	3337333

https://goo.gl/Vv1v3G

- **Embedded property**

13998993
707970739989937079707
19090917079707399899370797071909091
9173719190909170797073998993707970719090919173719
1643461
137473116434611374731
71363171374731164346113747317136317
9711179713631713747311643461137473171363179711179
7576757
348784375767573487843
32454233487843757675734878433245423
3337333324542334878437576757348784332454233337333

https://goo.gl/Vv1v3G

There are only 3 out of 621 palprimes of order 7 × 7. They are symmetric with respective rows and columns. Below are 3-digits palprimes of order 7 × 7, where each row, column, principal diagonals, and extended row are palprimes:

1. 1117111	2. 1333331	3. 7177717	4. 9919199
1117111	3223223	1777771	9199919
1114111	3223223	7722277	1941491
7747477	3321233	7722277	9919199
1114111	3223223	7722277	1941491
1117111	3223223	1777771	9199919
1117111	1333331	7177717	9919199

Extended Row Palprime Property:

1. 1117111111711111141111774747711141111171111117111
2. 1333331322322332232233321233322322332232231333331
3. 7177717177777177222777722277772227717777717177717
4. 9919199919991919414919919199194149191999199919199

https://goo.gl/Vv1v3G

These are only 4 possible 3-digits 7 × 7

Below are magic square type palprimes of order 7×7, embedded in rows, and palprime in rows, columns, principal diagonals, and extended rows and columns:

1.	2.	3.	4.	5.
1333331	7977797	9173719	9711179	3337333
9817189	9907099	1909091	7136317	3245423
9196919	3218123	7079707	1374731	3487843
3285823	3743473	3998993	1643461	7576757
9196919	3218123	7079707	1374731	3487843
9817189	9907099	1909091	7136317	3245423
1333331	7977797	9173719	9711179	3337333

https://goo.gl/Vv1v3G

Below are extended rows, columns and embedded properties:

1.
3285823
919691932858239196919
98171899196919328582391969199817189
1333331981718991969193285823919691998171891333331
1993991381218331989133765673319891338121831993991

2.
3743473
321812337434733218123
99070993218123374347332181239907099
7977797990709932181233743473321812399070997977797
7933397992729970141077783877701410799272997933397

3.
3998993
707970739989937079707
19090917079707399899370797071909091
9173719190909170797073998993707970719090919173719
9173719190909170797073998993707970719090919173719

https://goo.gl/Vv1v3G

4.

1643461
1374731164346113747 31
7136317137473116434611374731 7136317
97111797136317137473116434611374731713631797111 79
97111797136317137473116434611374731713631797111 79

5.

7576757
3487843757675734878 43
3245423348784375767573487843 3245423
33373333245423348784375767573487843324542333373 33
33373333245423348784375767573487843324542333373 33

https://goo.gl/Vv1v3G

23.3 Palprimes of Order 9 × 9

In [61], author worked with palprime distributions of order 9 × 9 in some particular cases. Since there are many examples of order 9 × 9, we restricted our study only to symmetric situations. Let us understand below the difference between general and symmetric distributions. Below are three magic square type palprimes, i.e, each row columns and principal diagonals are palindromic primes with embedded property. Each member in embedded pattern is also a palprime. Also there is symmetry in respective rows and columns.

1.	2.	3.
193191391	373171373	991737199
901606109	761969167	911868119
318181813	319909913	118686811
161535161	199515991	786848687
908363809	760111067	368414863
161535161	199515991	786848687
318181813	319909913	118686811
901606109	761969167	911868119
193191391	373171373	991737199

https://goo.gl/62syas
https://goo.gl/9tsBH0

Below are embedding property in each case.

1.

908363809

161535161908363809161535161

318181813161535161908363809161535161318181813

901606109318181813161535161908363809161535161318181813901606109

193191391901606109318181813161535161908363809161535161318181813901606109193191391

2.

760111067

199515991760111067199515991

319909913199515991760111067199515991319909913

761969167319909913199515991760111067199515991319909913761969167

373171373761969167319909913199515991760111067199515991319909913761969167373171373

3.

368414863

786848687368414863786848687

118686811786848687368414863786848687118686811

911868119118686811786848687368414863786848687118686811911868119

991737199911868119118686811786848687368414863786848687118686811911868119991737199

https://goo.gl/62syas
https://goo.gl/9tsBH0

There are only 3 out of 643338 palprimes of order 9×9. They are symmetric with respective rows and columns. For more details refer [60, 61].

Below are examples of 3-digits palprimes of order 7×7, where each row, column, principal diagonals, and extended row are palprimes:

1. 131111131	3. 331333133	5. 199999991	7. 331111133
322323223	314313413	989919989	333010333
121111121	141343141	998898899	131030131
131111131	333434333	998898899	100111001
121131121	314313413	919989919	113111311
131111131	333434333	998898899	100111001
121111121	141343141	998898899	131030131
322323223	314313413	989919989	333010333
131111131	331333133	199999991	331111133

2. 331333133	4. 111191111	6. 131333131	8. 991999199
313111313	188888881	331333133	900919009
131111131	188898881	113030311	100111001
311444113	188888881	330101033	991999199
311444113	989898989	333010333	911919119
311444113	188888881	330101033	991999199
131111131	188898881	113030311	100111001
313111313	188888881	331333133	900919009
331333133	111191111	131333131	991999199

https://goo.gl/62syas
https://goo.gl/9tsBH0

There are only 8 symmetric palprimes of order 9×9 with 3-digits. For more details refer author's complete work [51, 52, 53].

24 Palindromic-Type Numbers

A palindromic number is the number that remains the same when its digits are reversed, for example, 121, 3333, 161161, etc. The *palindromic-type* numbers are not palindromes but appears like palindromes. They are separated by operations of additions and/or multiplications, for example, 1343×3421, 1225×5221, $16+61$, $1825+5281$, etc. When we remove the sign of operation, they becomes palindromes. These types of numbers we call as *palindromic-type* numbers. Some studies in this direction can be seen in [**?**]. Let us separate these numbers in two categories:

- **Type 1:**

$$234 \times 111 + 111 \times 432 = 25974 + 47952 := 73926$$
$$10011 \times 11 + 11 \times 11001 = 110121 + 121011 := 231132$$

- **Type 2:**

$$1596 \times 6951 = 2793 \times 3972 := 11093796$$
$$616248 \times 842616 = 645408 \times 804546 := 519260424768.$$

https://goo.gl/PPqf2D

The difference is that the first type is with *addition* and *multiplication*, while second type is just with *multiplication*. This work is concentrated only on **Type 1** kind of numbers, i.e., *palindromic-type* numbers with *addition* and *multiplication*. Below are examples of *palindromic-type* numbers, where each digit appears once and multiplicative factors are of same width.

$$12 \times 21 + 21 \times 12 = 252 + 252 := 504$$
$$12 \times 12 + 21 \times 21 = 144 + 441 := 585$$

$$102 \times 210 + 201 \times 012 = 21420 + 02412 := 23832$$
$$201 \times 102 + 201 \times 102 = 20502 + 20502 := 41004$$
$$102 \times 012 + 210 \times 201 = 01224 + 42210 := 43434$$
$$102 \times 102 + 201 \times 201 = 10404 + 40401 := 50805$$

$$2031 \times 1032 + 2301 \times 1302 = 2095992 + 2995902 := 5091894$$
$$2103 \times 1203 + 3021 \times 3012 = 2529909 + 9099252 := 11629161$$
$$2013 \times 1023 + 3201 \times 3102 = 2059299 + 9929502 := 11988801$$

https://goo.gl/PPqf2D

Below are numbers, where squaring each multiplicative factor in each palindromic-type number lead us to final sum as a palindrome:

$$
\begin{aligned}
12^2+21^2 &= 144+441 &&:= 585\\
111^2+111^2 &= 12321+12321 &&:= 24642\\
1022^2+2201^2 &= 1044484+4844401 &&:= 5888885\\
10031^2+13001^2 &= 100620961+169026001 &&:= 269646962\\
100301^2+103001^2 &= 10060290601+10609206001 &&:= 20669496602.
\end{aligned}
$$

https://goo.gl/PPqf2D

For complete details refer to author's work [42]:

***https://goo.gl/PPqf2D*.**

24.1 Patterns in Palindromic-Type Numbers

Below are some examples of patterns in palindromic-type numbers:

$$
\begin{aligned}
101\times 44+44\times 101 &= 4444 &&+4444 &&:= 8888\\
202\times 22+22\times 202 &= 4444 &&+4444 &&:= 8888\\
404\times 11+11\times 404 &= 4444 &&+4444 &&:= 8888\\
\\
1001\times 444+444\times 1001 &= 444444 &&+444444 &&:= 888888\\
2002\times 222+222\times 2002 &= 444444 &&+444444 &&:= 888888\\
4004\times 111+111\times 4004 &= 444444 &&+444444 &&:= 888888\\
\\
10001\times 4444+4444\times 10001 &= 44444444 &&+44444444 &&:= 88888888\\
20002\times 2222+2222\times 20002 &= 44444444 &&+44444444 &&:= 88888888\\
40004\times 1111+1111\times 40004 &= 44444444 &&+44444444 &&:= 88888888.
\end{aligned}
$$

https://goo.gl/PPqf2D

$$
\begin{aligned}
1001\times 44+44\times 1001 &= 44044 + 44044 := 88088\\
2002\times 22+22\times 2002 &= 44044 + 44044 := 88088\\
4004\times 11+11\times 4004 &= 44044 + 44044 := 88088
\end{aligned}
$$

$$
\begin{aligned}
10001\times 404+404\times 10001 &= 4040404 + 4040404 := 8080808\\
20002\times 202+202\times 20002 &= 4040404 + 4040404 := 8080808\\
40004\times 101+101\times 40004 &= 4040404 + 4040404 := 8080808
\end{aligned}
$$

$$
\begin{aligned}
10001\times 4004+4004\times 10001 &= 40044004 + 40044004 := 80088008\\
20002\times 2002+2002\times 20002 &= 40044004 + 40044004 := 80088008\\
40004\times 1001+1001\times 40004 &= 40044004 + 40044004 := 80088008
\end{aligned}
$$

https://goo.gl/PPqf2D

- **Patterns with Power**

$$
\begin{aligned}
11^2+11^2 &= 121+121\\
110^2+011^2 &= 12100+00121\\
1100^2+0011^2 &= 1210000+0000121
\end{aligned}
$$

$$
\begin{aligned}
12^2+21^2 &= 144+441 := 585\\
102^2+201^2 &= 10404+40401 := 50805\\
1002^2+2001^2 &= 1004004+4004001 := 5008005
\end{aligned}
$$

$$
\begin{aligned}
13^2+31^2 &= 169+961 := 1130^{\star}\\
103^2+301^2 &= 10609+90601 := 101210\\
1003^2+3001^2 &= 1006009+9006001 := 10012010\\
10003^2+30001^2 &= 100060009+900060001 := 1000120010.
\end{aligned}
$$

The first pattern is not patterned with sum values. The last pattern is patterned with sum values, except first line ($\star$).

https://goo.gl/PPqf2D

For complete details refer to author's work [42].

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