Different Aspects of Magics Square of Order 20

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Abstract

This paper brings some different aspects of magic square of order 20. It includes constructions using block-wise system, palindromic-type, universal way, generated by pythagorean triple concatenation of prime mumbers, etc. The idea of superimposed double color pattern is also included.

Contents

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1 Introduction

This paper brings different aspects of magic square of order 20. It includes blocks constructions, colored pattern, palindromic, universal type are also included in the construction. Since we know that 20 can be written as 4×5, $5\times$ and 2×10. This means, we can construct, magic square of order 20 as blocks of magic squares of order 4, 5 and 10. Below are these three magic squares.

1.1 Magic Square of Order 4

Example 1. *Let's consider a **pan magic square** of order 4.*

		34	34	34	34
	7	12	1	14	34
34	2	13	8	11	34
34	16	3	10	5	34
34	9	6	15	4	34
	34	34	34	34	34

This magic square shall be used to bring 25 blocks of order 4 in magic square of order 20.

It is one of the most **perfect pan magic square of order 4**. Below are some properties in colors resulting magic square sum for each color:

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

It is also known by **compact magic square**. More studies can be seen at Taneja [6].

• History

The magic square given in Example 5 is one of the most **perfect magic square** of order 4 studied around 10th century. It is famous as **Khajuraho magic square**. Below is original plate of above magic square seen at **Parshvanath Jain temple in Khajuraho** - (*Link: Wikipedia - https://goo.gl/nsYn2j*):

Most-perfect magic square from
the Parshvanath Jain temple in Khajuraho

1.2 Magic Square of Order 5

Example 2. *Let's consider a **pan magic square** of order 5 is given by*

		65	65	65	65	65
	1	9	12	20	23	65
65	17	25	3	6	14	65
65	8	11	19	22	5	65
65	24	2	10	13	16	65
65	15	18	21	4	7	65
	65	65	65	65	65	65

This magic square shall be used to bring 16 blocks of order 5 in magic square of order 20.

1.3 Magic Square of Order 10

Example 3. *Let's consider a magic square of order 10 is given by*

										505
1	80	65	97	39	22	48	86	53	14	505
98	12	9	66	90	74	55	33	41	27	505
47	81	23	79	16	35	94	60	62	8	505
70	57	88	34	2	91	29	15	76	43	505
84	99	52	11	45	68	73	7	30	36	505
13	38	44	10	77	56	82	21	95	69	505
75	46	40	83	28	19	67	92	4	51	505
59	24	96	42	61	3	20	78	37	85	505
26	5	17	58	93	50	31	64	89	72	505
32	63	71	25	54	87	6	49	18	100	505
505	505	505	505	505	505	505	505	505	505	505

This magic square shall be used to bring 4 blocks of order 10 in magic square of order 20.

2 Magic Squares of Order 20

In this section, we shall present three different ways of writing **block-wise** magic square of order 20. One as 4×5, i.e., magic square formed by 25 blocks of equal magic sums of **pandiagonal** magic squares of order 4. The second as 5×4, i.e., magic square formed by 16 blocks of **pandiagonal** magic squares of order 5 with different magic sums. The third as 4 blocks of equal magic sums of magic squares of order 10. In the first two cases, the magic squares are **pandiagonals**, while in the third case is just a magic square of order 20.

2.1 First Approach: 25 Blocks of Order 4

Example 4. *25 blocks of magic squares of order 4 constructed according to Example 1 lead us to a* ***pandiagonal*** *magic square of order 20 is given by*

		4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
	151	300	1	350	152	299	2	349	153	298	3	348	154	297	4	347	155	296	5	346	4010
4010	50	301	200	251	49	302	199	252	48	303	198	253	47	304	197	254	46	305	196	255	4010
4010	400	51	250	101	399	52	249	102	398	53	248	103	397	54	247	104	396	55	246	105	4010
4010	201	150	351	100	202	149	352	99	203	148	353	98	204	147	354	97	205	146	355	96	4010
4010	156	295	6	345	157	294	7	344	158	293	8	343	159	292	9	342	160	291	10	341	4010
4010	45	306	195	256	44	307	194	257	43	308	193	258	42	309	192	259	41	310	191	260	4010
4010	395	56	245	106	394	57	244	107	393	58	243	108	392	59	242	109	391	60	241	110	4010
4010	206	145	356	95	207	144	357	94	208	143	358	93	209	142	359	92	210	141	360	91	4010
4010	161	290	11	340	162	289	12	339	163	288	13	338	164	287	14	337	165	286	15	336	4010
4010	40	311	190	261	39	312	189	262	38	313	188	263	37	314	187	264	36	315	186	265	4010
4010	390	61	240	111	389	62	239	112	388	63	238	113	387	64	237	114	386	65	236	115	4010
4010	211	140	361	90	212	139	362	89	213	138	363	88	214	137	364	87	215	136	365	86	4010
4010	166	285	16	335	167	284	17	334	168	283	18	333	169	282	19	332	170	281	20	331	4010
4010	35	316	185	266	34	317	184	267	33	318	183	268	32	319	182	269	31	320	181	270	4010
4010	385	66	235	116	384	67	234	117	383	68	233	118	382	69	232	119	381	70	231	120	4010
4010	216	135	366	85	217	134	367	84	218	133	368	83	219	132	369	82	220	131	370	81	4010
4010	171	280	21	330	172	279	22	329	173	278	23	328	174	277	24	327	175	276	25	326	4010
4010	30	321	180	271	29	322	179	272	28	323	178	273	27	324	177	274	26	325	176	275	4010
4010	380	71	230	121	379	72	229	122	378	73	228	123	377	74	227	124	376	75	226	125	4010
4010	221	130	371	80	222	129	372	79	223	128	373	78	224	127	374	77	225	126	375	76	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

In this case, the magic sum is $S_{20\times 20} = 4010$. All the 4×4 blocks are **pandiagonal** magic square of order 4 with the equal magic sums, $S_{4\times 4} = 802$.

2.1.1 Alternative Semi-Bimagic

Example 5. *The square values of magic square of order 20 given in Example 4 are given by*

																				1084870
181^2	211^2	10^2	400^2	162^2	232^2	29^2	379^2	143^2	253^2	48^2	358^2	124^2	274^2	67^2	337^2	105^2	295^2	86^2	316^2	*1060670*
20^2	390^2	191^2	201^2	39^2	369^2	172^2	222^2	58^2	348^2	153^2	243^2	77^2	327^2	134^2	264^2	96^2	306^2	115^2	285^2	*1080670*
391^2	1^2	220^2	190^2	372^2	22^2	239^2	169^2	353^2	43^2	258^2	148^2	334^2	64^2	277^2	127^2	315^2	85^2	296^2	106^2	*1060670*
210^2	200^2	381^2	11^2	229^2	179^2	362^2	32^2	248^2	158^2	343^2	53^2	267^2	137^2	324^2	74^2	286^2	116^2	305^2	95^2	*1080670*
123^2	273^2	68^2	338^2	104^2	294^2	87^2	317^2	185^2	215^2	6^2	396^2	161^2	231^2	30^2	380^2	142^2	252^2	49^2	359^2	*1060670*
78^2	328^2	133^2	263^2	97^2	307^2	114^2	284^2	16^2	386^2	195^2	205^2	40^2	370^2	171^2	221^2	59^2	349^2	152^2	242^2	*1080670*
333^2	63^2	278^2	128^2	314^2	84^2	297^2	107^2	395^2	5^2	216^2	186^2	371^2	21^2	240^2	170^2	352^2	42^2	259^2	149^2	*1060670*
268^2	138^2	323^2	73^2	287^2	117^2	304^2	94^2	206^2	196^2	385^2	15^2	230^2	180^2	361^2	31^2	249^2	159^2	342^2	52^2	*1080670*
165^2	235^2	26^2	376^2	141^2	251^2	50^2	360^2	122^2	272^2	69^2	339^2	103^2	293^2	88^2	318^2	184^2	214^2	7^2	397^2	*1060670*
36^2	366^2	175^2	225^2	60^2	350^2	151^2	241^2	79^2	329^2	132^2	262^2	98^2	308^2	113^2	283^2	17^2	387^2	194^2	204^2	*1080670*
375^2	25^2	236^2	166^2	351^2	41^2	260^2	150^2	332^2	62^2	279^2	129^2	313^2	83^2	298^2	108^2	394^2	4^2	217^2	187^2	*1060670*
226^2	176^2	365^2	35^2	250^2	160^2	341^2	51^2	269^2	139^2	322^2	72^2	288^2	118^2	303^2	93^2	207^2	197^2	384^2	14^2	*1080670*
102^2	292^2	89^2	319^2	183^2	213^2	8^2	398^2	164^2	234^2	27^2	377^2	145^2	255^2	46^2	356^2	121^2	271^2	70^2	340^2	*1060670*
99^2	309^2	112^2	282^2	18^2	388^2	193^2	203^2	37^2	367^2	174^2	224^2	56^2	346^2	155^2	245^2	80^2	330^2	131^2	261^2	*1080670*
312^2	82^2	299^2	109^2	393^2	3^2	218^2	188^2	374^2	24^2	237^2	167^2	355^2	45^2	256^2	146^2	331^2	61^2	280^2	130^2	*1060670*
289^2	119^2	302^2	92^2	208^2	198^2	383^2	13^2	227^2	177^2	364^2	34^2	246^2	156^2	345^2	55^2	270^2	140^2	321^2	71^2	*1080670*
144^2	254^2	47^2	357^2	125^2	275^2	66^2	336^2	101^2	291^2	90^2	320^2	182^2	212^2	9^2	399^2	163^2	233^2	28^2	378^2	*1060670*
57^2	347^2	154^2	244^2	76^2	326^2	135^2	265^2	100^2	310^2	111^2	281^2	19^2	389^2	192^2	202^2	38^2	368^2	173^2	223^2	*1080670*
354^2	44^2	257^2	147^2	335^2	65^2	276^2	126^2	311^2	81^2	300^2	110^2	392^2	2^2	219^2	189^2	373^2	23^2	238^2	168^2	*1060670*
247^2	157^2	344^2	54^2	266^2	136^2	325^2	75^2	290^2	120^2	301^2	91^2	209^2	199^2	382^2	12^2	228^2	178^2	363^2	33^2	*1080670*
1060670	*1080670*	*1060670*	*1080670*	*1060670*	*1080670*	*1060670*	*1080670*	*1060670*	*1080670*	*1060670*	*1080670*	*1060670*	*1080670*	*1060670*	*1080670*	*1060670*	*1080670*	*1060670*	*1080670*	*1097270*

In this case the above magic square is **alternate semi-bimagic**, i.e., square sum is in alternative way either 1060670 or 1080670. The diagonal sums are 1084870 and 1097270.

2.2 Second Approach: 16 Blocks of Order 5

Example 6. *16 blocks of magic squares of order 5 constructed according to Example 2 lead us to a* ***pandiagonal*** *magic square of order 20 given by*

		4010	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**
	7	135	183	311	359	12	140	188	316	364	1	129	177	305	353	14	142	190	318	366	**4010**
4010	263	391	39	87	215	268	396	44	92	220	257	385	33	81	209	270	398	46	94	222	**4010**
4010	119	167	295	343	71	124	172	300	348	76	113	161	289	337	65	126	174	302	350	78	**4010**
4010	375	23	151	199	247	380	28	156	204	252	369	17	145	193	241	382	30	158	206	254	**4010**
4010	231	279	327	55	103	236	284	332	60	108	225	273	321	49	97	238	286	334	62	110	**4010**
4010	2	130	178	306	354	13	141	189	317	365	8	136	184	312	360	11	139	187	315	363	**4010**
4010	258	386	34	82	210	269	397	45	93	221	264	392	40	88	216	267	395	43	91	219	**4010**
4010	114	162	290	338	66	125	173	301	349	77	120	168	296	344	72	123	171	299	347	75	**4010**
4010	370	18	146	194	242	381	29	157	205	253	376	24	152	200	248	379	27	155	203	251	**4010**
4010	226	274	322	50	98	237	285	333	61	109	232	280	328	56	104	235	283	331	59	107	**4010**
4010	16	144	192	320	368	3	131	179	307	355	10	138	186	314	362	5	133	181	309	357	**4010**
4010	272	400	48	96	224	259	387	35	83	211	266	394	42	90	218	261	389	37	85	213	**4010**
4010	128	176	304	352	80	115	163	291	339	67	122	170	298	346	74	117	165	293	341	69	**4010**
4010	384	32	160	208	256	371	19	147	195	243	378	26	154	202	250	373	21	149	197	245	**4010**
4010	240	288	336	64	112	227	275	323	51	99	234	282	330	58	106	229	277	325	53	101	**4010**
4010	9	137	185	313	361	6	134	182	310	358	15	143	191	319	367	4	132	180	308	356	**4010**
4010	265	393	41	89	217	262	390	38	86	214	271	399	47	95	223	260	388	36	84	212	**4010**
4010	121	169	297	345	73	118	166	294	342	70	127	175	303	351	79	116	164	292	340	68	**4010**
4010	377	25	153	201	249	374	22	150	198	246	383	31	159	207	255	372	20	148	196	244	**4010**
4010	233	281	329	57	105	230	278	326	54	102	239	287	335	63	111	228	276	324	52	100	**4010**
	4010	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**

We observe that it is not possible to have equal magic sums of order 5 as it is impossible to divide magic sum 4010 by 4, i.e., 4010/4:=1002.5. In this case, all the 5×5 blocks are magic squares of order 5 with different magic sums giving a **pandiagonal** magic square of order 4 given in example below.

Example 7. *A* ***pandiagonal*** *magic square of order 4, formed by magic sums of 16 magic squares of order 5 is given by*

		4010	4010	4010	4010
	995	1020	965	1030	4010
4010	970	1025	1000	1015	4010
4010	1040	975	1010	985	4010
4010	1005	990	1035	980	4010
	4010	4010	4010	4010	4010

2.3 Third Approach: 4 Blocks of Order 10

Example 8. *4 blocks of equal sums magic squares of order 10 constructed according to Example 3 lead us to a magic square of order 20 given by*

																				4010
1	177	72	384	336	88	309	153	240	245	2	178	71	383	335	87	310	154	239	246	**4010**
152	64	256	325	100	397	161	228	9	313	151	63	255	326	99	398	162	227	10	314	**4010**
244	236	85	180	392	13	157	321	308	69	243	235	86	179	391	14	158	322	307	70	**4010**
80	333	229	148	301	165	396	4	252	97	79	334	230	147	302	166	395	3	251	98	**4010**
388	81	17	233	169	304	332	260	65	156	387	82	18	234	170	303	331	259	66	155	**4010**
329	248	141	317	73	232	20	385	96	164	330	247	142	318	74	231	19	386	95	163	**4010**
176	5	320	92	224	149	253	77	381	328	175	6	319	91	223	150	254	78	382	327	**4010**
93	389	168	61	257	340	225	316	144	12	94	390	167	62	258	339	226	315	143	11	**4010**
305	160	393	249	8	76	84	172	337	221	306	159	394	250	7	75	83	171	338	222	**4010**
237	312	324	16	145	241	68	89	173	400	238	311	323	15	146	242	67	90	174	399	**4010**
21	197	52	364	356	108	289	133	220	265	22	198	51	363	355	107	290	134	219	266	**4010**
132	44	276	345	120	377	181	208	29	293	131	43	275	346	119	378	182	207	30	294	**4010**
264	216	105	200	372	33	137	341	288	49	263	215	106	199	371	34	138	342	287	50	**4010**
60	353	209	128	281	185	376	24	272	117	59	354	210	127	282	186	375	23	271	118	**4010**
368	101	37	213	189	284	352	280	45	136	367	102	38	214	190	283	351	279	46	135	**4010**
349	268	121	297	53	212	40	365	116	184	350	267	122	298	54	211	39	366	115	183	**4010**
196	25	300	112	204	129	273	57	361	348	195	26	299	111	203	130	274	58	362	347	**4010**
113	369	188	41	277	360	205	296	124	32	114	370	187	42	278	359	206	295	123	31	**4010**
285	140	373	269	28	56	104	192	357	201	286	139	374	270	27	55	103	191	358	202	**4010**
217	292	344	36	125	261	48	109	193	380	218	291	343	35	126	262	47	110	194	379	**4010**
4010	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**	**4010**

The above magic square is with magic sum $S_{20\times 20} = 4010$, and all the four blocks of order 10 are magic squares with equal magic sums $S_{10\times 10} := 2005$.

3 Self-Orthogonal Diagonal Latin Square

This section brings a **pair of self-orthogonal diagonal Latin squares** for the magic square given in Examples 4 and 6.

3.1 Blocks of Order 4

The magic square of order 20 with blocks of order 4 given in Example 4 is based on the following **pair-wise self-orthogonal diagonal Latin squares**

Example 9. ***Self-orthogonal diagonal Latin squares*** *of order 20 with blocks of order 4 are given by*

		210	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**
	10	11	1	20	9	12	2	19	8	13	3	18	7	14	4	17	6	15	5	16	**210**
210	1	20	10	11	2	19	9	12	3	18	8	13	4	17	7	14	5	16	6	15	**210**
210	20	1	11	10	19	2	12	9	18	3	13	8	17	4	14	7	16	5	15	6	**210**
210	11	10	20	1	12	9	19	2	13	8	18	3	14	7	17	4	15	6	16	5	**210**
210	7	14	4	17	6	15	5	16	10	11	1	20	9	12	2	19	8	13	3	18	**210**
210	4	17	7	14	5	16	6	15	1	20	10	11	2	19	9	12	3	18	8	13	**210**
210	17	4	14	7	16	5	15	6	20	1	11	10	19	2	12	9	18	3	13	8	**210**
210	14	7	17	4	15	6	16	5	11	10	20	1	12	9	19	2	13	8	18	3	**210**
210	9	12	2	19	8	13	3	18	7	14	4	17	6	15	5	16	10	11	1	20	**210**
210	2	19	9	12	3	18	8	13	4	17	7	14	5	16	6	15	1	20	10	11	**210**
210	19	2	12	9	18	3	13	8	17	4	14	7	16	5	15	6	20	1	11	10	**210**
210	12	9	19	2	13	8	18	3	14	7	17	4	15	6	16	5	11	10	20	1	**210**
210	6	15	5	16	10	11	1	20	9	12	2	19	8	13	3	18	7	14	4	17	**210**
210	5	16	6	15	1	20	10	11	2	19	9	12	3	18	8	13	4	17	7	14	**210**
210	16	5	15	6	20	1	11	10	19	2	12	9	18	3	13	8	17	4	14	7	**210**
210	15	6	16	5	11	10	20	1	12	9	19	2	13	8	18	3	14	7	17	4	**210**
210	8	13	3	18	7	14	4	17	6	15	5	16	10	11	1	20	9	12	2	19	**210**
210	3	18	8	13	4	17	7	14	5	16	6	15	1	20	10	11	2	19	9	12	**210**
210	18	3	13	8	17	4	14	7	16	5	15	6	20	1	11	10	19	2	12	9	**210**
210	13	8	18	3	14	7	17	4	15	6	16	5	11	10	20	1	12	9	19	2	**210**
	210	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**

A

		210	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**
	10	1	20	11	7	4	17	14	9	2	19	12	6	5	16	15	8	3	18	13	**210**
210	11	20	1	10	14	17	4	7	12	19	2	9	15	16	5	6	13	18	3	8	**210**
210	1	10	11	20	4	7	14	17	2	9	12	19	5	6	15	16	3	8	13	18	**210**
210	20	11	10	1	17	14	7	4	19	12	9	2	16	15	6	5	18	13	8	3	**210**
210	9	2	19	12	6	5	16	15	8	3	18	13	10	1	20	11	7	4	17	14	**210**
210	12	19	2	9	15	16	5	6	13	18	3	8	11	20	1	10	14	17	4	7	**210**
210	2	9	12	19	5	6	15	16	3	8	13	18	1	10	11	20	4	7	14	17	**210**
210	19	12	9	2	16	15	6	5	18	13	8	3	20	11	10	1	17	14	7	4	**210**
210	8	3	18	13	10	1	20	11	7	4	17	14	9	2	19	12	6	5	16	15	**210**
210	13	18	3	8	11	20	1	10	14	17	4	7	12	19	2	9	15	16	5	6	**210**
210	3	8	13	18	1	10	11	20	4	7	14	17	2	9	12	19	5	6	15	16	**210**
210	18	13	8	3	20	11	10	1	17	14	7	4	19	12	9	2	16	15	6	5	**210**
210	7	4	17	14	9	2	19	12	6	5	16	15	8	3	18	13	10	1	20	11	**210**
210	14	17	4	7	12	19	2	9	15	16	5	6	13	18	3	8	11	20	1	10	**210**
210	4	7	14	17	2	9	12	19	5	6	15	16	3	8	13	18	1	10	11	20	**210**
210	17	14	7	4	19	12	9	2	16	15	6	5	18	13	8	3	20	11	10	1	**210**
210	6	5	16	15	8	3	18	13	10	1	20	11	7	4	17	14	9	2	19	12	**210**
210	15	16	5	6	13	18	3	8	11	20	1	10	14	17	4	7	12	19	2	9	**210**
210	5	6	15	16	3	8	13	18	1	10	11	20	4	7	14	17	2	9	12	19	**210**
210	16	15	6	5	18	13	8	3	20	11	10	1	17	14	7	4	19	12	9	2	**210**
	210	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**

B

The matrix B is transpose of matrix A. The magic square given in Example 4 is obtained by using the operation $AB := 20 \times (A-1) + B$. A and B are well-known **pairwise self-orthogonal Latin squares**.

3.2 Blocks of Order 5

The magic square of order 20 with blocks of order 5 is based on the following **pair-wise self-orthogonal diagonal Latin squares**

Example 10. ***Pair of self-orthogonal diagonal Latin squares*** *of order 20 with blocks of order 5 are given by*

		210	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**
	2	7	10	15	18	3	6	11	14	19	1	8	9	16	17	4	5	12	13	20	**210**
210	15	18	2	7	10	14	19	3	6	11	16	17	1	8	9	13	20	4	5	12	**210**
210	7	10	15	18	2	6	11	14	19	3	8	9	16	17	1	5	12	13	20	4	**210**
210	18	2	7	10	15	19	3	6	11	14	17	1	8	9	16	20	4	5	12	13	**210**
210	10	15	18	2	7	11	14	19	3	6	9	16	17	1	8	12	13	20	4	5	**210**
210	1	8	9	16	17	4	5	12	13	20	2	7	10	15	18	3	6	11	14	19	**210**
210	16	17	1	8	9	13	20	4	5	12	15	18	2	7	10	14	19	3	6	11	**210**
210	8	9	16	17	1	5	12	13	20	4	7	10	15	18	2	6	11	14	19	3	**210**
210	17	1	8	9	16	20	4	5	12	13	18	2	7	10	15	19	3	6	11	14	**210**
210	9	16	17	1	8	12	13	20	4	5	10	15	18	2	7	11	14	19	3	6	**210**
210	4	5	12	13	20	1	8	9	16	17	3	6	11	14	19	2	7	10	15	18	**210**
210	13	20	4	5	12	16	17	1	8	9	14	19	3	6	11	15	18	2	7	10	**210**
210	5	12	13	20	4	8	9	16	17	1	6	11	14	19	3	7	10	15	18	2	**210**
210	20	4	5	12	13	17	1	8	9	16	19	3	6	11	14	18	2	7	10	15	**210**
210	12	13	20	4	5	9	16	17	1	8	11	14	19	3	6	10	15	18	2	7	**210**
210	3	6	11	14	19	2	7	10	15	18	4	5	12	13	20	1	8	9	16	17	**210**
210	14	19	3	6	11	15	18	2	7	10	13	20	4	5	12	16	17	1	8	9	**210**
210	6	11	14	19	3	7	10	15	18	2	5	12	13	20	4	8	9	16	17	1	**210**
210	19	3	6	11	14	18	2	7	10	15	20	4	5	12	13	17	1	8	9	16	**210**
210	11	14	19	3	6	10	15	18	2	7	12	13	20	4	5	9	16	17	1	8	**210**
	210	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**

A

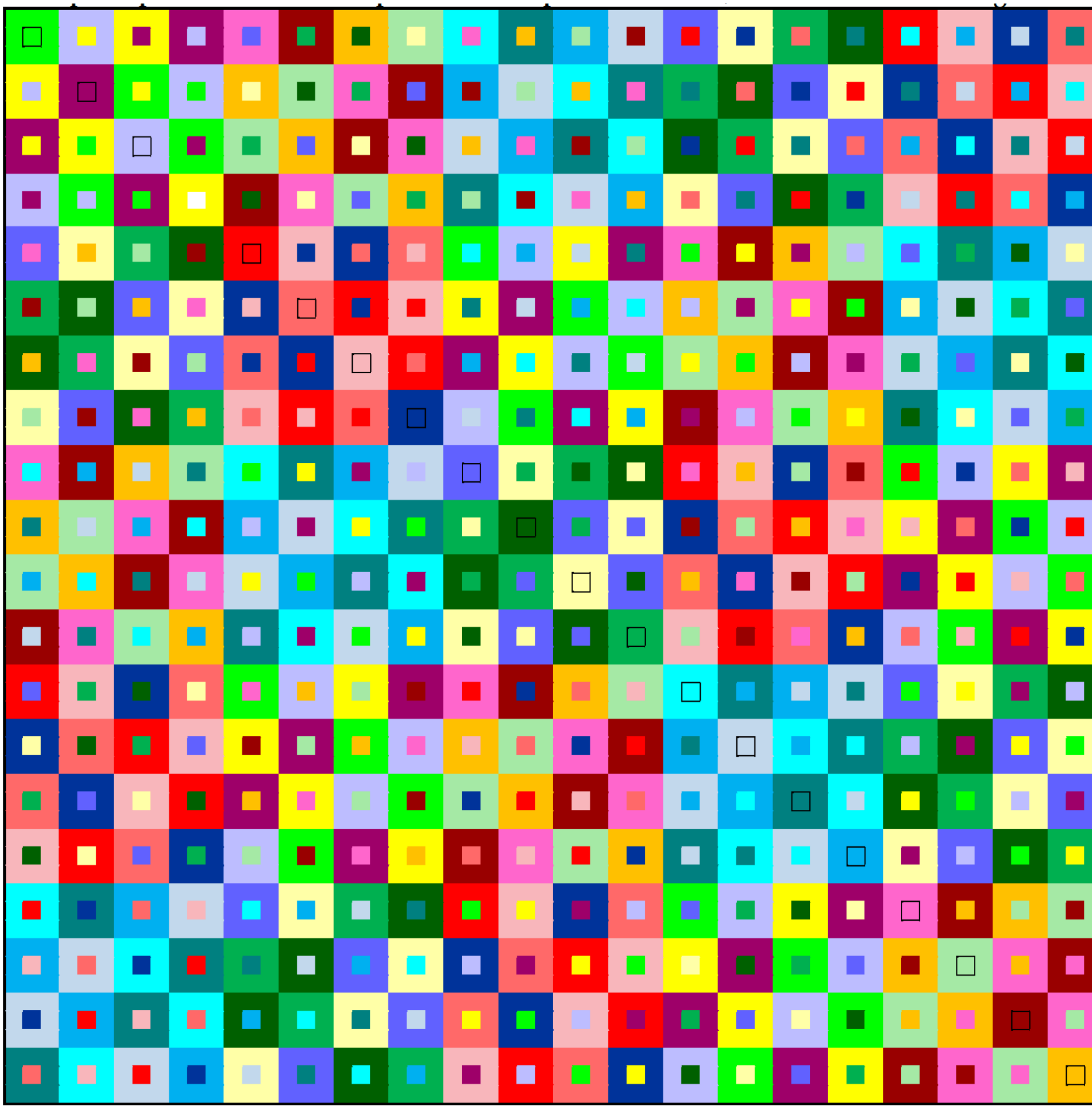
		210	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**
	2	15	7	18	10	1	16	8	17	9	4	13	5	20	12	3	14	6	19	11	**210**
210	7	18	10	2	15	8	17	9	1	16	5	20	12	4	13	6	19	11	3	14	**210**
210	10	2	15	7	18	9	1	16	8	17	12	4	13	5	20	11	3	14	6	19	**210**
210	15	7	18	10	2	16	8	17	9	1	13	5	20	12	4	14	6	19	11	3	**210**
210	18	10	2	15	7	17	9	1	16	8	20	12	4	13	5	19	11	3	14	6	**210**
210	3	14	6	19	11	4	13	5	20	12	1	16	8	17	9	2	15	7	18	10	**210**
210	6	19	11	3	14	5	20	12	4	13	8	17	9	1	16	7	18	10	2	15	**210**
210	11	3	14	6	19	12	4	13	5	20	9	1	16	8	17	10	2	15	7	18	**210**
210	14	6	19	11	3	13	5	20	12	4	16	8	17	9	1	15	7	18	10	2	**210**
210	19	11	3	14	6	20	12	4	13	5	17	9	1	16	8	18	10	2	15	7	**210**
210	1	16	8	17	9	2	15	7	18	10	3	14	6	19	11	4	13	5	20	12	**210**
210	8	17	9	1	16	7	18	10	2	15	6	19	11	3	14	5	20	12	4	13	**210**
210	9	1	16	8	17	10	2	15	7	18	11	3	14	6	19	12	4	13	5	20	**210**
210	16	8	17	9	1	15	7	18	10	2	14	6	19	11	3	13	5	20	12	4	**210**
210	17	9	1	16	8	18	10	2	15	7	19	11	3	14	6	20	12	4	13	5	**210**
210	4	13	5	20	12	3	14	6	19	11	2	15	7	18	10	1	16	8	17	9	**210**
210	5	20	12	4	13	6	19	11	3	14	7	18	10	2	15	8	17	9	1	16	**210**
210	12	4	13	5	20	11	3	14	6	19	10	2	15	7	18	9	1	16	8	17	**210**
210	13	5	20	12	4	14	6	19	11	3	15	7	18	10	2	16	8	17	9	1	**210**
210	20	12	4	13	5	19	11	3	14	6	18	10	2	15	7	17	9	1	16	8	**210**
	210	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**	**210**

B

The matrix B is transpose of matrix A. The magic square given in Example 6 is obtained by using the operation $AB := 20 \times (A-1) + B$. A and B are well-known **pairwise self-orthogonal diagonal Latin squares**.

3.3 Superimposed Double Colors Pattern

Example 11. *Below is a* ***double color pattern*** *written based SODLS and its transpose, where each block of order 4 is also a SODLS resulting in pandiagonal magic squares of order 4 and 20.*



4 Magic Square Generated by Pythagorean Triple

Example 12. *The Pythagorean triple (3600,16000,16400) generating a magic square of order 20 with consecutive odd numbers,* 32001, 32003, ..., 32797, 32799 *is given by*

		648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000
	32379	32401	32039	32781	32333	32447	32073	32747	32297	32483	32117	32703	32251	32529	32151	32669	32215	32565	32195	32625	648000
648000	32021	32799	32361	32419	32067	32753	32327	32453	32103	32717	32283	32497	32149	32671	32249	32531	32185	32635	32205	32575	648000
648000	32761	32019	32421	32399	32727	32053	32467	32353	32683	32097	32503	32317	32649	32131	32549	32271	32605	32175	32585	32235	648000
648000	32439	32381	32779	32001	32473	32347	32733	32047	32517	32303	32697	32083	32551	32269	32651	32129	32595	32225	32615	32165	648000
648000	32257	32523	32157	32663	32211	32569	32191	32629	32375	32405	32035	32785	32339	32441	32079	32741	32293	32487	32113	32707	648000
648000	32143	32677	32243	32537	32189	32631	32209	32571	32025	32795	32365	32415	32061	32759	32321	32459	32107	32713	32287	32493	648000
648000	32643	32137	32543	32277	32609	32171	32589	32231	32765	32015	32425	32395	32721	32059	32461	32359	32687	32093	32507	32313	648000
648000	32557	32263	32657	32123	32591	32229	32611	32169	32435	32385	32775	32005	32479	32341	32739	32041	32513	32307	32693	32087	648000
648000	32335	32445	32075	32745	32299	32481	32119	32701	32253	32527	32153	32667	32217	32563	32197	32623	32371	32409	32031	32789	648000
648000	32065	32755	32325	32455	32101	32719	32281	32499	32147	32673	32247	32533	32183	32637	32203	32577	32029	32791	32369	32411	648000
648000	32725	32055	32465	32355	32681	32099	32501	32319	32647	32133	32547	32273	32603	32177	32583	32237	32769	32011	32429	32391	648000
648000	32475	32345	32735	32045	32519	32301	32699	32081	32553	32267	32653	32127	32597	32223	32617	32163	32431	32389	32771	32009	648000
648000	32213	32567	32193	32627	32377	32403	32037	32783	32331	32449	32071	32749	32295	32485	32115	32705	32259	32521	32159	32661	648000
648000	32187	32633	32207	32573	32023	32797	32363	32417	32069	32751	32329	32451	32105	32715	32285	32495	32141	32679	32241	32539	648000
648000	32607	32173	32587	32233	32763	32017	32423	32397	32729	32051	32469	32351	32685	32095	32505	32315	32641	32139	32541	32279	648000
648000	32593	32227	32613	32167	32437	32383	32777	32003	32471	32349	32731	32049	32515	32305	32695	32085	32559	32261	32659	32121	648000
648000	32291	32489	32111	32709	32255	32525	32155	32665	32219	32561	32199	32621	32373	32407	32033	32787	32337	32443	32077	32743	648000
648000	32109	32711	32289	32491	32145	32675	32245	32535	32181	32639	32201	32579	32027	32793	32367	32413	32063	32757	32323	32457	648000
648000	32689	32091	32509	32311	32645	32135	32545	32275	32601	32179	32581	32239	32767	32013	32427	32393	32723	32057	32463	32357	648000
648000	32511	32309	32691	32089	32555	32265	32655	32125	32599	32221	32619	32161	32433	32387	32773	32007	32477	32343	32737	32043	648000
	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000	648000

Above magic square is **pandiagonal** with magic sum, $S_{20\times 20} := 648000$ with sum of entries a perfect square sum, i.e.,

$$T_{400} := 12960000 = 3600^2 = 16400^2 - 16000^2$$

.

All the 4×4 blocks are also **pandiagonal** magic squares of order 4 with the equal magic sums, $S_{4\times 4} = 129600$. For more details refer author's work [29].

5 Magic Squares with Concatenation of Prime and Palprime Numbers

Still, we don't have idea about magic square of order 20 with prime numbers. Here we shall present magic square of order 20 having composition of prime or palprime numbers.

5.1 Concatenation of Prime Numbers

In case of prime numbers, we known that there are 21 prime numbers of two digits:

$$11 \quad 13 \quad 17 \quad 19 \quad 23 \quad 29 \quad 31 \quad 37 \quad 41 \quad 43 \quad 47 \quad 53 \quad 59 \quad 61 \quad 67 \quad 71 \quad 73 \quad 79 \quad 83 \quad 89 \quad 97$$

We shall use the first 20 prime numbers and write them as $P := 100 \times A + B$, where A and B are prime numbers of two digits. Since this composition is not a prime number, we call it as **concatenation of prime numbers**. See below the magic square of order 20 with 25 blocks of order 4 constructed by the help of Example 9:

Example 13. *Let's consider following 20 prime numbers of two digits:*

11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89

A magic square of order 20 using ***concatenation of prime numbers*** *formed by above prime numbers is given by*

																				95546
1111	1317	1719	1913	2359	2967	3171	3761	4123	4331	4737	5329	5973	6183	6789	7179	7341	7947	8353	8943	95546
1713	1919	1117	1311	3161	3771	2367	2959	4729	5337	4131	4323	6779	7189	5983	6173	8343	8953	7347	7941	95546
1917	1711	1313	1119	3767	3159	2961	2371	5331	4723	4329	4137	7183	6773	6179	5989	8947	8341	7943	7353	95546
1319	1113	1911	1717	2971	2361	3759	3167	4337	4129	5323	4731	6189	5979	7173	6783	7953	7343	8941	8347	95546
5923	6131	6737	7129	7373	7983	8389	8979	1141	1347	1753	1943	2311	2917	3119	3713	4159	4367	4771	5361	95546
6729	7137	5931	6123	8379	8989	7383	7973	1743	1953	1147	1341	3113	3719	2317	2911	4761	5371	4167	4359	95546
7131	6723	6129	5937	8983	8373	7979	7389	1947	1741	1343	1153	3717	3111	2913	2319	5367	4759	4361	4171	95546
6137	5929	7123	6731	7989	7379	8973	8383	1353	1143	1941	1747	2919	2313	3711	3117	4371	4161	5359	4767	95546
2341	2947	3153	3743	4111	4317	4719	5313	5959	6167	6771	7161	7323	7931	8337	8929	1173	1383	1789	1979	95546
3143	3753	2347	2941	4713	5319	4117	4311	6761	7171	5967	6159	8329	8937	7331	7923	1779	1989	1183	1373	95546
3747	3141	2943	2353	5317	4711	4313	4119	7167	6759	6161	5971	8931	8323	7929	7337	1983	1773	1379	1189	95546
2953	2343	3741	3147	4319	4113	5311	4717	6171	5961	7159	6767	7937	7329	8923	8331	1389	1179	1973	1783	95546
7359	7967	8371	8961	1123	1331	1737	1929	2373	2983	3189	3779	4141	4347	4753	5343	5911	6117	6719	7113	95546
8361	8971	7367	7959	1729	1937	1131	1323	3179	3789	2383	2973	4743	5353	4147	4341	6713	7119	5917	6111	95546
8967	8359	7961	7371	1931	1723	1329	1137	3783	3173	2979	2389	5347	4741	4343	4153	7117	6711	6113	5919	95546
7971	7361	8959	8367	1337	1129	1923	1731	2989	2379	3773	3183	4353	4143	5341	4747	6119	5913	7111	6717	95546
4173	4383	4789	5379	5941	6147	6753	7143	7311	7917	8319	8913	1159	1367	1771	1961	2323	2931	3137	3729	95546
4779	5389	4183	4373	6743	7153	5947	6141	8313	8919	7317	7911	1761	1971	1167	1359	3129	3737	2331	2923	95546
5383	4773	4379	4189	7147	6741	6143	5953	8917	8311	7913	7319	1967	1759	1361	1171	3731	3123	2929	2337	95546
4389	4179	5373	4783	6153	5943	7141	6747	7919	7313	8911	8317	1371	1161	1959	1767	2937	2329	3723	3131	95546
95546	95546	95546	95546	95546	95546	95546	95546	95546	95546	95546	95546	95546	95546	95546	95546	95546	95546	95546	95546	95546

In this case, the magic square sum is $S_{20\times 20} := 95546$, and all the blocks of order 4 are magic squares with different magic sums giving a **pandiagonal** magic square of order 5 given in example below.

Example 14. *The* ***pandiagonal*** *magic square of order 5 formed by magic sums of order 4 of Example 13 is given by*

		95546	95546	95546	95546	95546
	6060	12258	18520	26124	32584	95546
95546	25920	32724	6184	12060	18658	95546
95546	12184	18460	26058	32520	6324	95546
95546	32658	6120	12324	18584	25860	95546
95546	18724	25984	32460	6258	12120	95546
	95546	95546	95546	95546	95546	95546

5.2 Concatenation of Palprime Numbers

In case of palprimes, we known that there are 93 palprimes of 5 digits. We shall use only first 20 palprimes of 5 digits. Let's write them as $P := 100000 \times A + B$, where A and B are palprimes of 5 digits. Let's call it **concatenation of palprimes**. See below the magic square of order 20 with 25 blocks of order 4:

Example 15. *Let's consider the first 20 palprimes of 5 digits:*

10301 10501 10601 11311 11411 12421 12721 12821 13331 13831
13931 14341 14741 15451 15551 16061 16361 16561 16561 17471

*A magic square of order 20 using **concatenation of palprimes** formed first 20 palprimes of 5 digits is given by*

1030110301	1050110601	1060111311	1131110501	1141114741	1242115551	1272116061	1282115451	1333111411	1383112721	1393112821	1434112421	1474116361	1545116561	1555117471	1606116561	1636113331	1656113931	1656114341	1747113831
1060110501	1131111311	1030110601	1050110301	1272115451	1282116061	1141115551	1242114741	1393112421	1434112821	1333112721	1383111411	1555116561	1606117471	1474116561	1545116361	1656113831	1747114341	1636113931	1656113331
1131110601	1060110301	1050110501	1030111311	1282115551	1272114741	1242115451	1141116061	1434112721	1393111411	1383112421	1333112821	1606116561	1555116361	1545116561	1474117471	1747113931	1656113331	1656113831	1636114341
1050111311	1030110501	1131110301	1060110601	1242116061	1141115451	1282114741	1272115551	1383112821	1333112421	1434111411	1393112721	1545117471	1474116561	1606116361	1555116561	1656114341	1636113831	1747113331	1656113931
1474111411	1545112721	1555112821	1606112421	1636116361	1656116561	1656117471	1747116561	1030113331	1050113931	1060114341	1131113831	1141110301	1242110601	1272111311	1282110501	1333114741	1383115551	1393116061	1434115451
1555112421	1606112821	1474112721	1545111411	1656116561	1747117471	1636116561	1656116361	1060113831	1131114341	1030113931	1050113331	1272110501	1282111311	1141110601	1242110301	1393115451	1434116061	1333115551	1383114741
1606112721	1555111411	1545112421	1474112821	1747116561	1656116361	1656116561	1636117471	1131113931	1060113331	1050113831	1030114341	1282110601	1272110301	1242110501	1141111311	1434115551	1393114741	1383115451	1333116061
1545112821	1474112421	1606111411	1555112721	1656117471	1636116561	1747116361	1656116561	1050114341	1030113831	1131113331	1060113931	1242111311	1141110501	1282110301	1272110601	1383116061	1333115451	1434114741	1393115551
1141113331	1242113931	1272114341	1282113831	1333110301	1383110601	1393111311	1434110501	1474114741	1545115551	1555116061	1606115451	1636111411	1656112721	1656112821	1747112421	1030116361	1050116561	1060117471	1131116561
1272113831	1282114341	1141113931	1242113331	1393110501	1434111311	1333110601	1383110301	1555115451	1606116061	1474115551	1545114741	1656112421	1747112821	1636112721	1656111411	1060116561	1131117471	1030116561	1050116361
1282113931	1272113331	1242113831	1141114341	1434110601	1393110301	1383110501	1333111311	1606115551	1555114741	1545115451	1474116061	1747112721	1656111411	1656112421	1636112821	1131116561	1060116361	1050116561	1030117471
1242114341	1141113831	1282113331	1272113931	1383111311	1333110501	1434110301	1393110601	1545116061	1474115451	1606114741	1555115551	1656112821	1636112421	1747111411	1656112721	1050117471	1030116561	1131116361	1060116561
1636114741	1656115551	1656116061	1747115451	1030111411	1050112721	1060112821	1131112421	1141116361	1242116561	1272117471	1282116561	1333113331	1383113931	1393114341	1434113831	1474110301	1545110601	1555111311	1606110501
1656115451	1747116061	1636115551	1656114741	1060112421	1131112821	1030112721	1050111411	1272116561	1282117471	1141116561	1242116361	1393113831	1434114341	1333113931	1383113331	1555110501	1606111311	1474110601	1545110301
1747115551	1656114741	1656115451	1636116061	1131112721	1060111411	1050112421	1030112821	1282116561	1272116361	1242116561	1141117471	1434113931	1393113331	1383113831	1333114341	1606110601	1555110301	1545110501	1474111311
1656116061	1636115451	1747114741	1656115551	1050112821	1030112421	1131111411	1060112721	1242117471	1141116561	1282116361	1272116561	1383114341	1333113831	1434113331	1393113931	1545111311	1474110501	1606110301	1555110601
1333116361	1383116561	1393117471	1434116561	1474113331	1545113931	1555114341	1606113831	1636110301	1656110601	1656111311	1747110501	1030114741	1050115551	1060116061	1131115451	1141111411	1242112721	1272112821	1282112421
1393116561	1434117471	1333116561	1383116361	1555113831	1606114341	1474113931	1545113331	1656110501	1747111311	1636110601	1656110301	1060115451	1131116061	1030115551	1050114741	1272112421	1282112821	1141112721	1242111411
1434116561	1393116361	1383116561	1333117471	1606113931	1555113331	1545113831	1474114341	1747110601	1656110301	1656110501	1636111311	1131115551	1060114741	1050115451	1030116061	1282112721	1272111411	1242112421	1141112821
1383117471	1333116561	1434116361	1393116561	1545114341	1474113831	1606113331	1555113931	1656111311	1636110501	1747110301	1656110601	1050116061	1030115451	1131114741	1060115551	1242112821	1141112421	1282111411	1272112721

In this case, the magic square sum is $S_{20\times20} := 27628276280$, and all the blocks of order 4 are magic squares with different magic sums giving a **pandiagonal** magic square of order 5 given in example below.

Example 16. *The **pandiagonal** magic square of order 5 formed by magic sums of order 4 of Example 15 is given by*

		27628276280	*27628276280*	*27628276280*	*27628276280*	*27628276280*
	4271442714	*4937461804*	*5543449374*	*6180466954*	*6695455434*	*27628276280*
27628276280	*6180449374*	*6695466954*	*4271455434*	*4937442714*	*5543461804*	*27628276280*
27628276280	*4937455434*	*5543442714*	*6180461804*	*6695449374*	*4271466954*	*27628276280*
27628276280	*6695461804*	*4271449374*	*4937466954*	*5543455434*	*6180442714*	*27628276280*
27628276280	*5543466954*	*6180455434*	*6695442714*	*4271461804*	*4937449374*	*27628276280*
	27628276280	*27628276280*	*27628276280*	*27628276280*	*27628276280*	*27628276280*

6 Palindromic-Type Magic Squares

In this section we shall write magic square of order 20 as blocks of order 4, 5 and 10 using palindromic numbers. If we analyse, we have exactly 400 palindrome of length 5 starting with numbers 1 to 4 or 5 to 8.

6.1 Blocks of Order 4

Example 17. *The block-wise **pandiagonal** palindromic magic square with palindromes of length 5, where the first digit is either 1, 2, 3 or 4 is given by*

		599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950
	28982	30003	11911	49094	26662	32323	13631	47374	24842	34143	15851	45154	22522	36463	17571	43434	20702	38283	19791	41214	599950
599950	11011	49994	28082	30903	13331	47674	26362	32623	15151	45854	24142	34843	17471	43534	22422	36563	19291	41714	20202	38783	599950
599950	48084	10901	31013	29992	46364	12621	33333	27672	44144	14841	35153	25852	42424	16561	37473	23532	40204	18781	39293	21712	599950
599950	31913	29092	48984	10001	33633	27372	46664	12321	35853	25152	44844	14141	37573	23432	42524	16461	39793	21212	40704	18281	599950
599950	22822	36163	17871	43134	20502	38483	19591	41414	28782	30203	11711	49294	26962	32023	13931	47074	24642	34343	15651	45354	599950
599950	17171	43834	22122	36863	19491	41514	20402	38583	11211	49794	28282	30703	13031	47974	26062	32923	15351	45654	24342	34643	599950
599950	42124	16861	37173	23832	40404	18581	39493	21512	48284	10701	31213	29792	46064	12921	33033	27972	44344	14641	35353	25652	599950
599950	37873	23132	42824	16161	39593	21412	40504	18481	31713	29292	48784	10201	33933	27072	46964	12021	35653	25352	44644	14341	599950
599950	26762	32223	13731	47274	24942	34043	15951	45054	22622	36363	17671	43334	20802	38183	19891	41114	28582	30403	11511	49494	599950
599950	13231	47774	26262	32723	15051	45954	24042	34943	17371	43634	22322	36663	19191	41814	20102	38883	11411	49594	28482	30503	599950
599950	46264	12721	33233	27772	44044	14941	35053	25952	42324	16661	37373	23632	40104	18881	39193	21812	48484	10501	31413	29592	599950
599950	33733	27272	46764	12221	35953	25052	44944	14041	37673	23332	42624	16361	39893	21112	40804	18181	31513	29492	48584	10401	599950
599950	20602	38383	19691	41314	28882	30103	11811	49194	26562	32423	13531	47474	24742	34243	15751	45254	22922	36063	17971	43034	599950
599950	19391	41614	20302	38683	11111	49894	28182	30803	13431	47574	26462	32523	15251	45754	24242	34743	17071	43934	22022	36963	599950
599950	40304	18681	39393	21612	48184	10801	31113	29892	46464	12521	33433	27572	44244	14741	35253	25752	42024	16961	37073	23932	599950
599950	39693	21312	40604	18381	31813	29192	48884	10101	33533	27472	46564	12421	35753	25252	44744	14241	37973	23032	42924	16061	599950
599950	24542	34443	15551	45454	22722	36263	17771	43234	20902	38083	19991	41014	28682	30303	11611	49394	26862	32123	13831	47174	599950
599950	15451	45554	24442	34543	17271	43734	22222	36763	19091	41914	20002	38983	11311	49694	28382	30603	13131	47874	26162	32823	599950
599950	44444	14541	35453	25552	42224	16761	37273	23732	40004	18981	39093	21912	48384	10601	31313	29692	46164	12821	33133	27872	599950
599950	35553	25452	44544	14441	37773	23232	42724	16261	39993	21012	40904	18081	31613	29392	48684	10301	33833	27172	46864	12121	599950
	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950

In this case, the magic sum is $S_{20\times 20} = 599950$. All the 4×4 blocks are **pandiagonal** magic square of order 4 with the equal magic sums, $S_{4\times 4} = 119990$. If we write 599950 as 0599950. In this case, the final sums also turns a palindrome.

6.2 Blocks of Order 5

Example 18. *The block-wise **pandiagonal** palindromic magic square of order 20 with palindromes of length 5, where starting digit in each block is either 1, 2, 3 or 4 is given by*

		599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950
	25052	25652	26262	26862	27472	37573	38183	38783	39393	39993	10001	10601	11211	11811	12421	42524	43134	43734	44344	44944	599950
599950	26762	27372	25452	25552	26162	39293	39893	37973	38083	38683	11711	12321	10401	10501	11111	44244	44844	42924	43034	43634	599950
599950	25952	26062	26662	27272	25352	38483	38583	39193	39793	37873	10901	11011	11611	12221	10301	43434	43534	44144	44744	42824	599950
599950	27172	25252	25852	26462	26562	39693	37773	38383	38983	39093	12121	10201	10801	11411	11511	44644	42724	43334	43934	44044	599950
599950	26362	26962	27072	25152	25752	38883	39493	39593	37673	38283	11311	11911	12021	10101	10701	43834	44444	44544	42624	43234	599950
599950	12521	13131	13731	14341	14941	40004	40604	41214	41814	42424	27572	28182	28782	29392	29992	35053	35653	36263	36863	37473	599950
599950	14241	14841	12921	13031	13631	41714	42324	40404	40504	41114	29292	29892	27972	28082	28682	36763	37373	35453	35553	36163	599950
599950	13431	13531	14141	14741	12821	40904	41014	41614	42224	40304	28482	28582	29192	29792	27872	35953	36063	36663	37273	35353	599950
599950	14641	12721	13331	13931	14041	42124	40204	40804	41414	41514	29692	27772	28382	28982	29092	37173	35253	35853	36463	36563	599950
599950	13831	14441	14541	12621	13231	41314	41914	42024	40104	40704	28882	29492	29592	27672	28282	36363	36963	37073	35153	35753	599950
599950	47574	48184	48784	49394	49994	15051	15651	16261	16861	17471	32523	33133	33733	34343	34943	20002	20602	21212	21812	22422	599950
599950	49294	49894	47974	48084	48684	16761	17371	15451	15551	16161	34243	34843	32923	33033	33633	21712	22322	20402	20502	21112	599950
599950	48484	48584	49194	49794	47874	15951	16061	16661	17271	15351	33433	33533	34143	34743	32823	20902	21012	21612	22222	20302	599950
599950	49694	47774	48384	48984	49094	17171	15251	15851	16461	16561	34643	32723	33333	33933	34043	22122	20202	20802	21412	21512	599950
599950	48884	49494	49594	47674	48284	16361	16961	17071	15151	15751	33833	34443	34543	32623	33233	21312	21912	22022	20102	20702	599950
599950	30003	30603	31213	31813	32423	22522	23132	23732	24342	24942	45054	45654	46264	46864	47474	17571	18181	18781	19391	19991	599950
599950	31713	32323	30403	30503	31113	24242	24842	22922	23032	23632	46764	47374	45454	45554	46164	19291	19891	17971	18081	18681	599950
599950	30903	31013	31613	32223	30303	23432	23532	24142	24742	22822	45954	46064	46664	47274	45354	18481	18581	19191	19791	17871	599950
599950	32123	30203	30803	31413	31513	24642	22722	23332	23932	24042	47174	45254	45854	46464	46564	19691	17771	18381	18981	19091	599950
599950	31313	31913	32023	30103	30703	23832	24442	24542	22622	23232	46364	46964	47074	45154	45754	18881	19491	19591	17671	18281	599950
	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950

We observe that it is not possible to have equal magic sums of order 5 as it is impossible to divide magic sum 599950 by 4, i.e., 599950/4:=149987.5. In this case, all the 5×5 blocks are **pandiagonal** magic squares of order 5 with different magic sums giving a **pandiagonal** magic square of order 4 given in example below.

Example 19. *A **pandiagonal** magic square of order 4, formed by magic sums of 16 magic squares of order 5 is given by*

		599950	599950	599950	599950
	131300	193925	56045	218680	599950
599950	68665	206060	143920	181305	599950
599950	243930	81295	168675	106050	599950
599950	156055	118670	231310	93915	599950
	599950	599950	599950	599950	599950

The above example of order 20 is with 16 blocks of order 5, where each blocks starts with same digit. Also, we have used only the five digits, i.e., 0, 1, 2, 3 and 4. Below is another example where each block is with five digits 1, 2, 3, 4 and 5 in the beginning of each block.

Example 20. *16 blocks of magic squares of order 5 constructed according to Example* **??** *lead us to a **pandiagonal** magic square of order 20 given by*

		699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960
	13131	26462	33633	46764	53935	15051	24542	35753	44644	55855	11311	28282	31413	48984	52125	17271	22322	37573	42824	58085	699960
699960	45654	54745	13931	25152	34443	43734	56665	15851	23032	36563	47474	52925	12121	27372	32223	41514	58885	18081	21212	38383	699960
699960	25952	33133	46464	53635	14741	23832	35053	44544	55755	16661	28182	31313	48284	51415	12921	22022	37273	42324	57575	18881	699960
699960	54445	13631	26762	33933	45154	56565	15751	24642	35853	43034	52225	11411	28982	32123	47374	58385	17571	22822	38083	41214	699960
699960	34743	45954	53135	14441	25652	36663	43834	55055	16561	23732	32923	48184	51315	12221	27472	38883	42024	57275	18381	21512	699960
699960	11211	28382	31513	48884	52025	17371	22222	37473	42924	58185	13031	26562	33733	46664	53835	15151	24442	35653	44744	55955	699960
699960	47574	52825	12021	27272	32323	41414	58985	18181	21312	38283	45754	54645	13831	25052	34543	43634	56765	15951	23132	36463	699960
699960	28082	31213	48384	51515	12821	22122	37373	42224	57475	18981	25852	33033	46564	53735	14641	23932	35153	44444	55655	16761	699960
699960	52325	11511	28882	32023	47274	58285	17471	22922	38183	41314	54545	13731	26662	33833	45054	56465	15651	24742	35953	43134	699960
699960	32823	48084	51215	12321	27572	38983	42124	57375	18281	21412	34643	45854	53035	14541	25752	36763	43934	55155	16461	23632	699960
699960	17071	22522	37773	42624	57875	11111	28482	31613	48784	51915	15251	24342	35553	44844	56065	13331	26262	33433	46964	54145	699960
699960	41714	58685	17871	21012	38583	47674	52725	11911	27172	32423	43534	56865	16061	23232	36363	45454	54945	14141	25352	34243	699960
699960	21812	37073	42524	57775	18681	27972	31113	48484	51615	12721	24042	35253	44344	55555	16861	26162	33333	46264	53435	14941	699960
699960	58585	17771	22622	37873	41014	52425	11611	28782	31913	47174	56365	15551	24842	36063	43234	54245	13431	26962	34143	45354	699960
699960	38683	41814	57075	18581	21712	32723	47974	51115	12421	27672	36863	44044	55255	16361	23532	34943	46164	53335	14241	25452	699960
699960	15351	24242	35453	44944	56165	13231	26362	33533	46864	54045	17171	22422	37673	42724	57975	11011	28582	31713	48684	51815	699960
699960	43434	56965	16161	23332	36263	45554	54845	14041	25252	34343	41614	58785	17971	21112	38483	47774	52625	11811	27072	32523	699960
699960	24142	35353	44244	55455	16961	26062	33233	46364	53535	14841	21912	37173	42424	57675	18781	27872	31013	48584	51715	12621	699960
699960	56265	15451	24942	36163	43334	54345	13531	26862	34043	45254	58485	17671	22722	37973	41114	52525	11711	28682	31813	47074	699960
699960	36963	44144	55355	16261	23432	34843	46064	53235	14341	25552	38783	41914	57175	18481	21612	32623	47874	51015	12521	27772	699960
	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960

In this case, all the 5×5 blocks are pandiagonal magic squares of order 5 with different magic sums giving a **pandiagonal** magic square of order 4 given in example below.

Example 21. *A **pandiagonal** magic square of order 4, formed by magic sums of 16 magic squares of order 5 is given by*

		699960	699960	699960	699960
	173925	175845	172115	178075	699960
699960	172015	178175	173825	175945	699960
699960	177865	171905	176055	174135	699960
699960	176155	174035	177965	171805	699960
	699960	699960	699960	699960	699960

7 Selfie and Upside Down Magic Squares

By **rotatable** or **upside down** magic squares, it is understood that when we make a rotation of 180^o, it remains the same. The **Mirror looking** magic square are when seen through mirror or their **reflection in water**, remains again as magic square. It may happen that when rotating to 180^o or looking through mirror, they are magic squares, but with different sums. In this case, they are not **Selfie magic square**. **Seflie** magic squares are also known by **Universal** magic squares.

More precisely, a **Selfie magic** square should have the following three properties:

(i) Upside down (180^o rotatable);

(ii) Mirror looking;

(iii) Having the same sum (sum of rows, columns and of principal diagonals) even after rotation and mirror lo.

See below two examples:

2552	5255	2222	5525
2225	5522	2555	5252
5555	2252	5225	2522
5222	2525	5552	2255

6996	9699	6666	9969
6669	9966	6999	9696
9999	6696	9669	6966
9666	6969	9996	6699

The first example is **upside down** and **mirror looking**, while second is only **upside down**. For more details refer author's work. Sometimes **Selfie** magic square with letters are famous as **IXOHOXI magic squares**.

We observe that, there are only five digits 0, 1, 2, 5 and 8 written in **digital form** give us the conditions to bring **Selfie** magic square. In case of 180^o rotation all the five remains the same, while in case of mirror looking, 0, 1 and 8 remains the same and 2 becomes 5, and vice versa. Also in case of 180^o the numbers 6 and 9 changes with each other. Below are two digits combinations of numbers 0, 1, 2, 5 and 8 in digital form. See below:

00	01	02	05	08
10	11	12	15	18
20	21	22	25	28
50	51	52	55	58
80	81	82	85	88

Below is a **Selfie** magic square of order 5 having all the above combinations:

88+88	88+88	88+88	88+88	88+88	88+88	88+88
88+88	00	11	22	55	88	88+88
88+88	52	85	08	10	21	88+88
88+88	18	20	51	82	05	88+88
88+88	81	02	15	28	50	88+88
88+88	25	58	80	01	12	88+88
88+88	88+88	88+88	88+88	88+88	88+88	88+88

This sections bring different types **Selfie** and **upside down** magic squares of order 20. Reduced digits magic squares are also studied. There are many ways to write 20 numbers, out of these 25. Let's consider the following 20 numbers:

00 01 02 05 10 11 12 15 20 21 22 25 28 50 51 52 55 58 82 85

7.1 Blocks of Order 4

Based on above numbers below are some magic squares of order 20, where each block is a magic square of order 4.

Example 22. *A Selfie magic square of order 20 using the digits 0, 1, 2, 5 and 8 formed by 25 blocks of magic squares of order 4 with different magic sums is given by*

0000	0110	1011	1101	0282	2085	0558	5028	1202	2105	1550	5120	8222	2825	8552	5855	2212	5515	2551	5221
1001	1111	0010	0100	0528	5058	0285	2082	1520	5150	1205	2102	8555	5852	8225	2822	2521	5251	2215	5512
1110	1000	0101	0011	5085	0582	2028	0258	5105	1502	2120	1250	5825	8522	2855	8252	5215	2512	5521	2251
0111	0001	1100	1010	2058	0228	5082	0585	2150	1220	5102	1505	2852	8255	5822	8525	5551	2221	5212	2515
8202	2805	8550	5820	2222	5525	2552	5255	0012	0115	1051	1121	0200	2010	0511	5001	1282	2185	1558	5128
8520	5850	8205	2802	2555	5252	2225	5522	1021	1151	0015	0112	0501	5011	0210	2000	1528	5158	1285	2182
5805	8502	2820	8250	5225	2522	5555	2252	1115	1012	0121	0051	5010	0500	2001	0211	5185	1582	2128	1258
2850	8220	5802	8505	5552	2255	5222	2525	0151	0021	1112	1015	2011	0201	5000	0510	2158	1228	5182	1585
0212	2015	0551	5021	1200	2110	1511	5101	8282	2885	8558	5828	2202	5505	2550	5220	0022	0125	1052	1155
0521	5051	0215	2012	1501	5111	1210	2100	8528	5858	8285	2882	2520	5250	2205	5502	1055	1152	0025	0122
5015	0512	2021	0251	5110	1500	2101	1211	5885	8582	2828	8258	5205	2502	5520	2250	1125	1022	0155	0052
2051	0221	5012	0515	2111	1201	5100	1510	2858	8228	5882	8585	5550	2220	5202	2505	0152	0055	1122	1025
2282	5585	2558	5228	0002	0105	1050	1120	0222	2025	0552	5055	1212	2115	1551	5121	8200	2810	8511	5801
2528	5258	2285	5582	1020	1150	0005	0102	0555	5052	0225	2022	1521	5151	1215	2112	8501	5811	8210	2800
5285	2582	5528	2258	1105	1002	0120	0050	5025	0522	2055	0252	5115	1512	2121	1251	5810	8500	2801	8211
5558	2228	5282	2585	0150	0020	1102	1005	2052	0255	5022	0525	2151	1221	5112	1515	2811	8201	5800	8510
1222	2125	1552	5155	8212	2815	8551	5821	2200	5510	2511	5201	0082	0185	1058	1128	0202	2005	0550	5020
1555	5152	1225	2122	8521	5851	8215	2812	2501	5211	2210	5500	1028	1158	0085	0182	0520	5050	0205	2002
5125	1522	2155	1252	5815	8512	2821	8251	5210	2500	5501	2211	1185	1082	0128	0058	5005	0502	2020	0250
2152	1255	5122	1525	2851	8221	5812	8515	5511	2201	5200	2510	0158	0028	1182	1085	2050	0220	5002	0505

In this case, the magic sum is $S_{20\times 20} = 61105$. All the 4×4 blocks are magic square of order 4 with the different magic sums forming again a **pandiagonal** magic square of order 5 given in example below:

Example 23. *The* ***pandiagonal*** *magic square of order 5 formed by magic sums of order 4 of Example 22 is given by*

		61105	*61105*	*61105*	*61105*	*61105*
	2222	*7953*	*9977*	*25454*	*15499*	*61105*
61105	*25377*	*15554*	*2299*	*7722*	*10153*	*61105*
61105	*7799*	*9922*	*25553*	*15477*	*2354*	*61105*
61105	*15653*	*2277*	*7854*	*9999*	*25322*	*61105*
61105	*10054*	*25399*	*15422*	*2453*	*7777*	*61105*
	61105	*61105*	*61105*	*61105*	*61105*	*61105*

Example 24. *A* ***upside down*** *magic square of order 20 using the digits 2, 5, 6, 8 and 9 formed by 25 blocks of magic squares of order 4 with different magic sums is given by*

6666	6996	9699	9969	6282	2685	6558	5628	9262	2965	9556	5926	8222	2825	8552	5855	2292	5595	2559	5229
9669	9999	6696	6966	6528	5658	6285	2682	9526	5956	9265	2962	8555	5852	8225	2822	2529	5259	2295	5592
9996	9666	6969	6699	5685	6582	2628	6258	5965	9562	2926	9256	5825	8522	2855	8252	5295	2592	5529	2259
6999	6669	9966	9696	2658	6228	5682	6585	2956	9226	5962	9565	2852	8255	5822	8525	5559	2229	5292	2595
8262	2865	8556	5826	2222	5525	2552	5255	6692	6995	9659	9929	6266	2696	6599	5669	9282	2985	9558	5928
8526	5856	8265	2862	2555	5252	2225	5522	9629	9959	6695	6992	6569	5699	6296	2666	9528	5958	9285	2982
5865	8562	2826	8256	5225	2522	5555	2252	9995	9692	6929	6659	5696	6566	2669	6299	5985	9582	2928	9258
2856	8226	5862	8565	5552	2255	5222	2525	6959	6629	9992	9695	2699	6269	5666	6596	2958	9228	5982	9585
6292	2695	6559	5629	9266	2996	9599	5969	8282	2885	8558	5828	2262	5565	2556	5226	6622	6925	9652	9955
6529	5659	6295	2692	9569	5999	9296	2966	8528	5858	8285	2882	2526	5256	2265	5562	9655	9952	6625	6922
5695	6592	2629	6259	5996	9566	2969	9299	5885	8582	2828	8258	5265	2562	5526	2256	9925	9622	6955	6652
2659	6229	5692	6595	2999	9269	5966	9596	2858	8228	5882	8585	5556	2226	5262	2565	6952	6655	9922	9625
2282	5585	2558	5228	6662	6965	9656	9926	6222	2625	6552	5655	9292	2995	9559	5929	8266	2896	8599	5869
2528	5258	2285	5582	9626	9956	6665	6962	6555	5652	6225	2622	9529	5959	9295	2992	8569	5899	8296	2866
5285	2582	5528	2258	9965	9662	6926	6656	5625	6522	2655	6252	5995	9592	2929	9259	5896	8566	2869	8299
5558	2228	5282	2585	6956	6626	9962	9665	2652	6255	5622	6525	2959	9229	5992	9595	2899	8269	5866	8596
9222	2925	9552	5955	8292	2895	8559	5829	2266	5596	2599	5269	6682	6985	9658	9928	6262	2665	6556	5626
9555	5952	9225	2922	8529	5859	8295	2892	2569	5299	2296	5566	9628	9958	6685	6982	6526	5656	6265	2662
5925	9522	2955	9252	5895	8592	2829	8259	5296	2566	5569	2299	9985	9682	6928	6658	5665	6562	2626	6256
2952	9255	5922	9525	2859	8229	5892	8595	5599	2269	5266	2596	6958	6628	9982	9685	2656	6226	5662	6565

In this case, the magic sum is $S_{20\times 20} = 123321$. All the 4×4 blocks are magic square of order 4 with the different magic sums forming again a **pandiagonal** magic square of order 5 given in example below:

Example 25. *The **pandiagonal** magic square of order 5 formed by magic sums of order 4 of Example 24 is given by*

		123321	123321	123321	123321	123321
	33330	21153	27709	25454	15675	123321
123321	25509	15554	33275	21230	27753	123321
123321	21175	27830	25553	15609	33154	123321
123321	15653	33209	21054	27775	25630	123321
123321	27654	25575	15730	33253	21109	123321
	123321	123321	123321	123321	123321	123321

Example 26. *A **upside down** magic square of order 20 using the digits 0, 1, 6, 8 and 9 formed by 25 blocks of magic squares of order 4 with different magic sums is given by*

0000	0110	1011	1101	0686	6089	0998	9068	1606	6109	1990	9160	8666	6869	8996	9899	6616	9919	6991	9661
1001	1111	0010	0100	0968	9098	0689	6086	1960	9190	1609	6106	8999	9896	8669	6866	6961	9691	6619	9916
1110	1000	0101	0011	9089	0986	6068	0698	9109	1906	6160	1690	9869	8966	6899	8696	9619	6916	9961	6691
0111	0001	1100	1010	6098	0668	9086	0989	6190	1660	9106	1909	6896	8699	9866	8969	9991	6661	9616	6919
8606	6809	8990	9860	6666	9969	6996	9699	0016	0119	1091	1161	0600	6010	0911	9001	1686	6189	1998	9168
8960	9890	8609	6806	6999	9696	6669	9966	1061	1191	0019	0116	0901	9011	0610	6000	1968	9198	1689	6186
9809	8906	6860	8690	9669	6966	9999	6696	1119	1016	0161	0091	9010	0900	6001	0611	9189	1986	6168	1698
6890	8660	9806	8909	9996	6699	9666	6969	0191	0061	1116	1019	6011	0601	9000	0910	6198	1668	9186	1989
0616	6019	0991	9061	1600	6110	1911	9101	8686	6889	8998	9868	6606	9909	6990	9660	0066	0169	1096	1199
0961	9091	0619	6016	1901	9111	1610	6100	8968	9898	8689	6886	6960	9690	6609	9906	1099	1196	0069	0166
9019	0916	6061	0691	9110	1900	6101	1611	9889	8986	6868	8698	9609	6906	9960	6690	1169	1066	0199	0096
6091	0661	9016	0919	6111	1601	9100	1910	6898	8668	9886	8989	9990	6660	9606	6909	0196	0099	1166	1069
6686	9989	6998	9668	0006	0109	1090	1160	0666	6069	0996	9099	1616	6119	1991	9161	8600	6810	8911	9801
6968	9698	6689	9986	1060	1190	0009	0106	0999	9096	0669	6066	1961	9191	1619	6116	8901	9811	8610	6800
9689	6986	9968	6698	1109	1006	0160	0090	9069	0966	6099	0696	9119	1916	6161	1691	9810	8900	6801	8611
9998	6668	9686	6989	0190	0060	1106	1009	6096	0699	9066	0969	6191	1661	9116	1919	6811	8601	9800	8910
1666	6169	1996	9199	8616	6819	8991	9861	6600	9910	6911	9601	0086	0189	1098	1168	0606	6009	0990	9060
1999	9196	1669	6166	8961	9891	8619	6816	6901	9611	6610	9900	1068	1198	0089	0186	0960	9090	0609	6006
9169	1966	6199	1696	9819	8916	6861	8691	9610	6900	9901	6611	1189	1086	0168	0098	9009	0906	6060	0690
6196	1699	9166	1969	6891	8661	9816	8919	9911	6601	9600	6910	0198	0068	1186	1089	6090	0660	9006	0909

In this case, the magic sum is $S_{20\times20} = 105545$. All the 4×4 blocks are magic square of order 4 with the different magic sums forming again a **pandiagonal** magic square of order 5 given in example below:

Example 27. *The **pandiagonal** magic square of order 5 formed by magic sums of order 4 of Example 26 is given by*

		105545	105545	105545	105545	105545
	2222	16841	18865	34430	33187	105545
105545	34265	33330	2387	16522	19041	105545
105545	16687	18722	34441	33165	2530	105545
105545	33341	2365	16830	18887	34122	105545
105545	19030	34287	33022	2541	16665	105545
	105545	105545	105545	105545	105545	105545

7.2 Blocks of Order 5

Example 28. *A Selfie magic square of order 20 formed by 16 blocks of magic squares of order 5 with different magic sums using the digits 0, 1, 2, 5 and 8 is given by*

2222	2518	5225	1881	8152	5511	2815	8212	5851	8521	1188	1208	2101	1580	5110	8855	0158	1028	0885	8082
1825	8181	2252	2522	5218	5812	8551	5521	2811	8215	1501	5180	1110	1288	2108	0828	8085	8882	0155	1058
2552	5222	1818	8125	2281	2821	8211	5815	8512	5551	1210	2188	1508	5101	1180	0182	1055	0858	8028	8885
8118	2225	2581	5252	1822	8515	5512	2851	8221	5811	5108	1101	1280	2110	1588	8058	8828	0185	1082	0855
5281	1852	8122	2218	2525	8251	5821	8511	5515	2812	2180	1510	5188	1108	1201	1085	0882	8055	8858	0128
1155	1258	2128	1585	5182	8888	0108	1001	0880	8010	2211	2515	5212	1851	8121	5522	2818	8225	5881	8552
1528	5185	1182	1255	2158	0801	8080	8810	0188	1008	1812	8151	2221	2511	5215	5825	8581	5552	2822	8218
1282	2155	1558	5128	1185	0110	1088	0808	8001	8880	2521	5211	1815	8112	2251	2852	8222	5818	8525	5581
5158	1128	1285	2182	1555	8008	8801	0180	1010	0888	8115	2212	2551	5221	1811	8518	5525	2881	8252	5822
2185	1582	5155	1158	1228	1080	0810	8088	8808	0101	5251	1821	8111	2215	2512	8281	5852	8522	5518	2825
8811	0115	1012	0851	8021	1122	1218	2125	1581	5152	5555	2858	8228	5885	8582	2288	2501	5210	1808	8180
0812	8051	8821	0111	1015	1525	5181	1152	1222	2118	5828	8585	5582	2855	8258	1810	8108	2280	2588	5201
0121	1011	0815	8012	8851	1252	2122	1518	5125	1181	2882	8255	5858	8528	5585	2580	5288	1801	8110	2208
8015	8812	0151	1021	0811	5118	1125	1281	2152	1522	8558	5528	2885	8282	5855	8101	2210	2508	5280	1888
1051	0821	8011	8815	0112	2181	1552	5122	1118	1225	8285	5882	8555	5558	2828	5208	1880	8188	2201	2510
5588	2808	8201	5880	8510	2255	2558	5228	1885	8182	8822	0118	1025	0881	8052	1111	1215	2112	1551	5121
5801	8580	5510	2888	8208	1828	8185	2282	2555	5258	0825	8081	8852	0122	1018	1512	5151	1121	1211	2115
2810	8288	5808	8501	5580	2582	5255	1858	8128	2285	0152	1022	0818	8025	8881	1221	2111	1515	5112	1151
8508	5501	2880	8210	5888	8158	2228	2585	5282	1855	8018	8825	0181	1052	0822	5115	1112	1251	2121	1511
8280	5810	8588	5508	2801	5285	1882	8155	2258	2528	1081	0852	8022	8818	0125	2151	1521	5111	1115	1212

In this case, the magic sum is $S_{20\times 20} = 81103$. All the 5×5 blocks are magic square of order 5 with the different magic sums forming again a magic square of order 4 given in example below:

Example 29. *The magic square of order 4 formed by magic sums of order 5 of Example 28 is given by*

				81103
19998	30910	11187	19008	81103
11308	18887	19910	30998	81103
18810	11198	31108	19987	81103
30987	20108	18898	11110	81103
81103	81103	81103	81103	81103

Example 30. *A Selfie magic square of order 20 formed by 16 blocks of magic squares of order 5 with different magic sums constructed according to Example 2 is given by*

2222	2598	5225	9889	8952	5599	2895	8292	5859	8529	9988	9268	2969	9586	5996	8855	6958	9628	6885	8682
9825	8989	2252	2522	5298	5892	8559	5529	2899	8295	9569	5986	9996	9288	2968	6828	8685	8882	6955	9658
2552	5222	9898	8925	2289	2829	8299	5895	8592	5559	9296	2988	9568	5969	9986	6982	9655	6858	8628	8885
8998	2225	2589	5252	9822	8595	5592	2859	8229	5899	5968	9969	9286	2996	9588	8658	8828	6985	9682	6855
5289	9852	8922	2298	2525	8259	5829	8599	5595	2892	2986	9596	5988	9968	9269	9685	6882	8655	8858	6928
9955	9258	2928	9585	5982	8888	6968	9669	6886	8696	2299	2595	5292	9859	8929	5522	2898	8225	5889	8552
9528	5985	9982	9255	2958	6869	8686	8896	6988	9668	9892	8959	2229	2599	5295	5825	8589	5552	2822	8298
9282	2955	9558	5928	9985	6996	9688	6868	8669	8886	2529	5299	9895	8992	2259	2852	8222	5898	8525	5589
5958	9928	9285	2982	9555	8668	8869	6986	9696	6888	8995	2292	2559	5229	9899	8598	5525	2889	8252	5822
2985	9582	5955	9958	9228	9686	6896	8688	8868	6969	5259	9829	8999	2295	2592	8289	5852	8522	5598	2825
8899	6995	9692	6859	8629	9922	9298	2925	9589	5952	5555	2858	8228	5885	8582	2288	2569	5296	9868	8986
6892	8659	8829	6999	9695	9525	5989	9952	9222	2998	5828	8585	5582	2855	8258	9896	8968	2286	2588	5269
6929	9699	6895	8692	8859	9252	2922	9598	5925	9989	2882	8255	5858	8528	5585	2586	5288	9869	8996	2268
8695	8892	6959	9629	6899	5998	9925	9289	2952	9522	8558	5528	2885	8282	5855	8969	2296	2568	5286	9888
9659	6829	8699	8895	6992	2989	9552	5922	9998	9225	8285	5882	8555	5558	2828	5268	9886	8988	2269	2596
5588	2868	8269	5886	8596	2255	2558	5228	9885	8982	8822	6998	9625	6889	8652	9999	9295	2992	9559	5929
5869	8586	5596	2888	8268	9828	8985	2282	2555	5258	6825	8689	8852	6922	9698	9592	5959	9929	9299	2995
2896	8288	5868	8569	5586	2582	5255	9858	8928	2285	6952	9622	6898	8625	8889	9229	2999	9595	5992	9959
8568	5569	2886	8296	5888	8958	2228	2585	5282	9855	8698	8825	6989	9652	6822	5995	9992	9259	2929	9599
8286	5896	8588	5568	2869	5285	9882	8955	2258	2528	9689	6852	8622	8898	6925	2959	9529	5999	9995	9292

In this case, the magic sum is $S_{20\times 20} = 138875$. All the 5×5 blocks are magic square of order 5 with the different magic sums forming again a magic square of order 4 given in example below:

Example 31. *The magic square of order 4 formed by magic sums of order 5 of Example 30 is given by*

				138875
28886	*31174*	*37807*	*41008*	*138875*
37708	*41107*	*28974*	*31086*	*138875*
41074	*37686*	*31108*	*29007*	*138875*
31207	*28908*	*40986*	*37774*	*138875*
138875	*138875*	*138875*	*138875*	*138875*

Example 32. *A upside down magic square of order 20 formed by 25 blocks of magic squares of order 4 with different magic sums constructed according to Example 2 having the digits 0, 1, 6, 8 and 9 is given by*

6666	6918	9669	1881	8196	9911	6819	8616	9891	8961	1188	1608	6101	1980	9110	8899	0198	1068	0889	8086
1869	8181	6696	6966	9618	9816	8991	9961	6811	8619	1901	9180	1110	1688	6108	0868	8089	8886	0199	1098
6996	9666	1818	8169	6681	6861	8611	9819	8916	9991	1610	6188	1908	9101	1180	0186	1099	0898	8068	8889
8118	6669	6981	9696	1866	8919	9916	6891	8661	9811	9108	1101	1680	6110	1988	8098	8868	0189	1086	0899
9681	1896	8166	6618	6969	8691	9861	8911	9919	6816	6180	1910	9188	1108	1601	1089	0886	8099	8898	0168
1199	1698	6168	1989	9186	8888	0108	1001	0880	8010	6611	6919	9616	1891	8161	9966	6818	8669	9881	8996
1968	9189	1186	1699	6198	0801	8080	8810	0188	1008	1816	8191	6661	6911	9619	9869	8981	9996	6866	8618
1686	6199	1998	9168	1189	0110	1088	0808	8001	8880	6961	9611	1819	8116	6691	6896	8666	9818	8969	9981
9198	1168	1689	6186	1999	8008	8801	0180	1010	0888	8119	6616	6991	9661	1811	8918	9969	6881	8696	9866
6189	1986	9199	1198	1668	1080	0810	8088	8808	0101	9691	1861	8111	6619	6916	8681	9896	8966	9918	6869
8811	0119	1016	0891	8061	1166	1618	6169	1981	9196	9999	6898	8668	9889	8986	6688	6901	9610	1808	8180
0816	8091	8861	0111	1019	1969	9181	1196	1666	6118	9868	8989	9986	6899	8698	1810	8108	6680	6988	9601
0161	1011	0819	8016	8891	1696	6166	1918	9169	1181	6886	8699	9898	8968	9989	6980	9688	1801	8110	6608
8019	8816	0191	1061	0811	9118	1169	1681	6196	1966	8998	9968	6889	8686	9899	8101	6610	6908	9680	1888
1091	0861	8011	8819	0116	6181	1996	9166	1118	1669	8689	9886	8999	9998	6868	9608	1880	8188	6601	6910
9988	6808	8601	9880	8910	6699	6998	9668	1889	8186	8866	0118	1069	0881	8096	1111	1619	6116	1991	9161
9801	8980	9910	6888	8608	1868	8189	6686	6999	9698	0869	8081	8896	0166	1018	1916	9191	1161	1611	6119
6810	8688	9808	8901	9980	6986	9699	1898	8168	6689	0196	1066	0818	8069	8881	1661	6111	1919	9116	1191
8908	9901	6880	8610	9888	8198	6668	6989	9686	1899	8018	8869	0181	1096	0866	9119	1116	1691	6161	1911
8680	9810	8988	9908	6801	9689	1886	8199	6698	6968	1081	0896	8066	8818	0169	6191	1961	9111	1119	1616

In this case, the magic sum is $S_{20\times 20} = 116655$. All the 5×5 blocks are magic square of order 5 with the different magic sums forming again a magic square of order 4 given in example below:

Example 33. *The magic square of order 4 formed by magic sums of order 5 of Example 32 is given by*

				116655
33330	44198	19987	19140	116655
20240	18887	33198	44330	116655
18898	20130	44440	33187	116655
44187	33440	19030	19998	116655
116655	116655	116655	116655	116655

7.3 Three Digits

Below are selfie and upside down magic squares of order 20 having only three digits. In case of selfie magic squares, we considered 0, 1 and 8. In case of upside down 1, 6 and 9 are considered. We have written only in terms of blocks of order 4. In the similar way, blocks of order 5 can be constructed.

Example 34. *A Selfie magic square of order 20 formed by 25 blocks of magic squares of order 4 with different magic sums having the digits 0, 1 and 8 is given by*

010010	080101	101808	808080	001180	100810	008018	800081	011001	110008	088800	880100	180108	081181	810818	018801	108011	801088	181880	818110
101080	808808	010101	080010	008081	800018	001810	100180	088100	880800	011008	110001	810801	018818	180181	081108	181110	818880	108088	801011
808101	101010	080080	010808	800810	008180	100081	001018	880008	088001	110100	011800	018181	810108	081801	180818	818088	181011	801110	108880
080808	010080	808010	101101	100018	001081	800180	008810	110800	011100	880001	088008	081818	180801	018108	810181	801880	108110	818011	181088
180001	081008	810800	018100	108108	801181	181818	818801	010011	080088	101880	808110	001010	100101	008808	800080	011180	110810	088018	880081
810100	018800	180008	081001	181801	818818	108181	801108	101110	808880	010088	080011	008080	800808	001101	100010	088081	880018	011810	110180
018008	810001	081100	180800	818181	181108	801801	108818	808088	101011	080110	010880	800101	008010	100080	001808	880810	088180	110081	011018
081800	180100	018001	810008	801818	108801	818108	181181	080880	010110	808011	101088	100808	001080	800010	008101	110018	011081	880180	088810
001011	100088	008880	800110	011010	110101	088808	880080	180180	081810	810018	018081	108001	801008	181800	818100	010108	080181	101818	808801
008110	800880	001088	100011	088080	880808	011101	110010	810081	018018	180810	081180	181100	818800	108008	801001	101801	808818	010181	080108
800088	008011	100110	001880	880101	088010	110080	011808	018810	810180	081081	180018	818008	181001	801100	108800	808181	101108	080801	010818
100880	001110	800011	008088	110808	011080	880010	088101	081018	180081	018180	810810	801800	108100	818001	181008	080818	010801	808108	101181
108180	801810	181018	818081	010001	080008	101800	808100	001108	100181	008818	800801	011011	110088	088880	880110	180010	081101	810808	018080
181081	818018	108810	801180	101100	808800	010008	080001	008801	800818	001181	100108	088110	880880	011088	110011	810080	018808	180101	081010
818810	181180	801081	108018	808008	101001	080100	010800	800181	008108	100801	001818	880088	088011	110110	011880	018101	810010	081080	180808
801018	108081	818180	181810	080800	010100	808001	101008	100818	001801	800108	008181	110880	011110	880011	088088	081808	180080	018010	810101
011108	110181	088818	880801	180011	081088	810880	018110	108010	801101	181808	818080	010180	080810	101018	808081	001001	100008	008800	800100
088801	880818	011181	110108	810110	018880	180088	081011	181080	818808	108101	801010	101081	808018	010810	080180	008100	800800	001008	100001
880181	088108	110801	011818	018088	810011	081110	180880	818101	181010	801080	108808	808810	101180	080081	010018	800008	008001	100100	001800
110818	011801	880108	088181	081880	180110	018011	810088	801808	108080	818010	181101	080018	010081	808180	101810	100800	001100	800001	008008

In this case, the magic sum is $S_{20\times 20} = 5999994$. All the 4×4 blocks are magic square of order 4 with the different magic sums forming again a **pandiagonal** magic square of order 5 given in example below:

Example 35. *The* ***pandiagonal*** *magic square of order 5 formed by magic sums of order 4 of Example 34 is given by*

		5999994	5999994	5999994	5999994	5999994
	999999	910089	1089909	1090908	1909089	5999994
5999994	1089909	1909908	1000089	909999	1090089	5999994
5999994	910089	1089999	1090089	1908909	1000908	5999994
5999994	1909089	999909	910908	1090089	1089999	5999994
5999994	1090908	1090089	1908999	1000089	909909	5999994
	5999994	5999994	5999994	5999994	5999994	5999994

Example 36. *A upside down magic square of order 20 formed by 25 blocks of magic squares of order 4 with different magic sums having the digits 1, 6 and 9 is given by*

161161	191616	616919	919191	116691	611961	119169	911196	166116	661119	199911	991611	691619	196696	961969	169916	619166	916199	696991	969661
616191	919919	161616	191161	119196	911169	116961	611691	199611	991911	166119	661116	961916	169969	691696	196619	696661	969991	619199	916166
919616	616161	191191	161919	911961	119691	611196	116169	991119	199116	661611	166911	169696	961619	196916	691969	969199	696166	916661	619991
191919	161191	919161	616616	611169	116196	911691	119961	661911	166611	991116	199119	196969	691916	169619	961696	916991	619661	969166	696199
691116	196119	961911	169611	619619	916696	696969	969916	161166	191199	616991	919661	116161	611616	119919	911191	166691	661961	199169	991196
961611	169911	691119	196116	696916	969969	619696	916619	616661	919991	161199	191166	119191	911919	116616	611161	199196	991169	166961	661691
169119	961116	196611	691911	969696	696619	916916	619969	919199	616166	191661	161991	911616	119161	611191	116919	991961	199691	661196	166169
196911	691611	169116	961119	916969	619916	969619	696696	191991	161661	919166	616199	611919	116191	911161	119616	661169	166196	991691	199961
116166	611199	119991	911661	166161	661616	199919	991191	691691	196961	961169	169196	619116	916119	696911	969611	161619	191696	616969	919916
119661	911991	116199	611166	199191	991919	166616	661161	961196	169169	691961	196691	696611	969911	619119	916116	616916	919969	161696	191619
911199	119166	611661	116991	991616	199161	661191	166919	169961	961691	196196	691169	969119	696116	916611	619911	919696	616619	191916	161969
611991	116661	911166	119199	661919	166191	991161	199616	196169	691196	169691	961961	916911	619611	969116	696119	191969	161916	919619	616696
619691	916961	696169	969196	161116	191119	616911	919611	116619	611696	119969	911916	166166	661199	199991	991661	691161	196616	961919	169191
696196	969169	619961	916691	616611	919911	161119	191116	119916	911969	116696	611619	199661	991991	166199	661166	961191	169919	691616	196161
969961	696691	916196	619169	919119	616116	191611	161911	911696	119619	611916	116969	991199	199166	661661	166991	169616	961161	196191	691919
916169	619196	969691	696961	191911	161611	919116	616119	611969	116916	911619	119696	661991	166661	991166	199199	196919	691191	169161	961616
166619	661696	199969	991916	691166	196199	961991	169661	619161	916616	696919	969191	161691	191961	616169	919196	116116	611119	119911	911611
199916	991969	166696	661619	961661	169991	691199	196166	696191	969919	619616	916161	616196	919169	161961	191691	119611	911911	116119	611116
991696	199619	661916	166969	169199	961166	196661	691991	969616	696161	916191	619919	919961	616691	191196	161169	911119	119116	611611	116911
661969	166916	991619	199696	196991	691661	169166	961199	916919	619191	969161	696616	191169	161196	919691	616961	611911	116611	911116	119119

In this case, the magic sum is $S_{20\times20} = 10888878$. All the 4×4 blocks are magic square of order 4 with the different magic sums forming again a **pandiagonal** magic square of order 5 given in example below:

Example 37. *The* ***pandiagonal*** *magic square of order 5 formed by magic sums of order 4 of Example 36 is given by*

		10888878	10888878	10888878	10888878	10888878
	1888887	1759017	2018757	2020200	3202017	10888878
10888878	2018757	3203200	1889017	1758887	2019017	10888878
10888878	1759017	2018887	2019017	3201757	1890200	10888878
10888878	3202017	1888757	1760200	2019017	2018887	10888878
10888878	2020200	2019017	3201887	1889017	1758757	10888878
	10888878	10888878	10888878	10888878	10888878	10888878

7.4 Two Digits

Below are selfie and upside down magic squares of order 20 having only three digits. In case of selfie magic squares, we considered 0 and 1 or 1 and 8. In case of upside down 6 and 9 are considered. We have written only in terms of blocks of order 4. In the similar way, blocks of order 5 can be constructed.

Example 38. *A Selfie magic square of order 20 formed by 25 blocks of magic squares of order 4 with different magic sums having the digits 0 and 1 is given by*

0000100001	1000001111	0111111110	1111010000	0001101000	0011100010	1110010111	1100011101	0011000011	1001111100	0101011000	1010100111	0100000101	1110111010	0001010100	1011101011	0010100110	0101101010	1101010101	1010010011
0111110000	1111011110	0000101111	1000000001	1110011101	1100010111	0001100010	0011101000	0101000111	1010111000	0011011100	1001100011	0001001011	1011110100	0100011010	1110100101	1101010011	1010010101	0010101010	0101100110
1111001111	0111100001	1000010000	0000111110	1100000010	1110001000	0011111101	0001110111	1010111100	0101000011	1001100111	0011011000	1011111010	0001000101	1110101011	0100010100	1010001010	1101000110	0101110011	0010110101
1000011110	0000110000	1111000001	0111101111	0011110111	0001111101	1100001000	1110000010	1001111000	0011000111	1010100011	0101011100	1110110100	0100001011	1011100101	0001011010	0101110101	0010110011	1010000110	1101001010
0100000011	1110111100	0001011000	1011100111	0010100101	0101111010	1101010100	1010001011	0000100110	1000001010	0111110101	1111010011	0001100001	0011101111	1110011110	1100010000	0011001000	1001100010	0101010111	1010111101
0001000111	1011111000	0100011100	1110100011	1101001011	1010010100	0010111010	0101100101	0111110011	1111010101	0000101010	1000000110	1110010000	1100011110	0001101111	0011100001	0101011101	1010110111	0011000010	1001101000
1011111100	0001000011	1110100111	0100011000	1010011010	1101000101	0101101011	0010110100	1111001010	0111100110	1000010011	0000110101	1100001111	1110000001	0011110000	0001111110	1010100010	0101001000	1001111101	0011010111
1110111000	0100000111	1011100011	0001011100	0101110100	0010101011	1010000101	1101011010	1000010101	0000110011	1111000110	0111101010	0011111110	0001110000	1100000001	1110001111	1001110111	0011011101	1010101000	0101000010
0001100110	0011101010	1110010101	1100010011	0011000001	1001101111	0101011110	1010110000	0100001000	1110100010	0001010111	1011111101	0010100011	0101111100	1101011000	1010000111	0000100101	1000011010	0111110100	1111001011
1110010011	1100010101	0001101010	0011100110	0101010000	1010111110	0011001111	1001100001	0001011101	1011110111	0100000010	1110101000	1101000111	1010011000	0010111100	0101100011	0111101011	1111010100	0000111010	1000000101
1100001010	1110000110	0011110011	0001110101	1010101111	0101000001	1001110000	0011011110	1011100010	0001001000	1110111101	0100010111	1010011100	1101000011	0101100111	0010111000	1111011010	0111100101	1000001011	0000110100
0011110101	0001110011	1100000110	1110001010	1001111110	0011010000	1010100001	0101001111	1110110111	0100011101	1011101000	0001000010	0101111000	0010100111	1010000011	1101011100	1000010100	0000101011	1111000101	0111111010
0010101000	0101100010	1101010111	1010011101	0000100011	1000011100	0111111000	1111000111	0001100101	0011111010	1110010100	1100001011	0011000110	1001101010	0101010101	1010110011	0100000001	1110101111	0001011110	1011110000
1101011101	1010010111	0010100010	0101101000	0111100111	1111011000	0000111100	1000000011	1110001011	1100010100	0001111010	0011100101	0101010011	1010110101	0011001010	1001100110	0001010000	1011111110	0100001111	1110100001
1010000010	1101001000	0101111101	0010110111	1111011100	0111100011	1000000111	0000111000	1100011010	1110000101	0011101011	0001110100	1010101010	0101000110	1001110011	0011010101	1011101111	0001000001	1110110000	0100011110
0101110111	0010111101	1010001000	1101000010	1000011000	0000100111	1111000011	0111111100	0011110100	0001101011	1100000101	1110011010	1001110101	0011010011	1010100110	0101001010	1110111110	0100010000	1011100001	0001001111
0011000101	1001111010	0101010100	1010101011	0100000110	1110101010	0001010101	1011110011	0010100001	0101101111	1101011110	1010010000	0000101000	1000000010	0111110111	1111011101	0001100011	0011111100	1110011000	1100000111
0101001011	1010110100	0011011010	1001100101	0001010011	1011110101	0100001010	1110100110	1101010000	1010011110	0010101111	0101100001	0111111101	1111010111	0000100010	1000001000	1110000111	1100011000	0001111100	0011100011
1010111010	0101000101	1001101011	0011010100	1011101010	0001000110	1110110011	0100010101	1010001111	1101000001	0101110000	0010111110	1111000010	0111101000	1000011101	0000110111	1100011100	1110000011	0011100111	0001111000
1001110100	0011001011	1010100101	0101011010	1110110101	0100010011	1011100110	0001001010	0101111110	0010110000	1010000001	1101001111	1000010111	0000111101	1111001000	0111100010	0011111000	0001100111	1100000011	1110011100

In this case, the magic sum is $S_{20\times 20} = 11012110120$. All the 4×4 blocks are magic square of order 4 with the different magic sums forming again a **pandiagonal** magic square of order 5 given in example below:

Example 39. *The **pandiagonal** magic square of order 5 formed by magic sums of order 4 of Example 39 is given by*

		11012110120	11012110120	11012110120	11012110120	11012110120
	2222222222	2222222222	2123222222	2222222222	2222221232	11012110120
11012110120	2222222222	2222222222	2222221232	2222222222	2123222222	11012110120
11012110120	2222221232	2123222222	2222222222	2222222222	2222222222	11012110120
11012110120	2222222222	2222222222	2222222222	2123221232	2222222222	11012110120
11012110120	2123222222	2222221232	2222222222	2222222222	2222222222	11012110120
	11012110120	11012110120	11012110120	11012110120	11012110120	11012110120

Example 40. *A Selfie magic square of order 20 formed by 25 blocks of magic squares of order 4 with different magic sums having the digits 1 and 8 is given by*

1111811118	8111118888	1888888881	8888181111	1118818111	1188811181	8881181888	8811188818	1188111188	8118888811	1818188111	8181811888	1811111818	8881888181	1118181811	8188818188	1181811881	1818818181	8818181818	8181181188
1888881111	8888188881	1111818888	8111111118	8881188818	8811181888	1118811181	1188818111	1818111888	8181888111	1188188811	8118811188	1118118188	8188881811	1811188181	8881811818	8818181188	8181181818	1181818181	1818811881
8888118888	1888811118	8111181111	1111888881	8811111181	8881118111	1188888818	1118881888	8181888811	1818111188	8118811888	1188188111	8188888181	1118111818	8881818188	1811181811	8181118181	8818111881	1818881188	1181881818
8111188881	1111881111	8888111118	1888818888	1188881888	1118888818	8811118111	8881111181	8118888111	1188111888	8181811188	1818188811	8881881811	1811118188	8188811818	1118188181	1818881818	1181881188	8181111881	8818118181
1811111188	8881888811	1118188111	8188811888	1181811818	1818888181	8818181811	8181118188	1111811881	8111118181	1888881818	8888181188	1118811118	1188818888	8881188881	8811181111	1188118111	8118811181	1818181888	8181888818
1118111888	8188888111	1811188811	8881811188	8818118188	8181181811	1181888181	1818811818	1888881188	8888181818	1111818181	8111111881	8881181111	8811188881	1118818888	1188811118	1818188818	8181881888	1188111181	8118818111
8188888811	1118111188	8881811888	1811188111	8181188181	8818111818	1818818188	1181881811	8888118181	1888811881	8111181188	1111881818	8811118888	8881111118	1188881111	1118888881	8181811181	1818118111	8118888818	1188181888
8881888111	1811111888	8188811188	1118188811	1818881811	1181818188	8181111818	8818188181	8111181818	1111881188	8888111881	1888818181	1188888881	1118881111	8811111118	8881118888	8118881888	1188188818	8181818111	1818111181
1118811881	1188818181	8881181818	8811181188	1188111118	8118818888	1818188881	8181881111	1811118111	8881811181	1118181888	8188888818	1181811188	1818888811	8818188111	8181111888	1111811818	8111188181	1888881811	8888118188
8881181188	8811181818	1118818181	1188811881	1818181111	8181888881	1188118888	8118811118	1118188818	8188881888	1811111181	8881818111	8818111888	8181188111	1181888811	1818811188	1888818188	8888181811	1111888181	8111111818
8811118181	8881111881	1188881188	1118881818	8181818888	1818111118	8118881111	1188188881	8188811181	1118118111	8881888818	1811181888	8181188811	8818111188	1818811888	1181888111	8888188181	1888811818	8111118188	1111881811
1188881818	1118881188	8811111881	8881118181	8118888881	1188181111	8181811118	1818118888	8881881888	1811188818	8188818111	1118111181	1818888111	1181811888	8181111188	8818188811	8111181811	1111818188	8888111818	1888888181
1181818111	1818811181	8818181888	8181188818	1111811188	8111188811	1888888111	8888111888	1118811818	1188888181	8881181811	8811118188	1188111881	8118818181	1818181818	8181881188	1811111118	8881818888	1118188881	8188881111
8818188818	8181181888	1181811181	1818818111	1888811888	8888188111	1111888811	8111111188	8881118188	8811181811	1118888181	1188811818	1818181188	8181881818	1188118181	8118811881	1118181111	8188888881	1811118888	8881811118
8181111181	8818118111	1818888818	1181881888	8888188811	1888811188	8111111888	1111888111	8811188181	8881111818	1188818188	1118881811	8181818181	1818111881	8118881188	1188181818	8188818888	1118111118	8881881111	1811188881
1818881888	1181888818	8181118111	8818111181	8111188111	1111811888	8888111188	1888888811	1188881811	1118818188	8811111818	8881188181	8118881818	1188181188	8181811881	1818118181	8881888881	1811181111	8188811118	1118118888
1188111818	8118888181	1818181811	8181818188	1811111881	8881818181	1118181818	8188881188	1181811118	1818818888	8818188881	8181181111	1111818111	8111111181	1888881888	8888188818	1118811188	1188888811	8881188111	8811111888
1818118188	8181881811	1188188181	8118811818	1118181188	8188881818	1811118181	8881811881	8818181111	8181188881	1181818888	1818811118	1888888818	8888181888	1111811181	8111118111	8881111888	8811188111	1118888811	1188811188
8181888181	1818111818	8118818188	1188181811	8188818181	1118111881	8881881188	1811181818	8181118888	8818111118	1818881111	1181888881	8888111181	1888818111	8111188818	1111881888	8811188811	8881111188	1188811888	1118888111
8118881811	1188118188	8181811818	1818188181	8881881818	1811181188	8188811881	1118118181	1818888881	1181881111	8181111118	8818118888	8111181888	1111888818	8888118111	1888811181	1188888111	1118811888	8811111188	8881188811

In this case, the magic sum is $S_{20\times20} = 99306993060$. All the 4×4 blocks are magic square of order 4 with the different magic sums forming again a **pandiagonal** magic square of order 5 given in example below:

Example 41. *The* ***pandiagonal*** *magic square of order 5 formed by magic sums of order 4 of Example 40 is given by*

		99306993060	99306993060	99306993060	99306993060	99306993060
	19999999998	19999999998	19306999998	19999999998	19999993068	99306993060
99306993060	19999999998	19999999998	19999993068	19999999998	19306999998	99306993060
99306993060	19999993068	19306999998	19999999998	19999999998	19999999998	99306993060
99306993060	19999999998	19999999998	19999999998	19306993068	19999999998	99306993060
99306993060	19306999998	19999993068	19999999998	19999999998	19999999998	99306993060
	99306993060	99306993060	99306993060	99306993060	99306993060	99306993060

Example 42. *A upside down magic square of order 20 formed by 25 blocks of magic squares of order 4 with different magic sums having the digits 6 and 9 is given by*

6666966669	9666669999	6999999996	9999696666	6669969666	6699966696	9996696999	9966699969	6699666699	9669999966	6969699666	9696966999	6966666969	9996999696	6669696966	9699969699	6696966996	6969969696	9969696969	9696696699
6999996666	9999699996	6666969999	9666666669	9996699969	9966696999	6669966696	6699969666	6969666999	9696999666	6699699966	9669966699	6669669699	9699996966	6966699696	9996966969	9969696699	9696696969	6696969696	6969966996
9999669999	6999966669	9666696666	6666999996	9966666696	9996669666	6699999969	6669996999	9696999966	6969666699	9669966999	6699699666	9699999696	6669666969	9996969699	6966696966	9696669696	9969666996	6969996699	6696996969
9666699996	6666996666	9999666669	6999969999	6699996999	6669999969	9966669666	9996666696	9669999666	6699666999	9696966699	6969699966	9996996966	6966669699	9699966969	6669699696	6969996969	6696996699	9696666996	9969669696
6966666699	9996999966	6669699666	9699966999	6696966969	6969999696	9969696966	9696669699	6666966996	9666669696	6999996969	9999696699	6669966669	6699969999	9996699996	9966696666	6699669666	9669966696	6969696999	9696999969
6669666999	9699999666	6966699966	9996966699	9969669699	9696696966	6696999696	6969966969	6999996699	9999696969	6666969696	9666666996	9996696666	9966699996	6669969999	6699966669	6969699969	9696996999	6699666696	9669969666
9699999966	6669666699	9996966999	6966699666	9696699696	9969666969	6969969699	6696996966	9999669696	6999966996	9666696699	6666996969	9966669999	9996666669	6699996666	6669999996	9696966696	6969669666	9669999969	6699696999
9996999666	6966666999	9699966699	6669699966	6969996966	6696969699	9696666969	9969699696	9666696969	6666996699	9999666996	6999969696	6699999996	6669996666	9966666669	9996669999	9669996999	6699699969	9696969666	6969666696
6669966996	6699969696	9996696969	9966696699	6699666669	9669969999	6969699996	9696996666	6966669666	9996966696	6669696999	9699999969	6696966699	6969999966	9969699666	9696666999	6666966969	9666699696	6999996966	9999669699
9996696699	9966696969	6669969696	6699966996	6969696666	9696999996	6699669999	9669966669	6669699969	9699996999	6966666696	9996969666	9969666999	9696699666	6696999966	6969966699	6999969699	9999696966	6666999696	9666666969
9966669696	9996666996	6699996699	6669996969	9696969999	6969666669	9669996666	6699699996	9699966696	6669669666	9996999969	6966696999	9696699966	9969666699	6969966999	6696999666	9999699696	6999966969	9666669699	6666996966
6699996969	6669996699	9966666996	9996669696	9669999996	6699696666	9696966669	6969669999	9996996999	6966699969	9699969666	6669666696	6969999666	6696966999	9696666699	9969699966	9666696966	6666969699	9999666969	6999999696
6696969666	6969966696	9969696999	9696699969	6666966699	9666699966	6999999666	9999666999	6669966969	6699999696	9996696966	9966669699	6699666996	9669969696	6969696969	9696996699	6966666669	9996969999	6669699996	9699996666
9969699969	9696696999	6696966696	6969969666	6999966999	9999699666	6666999966	9666666699	9996669699	9966696966	6669999696	6699966969	6969696699	9696996969	6699669696	9669966996	6669696666	9699999996	6966669999	9996966669
9696666696	9969669666	6969999969	6696996999	9999699966	6999966699	9666666999	6666999666	9966699696	9996666969	6699969699	6669996966	9696969696	6969666996	9669996699	6699696969	9699969999	6669666669	9996996666	6966699996
6969996999	6696999969	9696669666	9969666696	9666699666	6666966999	9999666699	6999999966	6699996966	6669969699	9966666969	9996699696	9669996969	6699696699	9696966996	6969669696	9996999996	6966696666	9699966669	6669669999
6699666969	9669999696	6969696966	9696969699	6966666996	9996969696	6669696969	9699996699	6696966669	6969969999	9969699996	9696696666	6666969666	9666666696	6999996999	9999699969	6669966699	6699999966	9996699666	9966666999
6969669699	9696996966	6699699696	9669966969	6669696699	9699996969	6966669696	9996966996	9969696666	9696699996	6696969999	6969966669	6999999969	9999696999	6666966696	9666669666	9996666999	9966699666	6669999966	6699966699
9696999696	6969666969	9669969699	6699696966	9699969696	6669666996	9996996699	6966696969	9696669999	9969666669	6969996666	6696999996	9999666696	6999969666	9666699969	6666996999	9966699966	9996666699	6699966999	6669999666
9669996966	6699669699	9696966969	6969699696	9996996969	6966696699	9699966996	6669669696	6969999996	6696996666	9696666669	9969669999	9666696999	6666999969	9999669666	6999966696	6699999666	6669966999	9966666699	9996699966

In this case, the magic sum is $S_{20\times20} = 166369663680$. All the 4×4 blocks are magic square of order 4 with the different magic sums forming again a **pandiagonal** magic square of order 5 given in example below:

Example 43. *The* ***pandiagonal*** *magic square of order 5 formed by magic sums of order 4 of Example 42 is given by*

		166369663680	166369663680	166369663680	166369663680	166369663680
	33333333330	33333333330	33036333330	33333333330	33333330360	166369663680
166369663680	33333333330	33333333330	33333330360	33333333330	33036333330	166369663680
166369663680	33333330360	33036333330	33333333330	33333333330	33333333330	166369663680
166369663680	33333333330	33333333330	33333333330	33036330360	33333333330	166369663680
166369663680	33036333330	33333330360	33333333330	33333333330	33333333330	166369663680
	166369663680	166369663680	166369663680	166369663680	166369663680	166369663680

8 Author's Contributions to Magic Squares

The item-wise author's contribution to magic squares:

(i) **Digital Numbers** Magic Squares - [2, 3, 4, 5, 6, 7];

(ii) Connections with **Genetic Tables** and **Shannon's entropy** - [9];

(iii) **Selfie** and **palindromic-type** Magic Squares - [10, 26];

(iv) **Intervally Distributed** and **Block-Wise** Magic Squares - [11, 12, 13, 27];

(v) **Multi-digits** and **Number Patterns** Magic Squares - [14, 26];

(vi) **Perfect Square Sum** Magic Squares with **Uniformity**, **Minimum Sum** and **Pythagorean Triples** - [15, 16];

(vii) **Block-Wise** Constructions of Magic and Bimagic Squares - [8, 17, 18, 19, 20, 22, 25, 28];

(viii) **Magic Crosses:** Repeated and Non Repeated Entries - [21];

(ix) Representations of **Letters** and **Numbers** With Equal Sums Magic Squares of Orders 4 and 6 - [23, 24].(x) Generating

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