Received 08/02/18

Palindromic, Patterned Magic Sums, Composite, and Colored Patterns in Magic Squares

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Abstract

There are many ways of representing magic squares with palindromic type entries. This paper works with magic squares of order 3 to 10. In each case, the magic squares written for the entries from the digits 3 to 8 with all palindromic numbers. These entries are arranged in such a way that their magic sum turns ***number patterns****. In some cases, using digital type letters, these* ***palindromic*** *magic squares are made in such a way that they becomes* ***upside down*** *and* ***mirror looking****. The study is extended to composite magic squares and* ***double colored patterns****. Some particular case resulting in* ***upside down*** *and* ***mirror looking*** *magic squares are also considered. In case of magic square of order 9, we have considered two different situations. One with* ***bimagic*** *and another with* ***pan diagonal*** *magic squares. In case of magic square of order 10, to make it* ***palindromic-type****, we considered some numbers as 010, 020, etc. In some other examples, we also used the idea of symmetry as 010, 020, etc.*

Contents

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RGMIA Res. Rep. Coll. 21 (2018), Art. 19, 81 pp.

1 Introduction

Magic squares are generally constructed using sequential or consecutive numbers such as $1, 2, \ldots, n^2$, etc. Here in this work, we shall write magic squares in consecutive numbers, and then transforming it in 3-digits **palindromic-type** magic squares. The extension to further digits is done in such a way that we get number patterns with magic sums. This work we have done for the magic squares of orders 3, to 10. From palindromic-type grid, we brought composite magic squares transforming in Latin squares decompositions. The idea of composite magic square is used to bring colored patterns. This work summarizes authors previous works [15, 16] This works brings extra as the idea of number patterns with magic sums. The same we did for different digits magic squares [31]. Before proceeding further let's see first the idea of palindromic numbers. In case of orders 8 and 9, we worked in two situations. One as **pan diagonal** and another as **bimagic** squares. The magic square of order 8 considered is both **pan diagonal** and **bimagic**, while the case of order 9 is worked in two subsections separately. In most of the situations, i.e., for the orders 3 to 10 we constructed **upside down** and/or **mirror looking**. Orders 3 and 6 are with little difficulty to write these kind of magic squares.

1.1 Palindromic Numbers

Palindromic numbers are numbers those read same as backwards and forwards, i.e, they remains the same when its digits are reversed, for example, 121, 3883, 19991, etc. Let's divide **palindromic numbers** in two parts, odd and even orders.

1.1.1 Odd Order Palindromes

Here is general way to write **odd order palindromes**, i.e., aba, $abcba$, etc.

$$\begin{aligned}
aba &:= (10^2+10^0)\times a + 10^1 \times b \\
abcba &:= (10^4+10^0)\times a + (10^3+10^1)\times b + 10^2 \times c \\
abcdcba &:= 10^6+10^0)\times a + (10^5+10^1)\times b + (10^4+10^2)\times c + 10^3 \times d \\
&\ldots \quad \ldots \quad \ldots \quad \ldots
\end{aligned}$$

Equivalently,

$$\begin{aligned}
aba &:= 101\times a + 10 \times b \\
abcba &:= 10001\times a + 1010\times b + 100 \times c \\
abcdcba &:= 1000001\times a + 1000010\times b + 10100\times c + 1000 \times d \\
&\ldots \quad \ldots \quad \ldots \quad \ldots
\end{aligned}$$

1.1.2 Even Order Palindromes

Here is general way to write **even order palindromes**, i.e., $abba$, $abccba$, etc.

$$\begin{aligned}
abba &:= (10^3+10^0)\times a + (10^2+10^1)\times b \\
abcba &:= (10^5+10^0)\times a + (10^4+10^1)\times b + (10^3+10^2)\times c \\
abcdcba &:= 10^7+10^0)\times a + (10^6+10^1)\times b + (10^5+10^2)\times c + (10^4+10^3)\times d \\
&\ldots \quad \ldots \quad \ldots \quad \ldots
\end{aligned}$$

Equivalently,

$$\begin{aligned} abba &:= 1001 \times a + 1110 \times b \\ abccba &:= 100001 \times a + 10010 \times b + 1100 \times c \\ abcdcba &:= 10000001 \times a + 1000010 \times b + 100100 \times c + 11000 \times d \\ \ldots & \quad \ldots \quad \ldots \quad \ldots \end{aligned}$$

Table below give the quantity of palindromes for each number of digit:

Table 1.1. *The palindrome quantity given by*

Digits	*Palindromes*	*Quantity*
1 and 2	a *and* aa	*9*
3 and 4	aba *and* $abba$	*90*
5 and 6	$abcba$ *and* $abccba$	*900*
7 and 8	$abcdcba$ *and* $abcddcba$	*9000*

If we want to write 3-digits palindromes using only 1 and 2 we have exactly four (2^2) palindromes, i.e., 111, 121, 212 and 222. The same is true for 4-digits. The table below give exact quantity in each case:

Table 1.2. *The 3-digits palindromes are:*

3-digits using only the numbers	*Total Palindromes*
1, 2	$4 := 2^2$
1, 2, 3	$9 := 3^2$
1, 2, 3, 4	$16 := 4^2$
1, 2, 3, 4, 5	$25 := 5^2$
1, 2, 3, 4, 5, 6	$36 := 6^2$
1, 2, 3, 4, 5, 6, 7	$49 := 7^2$
1, 2, 3, 4, 5, 6, 7, 8	$64 := 8^2$
1, 2, 3, 4, 5, 6, 7, 8, 9	$81 := 9^2$
0, 1, 2, 3, 4, 5, 6, 7, 8, 9	$90 := 10^2 - 10$

We shall use above table to construct **palindromic magic** squares in each case. In the last case we have written 90 instead of 100, the reason is that 10 numbers starting from 0, such as, 000, 010, 020, 030, 040, 050, 060, 070, 080 and 090 are excluded. In case we include them, we call them as **palindromic-type** numbers.

Below are magic squares of orders 3 to 10 written in terms of **palindromic numbers**. These numbers are considered in such a way that their magic sums brings **number patterns** in an increasing way. As a consequence, some **composite magic** squares with Latin squares decompositions are also given.

2 Magic Squares of Order 3

Let's consider a classical Lo-Shu magic square of order 3:

Example 2.1. *A magic square of order 3 is given by*

			15
8	1	6	15
3	5	7	15
4	9	2	15
15	15	15	15

2.1 Palindromic Representations

Let's consider three letters a, b and c, where $a,b,c \in \{1,2,3,4,5,6,7,8,9\}$. We can make exactly 9 palindromes of 3-digits using only these three letters:

Table 2.1. *The 9 palindromes with letters a, b and c are given by*

1	2	3	4	5	6	7	8	9
aaa	aba	aca	bab	bbb	bcb	cac	cbc	ccc

Grid 2.1. *Using three letters a, b and c, we have only 9 palindromes of 3-digits. This allows us to write following **palindromic grid**:*

bab	ccc	aba
aca	bbb	cac
cbc	aaa	bcb

where $a,b,c \in \{1,2,3,4,5,6,7,8,9\}$

Some particular examples of Grid 2.1 are as follows:

Example 2.2. *For $a = 1$, $b = 2$ and $c = 3$ in Grid 2.1, we have 3-digits **palindromic** magic square of order 3:*

			666
213	333	121	666
131	222	313	666
323	111	232	666
666	666	666	666

Example 2.3. *For $a = 2$, $b = 5$ and $c = 8$ in Grid 2.1, we have 3-digits **palindromic** magic square of order 3:*

			1665
525	888	252	1665
282	555	828	1665
858	222	585	1665
1665	1665	1665	1665

Example 2.4. *For $a = 1$, $b = 6$ and $c = 9$ in Grid 2.1, we have 3-digits **palindromic semi-magic** square of order 3:*

			1796
616	999	161	1665
191	666	919	1665
969	111	696	1665
1665	1665	1665	1978

Note 2.1. *We observe that the Examples 2.2 and 2.3 are magic squares, while Examples 2.4 is **semi-magic**. The reason is on the choices of a, b and c. If we choose them with the property that $b = \dfrac{a+c}{2}$, then we always get a magic square, otherwise it becomes **semi-magic**. This happens in two ways, one when we have consecutive numbers such as, {1,2,3}, {6,7,8}, etc. Second when there is uniform difference between the numbers, for example, {2,5,8}, {1,5,9}, etc.*

Note 2.2. *Writing in digital forms, the Example 2.3 becomes **upside down** and **mirror looking**, while the Example 2.4 is only **upside down**:*

525	888	252
282	555	828
858	222	585

616	999	161
191	666	919
969	111	696

*This is due to the fact that in mirror 2 become 5 and 5 becomes 2, while it doesn't happens with 6 and 9. We don't have mirror image of 6 and 9. Only **upside down**. In this case 6 become 9 and 9 as 6.*

Note 2.3. *In case of Example 2.3 when it is seen in the mirror 2 become 5 and 5 as 2. In case of mirror image, it not a magic square but **semi-magic**. See the **mirror looking** version of Example 2.3:*

			1575
252	888	525	1665
585	222	858	1665
828	555	282	1665
1665	1665	1665	756

2.2 Patterned Magic Sums

The the palindromic grid given in Grid 2.1 can be extended for the higher order palindromes in each cell. See below

Grid 2.2. *For all $a, b, c \in \{1,2,3,4,5,6,7,8,9\}$, we have following higher digits palindromic grids:*

baab	*cccc*	*abba*
acca	*bbbb*	*caac*
cbbc	*aaaa*	*bccb*

babab	*ccccc*	*ababa*
acaca	*bbbbb*	*cacac*
cbcbc	*aaaaa*	*bcacb*

babbab	*cccccc*	*abaaba*
acaaca	*bbbbbb*	*caccac*
cbccbc	*aaaaaa*	*bcbbcb*

bababab	*ccccccc*	*abababa*
acacaca	*bbbbbbb*	*cacacac*
cbcbcbc	*aaaaaaa*	*bcbcbcb*

babaabab	*cccccccc*	*ababbaba*
acaccaca	*bbbbbbbb*	*cacaacac*
cbcbbcbc	*aaaaaaaa*	*bcbccbcb*

If magic squares of order 3 exits, then magic sums are given by

Digits in each cell	Magic Sums
3	$S_{3\times 3}(a,b,c) := (a+b+c)\times 111$
4	$S_{3\times 3}(a,b,c) := (a+b+c)\times 1111$
5	$S_{3\times 3}(a,b,c) := (a+b+c)\times 11111$
6	$S_{3\times 3}(a,b,c) := (a+b+c)\times 111111$
7	$S_{3\times 3}(a,b,c) := (a+b+c)\times 1111111$
8	$S_{3\times 3}(a,b,c) := (a+b+c)\times 11111111$

As explained in the Note 2.1 These sums becomes magic of semi-magic depending upon the choices of letters a, b and c.

Below are some particular examples of Grids 2.1 and 2.2:

Example 2.5. . *Let's consider $a = 1$, $b = 2$ and $c = 3$ in Grids 2.1 and 2.2, we have 3 to 8 digits* ***palindromic*** *magic squares of order 3:*

			666
212	333	121	666
131	222	313	666
323	111	232	666
666	666	666	666

			6666
2112	3333	1221	6666
1331	2222	3113	6666
3223	1111	2332	6666
6666	6666	6666	6666

			66666
21212	33333	12121	66666
13131	22222	31313	66666
13131	22222	31313	66666
66666	66666	66666	66666

			666666
212212	333333	121121	666666
131131	222222	313313	666666
323323	111111	232232	666666
666666	666666	666666	666666

			6666666
2121212	3333333	1212121	6666666
1313131	2222222	3131313	6666666
3232323	1111111	2323232	6666666
6666666	6666666	6666666	6666666

			66666666
21211212	33333333	12122121	66666666
13133131	22222222	31311313	66666666
32322323	11111111	23233232	66666666
66666666	66666666	66666666	66666666

According to above six palindromic magic squares we have the following **number pattern** with magic squares sums. It increases as the number of digits in each cell increases. See below:

Digits in each cell	Magic Sums
3	666
4	6666
5	66666
6	666666
7	6666666
8	66666666

Example 2.6. . *Let's consider* $a = 1$, $b = 5$ *and* $c = 9$ *in Grids 2.1 and 2.2, we have 3 to 8 digits **palindromic** magic squares of order 3:*

			1665
515	999	151	1665
191	555	919	1665
959	111	595	1665
1665	1665	1665	1665

			16665
5115	9999	1551	16665
1991	5555	9119	16665
9559	1111	5995	16665
16665	16665	16665	16665

			166665
51515	99999	15151	166665
19191	55555	91919	166665
95959	11111	59595	166665
166665	166665	166665	166665

			1666665
515515	999999	151151	1666665
191191	555555	919919	1666665
959959	111111	595595	1666665
1666665	1666665	1666665	1666665

			16666665
5151515	9999999	1515151	16666665
1919191	5555555	9191919	16666665
9595959	1111111	5959595	16666665
16666665	16666665	16666665	16666665

			166666665
51511515	99999999	15155151	166666665
19199191	55555555	91911919	166666665
95955959	11111111	59599595	166666665
166666665	166666665	166666665	166666665

According to above six palindromic magic squares, we have the following **number pattern** with magic squares sums:

Digits in each cell	Magic Sums
3	1665
4	16665
5	166665
6	1666665
7	16666665
8	166666665

2.3 Composite Magic Squares

Eliminating the third value in Grid 2.1, and then splitting in two Latin squares, we get

b	c	a
a	b	c
c	a	b
	A	

a	c	b
c	b	a
b	a	c
	B	

ba	cc	ab
ac	bb	ca
cb	aa	bc
	AB	

The grid AB can be written as

$$AB := 10 \times A + B$$

In particular for $a = 1$, $b = 2$ and $c = 3$, we get

2	3	1
1	2	3
3	1	2
	A	

1	3	2
3	2	1
2	1	3
	B	

21	33	12
13	22	31
32	11	23
	AB	

Applying $3 \times (A-1) + B$ over the elements of A and B given above, we get a magic square of order 3 given in Example 2.1. Below are some examples **composite upside down** and **mirror looking semi-magic** squares.

Example 2.7. *The **composite semi-magic** squares, with the property that all these three are **upside down** and **mirror looking** are given by:*

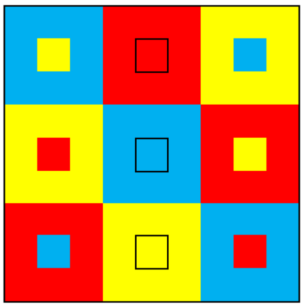
	88	88	88	
88	21	55	12	88
88	15	22	51	88
88	52	11	25	88
	88	88	88	

	55+22	55+22	55+22	
55+22	20	55	02	55+22
55+22	05	22	50	55+22
55+22	52	00	25	55+22
	55+22	55+22	55+22	

	55+88+22	55+88+22	55+88+22	
55+88+22	28	55	82	55+88+22
55+88+22	85	22	58	55+88+22
55+88+22	52	88	25	55+88+22
	55+88+22	55+88+22	55+88+22	

2.4 Superimposed Colored Pattern

The grid AB, we call **composite magic square**. Based on it, here below is **semi-superimposed double colored pattern**:



Looking from the above **superimposed colored pattern**, we observe that it don't have diagonal property. Due to this, we call it **semi superimposed colored pattern**.

3 Pan Diagonal Magic Squares of Order 4

Let consider a classical **Khajuraho magic** square of order 4.

Example 3.1. ***Khajuraho magic** square of order 4 is given by*

		34	34	34	34
	7	12	1	14	34
34	2	13	8	11	34
34	16	3	10	5	34
34	9	6	15	4	34
	34	34	34	34	34

This is considered as one of the most **perfect magic square** of order 4 studied around 10^{th} century. Some times it is called as **dense** or **fully distributed** magic square. Dense in the sense that there are so many blocks of order 2×2 has the same sum as of magic square. It is found in India in *Khajuraho in the Parshvanath Jain temple*. This magic square is not only a normal magic square, also a **pan diagonal**. It has lot of other properties. For details see Taneja [11].

3.1 Palindromic Representations

Let's consider four letters a, b, c and d, where $a, b, c, d \in \{1,2,3,4,5,6,7,8,9\}$. We can make exactly 16 palindromes of 3-digits with these four letters:

Table 3.1. *The palindromes are as follows:*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
aaa	aba	aca	ada	bab	bbb	bcb	bdb	cac	cbc	ccc	cdc	dad	dbd	dcd	ddd

Replacing the above values with their respective palindromes in Example 3.1, we get the following grid of order 4:

Grid 3.1. *Using three letters a, b, c and d, we have only 16 palindromes of 3-digits. This allows to write as the following **palindromic grid**:*

bcb	cdc	aaa	dbd
aba	dad	bdb	ccc
ddd	aca	cbc	bab
cac	bbb	dcd	ada

where $a, b, c, d \in \{1,2,3,4,5,6,7,8,9\}$

For all $a, b, c, d \in \{1,2,3,4,5,6,7,8,9\}$, the grid given in Grid 3.1 represents a palindromic magic square of order 4. If it exists, then its sum is given by

$$S_{4\times4}(a,b,c,d) := (a+b+c+d) \times 111.$$

Depending upon the choices of a, b, c and d, it may be magic or semi-magic square. Let's see below some examples:

Example 3.2. *For $a = 1$, $b = 2$, $c = 3$ and $d = 4$ in Grid 3.1, the 3-digits **palindromic** magic square of order 4 with magic sum $S_{4\times4}(1,2,3,4) := (1+2+3+4) \times 111 = 1110$ is given by*

		1110	1110	1110	1110
	232	343	111	424	1110
1110	121	414	242	333	1110
1110	444	131	323	212	1110
1110	313	222	434	141	1110
	1110	1110	1110	1110	1110

Example 3.3. *For $a=1$, $b=2$, $c=5$ and $d=8$ in Grid 3.1, the 3-digits* ***palindromic*** *magic square of order 4 with magic sum $S_{4\times 4}(1,2,5,8) := (1+2+5+8)\times 111 = 1776$ is given by.*

				1776
252	585	111	828	1776
121	818	282	555	1776
888	151	525	212	1776
515	222	858	181	1776
1776	1776	1776	1776	1776

Example 3.4. *For $a=1$, $b=6$, $c=8$ and $d=9$ in Grid 3.1, the 3-digits* ***palindromic*** *magic square of order 4 with magic sum $S_{4\times 4}(1,6,8,9) := (1+6+8+9)\times 111 = 2664$ is given by.*

				2664
686	898	111	969	2664
161	919	696	888	2664
999	181	868	616	2664
818	666	989	191	2664
2664	2664	2664	2664	2664

Note 3.1. *We observe that the Example 3.2 is a* **pan diagonal** *magic square, while the Examples 3.3 and 3.4 are just magic squares. It depends on the choices of letters a, b, c and d. If the choices are consecutive numbers such as, {1,2,3, 4}, {6,7,8, 9}, etc. or with equal differences, such as, {1, 3,5,7}, {3, 5,7,9}, etc. then we always have a* **pan diagonal** *magic square.*

The Examples 3.3 and 3.4 can be written as **upside down** and/or **mirror looking** see below:

888+888	888+888	888+888	888+888	888+888	888+888
888+888	252	585	111	828	888+888
888+888	121	818	282	555	888+888
888+888	888	151	525	212	888+888
888+888	515	222	858	181	888+888
888+888	888+888	888+888	888+888	888+888	888+888

686	898	111	969
161	919	696	888
999	181	868	616
818	666	989	191

The first example is **upside down** and **mirror looking** while second is only **upside down**.

Note 3.2. *In case of magic square of order 3, the mirror looking magic square (Note 2.2) turns as* ***semi-magic***. *In case of order 4, the Example 3.3 , it remains a magic square even in mirror looking.*

3.2 Patterned Magic Sums

The the palindromic grid given in 3.1 can be extended for the higher order palindromes in each cell. See below

Grid 3.2. *For all $a, b, c, d \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, we have following higher digits palindromic grids:*

bccb	cddc	aaaa	dbbd
abba	daad	bddb	cccc
dddd	acca	cbbc	baab
caac	bbbb	dccd	adda

bcbcb	cdcdc	aaaaa	dbdbd
ababa	dadad	bdbdb	ccccc
ddddd	acaca	cbcbc	babab
cacac	bbbbb	dcdcd	adada

bcbbcb	cdccdc	aaaaaa	dbddbd
abaaba	daddad	bdbbdb	cccccc
dddddd	acaaca	cbccbc	babbab
caccac	bbbbbb	dcddcd	adaada

bcbcbcb	cdcdcdc	aaaaaaa	dbdbdbd
abababa	dadadad	bdbdbdb	ccccccc
ddddddd	acacaca	cbcbcbc	bababab
cacacac	bbbbbbb	dcdcdcd	adadada

bcbccbcb	cdcddcdc	aaaaaaaa	dbdbbdbd
ababbaba	dadaadad	bdbddbdb	cccccccc
dddddddd	acaccaca	cbcbbcbc	babaabab
cacaacac	bbbbbbbb	dcdccdcd	adaddada

If magic squares of order 4 exists, then magic sums are given by

Digits in each cell	Magic Sums
3	$S_{4\times4}(a,b,c,d) := (a+b+c+d)\times 111$
4	$S_{4\times4}(a,b,c,d) := (a+b+c+d)\times 1111$
5	$S_{4\times4}(a,b,c,d) := (a+b+c+d)\times 11111$
6	$S_{4\times4}(a,b,c,d) := (a+b+c+d)\times 111111$
7	$S_{4\times4}(a,b,c,d) := (a+b+c+d)\times 1111111$
8	$S_{4\times4}(a,b,c,d) := (a+b+c+d)\times 11111111$

Depending upon the choices of a, b, c and d, we get always magic squares, but some times **pan diagonal**. **Pan diagonal** happens when we have consecutive or equal difference values of letters. Below are some magic squares based on the Grids 3.1 and 3.2:

Example 3.5. *For $a = 1$, $b = 2$, $c = 3$ and $d = 4$ in Grids 3.1 and 3.2, below are 3 to 8 digits **pan diagonal palindromic** magic square of order 4:*

		1110	1110	1110	1110
	232	343	111	424	1110
1110	121	414	242	333	1110
1110	444	131	323	212	1110
1110	313	222	434	141	1110
	1110	1110	1110	1110	1110

		11110	11110	11110	11110
	2332	3443	1111	4224	11110
11110	1221	4114	2442	3333	11110
11110	4444	1331	3223	2112	11110
11110	3113	2222	4334	1441	11110
	11110	11110	11110	11110	11110

		111110	111110	111110	111110
	23232	34343	11111	42424	111110
111110	12121	41414	24242	33333	111110
111110	44444	13131	32323	21212	111110
111110	31313	22222	43434	14141	111110
	111110	111110	111110	111110	111110

		1111110	1111110	1111110	1111110
	232232	343343	111111	424424	1111110
1111110	121121	414414	242242	333333	1111110
1111110	444444	131131	323323	212212	1111110
1111110	313313	222222	434434	141141	1111110
	1111110	1111110	1111110	1111110	1111110

		11111110	11111110	11111110	11111110
	2323232	3434343	1111111	4242424	11111110
11111110	1212121	4141414	2424242	3333333	11111110
11111110	4444444	1313131	3232323	2121212	11111110
11111110	3131313	2222222	4343434	1414141	11111110
	11111110	11111110	11111110	11111110	11111110

		111111110	111111110	111111110	111111110
	23233232	34344343	11111111	42422424	111111110
111111110	12122121	41411414	24244242	33333333	111111110
111111110	44444444	13133131	32322323	21211212	111111110
111111110	31311313	22222222	43433434	14144141	111111110
	111111110	111111110	111111110	111111110	111111110

According to above six palindromic magic squares of order 4, we have the following **number pattern** with magic sums:

Digits in each cell	Magic Sums
3	1110
4	11110
5	111110
6	1111110
7	11111110
8	111111110

Note 3.3. *We should observe that the way the letters a, b, c and d are written is not the unique way, except the first case. These extensions are done in such a way they always give us* ***number pattern*** *with magic square sums. See below example, that don't follow the number pattern:*

		51104	51104	51104	51104
	12321	13431	11111	14241	51104
51104	11211	14141	12421	13331	51104
51104	14441	11311	13231	12121	51104
51104	13131	12221	14341	11411	51104
	51104	51104	51104	51104	51104

		110510	110510	110510	110510
	23132	34143	11111	42124	110510
110510	12121	41114	24142	33133	110510
110510	44144	13131	32123	21112	110510
110510	31113	22122	43134	14141	110510
	110510	110510	110510	110510	110510

*The above two **pan diagonal** magic squares are with 5-digits palindromes, but the magic square sums are different from the one choose above. It show that not all the **palindromic grids** of same digits give same magic sums. It depends upon the way we choose letters.*

Example 3.6. *For $a = 1$, $b = 2$, $c = 5$ and $d = 8$ in Grids 3.1 and 3.2, below are **palindromic** magic square of order 4 for 3 to 8 digits:*

				1776
252	585	111	828	1776
121	818	282	555	1776
888	151	525	212	1776
515	222	858	181	1776
1776	1776	1776	1776	1776

				17776
2552	5885	1111	8228	17776
1221	8118	2882	5555	17776
8888	1551	5225	2112	17776
5115	2222	8558	1881	17776
17776	17776	17776	17776	17776

				177776
25252	58585	11111	82828	177776
12121	81818	28282	55555	177776
88888	15151	52525	21212	177776
51515	22222	85858	18181	177776
177776	177776	177776	177776	177776

				1777776
252252	585585	111111	828828	1777776
121121	818818	282282	555555	1777776
888888	151151	525525	212212	1777776
515515	222222	858858	181181	1777776
1777776	1777776	1777776	1777776	1777776

				17777776
2525252	5858585	1111111	8282828	17777776
1212121	8181818	2828282	5555555	17777776
8888888	1515151	5252525	2121212	17777776
5151515	2222222	8585858	1818181	17777776
17777776	17777776	17777776	17777776	17777776

				177777776
25255252	58588585	11111111	82822828	177777776
12122121	81811818	28288282	55555555	177777776
88888888	15155151	52522525	21211212	177777776
51511515	22222222	85855858	18188181	177777776
177777776	177777776	177777776	177777776	177777776

According to above six palindromic magic squares of order 4, we have the following **number pattern** with magic sums. It increases as the number of digits in each cell increases. See below:

Digits in each cell	Magic Sums
3	1776
4	17776
5	177776
6	1777776
7	17777776
8	177777776

Note 3.4. *We observe that the above magic squares given in Example 3.7 are not **pan diagonal**, as we have chosen non consecutive numbers. This gives us that not all the particular cases of Grids 3.1 and 3.2 are **pan diagonal**. It depends on the choice of numbers. These are **pan diagonal**, only when se choose the numbers in consecutive way or with equal differences, for examples, (1,2,3,4); (2,4,6,8), (1,3,5,7), etc.*

3.3 7-digit Palindromic Magic Squares with 2 Letters

We have only 16 choices of 7-digits palindromes for the two letters, a and b. This allows us to write following grid of order 4:

Grid 3.3. *We have following palindromic grid of order 4 with 7-digits and 2 letters a and b:*

abbabba	*babbbab*	*aaaaaaa*	*bbababb*
aaabaaa	*bbaaabb*	*abbbbba*	*bababab*
bbbbbbb	*aababaa*	*baabaab*	*abaaaba*
baaaaab	*abababa*	*bbbabbb*	*aabbbaa*

For all where $a, b \in \{1,2,3,4,5,6,7,8,9\}$, above grid represent a **palindromic** magic square of order 4. If it exists, then it magic sum is given by

$$S_{4\times4}(a,b) := 2 \times (aaaaaaa + bbbbbbb) = (a+b) \times 2222222.$$

Let's apply the Grid 3.4 to bring interesting magic squares, such as **upside down** and **mirror looking**. See below examples.

Example 3.7. *For $a = 2$ and $b = 5$ in 3.4 we have 7-digits palindromic **pan diagonal upside down** and **mirror looking** magic square of order 4 with sum $S_{4\times4}(2,5) := (2+5) \times 2222222 = 15555554$:*

2552552	5255525	2222222	5525255
2225222	5522255	2555552	5252525
5555555	2252522	5225225	2522252
5222225	2525252	5552555	2255522

Example 3.8. *For $a = 1$ and $b = 8$ in 3.4 we have 7-digits palindromic **pan diagonal upside down** and **mirror looking** magic square of order 4 with sum $S_{4\times4}(1,8) := (1+8) \times 2222222 = 19999998$:*

1881881	8188818	1111111	8818188
1118111	8811188	1888881	8181818
8888888	1181811	8118118	1811181
8111118	1818181	8881888	1188811

Example 3.9. *For $a = 6$ and $b = 9$ in 3.4 we have 7-digits palindromic **pan diagonal upside down** and **mirror looking** magic square of order 4 with sum $S_{4\times4}(6,9) := (6+9) \times 2222222 = 33333330$:*

6996996	9699969	6666666	9969699
6669666	9966699	6999996	9696969
9999999	6696966	9669669	6966696
9666669	6969696	9996999	6699966

Note 3.5. *Above three examples give us following symmetry in magic sums:*

$$\frac{S_{4\times4}(2,5)}{2+5} = \frac{S_{4\times4}(1,8)}{1+8} = \frac{S_{4\times4}(6,9)}{6+9} = 2222222.$$

3.4 Composite Magic Squares

Grid 3.4. *Eliminating the third value in Grid 3.1, and then splitting in two Latin squares, we get*

b	*c*	*a*	*d*
a	*d*	*b*	*c*
d	*a*	*c*	*b*
c	*b*	*d*	*a*
	A		

c	*d*	*a*	*b*
b	*a*	*d*	*c*
d	*c*	*b*	*a*
a	*b*	*c*	*d*
	B		

bc	*cd*	*aa*	*db*
ab	*da*	*bd*	*cc*
dd	*ac*	*cb*	*ba*
ca	*bb*	*dc*	*ad*
	AB		

The grid AB can be written as

$$AB := 10 \times A + B$$

Example 3.10. *In particular for* $a = 1$*,* $b = 2$*,* $c = 3$ *and* $d = 4$*, we get*

2	3	1	4
1	4	2	3
4	1	3	2
3	2	4	1
	A		

3	4	1	2
2	1	4	3
4	3	2	1
1	2	3	4
	B		

23	34	11	42
12	41	24	33
44	13	32	21
31	22	43	14
	AB		

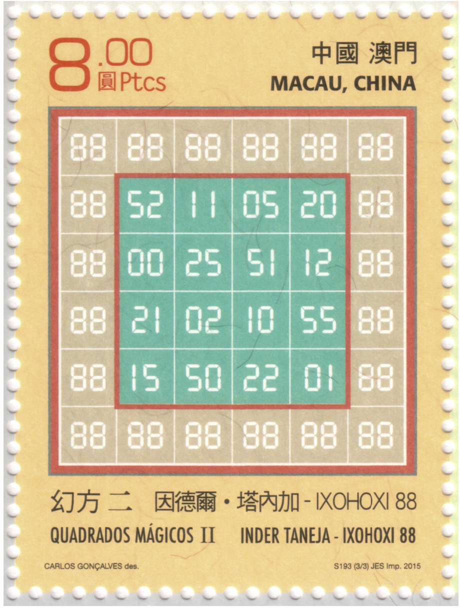
Note 3.6. *Applying* $4 \times (A-1) + B$ *over the members of A and B, we get a magic square of order 4 given in Example 3.1. Below are some more examples in composite form giving* ***upside down*** *and* ***mirror looking*** *magic squares. Moreover, A and B are* ***pairwise mutually orthogonal diagonal Latin squares****.*

Example 3.11. *The following two examples are* ***upside down*** *and* ***mirror looking*** *magic squares just with four numbers with sums* $S_{4\times4}(1,6,8,9) := (1+6+8+9) \times 11 = 264$ *and* $S_{4\times4}(0,1,2,5) := (0+1+2+5) \times 11 = 88$ *respectively:*

88+88+88	88+88+88	88+88+88	88+88+88	88+88+88	88+88+88
88+88+88	61	86	99	18	88+88+88
88+88+88	19	98	81	66	88+88+88
88+88+88	88	69	16	91	88+88+88
88+88+88	96	11	68	89	88+88+88
88+88+88	88+88+88	88+88+88	88+88+88	88+88+88	88+88+88

88	88	88	88	88	88
88	10	21	55	02	88
88	05	52	20	11	88
88	22	15	01	50	88
88	51	00	12	25	88
88	88	88	88	88	88

The first example is ***upside down****, while the second example is* ***upside down*** *and* ***mirror looking****. Moreover, second example is well-known author's stamp published at Macau - China - 2015:*



Example 3.12. *The following two examples are* ***upside down*** *and* ***mirror looking*** *just with two numbers with magic sums* $S_{4\times 4}(6,9) := (6+9)\times 2222 = 33330$ *and* $S_{4\times 4}(2,5) := (2+5)\times 2222 = 15554$ *respectively.*

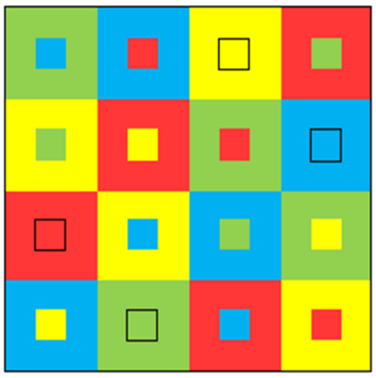
6996	9699	6666	9969
6669	9966	6999	9696
9999	6696	9669	6966
9666	6969	9996	6699

2552	5255	2222	5525
2225	5522	2555	5252
5555	2252	5225	2522
5222	2525	5552	2255

The magic square is ***upside down****, while second is* ***upside down*** *and* ***mirror looking****. Both are made from composite magic square AB, where we have chosen double digits instead of single. Both are* ***pan diagonal****. The first example is obtained considering a=66, b=69, c=96 and d=99 in 3.4. The second is obtained by considering a= 22, b=25, c=52 and d=55 in 3.4. In the second case, even in the mirror looking situation, it remains as pan diagonal with same magic sum.*

3.5 Superimposed Colored Pattern

The grid *AB* is a **composite** magic square of order 4. Based on it, here below is a **superimposed double colored pattern**.



Looking from the above **superimposed colored pattern** of order 4, we observe that it has **diagonal property**, as it is made from **pairwise mutually orthogonal diagonal Latin squares**

4 Pan Diagonal Magic Squares of Order 5

Example 4.1. *Let's consider a **pan diagonal** magic square of order 5 given by*

		65	65	65	65	65
	1	9	12	20	23	65
65	17	25	3	6	14	65
65	8	11	19	22	5	65
65	24	2	10	13	16	65
65	15	18	21	4	7	65
	65	65	65	65	65	65

4.1 Palindromic Representations

Let's consider five letters a, b, c, d and e, where $a, b, c, d, e \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. We can make exactly 25 palindromes of 3-digits with these five letters:

Table 4.1. *The palindromes are as follows:*

1	2	3	4	5	6	7	8	9	10	11	12	13
aaa	aba	aca	ada	aea	bab	bbb	bcb	bdb	beb	cac	cbc	ccc
14	15	16	17	18	19	20	21	22	23	24	25	
cdc	cec	dad	dbd	dcd	ddd	ded	eae	ebe	ece	ede	eee	

Replacing the above values with their respective palindromes in a magic square of order 5 given in Example 4.1 , we get a **palindromic grid** given below:

Grid 4.1. *Using five letters a, b, c, d and e, we have only 25 palindromes of 3-digits. This allows us to write as the following grid:*

aaa	bbb	ccc	ddd	eee
dcd	ede	aea	bab	cbc
beb	cac	dbd	ece	ada
ebe	aca	bdb	cec	dad
cdc	ded	eae	aba	bcb

where $a,b,c,d,e \in \{1,2,3,4,5,6,7,8,9\}$

For all $a,b,c,d,e \in \{1,2,3,4,5,6,7,8,9\}$, the grid given in 4.1 represents a palindromic magic square of order 5. If it exists, then its magic sum is given by

$$S_{5\times5}(a,b,c,d,e) := (a+b+c+d+e)\times 111.$$

Let's consider some examples of Grid 4.1

Example 4.2. *For* $a=1, b=2, c=3, d=4$ *and* $e=5$ *in Grid 4.1, the 3-digits* ***palindromic*** *magic square of order 5 with magic sum* $S_{5\times5}(1,2,3,4,5) := (1+2+3+4+5)\times 111 = 1665$ *is given by.*

		1665	1665	1665	1665	1665
	111	222	333	444	555	1665
1665	434	545	151	212	323	1665
1665	252	313	424	535	141	1665
1665	525	131	242	353	414	1665
1665	343	454	515	121	232	1665
	1665	1665	1665	1665	1665	1665

The above Example 4.1 is with consecutive numbers 1 to 5 that gives us a **pan diagonal** magic square. Below is another example, with non consecutive numbers.

Example 4.3. *For* $a=1, b=2, c=5, d=6$ *and* $e=9$ *in Grid 4.1, the 3-digits* ***palindromic*** *magic square of order 5 with magic sum* $S_{5\times5}(1,2,5,6,9) := (1+2+5+6+9)\times 111 = 2553$ *is given by.*

		2553	2553	2553	2553	2553
	111	222	555	666	999	2553
2553	656	969	191	212	525	2553
2553	292	515	626	959	161	2553
2553	929	151	262	595	616	2553
2553	565	696	919	121	252	2553
	2553	2553	2553	2553	2553	2553

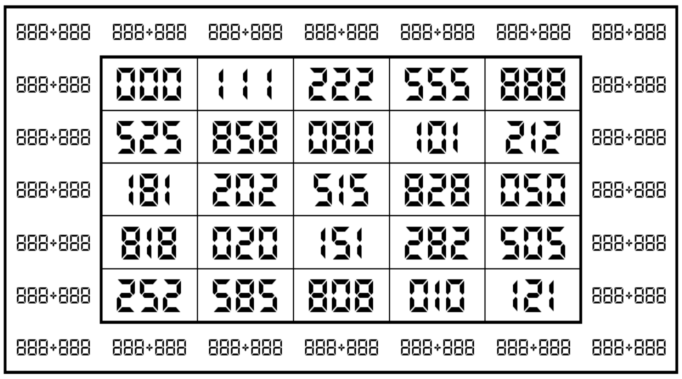
The **upside down** version of above example is given by

666+888+999	666+888+999	666+888+999	666+888+999	666+888+999	666+888+999	666+888+999
666+888+999	111	222	555	666	999	666+888+999
666+888+999	656	969	191	212	525	666+888+999
666+888+999	292	515	626	959	161	666+888+999
666+888+999	929	151	262	595	616	666+888+999
666+888+999	565	696	919	121	252	666+888+999
666+888+999	666+888+999	666+888+999	666+888+999	666+888+999	666+888+999	666+888+999

Example 4.4. *For* $a=0, b=1, c=2, d=5$ *and* $e=8$ *in Grid 4.1, the 3-digits* ***palindromic*** *magic square of order 5 with magic sum* $S_{5\times5}(0,1,2,5,8) := (0+1+2+5+8)\times 111 = 1776$ *is given by.*

		1776	1776	1776	1776	1776
	000	111	222	555	888	1776
1776	525	858	080	101	212	1776
1776	181	202	515	828	050	1776
1776	818	020	151	282	505	1776
1776	252	585	808	010	121	1776
	1776	1776	1776	1776	1776	1776

The **upside down** and **mirror looking** version of above example is given by



4.2 Patterned Magic Sums

The the palindromic grid given in 4.1 can be extended for the higher order palindromes in each cell. See below

Grid 4.2. *For all* $a, b, c, d, e \in \{1,2,3,4,5,6,7,8,9\}$*, we have following higher digits palindromic grids:*

aaaa	bbbb	cccc	dddd	eeee
dccd	edde	aeea	baab	cbbc
beeb	caac	dbbd	ecce	adda
ebbe	acca	bddb	ceec	daad
cddc	deed	eaae	abba	bccb

aaaaa	bbbbb	ccccc	ddddd	eeeee
dcdcd	edede	aeaea	babab	cbcbc
bebeb	cacac	dbdbd	ecece	adada
ebebe	acaca	bdbdb	cecec	dadad
cdcdc	deded	eaeae	ababa	bcbcb

aaaaaa	bbbbbb	cccccc	dddddd	eeeeee
dcddcd	edeede	aeaaea	babbab	cbccbc
bebbeb	caccac	dbddbd	eceece	adaada
ebeebe	acaaca	bdbbdb	ceccec	daddad
cdccdc	dedded	eaeeae	abaaba	bcbbcb

aaaaaaa	bbbbbbb	ccccccc	ddddddd	eeeeeee
dcdcdcd	ededede	aeaeaea	bababab	cbcbcbc
bebebeb	cacacac	dbdbdbd	ececece	adadada
ebebebe	acacaca	bdbdbdb	cececec	dadadad
cdcdcdc	dededed	eaeaeae	abababa	bcbcbcb

aaaaaaaa	*bbbbbbbb*	*cccccccc*	*dddddddd*	*eeeeeeee*
dcdccdcd	*ededdede*	*aeaeeaea*	*babaabab*	*cbcbbcbc*
bebeebeb	*cacaacac*	*dbdbbdbd*	*ececcece*	*adaddada*
ebebbebe	*acaccaca*	*bdbddbdb*	*cecеecec*	*dadaadad*
cdcddcdc	*dedeeded*	*eaeaaeae*	*ababbaba*	*bcbccbcb*

If magic squares of order 5 exits, then magic sums are given by

Digits in each cell	Magic Sums
3	$S_{5\times5}(a,b,c,d,e) := (a+b+c+d+e)\times 111$
4	$S_{5\times5}(a,b,c,d,e) := (a+b+c+d+e)\times 1111$
5	$S_{5\times5}(a,b,c,d,e) := (a+b+c+d+e)\times 11111$
6	$S_{5\times5}(a,b,c,d,e) := (a+b+c+d+e)\times 111111$
7	$S_{5\times5}(a,b,c,d,e) := (a+b+c+d+e)\times 1111111$
8	$S_{5\times5}(a,b,c,d,e) := (a+b+c+d+e)\times 11111111$

Making proper choices of $a,b,c,d,e \in \{1,2,3,4,5,6,7,8,9\}$, the grids given in 4.1 and 4.2 lead **pan diagonal palindromic** magic squares of order 5 resulting in an interesting **number pattern** in magic sums. See below examples.

Example 4.5. *For $a=1, b=2, c=3, d=4$ and $e=5$ in Grids 4.1 and 3.2, below are* ***pan diagonal palindromic*** *magic square of order 5 for 3 to 8 digits:*

		1665	1665	1665	1665	1665
	111	222	333	444	555	1665
1665	434	545	151	212	323	1665
1665	252	313	424	535	141	1665
1665	525	131	242	353	414	1665
1665	343	454	515	121	232	1665
	1665	1665	1665	1665	1665	1665

		16665	16665	16665	16665	16665
	1111	2222	3333	4444	5555	16665
16665	4334	5445	1551	2112	3223	16665
16665	2552	3113	4224	5335	1441	16665
16665	5225	1331	2442	3553	4114	16665
16665	3443	4554	5115	1221	2332	16665
	16665	16665	16665	16665	16665	16665

		166665	166665	166665	166665	166665
	11111	22222	33333	44444	55555	166665
166665	43434	54545	15151	21212	32323	166665
166665	25252	31313	42424	53535	14141	166665
166665	52525	13131	24242	35353	41414	166665
166665	34343	45454	51515	12121	23232	166665
	166665	166665	166665	166665	166665	166665

		1666665	1666665	1666665	1666665	1666665
	111111	222222	333333	444444	555555	1666665
1666665	434434	545545	151151	212212	323323	1666665
1666665	252252	313313	424424	535535	141141	1666665
1666665	525525	131131	242242	353353	414414	1666665
1666665	343343	454454	515515	121121	232232	1666665
	1666665	1666665	1666665	1666665	1666665	1666665

		16666665	16666665	16666665	16666665	16666665
	1111111	2222222	3333333	4444444	5555555	16666665
16666665	4343434	5454545	1515151	2121212	3232323	16666665
16666665	2525252	3131313	4242424	5353535	1414141	16666665
16666665	5252525	1313131	2424242	3535353	4141414	16666665
16666665	3434343	4545454	5151515	1212121	2323232	16666665
	16666665	16666665	16666665	16666665	16666665	16666665

		166666665	166666665	166666665	166666665	166666665
	11111111	22222222	33333333	44444444	55545555	166666665
166666665	43433434	54544545	15155151	21211212	32322323	166666665
166666665	25255252	31311313	42422424	53533535	14144141	166666665
166666665	52522525	13133131	24244242	35355353	41411414	166666665
166666665	34344343	45455454	51511515	12122121	23233232	166666665
	166666665	166666665	166666665	166666665	166666665	166666665

According to above six palindromic magic squares of order 5, we have the following **number pattern** with magic sums. It increases as the number of digits in each cell increases. See below:

Digits in each cell	Magic Sums
3	1665
4	16665
5	166665
6	1666665
7	16666665
8	166666665

Let's see another example with non consecutive numbers.

Example 4.6. *For $a = 1, b = 2, c = 5, d = 8$ and $e = 9$ in Grids 4.1 and 3.2, below are **pan diagonal palindromic** magic square of order 5 for 3 to 8 digits:*

		2775	2775	2775	2775	2775
	111	222	555	888	999	2775
2775	858	989	191	212	525	2775
2775	292	515	828	959	181	2775
2775	929	151	282	595	818	2775
2775	585	898	919	121	252	2775
	2775	2775	2775	2775	2775	2775

		27775	27775	27775	27775	27775
	1111	2222	5555	8888	9999	27775
27775	8558	9889	1991	2112	5225	27775
27775	2992	5115	8228	9559	1881	27775
27775	9229	1551	2882	5995	8118	27775
27775	5885	8998	9119	1221	2552	27775
	27775	27775	27775	27775	27775	27775

		277775	277775	277775	277775	277775
	11111	22222	55555	88888	99999	277775
277775	85858	98989	19191	21212	52525	277775
277775	29292	51515	82828	95959	18181	277775
277775	92929	15151	28282	59595	81818	277775
277775	58585	89898	91919	12121	25252	277775
	277775	277775	277775	277775	277775	277775

		2777775	2777775	2777775	2777775	2777775
	111111	222222	555555	888888	999999	2777775
2777775	858858	989989	191191	212212	525525	2777775
2777775	292292	515515	828828	959959	181181	2777775
2777775	929929	151151	282282	595595	818818	2777775
2777775	585585	898898	919919	121121	252252	2777775
	2777775	2777775	2777775	2777775	2777775	2777775

		27777775	27777775	27777775	27777775	27777775
	1111111	2222222	5555555	8888888	9999999	27777775
27777775	8585858	9898989	1919191	2121212	5252525	27777775
27777775	2929292	5151515	8282828	9595959	1818181	27777775
27777775	9292929	1515151	2828282	5959595	8181818	27777775
27777775	5858585	8989898	9191919	1212121	2525252	27777775
	27777775	27777775	27777775	27777775	27777775	27777775

		277777775	277777775	277777775	277777775	277777775
	11111111	22222222	55555555	88888888	99999999	277777775
277777775	85855858	98988989	19199191	21211212	52522525	277777775
277777775	29299292	51511515	82822828	95955959	18188181	277777775
277777775	92922929	15155151	28288282	59599595	81811818	277777775
277777775	58588585	89899898	91911919	12122121	25255252	277777775
	277777775	277777775	277777775	277777775	277777775	277777775

According to above six palindromic magic squares of order 5, we have the following **number pattern** with magic sums. See below:

Digits in each cell	Magic Sums
3	2775
4	27775
5	277775
6	2777775
7	27777775
8	277777775

We observe that independent of choice of a, b, c, d and e, consecutive or not, we always get a **pan diagonal** magic squares, obviously, if it exists.

4.3 Composite Magic Squares

Grid 4.3. *Eliminating the third value in Grid 4.1, and then splitting in two Latin squares, we get*

a	b	c	d	e
d	e	a	b	c
b	c	d	e	a
e	a	b	c	d
c	d	e	a	b
		A		

a	b	c	d	e
c	d	e	a	b
e	a	b	c	d
b	c	d	e	a
d	e	a	b	c
		B		

aa	bb	cc	dd	ee
dc	ed	ae	ba	cb
be	ca	db	ec	ad
eb	ac	bd	ce	da
cd	de	ea	ab	bc
		AB		

The grid AB can be written as

$$AB := 10 \times A + B$$

As a particular case, we have following example.

Example 4.7. *In particular for* $a = 1, b = 2, c = 3, d = 4$ *and* $e = 5$, *we get*

1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2
		A		

1	2	3	4	5
3	4	5	1	2
5	1	2	3	4
2	3	4	5	1
4	5	1	2	3
		B		

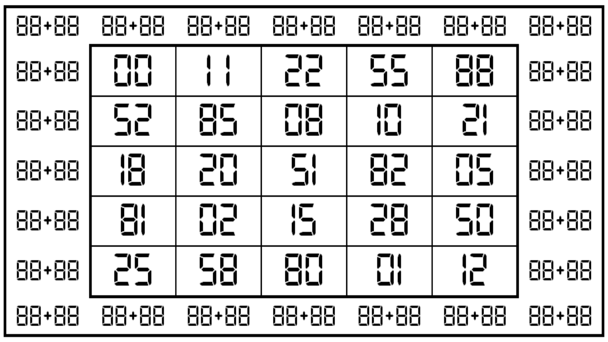
11	22	33	44	55
43	54	15	21	32
25	31	42	53	14
52	13	24	35	41
34	45	51	12	23
		AB		

Note 4.1. *Applying* $5 \times (A - 1) + B$ *over the elements of A and B given above, we get a magic square of order 5 given in Example 4.1. Also, A and B are* ***pairwise mutually orthogonal diagonal Latin squares****. Below are some examples of composite* ***upside down*** *and* ***mirror looking*** *magic squares. The above Grid 4.3 is written for single letters* a, b, c, d *and* e*. We can choose double digits numbers to write composite examples.*

Example 4.8. *Let's consider* $a = 0, b = 1, c = 2, d = 5$ *and* $e = 8$ *in Grid 4.3 , we get following* ***pan diagonal upside down*** *magic square:*

		176	176	176	176	176
	00	11	22	55	88	176
176	52	85	08	10	21	176
176	18	20	51	82	05	176
176	81	02	15	28	50	176
176	25	58	80	01	12	176
	176	176	176	176	176	176

The **upside down** and **mirror looking** version of above example is given by



Example 4.9. *Let's consider $a = 0, b = 1, c = 6, d = 8$ and $e = 9$ in Grid 4.3 , we get following* ***pan diagonal upside down*** *magic square:*

		264	264	264	264	264
	00	11	66	88	99	264
264	86	98	09	10	61	264
264	19	60	81	96	08	264
264	91	06	18	69	80	264
264	68	89	90	01	16	264
	264	264	264	264	264	264

The **upside down** version of above example is given by

00	11	66	88	99
86	98	09	10	61
19	60	81	96	08
91	06	18	69	80
68	89	90	01	16

Example 4.10. *Let's consider $a = 2, b = 5, c = 6, d = 8$ and $e = 9$ in Grid 4.3 , we get following* ***pan diagonal upside down*** *magic square:*

		330	330	330	330	330
	22	55	66	88	99	330
330	86	98	29	52	65	330
330	59	62	85	96	28	330
330	95	26	58	69	82	330
330	68	89	92	25	56	330
	330	330	330	330	330	330

The **upside down** version of above example is given by

22	55	66	88	99
86	98	29	52	65
59	62	85	96	28
95	26	58	69	82
68	89	92	25	56

Example 4.11. *Let's consider $a = 22, b = 25, c = 52, d = 55$ and $e = 88$ in Grid 4.3 , we get following* ***pan diagonal upside down*** *and* ***mirror looking*** *magic square:*

		24442	24442	24442	24442	24442
	2222	2525	5252	5555	8888	24442
24442	5552	8855	2288	2522	5225	24442
24442	2588	5222	5525	8852	2255	24442
24442	8825	2252	2555	5288	5522	24442
24442	5255	5588	8822	2225	2552	24442
	24442	24442	24442	24442	24442	24442

The **upside down** and **mirror looking** version of above example is given by

2222	2525	5252	5555	8888
5552	8855	2288	2522	5225
2588	5222	5525	8852	2255
8825	2252	2555	5288	5522
5255	5588	8822	2225	2552

Example 4.12. *Let's consider* $a = 11, b = 66, c = 69, d = 96$ *and* $e = 99$ *in Grid 4.3 , we get following* ***pan diagonal upside down*** *magic square:*

		34441	34441	34441	34441	34441
	1111	6666	6969	9696	9999	34441
34441	9669	9996	1199	6611	6966	34441
34441	6699	6911	9666	9969	1196	34441
34441	9966	1169	6696	6999	9611	34441
34441	6996	9699	9911	1166	6669	34441
	34441	34441	34441	34441	34441	34441

The **upside down** version of above example is given by

1111	6666	6969	9696	9999
9669	9996	1199	6611	6966
6699	6911	9666	9969	1196
9966	1169	6696	6999	9611
6996	9699	9911	1166	6669

Example 4.13. *Let's consider* $a = 66, b = 69, c = 88, d = 96$ *and* $e = 99$ *in Grid 4.3 , we get following* ***pan diagonal upside down*** *magic square:*

		42218	*42218*	*42218*	*42218*	*42218*
	6666	*6969*	*8888*	*9696*	*9999*	*42218*
42218	*9688*	*9996*	*6699*	*6966*	*8869*	*42218*
42218	*6999*	*8866*	*9669*	*9988*	*6696*	*42218*
42218	*9969*	*6688*	*6996*	*8899*	*9666*	*42218*
42218	*8896*	*9699*	*9966*	*6669*	*6988*	*42218*
	42218	*42218*	*42218*	*42218*	*42218*	*42218*

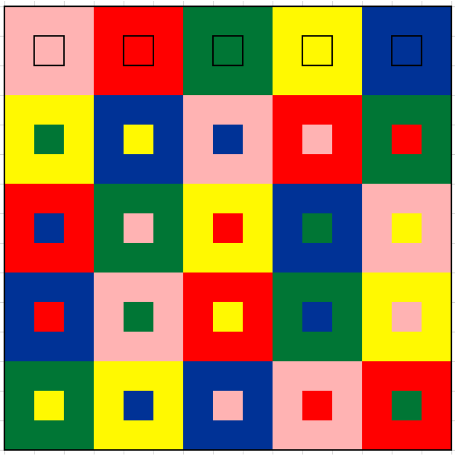
The **upside down** version of above example is given by

6666	6969	8888	9696	9999
9688	9996	6699	6966	8869
6999	8866	9669	9988	6696
9969	6688	6996	8899	9666
8896	9699	9966	6669	6988

The first magic square is **upside down** and **mirror looking** while the second and third are only **upside down**.

4.4 Superimposed Colored Pattern

The grid *AB* given in 4.7 is a **composite** magic square of order 5. Based on it, here below is **double colored pattern**.



Looking from the above **superimposed colored pattern** of order 5, we observe that it has **diagonal property**, as it is made from **pairwise mutually orthogonal diagonal Latin squares**.

5 Magic Squares of Order 6

Example 5.1. *Let's consider a magic square of order 6.*

						111
1	35	34	33	2	6	111
30	8	28	9	11	25	111
24	23	15	16	20	13	111
18	14	21	22	17	19	111
7	26	10	27	29	12	111
31	5	3	4	32	36	111
111	111	111	111	111	111	111

5.1 Palindromic Representations

Let's consider six letters a, b, c, d, e and f, where $a, b, c, d, e, f \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. We can make exactly 36 palindromes of 3-digits with these six letters:

Table 5.1. *The palindromes are as follows:*

1	2	3	4	5	6	7	8	9	10	11	12
aaa	aba	aca	ada	aea	afa	bab	bbb	bcb	bdb	beb	bfb
13	14	15	16	17	18	19	20	21	22	23	24
cac	cbc	ccc	cdc	cec	cfc	dad	dbd	dcd	ddd	ded	dfd
25	26	27	28	29	30	31	32	33	34	35	36
eae	ebe	ece	ede	eee	efe	faf	fbf	fcf	fdf	fef	fff

Replacing the above values with their respective palindromes in a magic square of order 6 given in Example 5.1 , we get a **palindromic** grid given below:

Grid 5.1. *Using six letters a, b, c, d, e and f, we have only 36 palindromes of 3-digits. This allows us to write as the following palindromic grid:*

aaa	fef	fdf	fcf	aba	afa
efe	bbb	ede	bcb	beb	eae
dfd	ded	ccc	cdc	dbd	cac
cfc	cbc	dcd	ddd	cec	dad
bab	ebe	bdb	ece	eee	bfb
faf	aea	aca	ada	fbf	fff

where $a, b, c, d, e, f \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

For all $a, b, c, d, e, f \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the grid given in 5.1 represents a palindromic magic square of order 6, if exists, then its sum is given by

$$S_{6\times 6}(a, b, c, d, e, f) := (a + b + c + d + e + f) \times 111.$$

Let's see below some examples:

Example 5.2. *For $a = 1, b = 2, c = 3, d = 4, e = 5$ and $f = 6$ in Grid 5.1, the 3-digits **palindromic** magic square of order 6 with magic sum $S_{6\times 6}(1, 2, 3, 4, 5, 6) := (1 + 2 + 3 + 4 + 5 + 6) \times 111 = 2331$ is given by.*

						2331
111	656	646	636	121	161	2331
565	222	545	232	252	515	2331
464	454	333	343	424	313	2331
363	323	434	444	353	414	2331
212	525	242	535	555	262	2331
616	151	131	141	626	666	2331
2331	2331	2331	2331	2331	2331	2331

Here we are not sure whether we shall have a magic square of order 6 for non-sequential numbers. See the example below:

Example 5.3. *For $a = 1, b = 2, c = 3, d = 5, e = 7$ and $f = 9$ in Grid 5.1, we following grid of order 6:*

						2997
111	979	959	939	121	191	3300
797	222	757	232	272	717	2997
595	575	333	353	525	313	2694
393	323	535	555	373	515	2694
212	727	252	737	777	292	2997
919	171	131	151	929	999	3300
3027	2997	2997	2997	2997	3027	2997

Thus we observe that considering no consecutive entries, we are unable to bring a magic square. Below is another example of consecutive entries giving a magic square of order 6.

Example 5.4. *For $a = 3, b = 4, c = 5, d = 6, e = 7$ and $f = 8$ in Grid 5.1, the 3-digits **palindromic** magic square of order 6 with magic sum $S_{6\times 6}(1,2,3,4,5,6) := (3+4+5+6+7+8)\times 111 = 3663$ is given by*

						3663
333	878	868	858	343	383	3663
787	444	767	454	474	737	3663
686	676	555	565	646	535	3663
585	545	656	666	575	636	3663
434	747	464	757	777	484	3663
838	373	353	363	848	888	3663
3663	3663	3663	3663	3663	3663	3663

5.2 Patterned Magic Sums

The the palindromic grid given in 5.1 can be extended for the higher order palindromes. See below

Grid 5.2. *For all $a, b, c, d, e, f \in \{1,2,3,4,5,6,7,8,9\}$, we have following higher order palindromic grids:*

aaaa	*feef*	*fddf*	*fccf*	*abba*	*affa*
effe	*bbbb*	*edde*	*bccb*	*beeb*	*eaae*
dffd	*deed*	*cccc*	*cddc*	*dbbd*	*caac*
cffc	*cbbc*	*dccd*	*dddd*	*ceec*	*daad*
baab	*ebbe*	*bddb*	*ecce*	*eeee*	*bffb*
faaf	*aeea*	*acca*	*adda*	*fbbf*	*ffff*

aaaaa	*fefef*	*fdfdf*	*fcfcf*	*ababa*	*afafa*
efefe	*bbbbb*	*edede*	*bcbcb*	*bebeb*	*eaeae*
dfdfd	*deded*	*ccccc*	*cdcdc*	*dbdbd*	*cacac*
cfcfc	*cbcbc*	*dcdcd*	*ddddd*	*cecec*	*dadad*
babab	*ebebe*	*bdbdb*	*ecece*	*eeeee*	*bfbfb*
fafaf	*aeaea*	*acaca*	*adada*	*fbfbf*	*fffff*

aaaaaa	*feffef*	*fdffdf*	*fcffcf*	*abaaba*	*afaafa*
efeefe	*bbbbbb*	*edeede*	*bcbbcb*	*bebbeb*	*eaeeae*
dfddfd	*dedded*	*cccccc*	*cdccdc*	*dbddbd*	*caccac*
cfccfc	*cbccbc*	*dcddcd*	*dddddd*	*ceccec*	*daddad*
babbab	*ebeebe*	*bdbbdb*	*eceece*	*eeeeee*	*bfbbfb*
faffaf	*aeaaea*	*acaaca*	*adaada*	*fbffbf*	*ffffff*

aaaaaaa	*fefefef*	*fdfdfdf*	*fcfcfcf*	*abababa*	*afafafa*
efefefe	*bbbbbbb*	*ededede*	*bcbcbcb*	*bebebeb*	*eaeaeae*
dfdfdfd	*dededed*	*ccccccc*	*cdcdcdc*	*dbdbdbd*	*cacacac*
cfcfcfc	*cbcbcbc*	*dcdcdcd*	*ddddddd*	*cececec*	*dadadad*
bababab	*ebebebe*	*bdbdbdb*	*ececece*	*eeeeeee*	*bfbfbfb*
fafafaf	*aeaeaea*	*acacaca*	*adadada*	*fbfbfbf*	*fffffff*

aaaaaaaa	*fefeefef*	*fdfddfdf*	*fcfccfcf*	*ababbaba*	*afaffafa*
efeffefe	*bbbbbbbb*	*ededdede*	*bcbccbcb*	*bebeebeb*	*eaeaaeae*
dfdffdfd	*dedeeded*	*cccccccc*	*cdcddcdc*	*dbdbbdbd*	*cacaacac*
cfcffcfc	*cbcbbcbc*	*dcdccdcd*	*dddddddd*	*ceceecec*	*dadaadad*
babaabab	*ebebbebe*	*bdbddbdb*	*ececcece*	*eeeeeeee*	*bfbffbfb*
fafaafaf	*aeaeeaea*	*acaccaca*	*adaddada*	*fbfbbfbf*	*ffffffff*

If magic squares exist, then the magic sums are given by

Digits in each cell	Magic Sums
3	$S_{6\times6}(a,b,c,d,e,f) := (a+b+c+d+e+f)\times 111$
4	$S_{6\times6}(a,b,c,d,e,f) := (a+b+c+d+e+f)\times 1111$
5	$S_{6\times6}(a,b,c,d,e,f) := (a+b+c+d+e+f)\times 11111$
6	$S_{6\times6}(a,b,c,d,e,f) := (a+b+c+d+e+f)\times 111111$
7	$S_{6\times6}(a,b,c,d,e,f) := (a+b+c+d+e+f)\times 1111111$
8	$S_{6\times6}(a,b,c,d,e,f) := (a+b+c+d+e+f)\times 11111111$

Making proper choices of $a,b,c,d,e,f \in \{1,2,3,4,5,6,7,8,9\}$, the grids given in 5.1 and 5.2 lead us to **palindromic** magic squares of order 6 resulting in an interesting **number pattern** in magic sums. See below examples.

Example 5.5. *For $a=1, b=2, c=3, d=4, e=5$ and $f=6$ in Grids 5.1 and 5.2 , the* ***palindromic*** *magic square of order 6 for the digits 3 to 8 are given by*

						2331
111	656	646	636	121	161	2331
565	222	545	232	252	515	2331
464	454	333	343	424	313	2331
363	323	434	444	353	414	2331
212	525	242	535	555	262	2331
616	151	131	141	626	666	2331
2331	2331	2331	2331	2331	2331	2331

						23331
1111	6556	6446	6336	1221	1661	23331
5665	2222	5445	2332	2552	5115	23331
4664	4554	3333	3443	4224	3113	23331
3663	3223	4334	4444	3553	4114	23331
2112	5225	2442	5335	5555	2662	23331
6116	1551	1331	1441	6226	6666	23331
23331	23331	23331	23331	23331	23331	23331

						233331
11111	65656	64646	63636	12121	16161	233331
56565	22222	54545	23232	25252	51515	233331
46464	45454	33333	34343	42424	31313	233331
36363	32323	43434	44444	35353	41414	233331
21212	52525	24242	53535	55555	26262	233331
61616	15151	13131	14141	62626	66666	233331
233331	233331	233331	233331	233331	233331	233331

						2333331
111111	656656	646646	636636	121121	161161	2333331
565565	222222	545545	232232	252252	515515	2333331
464464	454454	333333	343343	424424	313313	2333331
363363	323323	434434	444444	353353	414414	2333331
212212	525525	242242	535535	555555	262262	2333331
616616	151151	131131	141141	626626	666666	2333331
2333331	2333331	2333331	2333331	2333331	2333331	2333331

						23333331
1111111	6565656	6464646	6363636	1212121	1616161	23333331
5656565	2222222	5454545	2323232	2525252	5151515	23333331
4646464	4545454	3333333	3434343	4242424	3131313	23333331
3636363	3232323	4343434	4444444	3535353	4141414	23333331
2121212	5252525	2424242	5353535	5555555	2626262	23333331
6161616	1515151	1313131	1414141	6262626	6666666	23333331
23333331	23333331	23333331	23333331	23333331	23333331	23333331

						233333331
11111111	65655656	64644646	63633636	12122121	16166161	233333331
56566565	22222222	54544545	23233232	25255252	51511515	233333331
46466464	45455454	33333333	34344343	42422424	31311313	233333331
36366363	32322323	43433434	44444444	35355353	41411414	233333331
21211212	52522525	24244242	53533535	55555555	26266262	233333331
61611616	15155151	13133131	14144141	62622626	66666666	233333331
233333331	233333331	233333331	233333331	233333331	233333331	233333331

According to above six magic squares of order 6, we have the following **number pattern** with magic sums. See below:

Digits in each cell	Magic Sums
3	2331
4	23331
5	233331
6	2333331
7	23333331
8	233333331

5.3 Composite Magic Squares

Grid 5.3. *Eliminating the third values in Grid 5.1, and then splitting in two single digits grids, we get*

a	f	f	f	a	a
e	b	e	b	b	e
d	d	c	c	d	c
c	c	d	d	c	d
b	e	b	e	e	b
f	a	a	a	f	f
			A		

a	e	d	c	b	f
f	b	d	c	e	a
f	e	c	d	b	a
f	b	c	d	e	a
a	b	d	c	e	f
a	e	c	d	b	f
			B		

aa	fe	fd	fc	ab	af
ef	bb	ed	bc	be	ea
df	de	cc	cd	db	ca
cf	cb	dc	dd	ce	da
ba	eb	bd	ec	ee	bf
fa	ae	ac	ad	fb	ff
			A		

The grid AB can be written as

$$AB := 10 \times A + B$$

Example 5.6. *In particular for* $a = 1, b = 2, c = 3, d = 4, e = 5$ *and* $e = 6$*, we get*

1	6	6	6	1	1
5	2	5	2	2	5
4	4	3	3	4	3
3	3	4	4	3	4
2	5	2	5	5	2
6	1	1	1	6	6
			A		

1	5	4	3	2	6
6	2	4	3	5	1
6	5	3	4	2	1
6	2	3	4	5	1
1	2	4	3	5	6
1	5	3	4	2	6
			B		

11	65	64	63	12	16
56	22	54	23	25	51
46	45	33	34	42	31
36	32	43	44	35	41
21	52	24	53	55	26
61	15	13	14	62	66
			AB		

Note 5.1. *Applying* $6 \times (A-1) + B$ *over the elements of* A *and* B *given above, we get a magic square of order 6 given in Example 5.1. We observe that both the grid* A *and* B *don't obey very much the Latin square rules. Below is another example, where at least one of the grid is diagonal Latin square.*

Example 5.7. *Alternative way of composite magic square is given by*

2	*1*	*4*	*5*	*6*	*3*
1	*3*	*6*	*4*	*5*	*2*
3	*6*	*5*	*2*	*4*	*1*
4	*2*	*1*	*6*	*3*	*5*
5	*4*	*2*	*3*	*1*	*6*
6	*5*	*3*	*1*	*2*	*4*
			A		

5	*2*	*3*	*4*	*2*	*5*
1	*3*	*5*	*5*	*1*	*6*
1	*3*	*3*	*2*	*6*	*6*
2	*4*	*3*	*4*	*6*	*2*
6	*4*	*3*	*2*	*5*	*1*
6	*5*	*4*	*4*	*1*	*1*
			B		

25	*12*	*43*	*54*	*62*	*35*
11	*33*	*65*	*45*	*51*	*26*
31	*63*	*53*	*22*	*46*	*16*
42	*24*	*13*	*64*	*36*	*52*
56	*44*	*23*	*32*	*15*	*61*
66	*55*	*34*	*14*	*21*	*41*
			AB		

Note 5.2. *In this example, the grid A is diagonal Latin square. The Example 5.6 helps in bringing block-wise magic squares multiple of* $6k$ *(ref. Taneja [27]). Since, there don't exists regular pair of mutually diagonal Latin squares resulting in a magic square of order 6, we don't have* ***superimposed colored pattern****.*

Below is exceptional example of composite magic square in terms of three letters 2, 5 and 8.

Example 5.8. *For* $a = 25, b = 28, c = 52, d = 58, e = 82$ *and* $f = 85$ *in Grid 5.3, we get following* ***upside down*** *magic square of order 6:*

						33330
2525	*8582*	*8558*	*8552*	*2528*	*2585*	*33330*
8285	*2828*	*8258*	*2852*	*2882*	*8225*	*33330*
5885	*5882*	*5252*	*5258*	*5828*	*5225*	*33330*
5285	*5228*	*5852*	*5858*	*5282*	*5825*	*33330*
2825	*8228*	*2858*	*8252*	*8282*	*2885*	*33330*
8525	*2582*	*2552*	*2558*	*8528*	*8585*	*33330*
33330	*33330*	*33330*	*33330*	*33330*	*33330*	*33330*

Below is digital version of above magic square:

2525	8582	8558	8552	2528	2585
8285	2828	8258	2852	2882	8225
5885	5882	5252	5258	5828	5225
5285	5228	5852	5858	5282	5825
2825	8228	2858	8252	8282	2885
8525	2582	2552	2558	8528	8585

- 180^o **Rotation**

Rotating above magic square in 180^o, we get following magic square:

5858	8258	8552	2552	2852	5258
5882	2828	2528	8582	8228	5282
5285	2825	8585	2585	8225	5825
5225	8285	8525	2525	2885	5885
5228	2882	2582	8528	8282	5828
5852	8252	2558	8558	2858	5252

Note 5.3. *As we have seen above the magic square is* ***upside down****. It is not* ***mirror looking*** *magic square. In mirror 2 becomes 5 and 5 as 2, 8 remains 8. Replacing the positions if 2 by 5 and 5 by 2 the result that we get is not a magic square. See below:*

						33330
5252	8285	8228	8225	5258	5282	40530
8582	5858	8528	5825	5885	8552	43230
2882	2885	2525	2528	2858	2552	16230
2582	2558	2825	2828	2585	2852	16230
5852	8558	5828	8525	8585	5882	43230
8252	5285	5225	5228	8258	8282	40530
33402	33429	33159	33159	33429	33402	33330

Note 5.4. *Thus we observe that it is not a magic square. The same happens with the magic square of order 3 with three digits given in Example 2.3. It is* ***upside down*** *but in the mirror it becomes* ***semi-magic*** *square.*

6 Pan Diagonal Magic Squares of Order 7

Example 6.1. *A* ***pan diagonal*** *magic square of order 7 is given by*

		175	175	175	175	175	175	175
	1	9	17	25	33	41	49	15
175	40	48	7	8	16	24	32	175
175	23	31	39	47	6	14	15	175
175	13	21	22	30	38	46	5	175
175	45	4	12	20	28	29	37	175
175	35	36	44	3	11	19	27	175
175	18	26	34	42	43	2	10	175
	175	175	175	175	175	175	175	175

6.1 Palindromic Representations

Let's consider seven letters a, b, c, d, e, f and g, where $a, b, c, d, e, f, g \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. We can make exactly 49 palindromes of 3-digits with these seven letters. See the table below:

Table 6.1. *The palindromes are as follows:*

1	2	3	4	5	6	7	8	9	10	11	12	13
aaa	aba	aca	ada	aea	afa	aga	bab	bbb	bcb	bdb	beb	bfb
14	15	16	17	18	19	20	21	22	23	24	25	26
bgb	cac	cbc	ccc	cdc	cec	cfc	cgc	dad	dbd	dcd	ddd	ded
27	28	29	30	31	32	33	34	35	36	37	38	39
dfd	dgd	faf	fbf	fcf	fdf	fef	fff	fgf	faf	fbf	fcf	fdf
40	41	42	43	44	45	46	47	48	49			
fef	fff	fgf	gag	gbg	gcg	gdg	geg	gfg	ggg			

Replacing the above values with their respective palindromes in a magic square of order 7 given in Example 6.1 , we get a **palindromic-type** grid given in grid below:

Grid 6.1. *Using seven letters a, b, c, d, e, f and g, we have only 49 palindromes of 3-digits. This allows us to write as the following palindromic grid:*

aaa	bbb	ccc	ddd	eee	fff	ggg
fef	gfg	aga	bab	cbc	dcd	ede
dbd	ece	fdf	geg	afa	bgb	cac
bfb	cgc	dad	ebe	fcf	gdg	aea
gcg	ada	beb	cfc	dgd	eae	fbf
ege	faf	gbg	aca	bdb	cec	dfd
cdc	ded	efe	fgf	gag	aba	bcb

where $a, b, c, d, e, f, g \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

For all $a, b, c, d, e, f, g \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the grid given in 6.1 represents a **palindromic** magic square of order 7, if exists. In this case the magic sum is given by

$$S_{7\times 7}(a, b, c, d, e) := (a + b + c + d + e + f + g) \times 111.$$

Let's see some examples below:

Example 6.2. *For $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 5$, $f = 6$ and $g = 7$ in Grid 6.1, the 3-digits **palindromic** magic square of order 7 with magic sum $S_{7\times 7}(1, 2, 3, 4, 5, 6, 7) := (1 + 2 + 3 + 4 + 5 + 6 + 7) \times 111 = 3108$ is given by.*

		3108	3108	3108	3108	3108	3108	3108
	111	222	333	444	555	666	777	3108
3108	656	767	171	212	323	434	545	3108
3108	424	535	646	757	161	272	313	3108
3108	262	373	414	525	636	747	151	3108
3108	737	141	252	363	474	515	626	3108
3108	575	616	727	131	242	353	464	3108
3108	343	454	565	676	717	121	232	3108
	3108	3108	3108	3108	3108	3108	3108	3108

The above Example 6.2 we consider with consecutive numbers 1 to 7 that gives us a **pan diagonal** magic square. Below is another example, with non consecutive numbers.

Example 6.3. *For $a = 1$, $b = 2$, $c = 3$, $d = 5$, $e = 6$, $f = 8$ and $g = 9$ in Grid 6.1, the 3-digits* ***pan diagonal palindromic*** *magic square of order 7 with magic sum* $S_{7\times7}(1,2,3,5,6,8,9) := (1+2+3+5+6+8+9)\times 111 = 3774$ *is given by.*

		3774	3774	3774	3774	3774	3774	3774
	111	222	333	555	666	888	999	3774
3774	868	989	191	212	323	535	656	3774
3774	525	636	858	969	181	292	313	3774
3774	282	393	515	626	838	959	161	3774
3774	939	151	262	383	595	616	828	3774
3774	696	818	929	131	252	363	585	3774
3774	353	565	686	898	919	121	232	3774
	3774	3774	3774	3774	3774	3774	3774	3774

Example 6.4. *For $a = 0$, $b = 1$, $c = 2$, $d = 5$, $e = 6$, $f = 8$ and $g = 9$ in Grid 6.1, the 3-digits* ***pan diagonal palindromic-type*** *magic square of order 7 with magic sum* $S_{7\times7}(0,1,2,5,6,8,9) := (0+1+2+5+6+8+9)\times 111 = 3441$ *is given by*

		3441	3441	3441	3441	3441	3441	3441
	000	111	222	555	666	888	999	3441
3441	868	989	090	101	212	525	656	3441
3441	515	626	858	969	080	191	202	3441
3441	181	292	505	616	828	959	060	3441
3441	929	050	161	282	595	606	818	3441
3441	696	808	919	020	151	262	585	3441
3441	252	565	686	898	909	010	121	3441
	3441	3441	3441	3441	3441	3441	3441	3441

We observe that 010, 020, etc. are not palindromes, but they written here just to have symmetry in a magic square. The above Example 4.3 is **upside down** but not **mirror looking**. See below:

000	111	222	555	666	888	999
868	989	090	101	212	525	656
515	626	858	969	080	191	202
181	292	505	616	828	959	060
929	050	161	282	595	606	818
696	808	919	020	151	262	585
252	565	686	898	909	010	121

6.2 Patterned Magic Sums

The the palindromic grid 6.1 can be extended for the higher order palindromes. See below

Grid 6.2. *For all* $a,b,c,d,e,f,g \in \{1,2,3,4,5,6,7,8,9\}$, *we have following higher digits palindromic grids:*

aaaa	bbbb	cccc	dddd	eeee	ffff	gggg
feef	gffg	agga	baab	cbbc	dccd	edde
dbbd	ecce	fddf	geeg	affa	bggb	caac
bffb	cggc	daad	ebbe	fccf	gddg	aeea
gccg	adda	beeb	cffc	dggd	eaae	fbbf
egge	faaf	gbbg	acca	bddb	ceec	dffd
cddc	deed	effe	fggf	gaag	abba	bccb

aaaaa	bbbbb	ccccc	ddddd	eeeee	fffff	ggggg
fefef	gfgfg	agaga	babab	cbcbc	dcdcd	edede
dbdbd	ecece	fdfdf	gegeg	afafa	bgbgb	cacac
bfbfb	cgcgc	dadad	ebebe	fcfcf	gdgdg	aeaea
gcgcg	adada	bebeb	cfcfc	dgdgd	eaeae	fbfbf
egege	fafaf	gbgbg	acaca	bdbdb	cecec	dfdfd
cdcdc	deded	efefe	fgfgf	gagag	ababa	bcbcb

aaaaaa	bbbbbb	cccccc	dddddd	eeeeee	ffffff	gggggg
feffef	gfggfg	agaaga	babbab	cbccbc	dcddcd	edeede
dbddbd	eceece	fdffdf	geggeg	afaafa	bgbbgb	caccac
bfbbfb	cgccgc	daddad	ebeebe	fcffcf	gdggdg	aeaaea
gcggcg	adaada	bebbeb	cfccfc	dgddgd	eaeeae	fbffbf
egeege	faffaf	gbggbg	acaaca	bdbbdb	ceccec	dfddfd
cdccdc	dedded	efeefe	fgffgf	gaggag	abaaba	bcbbcb

aaaaaaa	bbbbbbb	ccccccc	ddddddd	eeeeeee	fffffff	ggggggg
fefefef	gfgfgfg	agagaga	bababab	cbcbcbc	dcdcdcd	ededede
dbdbdbd	ececece	fdfdfdf	gegegeg	afafafa	bgbgbgb	cacacac
bfbfbfb	cgcgcgc	dadadad	ebebebe	fcfcfcf	gdgdgdg	aeaeaea
gcgcgcg	adadada	bebebeb	cfcfcfc	dgdgdgd	eaeaeae	fbfbfbf
egegege	fafafaf	gbgbgbg	acacaca	bdbdbdb	cececec	dfdfdfd
cdcdcdc	dededed	efefefe	fgfgfgf	gagagag	abababa	bcbcbcb

aaaaaaaa	bbbbbbbb	cccccccc	dddddddd	eeeeeeee	ffffffff	gggggggg
fefeefef	gfgffgfg	agaggaga	babaabab	cbcbbcbc	dcdccdcd	ededdede
dbdbbdbd	ececcece	fdfddfdf	gegeegeg	afaffafa	bgbggbgb	cacaacac
bfbffbfb	cgcggcgc	dadaadad	ebebbebe	fcfccfcf	gdgddgdg	aeaeeaea
gcgccgcg	adaddada	bebeebeb	cfcffcfc	dgdggdgd	eaeaaeae	fbfbbfbf
egeggege	fafaafaf	gbgbbgbg	acaccaca	bdbddbdb	ceceecec	dfdffdfd
cdcddcdc	dedeeded	efeffefe	fgfggfgf	gagaagag	ababbaba	bcbccbcb

If magic squares of order 7 exits, then magic sums are given by

Digits	Magic Sums
3	$S_{5\times5}(a,b,c,d,e,f,g) := (a+b+c+d+e+f+g)\times 111$
4	$S_{5\times5}(a,b,c,d,e,f,g) := (a+b+c+d+e+f+g)\times 1111$
5	$S_{5\times5}(a,b,c,d,e,f,g) := (a+b+c+d+e+f+g)\times 11111$
6	$S_{5\times5}(a,b,c,d,e,f,g) := (a+b+c+d+e+f+g)\times 111111$
7	$S_{5\times5}(a,b,c,d,e,f,g) := (a+b+c+d+e+f+g)\times 1111111$
8	$S_{5\times5}(a,b,c,d,e,f,g) := (a+b+c+d+e+f+g)\times 11111111$

Making proper choices of $a, b, c, d, e \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the grids given in 6.1 and 6.2 lead **pan diagonal palindromic** magic squares of order 7 resulting in an interesting **number pattern** in magic sums. See below examples.

Example 6.5. *For $a = 1, b = 2, c = 4, d = 5, e = 6, f = 8$ and $g = 9$ in Grids 6.1 and 6.2, below are* ***pan diagonal palindromic*** *magic square of order 7 for 3 to 8 digits:*

		3885	3885	3885	3885	3885	3885	3885
	111	222	444	555	666	888	999	3885
3885	868	989	191	212	424	545	656	3885
3885	525	646	858	969	181	292	414	3885
3885	282	494	515	626	848	959	161	3885
3885	949	151	262	484	595	616	828	3885
3885	696	818	929	141	252	464	585	3885
3885	454	565	686	898	919	121	242	3885
	3885	3885	3885	3885	3885	3885	3885	3885

		38885	38885	38885	38885	38885	38885	38885
	1111	2222	4444	5555	6666	8888	9999	38885
38885	8668	9889	1991	2112	4224	5445	6556	38885
38885	5225	6446	8558	9669	1881	2992	4114	38885
38885	2882	4994	5115	6226	8448	9559	1661	38885
38885	9449	1551	2662	4884	5995	6116	8228	38885
38885	6996	8118	9229	1441	2552	4664	5885	38885
38885	4554	5665	6886	8998	9119	1221	2442	38885
	38885	38885	38885	38885	38885	38885	38885	38885

		388885	388885	388885	388885	388885	388885	388885
	11111	22222	44444	55555	66666	88888	99999	388885
388885	86868	98989	19191	21212	42424	54545	65656	388885
388885	52525	64646	85858	96969	18181	29292	41414	388885
388885	28282	49494	51515	62626	84848	95959	16161	388885
388885	94949	15151	26262	48484	59595	61616	82828	388885
388885	69696	81818	92929	14141	25252	46464	58585	388885
388885	45454	56565	68686	89898	91919	12121	24242	388885
	388885	388885	388885	388885	388885	388885	388885	388885

		3888885	3888885	3888885	3888885	3888885	3888885	3888885
	111111	222222	444444	555555	666666	888888	999999	3888885
3888885	868868	989989	191191	212212	424424	545545	656656	3888885
3888885	525525	646646	858858	969969	181181	292292	414414	3888885
3888885	282282	494494	515515	626626	848848	959959	161161	3888885
3888885	949949	151151	262262	484484	595595	616616	828828	3888885
3888885	696696	818818	929929	141141	252252	464464	585585	3888885
3888885	454454	565565	686686	898898	919919	121121	242242	3888885
	3888885	3888885	3888885	3888885	3888885	3888885	3888885	3888885

		38888885	38888885	38888885	38888885	38888885	38888885	38888885
	1111111	2222222	4444444	5555555	6666666	8888888	9999999	38888885
38888885	8686868	9898989	1919191	2121212	4242424	5454545	6565656	38888885
38888885	5252525	6464646	8585858	9696969	1818181	2929292	4141414	38888885
38888885	2828282	4949494	5151515	6262626	8484848	9595959	1616161	38888885
38888885	9494949	1515151	2626262	4848484	5959595	6161616	8282828	38888885
38888885	6969696	8181818	9292929	1414141	2525252	4646464	5858585	38888885
38888885	4545454	5656565	6868686	8989898	9191919	1212121	2424242	38888885
	38888885	38888885	38888885	38888885	38888885	38888885	38888885	38888885

		388888885	388888885	388888885	388888885	388888885	388888885	388888885
	11111111	22222222	44444444	55555555	66666666	88888888	99999999	388888885
388888885	86866868	98988989	19199191	21211212	42422424	54544545	65655656	388888885
388888885	52522525	64644646	85855858	96966969	18188181	29299292	41411414	388888885
388888885	28288282	49499494	51511515	62622626	84844848	95955959	16166161	388888885
388888885	94944949	15155151	26266262	48488484	59599595	61611616	82822828	388888885
388888885	69699696	81811818	92922929	14144141	25255252	46466464	58588585	388888885
388888885	45455454	56566565	68688686	89899898	91911919	12122121	24244242	388888885
	388888885	388888885	388888885	388888885	388888885	388888885	388888885	388888885

According to above six magic squares of order 7, we have the following **number pattern** with magic sums. See below:

Digits in each cell	Magic Sums
3	3885
4	38885
5	388885
6	3888885
7	38888885
8	388888885

6.3 Composite Magic Squares

Grid 6.3. *Eliminating the third value in Grid 6.1, and then splitting in two Latin squares, we get*

a	b	c	d	e	f	g
f	g	a	b	c	d	e
d	e	f	g	a	b	c
b	c	d	e	f	g	a
g	a	b	c	d	e	g
e	f	g	a	b	c	d
c	d	e	f	g	a	b
			A			

a	b	c	d	e	f	g
e	f	g	a	b	c	d
b	c	d	e	f	g	a
f	g	a	b	c	d	e
c	d	e	f	g	a	b
g	a	b	c	d	e	f
d	e	f	g	a	b	c
			B			

aa	bb	cc	dd	ee	ff	gg
fe	gf	ag	ba	cb	dc	ed
db	ec	fd	ge	af	bg	ca
bf	cg	da	eb	fc	gd	ae
gc	ad	be	cf	dg	ea	fb
eg	fa	gb	ac	bd	ce	df
cd	de	ef	fg	ga	ab	bc
			AB			

The grid AB can be written as

$$AB := 10 \times A + B$$

Example 6.6. *In particular for* $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 5$, $f = 6$ *and* $g = 7$, *we get*

1	2	3	4	5	6	7
6	7	1	2	3	4	5
4	5	6	7	1	2	3
2	3	4	5	6	7	1
7	1	2	3	4	5	6
5	6	7	1	2	3	4
3	4	5	6	7	1	2
			A			

1	2	3	4	5	6	7
5	6	7	1	2	3	4
2	3	4	5	6	7	1
6	7	1	2	3	4	5
3	4	5	6	7	1	2
7	1	2	3	4	5	6
4	5	6	7	1	2	3
			B			

11	22	33	44	55	66	77
65	76	17	21	32	43	54
42	53	64	75	16	27	31
26	37	41	52	63	74	15
73	14	25	36	47	51	62
57	61	72	13	24	35	46
34	45	56	67	71	12	23
			AB			

Note 6.1. *Applying* $7\times(A-1)+B$ *over the entries of* A *and* B *given above, we get a magic square of order 7 given in Example 6.1. Below are some examples of composite* ***upside down*** *and* ***mirror looking*** *magic squares. Moreover,* A *and* B *are* ***pairwise mutually orthogonal diagonal Latin squares****. The above Grid 6.3 is written for single letters* a, b, c, d, e, f *and* g*. We can choose double digits numbers to write composite examples. See the examples below.*

Example 6.7. *For* $a = 0, b = 1, c = 2, d = 5, e = 6, f = 8$ *and* $g = 9$ *in Grid 6.3, the* ***pan diagonal upside down*** *magic square of order 7 is given by*

		341	341	341	341	341	341	341
	00	11	22	55	66	88	99	341
341	86	98	09	10	21	52	65	341
341	51	62	85	96	08	19	20	341
341	18	29	50	61	82	95	06	341
341	92	05	16	28	59	60	81	341
341	69	80	91	02	15	26	58	341
341	25	56	68	89	90	01	12	341
	341	341	341	341	341	341	341	341

The above magic square is **upside down**. See below:

00	11	22	55	66	88	99
86	98	09	10	21	52	65
51	62	85	96	08	19	20
18	29	50	61	82	95	06
92	05	16	28	59	60	81
69	80	91	02	15	26	58
25	56	68	89	90	01	12

The above magic square is with single choice of letters, below are example with two digits choices. In this case we require less number of digits, for example, (1,6,9), (2,5,8), etc.

Example 6.8. *Let's consider* $a = 11$, $b = 16$, $c = 19$, $d = 61$, $e = 69$, $f = 91$ *and* $g = 96$ *in Grid 6.3 , we get following* ***upside down*** *magic square:*

		36663	36663	36663	36663	36663	36663	36663
	1111	1616	1919	6161	6969	9191	9696	36663
36663	9169	9691	1196	1611	1916	6119	6961	36663
36663	6116	6919	9161	9669	1191	1696	1911	36663
36663	1691	1996	6111	6916	9119	9661	1169	36663
36663	9619	1161	1669	1991	6196	6911	9116	36663
36663	6996	9111	9616	1119	1661	1969	6191	36663
36663	1961	6169	6991	9196	9611	1116	1619	36663
	36663	36663	36663	36663	36663	36663	36663	36663

The above magic square is **upside down**. See below:

1111	1616	1919	6161	6969	9191	9696
9169	9691	1196	1611	1916	6119	6961
6116	6919	9161	9669	1191	1696	1911
1691	1996	6111	6916	9119	9661	1169
9619	1161	1669	1991	6196	6911	9116
6996	9111	9616	1119	1661	1969	6191
1961	6169	6991	9196	9611	1116	1619

Example 6.9. *Let's consider* $a = 25, b = 28, c = 52, d = 58, e = 82, f = 85$ *and* $e = 88$ *in Grid* 6.3 *, we get following* ***upside down*** *magic square:*

		42218	42218	42218	42218	42218	42218	42218
	2525	2828	5252	5858	8282	8585	8888	42218
42218	8582	8885	2588	2825	5228	5852	8258	42218
42218	5828	8252	8558	8882	2585	2888	5225	42218
42218	2885	5288	5825	8228	8552	8858	2582	42218
42218	8852	2558	2882	5285	5888	8225	8528	42218
42218	8288	8525	8828	2552	2858	5282	5885	42218
42218	5258	5882	8285	8588	8825	2528	2852	42218
	42218	42218	42218	42218	42218	42218	42218	42218

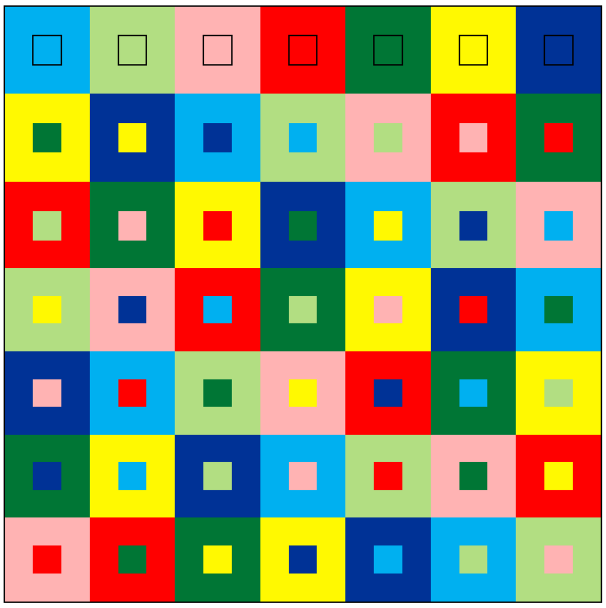
The **upside down** and **mirror looking** version of above magic square is givenby

2525	2828	5252	5858	8282	8585	8888
8582	8885	2588	2825	5228	5852	8258
5828	8252	8558	8882	2585	2888	5225
2885	5288	5825	8228	8552	8858	2582
8852	2558	2882	5285	5888	8225	8528
8288	8525	8828	2552	2858	5282	5885
5258	5882	8285	8588	8825	2528	2852

The first magic square is **upside down**, while second is **upside down** and **mirror looking**.

6.4 Superimposed Colored Pattern

The grid AB is a **composite** magic square of order 7. Based on it, here below is **double colored pattern**.



Looking from the above the **superimposed colored pattern** of order 7 has **diagonal property**, as it is made from **pairwise mutually orthogonal diagonal Latin squares**.

7 Pan diagonal Bimagic Squares of Order 8

It is well known in the literature that the first **bimagic** square of order 8 was constructed by G. Pfeffermann [6] in 1891. For more details ref C. Boyer [2]. Below is an example of a **pan diagonal bimagic** square of order 8.

Example 7.1. *Let's consider a **pan diagonal bimagic** square of order 8:*

										11180
		260	*260*	*260*	*260*	*260*	*260*	*260*	*260*	
	16	*41*	*36*	*5*	*27*	*62*	*55*	*18*	*260*	*11180*
260	*26*	*63*	*54*	*19*	*13*	*44*	*33*	*8*	*260*	*11180*
260	*1*	*40*	*45*	*12*	*22*	*51*	*58*	*31*	*260*	*11180*
260	*23*	*50*	*59*	*30*	*4*	*37*	*48*	*9*	*260*	*11180*
260	*38*	*3*	*10*	*47*	*49*	*24*	*29*	*60*	*260*	*11180*
260	*52*	*21*	*32*	*57*	*39*	*2*	*11*	*46*	*260*	*11180*
260	*43*	*14*	*7*	*34*	*64*	*25*	*20*	*53*	*260*	*11180*
260	*61*	*28*	*17*	*56*	*42*	*15*	*6*	*35*	*260*	*11180*
	260	*260*	*260*	*260*	*260*	*260*	*260*	*260*	*260*	
	11180	*11180*	*11180*	*11180*	*11180*	*11180*	*11180*	*11180*		*11180*

In this case the **magic** and **bimagic** sums are $S_{8\times 8} := 260$ and $Sb_{8\times 8} := 11180$ respectively. By **bimagic** sum we understand as the sum of square of each term.

7.1 Palindromic Representations

Let's consider eight letters a, b, c, d, e, f, g and h, where $a, b, c, d, e, f, g, h \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. We can make exactly 64 palindromes of 3-digits with these seven letters. See the table below:

Table 7.1. *The palindromes are as follows:*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
aaa	aba	aca	ada	aea	afa	aga	aha	bab	bbb	bcb	bdb	beb	bfb	bgb	bhb
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
cac	cbc	ccc	cdc	cec	cfc	cgc	chc	dad	dbd	dcd	ddd	ded	dfd	dgd	dhd
33	34	35	36	27	38	39	40	41	42	43	44	45	46	47	48
eae	ebe	ece	ede	eee	efe	ege	ehe	faf	fbf	fcf	fdf	fef	fff	fgf	fhf
49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64
gag	gbg	gcg	gdg	geg	gfg	ggg	ghg	hah	hbh	hch	hdh	heh	hfh	hgh	hhh

Replacing the above values with their respective palindromes in a magic square of order 8 given in Example 7.14 , we get a **palindromic grid** given below:

Grid 7.1. *Using eight letters a, b, c, d, e, f, g and h, we have only 49 palindromes of 3-digits. This allow us to write as the following palindromic grid:*

bhb	faf	ede	aea	dcd	hfh	ggg	cbc
dbd	hgh	gfg	ccc	beb	fdf	eae	aha
aaa	ehe	fef	bdb	cfc	gcg	hbh	dgd
cgc	gbg	hch	dfd	ada	eee	fhf	bab
efe	aca	bbb	fgf	gag	chc	ded	hdh
gdg	cec	dhd	hah	ege	aba	bcb	fff
fcf	bfb	aga	ebe	hhh	dad	cdc	geg
heh	ddd	cac	ghg	fbf	bgb	afa	ece

where $a, b, c, d, e, f, g, h \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

For all $a, b, c, d, e, f, g, h \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the grid given in 7.1 represents a **palindromic** magic square of order 8, if exists. In this case the magic sum is given by

$$S_{7\times 7}(a, b, c, d, e, f, g, h) := (a + b + c + d + e + f + g + h) \times 111.$$

Let's see some examples of Grid 7.1:

Example 7.2. *For $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 5$, $f = 6$, $g = 7$ and $h = 8$ in Grid 7.1, the 3-digits **pan diagonal palindromic bimagic** square of order 8 with magic sum $S_{8\times 8}(1, 2, 3, 4, 5, 6, 7, 8) := (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) \times 111 = 3996$ is given by.*

										2428644
		3996	3996	3996	3996	3996	3996	3996	3996	
	282	616	545	151	434	868	777	323	3996	2428644
3996	424	878	767	333	252	646	515	181	3996	2428644
3996	111	585	656	242	363	737	828	474	3996	2428644
3996	373	727	838	464	141	555	686	212	3996	2428644
3996	565	131	222	676	717	383	454	848	3996	2428644
3996	747	353	484	818	575	121	232	666	3996	2428644
3996	636	262	171	525	888	414	343	757	3996	2428644
3996	858	444	313	787	626	272	161	535	3996	2428644
	3996	3996	3996	3996	3996	3996	3996	3996	3996	
	2428644	2428644	2428644	2428644	2428644	2428644	2428644	2428644		2428644

Example 7.3. *For $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 6$, $f = 7$, $g = 8$ and $h = 9$ in Grid 7.1, the 3-digits* ***pan diagonal palindromic bimagic*** *square of order 8 with magic sum $S_{8\times8}(1,2,3,4,5,6,7,8) := (1+2+3+4+6+7+8+9)\times111 = 4440$ is given by.*

										3082260
		4440	4440	4440	4440	4440	4440	4440	4440	
	292	717	646	161	434	979	888	323	4440	3082260
4440	424	989	878	333	262	747	616	191	4440	3082260
4440	111	696	767	242	373	838	929	484	4440	3082260
4440	383	828	939	474	141	666	797	212	4440	3082260
4440	676	131	222	787	818	393	464	949	4440	3082260
4440	848	363	494	919	686	121	232	777	4440	3082260
4440	737	272	181	626	999	414	343	868	4440	3082260
4440	969	444	313	898	727	282	171	636	4440	3082260
	4440	4440	4440	4440	4440	4440	4440	4440	4440	
	3082260	3082260	3082260	3082260	3082260	3082260	3082260	3082260		3082260

Example 7.4. *For $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 5$, $f = 7$, $g = 8$ and $h = 9$ in Grid 7.1, the 3-digits magic square of order 8 with magic sum $S_{8\times8}(1,2,3,4,5,6,7,8) := (1+2+3+4+5+7+8+9)\times111 = 4329$ is given by.*

										2948749
		4329	4238	4329	4329	4329	4420	4329	4329	
	292	717	545	151	434	979	888	323	4329	2958849
4238	424	989	878	333	252	747	515	191	4329	2962889
4440	111	595	757	242	373	838	929	484	4329	2936629
4238	383	828	939	474	141	555	797	212	4329	2946729
4329	575	131	222	787	818	393	454	949	4329	2946729
4420	848	353	494	919	585	121	232	777	4329	2946729
4218	737	272	181	525	999	414	343	858	4329	2948749
4420	959	444	313	898	727	282	171	535	4329	2944709
	4329	4329	4329	4329	4329	4329	4329	4329	4329	
	2936629	2944709	2946729	2962889	2948749	2946729	2958849	2946729		2948749

This example is just a magic square. It is neither ***pan diagonal*** *nor* ***bimagic****. All it depends upon the choice of numbers.*

As in the previous cases, here we don't have examples for **upside down** magic square. The reason is that we have only 7 digits, i.e., 0, 1, 2, 5, 6, 8 and 9 with the possibility of **upside down** and/or **mirror looking**, with single choice of letters. If we consider double or more digits, we can have these types of magic squares. This we shall see later.

7.2 Patterned Magic Sums

The the palindromic grid 7.1 can be extended for the higher order palindromes in each cell. See below

Grid 7.2. *For all* $a,b,c,d,e,f,g,h \in \{1,2,3,4,5,6,7,8,9\}$, *we have following higher digits palindromic grids:*

bhhb	*faaf*	*edde*	*aeea*	*dccd*	*hffh*	*gggg*	*cbbc*
dbbd	*hggh*	*gffg*	*cccc*	*beeb*	*fddf*	*eaae*	*ahha*
aaaa	*ehhe*	*feef*	*bddb*	*cffc*	*gccg*	*hbbh*	*dggd*
cggc	*gbbg*	*hcch*	*dffd*	*adda*	*eeee*	*fhhf*	*baab*
effe	*acca*	*bbbb*	*fggf*	*gaag*	*chhc*	*deed*	*hddh*
gddg	*ceec*	*dhhd*	*haah*	*egge*	*abba*	*bccb*	*ffff*
fccf	*bffb*	*agga*	*ebbe*	*hhhh*	*daad*	*cddc*	*geeg*
heeh	*dddd*	*caac*	*ghhg*	*fbbf*	*bggb*	*affa*	*ecce*

bhbhb	*fafaf*	*edede*	*aeaea*	*dcdcd*	*hfhfh*	*ggggg*	*cbcbc*
dbdbd	*hghgh*	*gfgfg*	*ccccc*	*bebeb*	*fdfdf*	*eaeae*	*ahaha*
aaaaa	*ehehe*	*fefef*	*bdbdb*	*cfcfc*	*gcgcg*	*hbhbh*	*dgdgd*
cgcgc	*gbgbg*	*hchch*	*dfdfd*	*adada*	*eeeee*	*fhfhf*	*babab*
efefe	*acaca*	*bbbbb*	*fgfgf*	*gagag*	*chchc*	*deded*	*hdhdh*
gdgdg	*cecec*	*dhdhd*	*hahah*	*egege*	*ababa*	*bcbcb*	*fffff*
fcfcf	*bfbfb*	*agaga*	*ebebe*	*hhhhh*	*dadad*	*cdcdc*	*gegeg*
heheh	*ddddd*	*cacac*	*ghghg*	*fbfbf*	*bgbgb*	*afafa*	*ecece*

bhbbhb	*faffaf*	*edeede*	*aeaaea*	*dcddcd*	*hfhhfh*	*gggggg*	*cbccbc*
dbddbd	*hghhgh*	*gfggfg*	*cccccc*	*bebbeb*	*fdffdf*	*eaeeae*	*ahaaha*
aaaaaa	*eheehe*	*feffef*	*bdbbdb*	*cfccfc*	*gcggcg*	*hbhhbh*	*dgddgd*
cgccgc	*gbggbg*	*hchhch*	*dfddfd*	*adaada*	*eeeeee*	*fhffhf*	*babbab*
efeefe	*acaaca*	*bbbbbb*	*fgffgf*	*gaggag*	*chcchc*	*dedded*	*hdhhdh*
gdggdg	*ceccec*	*dhddhd*	*hahhah*	*egeege*	*abaaba*	*bcbbcb*	*ffffff*
fcffcf	*bfbbfb*	*agaaga*	*ebeebe*	*hhhhhh*	*daddad*	*cdccdc*	*geggeg*
hehheh	*dddddd*	*caccac*	*ghgghg*	*fbffbf*	*bgbbgb*	*afaafa*	*eceece*

bhbhbhb	*fafafaf*	*ededede*	*aeaeaea*	*dcdcdcd*	*hfhfhfh*	*ggggggg*	*cbcbcbc*
dbdbdbd	*hghghgh*	*gfgfgfg*	*ccccccc*	*bebebeb*	*fdfdfdf*	*eaeaeae*	*ahahaha*
aaaaaaa	*ehehehe*	*fefefef*	*bdbdbdb*	*cfcfcfc*	*gcgcgcg*	*hbhbhbh*	*dgdgdgd*
cgcgcgc	*gbgbgbg*	*hchchch*	*dfdfdfd*	*adadada*	*eeeeeee*	*fhfhfhf*	*bababab*
efefefe	*acacaca*	*bbbbbbb*	*fgfgfgf*	*gagagag*	*chchchc*	*dededed*	*hdhdhdh*
gdgdgdg	*cececec*	*dhdhdhd*	*hahahah*	*egegege*	*abababa*	*bcbcbcb*	*fffffff*
fcfcfcf	*bfbfbfb*	*agagaga*	*ebebebe*	*hhhhhhh*	*dadadad*	*cdcdcdc*	*gegegeg*
heheheh	*ddddddd*	*cacacac*	*ghghghg*	*fbfbfbf*	*bgbgbgb*	*afafafa*	*ececece*

bhbhhbhb	*fafaafaf*	*ededdede*	*aeaeeaea*	*dcdccdcd*	*hfhffhfh*	*gggggggg*	*cbcbbcbc*
dbdbbdbd	*hghgghgh*	*gfgffgfg*	*cccccccc*	*bebeebeb*	*fdfddfdf*	*eaeaaeae*	*ahahhaha*
aaaaaaaa	*ehehhehe*	*fefeefef*	*bdbddbdb*	*cfcffcfc*	*gcgccgcg*	*hbhbbhbh*	*dgdggdgd*
cgcggcgc	*gbgbbgbg*	*hchcchch*	*dfdffdfd*	*adaddada*	*eeeeeeee*	*fhfhhfhf*	*babaabab*
efeffefe	*acaccaca*	*bbbbbbbb*	*fgfggfgf*	*gagaagag*	*chchhchc*	*dedeeded*	*hdhddhdh*
gdgddgdg	*ceceecec*	*dhdhhdhd*	*hahaahah*	*egeggege*	*ababbaba*	*bcbccbcb*	*ffffffff*
fcfccfcf	*bfbffbfb*	*agaggaga*	*ebebbebe*	*hhhhhhhh*	*dadaadad*	*cdcddcdc*	*gegeegeg*
heheeheh	*dddddddd*	*cacaacac*	*ghghhghg*	*fbfbbfbf*	*bgbggbgb*	*afaffafa*	*ececcece*

If magic squares of order 8 exists, then magic sums are given by

Digits	Magic Sums
3	$S_{8\times 8}(a,b,c,d,e,f,g,h) := (a+b+c+d+e+f+g+h)\times 111$
4	$S_{8\times 8}(a,b,c,d,e,f,g,h) := (a+b+c+d+e+f+g+h)\times 1111$
5	$S_{8\times 8}(a,b,c,d,e,f,g,h) := (a+b+c+d+e+f+g+h)\times 11111$
6	$S_{8\times 8}(a,b,c,d,e,f,g,h) := (a+b+c+d+e+f+g+h)\times 111111$
7	$S_{8\times 8}(a,b,c,d,e,f,g,h) := (a+b+c+d+e+f+g+h)\times 1111111$
8	$S_{8\times 8}(a,b,c,d,e,f,g,h) := (a+b+c+d+e+f+g+h)\times 11111111$

Making proper choices of $a,b,c,d,e,f,g,h \in \{1,2,3,4,5,6,7,8,9\}$, the grids given in 7.1 and 7.2 lead us to a **pan diagonal palindromic bimagic** squares of order 8 resulting in a **number pattern** in magic sums. See below examples.

Example 7.5. *For $a=1$, $b=2$, $c=3$, $d=5$, $e=6$, $f=7$, $g=8$ and $8=9$ in Grids 7.1 and 7.2 , the 3 to 8 digits* ***pan diagonal palindromic bimagic*** *square of order 8 are given by*

		4440	4440	4440	4440	4440	4440	4440	4440
	292	717	646	161	434	979	888	323	4440
4440	424	989	878	333	262	747	616	191	4440
4440	111	696	767	242	373	838	929	484	4440
4440	383	828	939	474	141	666	797	212	4440
4440	676	131	222	787	818	393	464	949	4440
4440	848	363	494	919	686	121	232	777	4440
4440	737	272	181	626	999	414	343	868	4440
4440	969	444	313	898	727	282	171	636	4440
	4440	4440	4440	4440	4440	4440	4440	4440	4440

Its magic and ***bimagic*** *sums are $S_{8\times 8} := 4440$ and $Sb_{8\times 8} := 3082260$ respectively.*

		44440	44440	44440	44440	44440	44440	44440	44440
	2992	7117	6446	1661	4334	9779	8888	3223	44440
44440	4224	9889	8778	3333	2662	7447	6116	1991	44440
44440	1111	6996	7667	2442	3773	8338	9229	4884	44440
44440	3883	8228	9339	4774	1441	6666	7997	2112	44440
44440	6776	1331	2222	7887	8118	3993	4664	9449	44440
44440	8448	3663	4994	9119	6886	1221	2332	7777	44440
44440	7337	2772	1881	6226	9999	4114	3443	8668	44440
44440	9669	4444	3113	8998	7227	2882	1771	6336	44440
	44440	44440	44440	44440	44440	44440	44440	44440	44440

Its magic and ***bimagic*** *sums are $S_{8\times 8} := 44440$ and $Sb_{8\times 8} := 307710260$ respectively.*

		444440	444440	444440	444440	444440	444440	444440	444440
	29292	71717	64646	16161	43434	97979	88888	32323	444440
444440	42424	98989	87878	33333	26262	74747	61616	19191	444440
444440	11111	69696	76767	24242	37373	83838	92929	48484	444440
444440	38383	82828	93939	47474	14141	66666	79797	21212	444440
444440	67676	13131	22222	78787	81818	39393	46464	94949	444440
444440	84848	36363	49494	91919	68686	12121	23232	77777	444440
444440	73737	27272	18181	62626	99999	41414	34343	86868	444440
444440	96969	44444	31313	89898	72727	28282	17171	63636	444440
	444440	444440	444440	444440	444440	444440	444440	444440	444440

Its magic and **bimagic** *sums are* $S_{8\times 8} := 444440$ *and* $Sb_{8\times 8} := 30873882260$ *respectively.*

		4444440	4444440	4444440	4444440	4444440	4444440	4444440	4444440
	292292	717717	646646	161161	434434	979979	888888	323323	4444440
4444440	424424	989989	878878	333333	262262	747747	616616	191191	4444440
4444440	111111	696696	767767	242242	373373	838838	929929	484484	4444440
4444440	383383	828828	939939	474474	141141	666666	797797	212212	4444440
4444440	676676	131131	222222	787787	818818	393393	464464	949949	4444440
4444440	848848	363363	494494	919919	686686	121121	232232	777777	4444440
4444440	737737	272272	181181	626626	999999	414414	343343	868868	4444440
4444440	969969	444444	313313	898898	727727	282282	171171	636636	4444440
	4444440	4444440	4444440	4444440	4444440	4444440	4444440	4444440	4444440

Its magic and **bimagic** *sums are* $S_{8\times 8} := 4444440$ *and* $Sb_{8\times 8} := 3088427602260$ *respectively.*

		44444440	44444440	44444440	44444440	44444440	44444440	44444440	44444440
	2929292	7171717	6464646	1616161	4343434	9797979	8888888	3232323	44444440
44444440	4242424	9898989	8787878	3333333	2626262	7474747	6161616	1919191	44444440
44444440	1111111	6969696	7676767	2424242	3737373	8383838	9292929	4848484	44444440
44444440	3838383	8282828	9393939	4747474	1414141	6666666	7979797	2121212	44444440
44444440	6767676	1313131	2222222	7878787	8181818	3939393	4646464	9494949	44444440
44444440	8484848	3636363	4949494	9191919	6868686	1212121	2323232	7777777	44444440
44444440	7373737	2727272	1818181	6262626	9999999	4141414	3434343	8686868	44444440
44444440	9696969	4444444	3131313	8989898	7272727	2828282	1717171	6363636	44444440
	44444440	44444440	44444440	44444440	44444440	44444440	44444440	44444440	44444440

Its magic and **bimagic** *sums are* $S_{8\times 8} := 44444440$ *and* $Sb_{8\times 8} := 308743953882260$ *respectively.*

		444444440	444444440	444444440	444444440	444444440	444444440	444444440	444444440
	92922929	17177171	46466464	61611616	34344343	79799797	88888888	23233232	444444440
444444440	24244242	89899898	78788787	33333333	62622626	47477474	16166161	91911919	444444440
444444440	11111111	96966969	67677676	42422424	73733737	38388383	29299292	84844848	444444440
444444440	83833838	28288282	39399393	74744747	41411414	66666666	97977979	12122121	444444440
444444440	76766767	31311313	22222222	87877878	18188181	93933939	64644646	49499494	444444440
444444440	48488484	63633636	94944949	19199191	86866868	21211212	32322323	77777777	444444440
444444440	37377373	72722727	81811818	66266266	99999999	14144141	43433434	68688686	444444440
444444440	69699696	44444444	13133131	98988989	27277272	82822828	71711717	36366363	444444440
	444444440	444444440	444444440	444444440	444444440	444444440	444444440	444444440	444444440

Its magic and **bimagic** sums are $S_{8\times 8} := 444444440$ and $Sb_{8\times 8} := 30873408416682260$ respectively.

According to above six magic squares of order 8, we have the following **number pattern** with magic sums. See below:

Digits	Magic Sums	Bimagic Sums
3	4440	**308 2260**
4	44440	307 71 0260
5	444440	**308** 7388 **2260**
6	4444440	**308** 842760 **2260**
7	44444440	**308** 74395388 **2260**
8	444444440	**308** 7340841668 **2260**

We observe that there is a good pattern with magic sums, but in case **bimagic** sums, the pattern is not so good. Moreover, the 4-digits case is totally different.

7.3 5-Digit Palindromic Magic Squares with 4 Letters

We have exactly 64 choices of palindromes of 5-digits if we choose 4 letters a, b, c and d. This allows us to write following grid of order 8:

Grid 7.3. *We have following palindromic grid of order 8 with 5-digits and 4 letters a, b, c and d:*

addda	*ccacc*	*cadac*	*ababa*	*bcccb*	*ddbdd*	*dbcbd*	*babab*
bcbcb	*ddcdd*	*dbbbd*	*bacab*	*adada*	*ccdcc*	*caaac*	*abdba*
aaaaa	*cbdbc*	*cdadc*	*acdca*	*bbbbb*	*dacad*	*dcbcd*	*bdcdb*
bbcbb	*dabad*	*dcccd*	*bdbdb*	*aadaa*	*cbabc*	*cdddc*	*acaca*
cbbbc	*aacaa*	*acbca*	*cdcdc*	*daaad*	*bbdbb*	*bdadb*	*dcdcd*
dadad	*bbabb*	*bdddb*	*dcacd*	*cbcbc*	*aabaa*	*accca*	*cdbdc*
ccccc	*adbda*	*abcba*	*cabac*	*ddddd*	*bcacb*	*badab*	*dbabd*
ddadd	*bcdcb*	*baaab*	*dbdbd*	*ccbcc*	*adcda*	*abbba*	*cacac*

If exists, the above grid represent a **palindromic** magic square of order 8 with magic sum

$$S_{8\times 8}(a,b,c,d) := (a+b+c+d) \times 22222,$$

where $a, b, c, d \in \{1,2,3,4,5,6,7,8,9\}$,

We shall apply the Grid 3.4 to bring interesting magic squares, such as **upside down** and **mirror looking**. See below examples.

Example 7.6. *For $a = 1$, $b = 2$, $c = 3$ and $d = 4$ in 7.3 we have 5-digits palindromic **pan diagonal bimagic** square of order 8 with sum $S_{8\times 8}(1,2,3,4) := (1+2+3+4) \times 22222 = 222220$:*

										7183377060
		222220	222220	222220	222220	222220	222220	222220	222220	
	14441	33133	31413	12121	23332	44244	42324	21212	222220	7183377060
222220	23232	44344	42224	21312	14141	33433	31113	12421	222220	7183377060
222220	11111	32423	34143	13431	22222	41314	43234	24342	222220	7183377060
222220	22322	41214	43334	24242	11411	32123	34443	13131	222220	7183377060
222220	32223	11311	13231	34343	41114	22422	24142	43434	222220	7183377060
222220	41414	22122	24442	43134	32323	11211	13331	34243	222220	7183377060
222220	33333	14241	12321	31213	44444	23132	21412	42124	222220	7183377060
222220	44144	23432	21112	42424	33233	14341	12221	31313	222220	7183377060
	222220	222220	222220	222220	222220	222220	222220	222220	222220	
	7183377060	7183377060	7183377060	7183377060	7183377060	7183377060	7183377060	7183377060		7183377060

Example 7.7. *For $a = 2$, $b = 5$, $c = 6$ and $d = 9$ in 7.3 we have 5-digits palindromic **pan diagonal semi-bimagic** square of order 8 with sum $S_{8\times 8}(2,5,6,9) := (2+5+6+9) \times 22222 = 488884$:*

										34929410732
		488884	488884	488884	488884	488884	488884	488884	488884	
	29992	66266	62926	25252	56665	99599	95659	52525	488884	34928450732
488884	56565	99699	95559	52625	29292	66966	62226	25952	488884	34928450732
488884	22222	65956	69296	26962	55555	92629	96569	59695	488884	34928450732
488884	55655	92529	96669	59595	22922	65256	69996	26262	488884	34928450732
488884	65556	22622	26562	69696	92229	55955	59295	96969	488884	34928450732
488884	92929	55255	59995	96269	65656	22522	26662	69596	488884	34928450732
488884	66666	29592	25652	62526	99999	56265	52925	95259	488884	34928450732
488884	99299	56965	52225	95959	66566	29692	25552	62626	488884	34928450732
	488884	488884	488884	488884	488884	488884	488884	488884	488884	
	34928450732	34928450732	34928450732	34928450732	34928450732	34928450732	34928450732	34928450732		34929410732

The above magic square is **pan diagonal** and **semi-bimagic**. It can be written as **upside down**. See below:

29992	66266	62926	25252	56665	99599	95659	52525
56565	99699	95559	52625	29292	66966	62226	25952
22222	65956	69296	26962	55555	92629	96569	59695
55655	92529	96669	59595	22922	65256	69996	26262
65556	22622	26562	69696	92229	55955	59295	96969
92929	55255	59995	96269	65656	22522	26662	69596
66666	29592	25652	62526	99999	56265	52925	95259
99299	56965	52225	95959	66566	29692	25552	62626

Example 7.8. *For* $a = 0$, $b = 2$, $c = 5$ *and* $d = 8$ *in 7.3 we have 5-digits* ***palindromic-type semi-magic*** *square of order 8 with sum* $S_{8\times 8}(0,2,5,8) := (0+2+5+8)\times 22222 = 333330$*:*

								335350
08880	55055	50805	02020	25552	88288	82528	20202	333330
25252	88588	82228	20502	08080	55855	50005	02820	333330
00000	52825	58085	05850	22222	80508	85258	28582	333330
22522	80208	85558	28282	00800	52025	58885	05050	333330
52225	00500	05250	58585	80008	22822	28082	85858	333330
80808	22022	28882	85058	52525	00200	05550	58285	333330
55555	08280	02520	50205	88888	25052	20802	82028	333330
88088	25852	20002	82828	55255	08580	02220	50505	333330
333330	333330	333330	333330	333330	333330	333330	333330	335350

The above **semi-magic** square is not **bimagic**. Also, is it **palindromic-type** due to number 0. It is **upside down** and **mirror looking**. See below:

08880	55055	50805	02020	25552	88288	82528	20202
25252	88588	82228	20502	08080	55855	50005	02820
00000	52825	58085	05850	22222	80508	85258	28582
22522	80208	85558	28282	00800	52025	58885	05050
52225	00500	05250	58585	80008	22822	28082	85858
80808	22022	28882	85058	52525	00200	05550	58285
55555	08280	02520	50205	88888	25052	20802	82028
88088	25852	20002	82828	55255	08580	02220	50505

Instead of 0 if we choose 1, still it is **semi-magic** and not **bimagic**.

7.4 11-Digit Palindromic Magic Squares with 2 Letters

Let us consider the following 11-digits palindromic grid of order 8 only with two letters a and b:

Grid 7.4. *We have following palindromic grid of order 8 with 11-digits and 2 letters a and b:*

aabbbbbbbaa	*babaaaaabab*	*baaabbbaaab*	*aaabaaabaaa*	*abbabababba*	*bbbbababbbb*	*bbabbabbabb*	*abaaabaaaba*
abbaabaabba	*bbbbbabbbbb*	*bbababababb*	*abaababaaba*	*aabbaaabbaa*	*bababbbabab*	*baaaaaaaaab*	*aaabbbbbaaa*
aaaaaaaaaaa	*baabbbbbaab*	*babbaaabbab*	*aababbbabaa*	*ababababa ba*	*bbaababaabb*	*bbbaabaabbb*	*abbbbabbbba*
ababbabbaba	*bbaaabaaabb*	*bbbababab bb*	*abbbababbba*	*aaaabbbaaaa*	*baabaaabaab*	*babbbbbbbab*	*aabaaaaabaa*
baababab aab	*aaaababaaaa*	*aabaabaabaa*	*babbbabbbab*	*bbaaaaaaabb*	*ababbbbbaba*	*abbbaaabbba*	*bbbabbbabbb*
bbaabbbaabb	*ababaaababa*	*abbbbbbbbba*	*bbbaaaaabbb*	*baabbabbaab*	*aaaaabaaaaa*	*aabababa baa*	*babbababbab*
babababab ab	*aabbababbaa*	*aaabbabbaaa*	*baaaabaaaab*	*bbbbbbbbbbb*	*abbaaaaabba*	*abaabbbaaba*	*bbabaaababb*
bbbbaaabbbb	*abbabbbabba*	*abaaaaaaaba*	*bbabbbbbabb*	*babaabaabab*	*aabbbabbbaa*	*aaabababaaa*	*baaababaaab*

For all $a, b \in \{1,2,3,4,5,6,7,8,9\}$, the above palindromic grid represents a magic square (if exists), with magic sum

$$S_{8\times 8} := 4 \times (a+b) \times 11111111111 = (a+b) \times 44444444444.$$

Below are some examples of above Grid 7.4. Since we need only two digits. The examples are **upside down** and/or **mirror looking**.

Example 7.9. *For $a = 6$ and $b = 9$ in Grid 7.4, we have 11-digits palindromic* ***upside down pan diagonal semi-bimagic*** *square of order 8 with sum* $S_{8\times 8}(6,9) := (6+9) \times 44444444444 = 666.666.666.660$ *and the* ***semi-bimagic*** *sum* $Sb_{8\times 8}(1,8) := 5737373737442626262626 8$ *(only rows and columns).*

66999999966	96966666969	96669996669	66696669666	69969696996	99996969999	99699699699	69666966696
69966966996	99999699999	99696969699	69669696696	66996669966	96969996969	96666666669	66699999666
66666666666	96699999669	96996669969	66969996966	69696969696	99669696699	99966966999	69999699996
69699699696	99666966699	99969696999	69996969996	66669996666	96696669669	96999999969	66966666966
96696969669	66669696666	66966966966	96999699969	99666666699	69699999696	69996669996	99969996999
99669996699	69696669696	69999999996	99966666999	96699699669	66666966666	66969696966	96996969969
96969696969	66996969966	66699699666	96666966669	99999999999	69966666996	69669996696	99696669699
99996669999	69969996996	69666666696	99699999699	96966966969	66999699966	66696969666	96669696669

Example 7.10. *For $a = 1$ and $b = 8$ in Grid 7.4, we have 11-digits palindromic **upside down pan diagonal semi-bimagic** square of order 8 with sum* $S_{8\times8}(1,8) := (1+8)\times 44444444444 = 399.999.999.996$ *and the **semi-bimagic** sum* $Sb_{8\times8}(6,9) := 2989898990838989898989900$ *(only rows and columns).*

11888888811	81811111818	81118881118	11181118111	18818181881	88881818888	88188188188	18111811181
18811811881	88888188888	88181818188	18118181181	11881118811	81818881818	81111111118	11188888111
11111111111	81188888118	81881118818	11818881811	18181818181	88118181188	88811811888	18888188881
18188188181	88111811188	88818181888	18881818881	11118881111	81181118118	81888888818	11811111811
81181818118	11118181111	11811811811	81888188818	88111111188	18188888181	18881118881	88818881888
88118881188	18181118181	18888888881	88811111888	81188188118	11111811111	11818181811	81881818818
81818181818	11881818811	11188188111	81111811118	88888888888	18811111881	18118881181	88181118188
88881118888	18818881881	18111111181	88188888188	81811811818	11888188811	11181818111	81118181118

Example 7.11. *For $a = 2$ and $b = 5$ in Grid 7.4, we have 11-digits palindromic **upside down pan diagonal semi-bimagic** square of order 8 with sum* $S_{8\times8}(2,5) := (2+5)\times 44444444444 = 311.111.111.108$ *and the **semi-bimagic** sum* $Sb_{8\times8}(6,9) := 1391694725183860830527 6$ *(only rows and columns).*

22555555522	52522222525	52225552225	22252225222	25525252552	55552525555	55255255255	25222522252
25522522552	55555255555	55252525255	25225252252	22552225522	52525552525	52222222225	22255555222
22222222222	52255555225	52552225525	22525552522	25252525252	55225252255	55522522555	25555255552
25255255252	55222522255	55525252555	25552525552	22225552222	52252225225	52555555525	22522222522
52252525225	22225252222	22522522522	52555255525	55222222255	25255555252	25552225552	55525552555
55225552255	25252225252	25555555552	55522222555	52255255225	22222522222	22525252522	52552525525
52525252525	22552525522	22255255222	52222522225	55555555555	25522222552	25225552252	55252225255
55552225555	25525552552	25222222252	55255555255	52522522525	22555255522	22252525222	52225252225

Note 7.1. *Above three examples give us following symmetry in magic sums:*

$$\frac{S_{8\times8}(6,9)}{6+9} = \frac{S_{8\times8}(1,8)}{1+8} = \frac{S_{8\times8}(2,5)}{2+5} = 44444444444.$$

7.5 Composite Magic Squares

Grid 7.5. *Eliminating the third value in Grid 7.1, and then splitting in two Latin squares, we get*

b	f	e	a	d	h	g	c
d	h	g	c	b	f	e	a
a	e	f	b	c	g	h	d
c	g	h	d	a	e	f	b
e	a	b	f	g	c	d	h
g	c	d	h	e	a	b	f
f	b	a	e	h	d	c	g
h	d	c	g	f	b	a	e
				A			

h	a	d	e	c	f	g	b
b	g	f	c	e	d	a	h
a	h	e	d	f	c	b	g
g	b	c	f	d	e	h	a
f	c	b	g	a	h	e	d
d	e	h	a	g	b	c	f
c	f	g	b	h	a	d	e
e	d	a	h	b	g	f	c
				B			

bh	fa	ed	ae	dc	hf	gg	cb
db	hg	gf	cc	be	fd	ea	ah
aa	eh	fe	bd	cf	gc	hb	dg
cg	gb	hc	df	ad	ee	fh	ba
ef	ac	bb	fg	ga	ch	de	hd
gd	ce	dh	ha	eg	ab	bc	ff
fc	bf	ag	eb	hh	da	cd	ge
he	dd	ca	gh	fb	bg	af	ec
				AB			

The grid AB can be written as

$$AB := 10 \times A + B$$

Example 7.12. *In particular for $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 5$, $f = 6$, $g = 7$ and $h = 8$, we get*

2	6	5	1	4	8	7	3
4	8	7	3	2	6	5	1
1	5	6	2	3	7	8	4
3	7	8	4	1	5	6	2
5	1	2	6	7	3	4	8
7	3	4	8	5	1	2	6
6	2	1	5	8	4	3	7
8	4	3	7	6	2	1	5
				A			

8	1	4	5	3	6	7	2
2	7	6	3	5	4	1	8
1	8	5	4	6	3	2	7
7	2	3	6	4	5	8	1
6	3	2	7	1	8	5	4
4	5	8	1	7	2	3	6
3	6	7	2	8	1	4	5
5	4	1	8	2	7	6	3
				B			

28	61	54	15	43	86	77	32
42	87	76	33	25	64	51	18
11	58	65	24	36	73	82	47
37	72	83	46	14	55	68	21
56	13	22	67	71	38	45	84
74	35	48	81	57	12	23	66
63	26	17	52	88	41	34	75
85	44	31	78	62	27	16	53
				AB			

Applying $8 \times (A-1) + B$ over the entries of A and B given above, we get a magic square of order 8 given in Example 7.14. Moreover, A and B are **pairwise mutually orthogonal diagonal Latin squares**. Also the grid AB a bimagic square, see below:

Example 7.13. *The grid AB given in Example 7.12 is a* ***pan diagonal bimagic*** *square of order 8 given by*

										23844
		396	396	396	396	396	396	396	396	
	28	61	54	15	43	86	77	32	396	23844
396	42	87	76	33	25	64	51	18	396	23844
396	11	58	65	24	36	73	82	47	396	23844
396	37	72	83	46	14	55	68	21	396	23844
396	56	13	22	67	71	38	45	84	396	23844
396	74	35	48	81	57	12	23	66	396	23844
396	63	26	17	52	88	41	34	75	396	23844
396	85	44	31	78	62	27	16	53	396	23844
	396	396	396	396	396	396	396	396	396	
	23844	23844	23844	23844	23844	23844	23844	23844		23844

The above example is formed by numbers 1 to 8 written in composite way giving a **bimagic** square. It is not necessary that it always happens in case the numbers are not in consecutive way. The example below shows that considering non consecutive numbers we just a magic square.

Example 7.14. *Let's consider, $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 5$, $f = 7$, $g = 8$ and $h = 9$ in Grid 7.5, we get following magic square in composite form:*

								429
29	71	54	15	43	97	88	32	429
42	98	87	33	25	74	51	19	429
11	59	75	24	37	83	92	48	429
38	82	93	47	14	55	79	21	429
57	13	22	78	81	39	45	94	429
84	35	49	91	58	12	23	77	429
73	27	18	52	99	41	34	85	429
95	44	31	89	72	28	17	53	429
429	429	429	429	429	429	429	429	429

The above magic square is neither **pan diagonal** nor **bimagic**. Actually, it is the same Example 7.4 just removing the third member in each cell. Let's see below more examples with double digit choice of letters of Grid 7.5.

Example 7.15. *Let's consider* $a = 16$, $b = 19$, $c = 61$, $d = 66$, $e = 69$, $f = 91$, $g = 96$ *and* $h = 99$ *in Grid 7.5 , we get following magic square:*

								52217
1999	9116	6966	1669	6661	9991	9696	6119	52217
6619	9996	9691	6161	1969	9166	6916	1699	52217
1616	6999	9169	1966	6191	9661	9919	6696	52217
6196	9619	9961	6691	1666	6969	9199	1916	52217
6991	1661	1919	9196	9616	6199	6669	9966	52217
9666	6169	6699	9916	6996	1619	1961	9191	52217
9161	1991	1696	6919	9999	6616	6166	9669	52217
9969	6666	6116	9699	9119	1996	1691	6961	52217
52217	52217	52217	52217	52217	52217	52217	52217	52217

*The above magic square is neither **pan diagonal** nor **bimagic**, but is **upside down**. See below*

1999	9116	6966	1669	6661	9991	9696	6119
6619	9996	9691	6161	1969	9166	6916	1699
1616	6999	9169	1966	6191	9661	9919	6696
6196	9619	9961	6691	1666	6969	9199	1916
6991	1661	1919	9196	9616	6199	6669	9966
9666	6169	6699	9916	6996	1619	1961	9191
9161	1991	1696	6919	9999	6616	6166	9669
9969	6666	6116	9699	9119	1996	1691	6961

Example 7.16. *Let's consider* $a = 22$, $b = 25$, $c = 52$, $d = 55$ *and* $e = 88$ *in Grid 7.5 , we get following* ***upside down*** *and* ***mirror looking*** *magic square:*

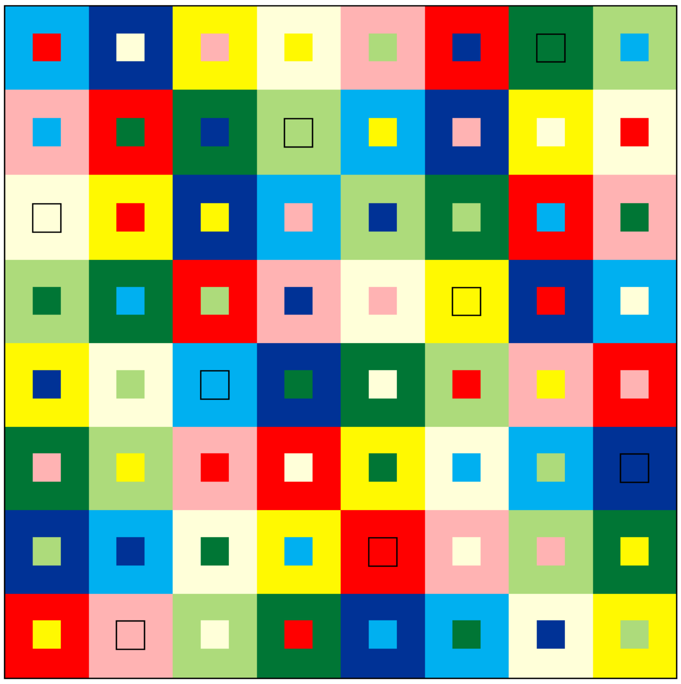
								41107
2585	5822	5552	2255	5228	8558	8282	2825	41107
5225	8582	8258	2828	2555	5852	5522	2285	41107
2222	5585	5855	2552	2858	8228	8525	5282	41107
2882	8225	8528	5258	2252	5555	5885	2522	41107
5558	2228	2525	5882	8222	2885	5255	8552	41107
8252	2855	5285	8522	5582	2225	2528	5858	41107
5828	2558	2282	5525	8585	5222	2852	8255	41107
8555	5252	2822	8285	5825	2582	2258	5528	41107
41107	41107	41107	41107	41107	41107	41107	41107	41107

The above magic square is neither **pan diagonal** nor **bimagic**, but is **upside down** and **mirror looking**. See below

2585	5822	5552	2255	5228	8558	8282	2825
5225	8582	8258	2828	2555	5852	5522	2285
2222	5585	5855	2552	2858	8228	8525	5282
2882	8225	8528	5258	2252	5555	5885	2522
5558	2228	2525	5882	8222	2885	5255	8552
8252	2855	5285	8522	5582	2225	2528	5858
5828	2558	2282	5525	8585	5222	2852	8255
8555	5252	2822	8285	5825	2582	2258	5528

7.6 Superimposed Colored Pattern

The grid *AB* is a **composite** magic square of order 8. Based on it, here below is **double colored pattern**.



Looking from the above **superimposed colored pattern** of order 8, we observe that it has **diagonal property**, as it is made from **pairwise mutually orthogonal diagonal Latin squares**.

8 Bimagic and Pan diagonal Magic Squares of Order 9

We observe that in case of magic square of order 8, we have both possibilities, i.e., pan diagonal and bimagic. In case of magic square of order 9, we don't have these two possibilities in a single magic square. Either we have pan diagonal or **bimagic** square. Initially, we shall work with bimagic magic square of order 9. It was first constructed by Pfeffermann [6] in 1891. We shall devide this section in two subsections one with **bimagic** cases another with **pan diagonal** examples.

8.1 Bimagic Magic Squares of Order 9

Example 8.1. *Let's consider a* ***bimagic*** *square of order 9 given by*

										20049
									369	
1	18	23	35	40	48	60	65	79	369	20049
33	38	52	55	72	77	8	13	21	369	20049
62	67	75	6	11	25	28	45	50	369	20049
27	5	10	49	30	44	74	61	69	369	20049
47	34	42	81	59	64	22	3	17	369	20049
76	57	71	20	7	15	54	32	37	369	20049
14	19	9	39	53	31	70	78	56	369	20049
43	51	29	68	73	63	12	26	4	369	20049
66	80	58	16	24	2	41	46	36	369	20049
369	369	369	369	369	369	369	369	369	369	
20049	20049	20049	20049	20049	20049	20049	20049	20049		20049

The above magic square is not a **pan diagonal**. Blocks of order 3 are of equal sums entries as of magic square. We shall apply this magic square to bring palindromic magic squares of order 9.

8.1.1 Palindromic Representations

Let's consider nine letters a, b, c, d, e, f, g, h and k, where $a, b, c, d, e, f, g, h, k \in \{1,2,3,4,5,6,7,8,9\}$. We can make exactly 81 palindromes of 3-digits with these nine letters. See the table below:

Table 8.1. *The palindromes are as follows:*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
aaa	aba	aca	ada	aea	afa	aga	aha	aka	bab	bbb	bcb	bdb	beb	bfb	bgb	bhb
18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
bkb	cac	cbc	ccc	cdc	cec	cfc	cgc	chc	ckc	dad	dbd	dcd	ddd	ded	dfd	dgd
35	36	27	38	39	40	41	42	43	44	45	46	47	48	49	50	51
dhd	dkd	eae	ebe	ece	ede	eee	efe	ege	ehe	eke	faf	fbf	fcf	fdf	fef	fff
52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68
fgf	fhf	fkf	gag	gbg	gcg	gdg	geg	gfg	ggg	ghg	gkg	hah	hbh	hch	hdh	heh
69	70	71	72	73	74	75	76	77	78	79	80	81				
hfh	hgh	hhh	hkh	kak	kbk	kck	kdk	kek	kfk	kgk	khk	kkk				

Replacing the above values with their respective palindromes in a magic square of order 9 given in Example 8.1 , we get a **palindromic grid** given below:

Grid 8.1. *Using nine letters a, b, c, d, e, f, g and h, we have only 81 palindromes of 3-digits. This allows us to write the following palindromic grid:*

aaa	bkb	cec	dhd	ede	fcf	gfg	hbh	kgk
dfd	ebe	fgf	gag	hkh	kek	aha	bdb	ccc
ghg	hdh	kck	afa	bbb	cgc	dad	eke	fef
ckc	aea	bab	fdf	dcd	ehe	kbk	ggg	hfh
fbf	dgd	efe	kkk	geg	hah	cdc	aca	bhb
kdk	gcg	hhh	cbc	aga	bfb	fkf	ded	eae
beb	cac	aka	ece	fhf	ddd	hgh	kfk	gbg
ege	fff	dbd	heh	kak	gkg	bcb	chc	ada
hch	khk	gdg	bgb	cfc	aba	eee	faf	dkd

where $a,b,c,d,e,f,g,h,k \in \{1,2,3,4,5,6,7,8,9\}$

The grid given in 8.1 represents a **palindromic** magic square of order 9. If exists, then the magic sum is given by

$$S_{9\times9}(a,b,c,d,e,f,g,h,k) := (a+b+c+d+e+f+g+h+k)\times 111.$$

Let's see some examples below:

Example 8.2. *For* $a=1, b=2, c=3, d=4, e=5, f=6, g=7, h=8$ *and* $h=9$ *in Grid 8.1, the 3-digits* ***palindromic bimagic*** *square of order 9 with magic sum* $S_{9\times9}(1,2,3,4,5,6,7,8,9) := (1+2+3+4+5+6+7+8+9)\times 111 = 4995$ *is given by*

										3390285
									4995	
111	292	353	484	545	636	767	828	979	4995	3390285
464	525	676	717	898	959	181	242	333	4995	3390285
787	848	939	161	222	373	414	595	656	4995	3390285
393	151	212	646	434	585	929	777	868	4995	3390285
626	474	565	999	757	818	343	131	282	4995	3390285
949	737	888	323	171	262	696	454	515	4995	3390285
252	313	191	535	686	444	878	969	727	4995	3390285
575	666	424	858	919	797	232	383	141	4995	3390285
838	989	747	272	363	121	555	616	494	4995	3390285
4995	4995	4995	4995	4995	4995	4995	4995	4995	4995	
3390285	3390285	3390285	3390285	3390285	3390285	3390285	3390285	3390285		3390285

Example 8.3. *For* $a=0,\ b=1,\ c=2,\ d=3,\ e=4,\ f=5,\ g=7,\ h=8$ *and* $h=9$ *in Grid 8.1, the 3-digits* ***palindromic-type bimagic*** *square of order 9 with magic sum* $S_{9\times9}(0,1,2,3,4,5,7,8,9) := (0+1+2+3+4+5+7+8+9)\times 111 = 4329$ *is given by.*

										2906329
									4329	
000	191	242	383	434	525	757	818	979	4329	2906329
353	414	575	707	898	949	080	131	222	4329	2906329
787	838	929	050	111	272	303	494	545	4329	2906329
292	040	101	535	323	484	919	777	858	4329	2906329
515	373	454	999	747	808	232	020	181	4329	2906329
939	727	888	212	070	151	595	343	404	4329	2906329
141	202	090	424	585	333	878	959	717	4329	2906329
474	555	313	848	909	797	121	282	030	4329	2906329
828	989	737	171	252	010	444	505	393	4329	2906329
4329	4329	4329	4329	4329	4329	4329	4329	4329	4329	
2906329	2906329	2906329	2906329	2906329	2906329	2906329	2906329	2906329		2906329

We called this example as **palindromic-type** as it has terms like: 000, 010, 020, etc.

8.1.2 7-Digits Palindromic Magic Squares with 3 Letters

Grid 8.2. *Using three letters a, b, c and d, we have exactly 81 palindromes of 5-digits. This allows to write as the following palindromic grid:*

aaaaaaa	abcccba	acbbbca	bacbcab	bbbabbb	bcacacb	cabcbac	cbababc	cccaccc
babcbab	bbababb	bccaccb	caaaaac	cbcccbc	ccbbbcc	aacbcaa	abbabba	acacaca
cacbcac	cbbabbc	ccacacc	aabcbaa	abababa	accacca	baaaaab	bbcccbb	bcbbbcb
accccca	aabbbaa	abaaaba	bcbabcb	baacaab	bbcbcbb	ccabacc	cacacac	cbbcbbc
bcabacb	bacacab	bbbcbbb	ccccccc	cabbbac	cbaaabc	acbabca	aaacaaa	abcbcba
ccbabcc	caacaac	cbcbcbc	acabaca	aacacaa	abbcbba	bcccccb	babbbab	bbaaabb
abbbbba	acaaaca	aacccaa	bbacabb	bccbccb	bababab	cbcacbc	ccbcbcc	caabaac
bbcacbb	bcbcbcb	baabaab	cbbbbbc	ccaaacc	cacccac	abacaba	accbcca	aababaa
cbacabc	cccbccc	cababac	abcacba	acbcbca	aaabaaa	bbbbbbb	bcaaacb	bacccab

where $a, b, c \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

For all $a, b, c \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, the grid given in 8.2 represents a **palindromic** magic square of order 9. If it exists, then the magic sum is given by

$$S_{9\times9}(a, b, c) := 3 \times (a + b + c) \times 1111111 = (a + b + c) \times 3333333.$$

Let's see some examples below.

Example 8.4. *Let's consider $a = 1$, $b = 2$ and $c = 3$ in Grid 8.2, we get following* ***palindromic bimagic*** *square just with three digits 1, 2 and 3:*

										50505077616162
									19999998	
1111111	1233321	1322231	2132312	2221222	2313132	3123213	3212123	3331333	19999998	50505077616162
2123212	2212122	2331332	3111113	3233323	3322233	1132311	1221221	1313131	19999998	50505077616162
3132313	3221223	3313133	1123211	1212121	1331331	2111112	2233322	2322232	19999998	50505077616162
1333331	1122211	1211121	2321232	2113112	2232322	3312133	3131313	3223223	19999998	50505077616162
2312132	2131312	2223222	3333333	3122213	3211123	1321231	1113111	1232321	19999998	50505077616162
3321233	3113113	3232323	1312131	1131311	1223221	2333332	2122212	2211122	19999998	50505077616162
1222221	1311131	1133311	2213122	2332332	2121212	3231323	3323233	3112113	19999998	50505077616162
2231322	2323232	2112112	3222223	3311133	3133313	1213121	1332331	1121211	19999998	50505077616162
3213123	3332333	3121213	1231321	1323231	1112111	2222222	2311132	2133312	19999998	50505077616162
19999998	19999998	19999998	19999998	19999998	19999998	19999998	19999998	19999998	19999998	
50505077616162	50505077616162	50505077616162	50505077616162	50505077616162	50505077616162	50505077616162	50505077616162	50505077616162		50505077616162

Example 8.5. *Let's consider $a = 1$, $b = 6$ and $c = 9$ in Grid 8.2 , we get following* ***upside down palindromic bimagic*** *square:*

1111111	1699961	1966691	6196916	6661666	6919196	9169619	9616169	9991999
6169616	6616166	6991996	9111119	9699969	9966699	1196911	1661661	1919191
9196919	9661669	9919199	1169611	1616161	1991991	6111116	6699966	6966696
1999991	1166611	1611161	6961696	6119116	6696966	9916199	9191919	9669669
6916196	6191916	6669666	9999999	9166619	9611169	1961691	1119111	1696961
9961699	9119119	9696969	1916191	1191911	1669661	6999996	6166616	6611166
1666661	1911191	1199911	6619166	6996996	6161616	9691969	9969699	9116119
6691966	6969696	6116116	9666669	9911199	9199919	1619161	1996991	1161611
9619169	9996999	9161619	1691961	1969691	1116111	6666666	6911196	6199916

In this case, the magic and **bimagic** sums are $S_{9\times 9} := 53333328$ and $Sb_{9\times 9} := 415039806496074$ respectively.

Example 8.6. *Let's consider $a = 2$, $b = 5$ and $c = 8$ in Grid* **??** *, we get following* ***upside down*** *and* ***mirror looking palindromic bimagic*** *square:*

2222222	2588852	2855582	5285825	5552555	5828285	8258528	8525258	8882888
5258525	5525255	5882885	8222228	8588858	8855588	2285822	2552552	2828282
8285828	8552558	8828288	2258522	2525252	2882882	5222225	5588855	5855585
2888882	2255522	2522252	5852585	5228225	5585855	8825288	8282828	8558558
5825285	5282825	5558555	8888888	8255528	8522258	2852582	2228222	2585852
8852588	8228228	8585858	2825282	2282822	2558552	5888885	5255525	5522255
2555552	2822282	2288822	5528255	5885885	5252525	8582858	8858588	8225228
5582855	5858585	5225225	8555558	8822288	8288828	2528252	2885882	2252522
8528258	8885888	8252528	2582852	2858582	2225222	5555555	5822285	5288825

In this case, the magic and **bimagic** sums are $S_{9\times 9} := 49999995$ and $Sb_{9\times 9} := 332323500767679$ respectively.

Above two Examples 8.5 and 8.6 give us very interesting relation:

$$\frac{S_{9\times 9}(1,6,9)}{1+6+9} = \frac{S_{9\times 9}(2,5,8)}{2+5+8} = 3333333.$$

8.1.3 Patterned Magic Sums

The palindromic Grid 8.1 can be extended for the higher order palindromes in each cell. See below

Grid 8.3. *For all $a, b, c, d, e, f, g, h, k \in \{1,2,3,4,5,6,7,8,9\}$, we have following higher digits palindromic grids:*

aaaa	bkkb	ceec	dhhd	edde	fccf	gffg	hbbh	kggk
dffd	ebbe	fggf	gaag	hkkh	keek	ahha	bddb	cccc
ghhg	hddh	kcck	affa	bbbb	cggc	daad	ekke	feef
ckkc	aeea	baab	fddf	dccd	ehhe	kbbk	gggg	hffh
fbbf	dggd	effe	kkkk	geeg	haah	cddc	acca	bhhb
kddk	gccg	hhhh	cbbc	agga	bffb	fkkf	deed	eaae
beeb	caac	akka	ecce	fhhf	dddd	hggh	kffk	gbbg
egge	ffff	dbbd	heeh	kaak	gkkg	bccb	chhc	adda
hcch	khhk	gddg	bggb	cffc	abba	eeee	faaf	dkkd

aaaaa	bkbkb	cecec	dhdhd	edede	fcfcf	gfgfg	hbhbh	kgkgk
dfdfd	ebebe	fgfgf	gagag	hkhkh	kekek	ahaha	bdbdb	ccccc
ghghg	hdhdh	kckck	afafa	bbbbb	cgcgc	dadad	ekeke	fefef
ckckc	aeaea	babab	fdfdf	dcdcd	ehehe	kbkbk	ggggg	hfhfh
fbfbf	dgdgd	efefe	kkkkk	gegeg	hahah	cdcdc	acaca	bhbhb
kdkdk	gcgcg	hhhhh	cbcbc	agaga	bfbfb	fkfkf	deded	eaeae
bebeb	cacac	akaka	ecece	fhfhf	ddddd	hghgh	kfkfk	gbgbg
egege	fffff	dbdbd	heheh	kakak	gkgkg	bcbcb	chchc	adada
hchch	khkhk	gdgdg	bgbgb	cfcfc	ababa	eeeee	fafaf	dkdkd

aaaaaa	bkbbkb	ceccec	dhddhd	edeede	fcffcf	gfggfg	hbhhbh	kgkkgk
dfddfd	ebeebe	fgffgf	gaggag	hkhhkh	kekkek	ahaaha	bdbbdb	cccccc
ghgghg	hdhhdh	kckkck	afaafa	bbbbbb	cgccgc	daddad	ekeeke	feffef
ckcckc	aeaaea	babbab	fdffdf	dcddcd	eheehe	kbkkbk	gggggg	hfhhfh
fbffbf	dgddgd	efeefe	kkkkkk	geggeg	hahhah	cdccdc	acaaca	bhbbhb
kdkkdk	gcggcg	hhhhhh	cbccbc	agaaga	bfbbfb	fkffkf	dedded	eaeeae
bebbeb	caccac	akaaka	eceece	fhffhf	dddddd	hghhgh	kfkkfk	gbggbg
egeege	ffffff	dbddbd	hehheh	kakkak	gkggkg	bcbbcb	chcchc	adaada
hchhch	khkkhk	gdggdg	bgbbgb	cfccfc	abaaba	eeeeee	faffaf	dkddkd

aaaaaaa	bkbkbkb	cececec	dhdhdhd	ededede	fcfcfcf	gfgfgfg	hbhbhbh	kgkgkgk
dfdfdfd	ebebebe	fgfgfgf	gagagag	hkhkhkh	kekekek	ahahaha	bdbdbdb	ccccccc
ghghghg	hdhdhdh	kckckck	afafafa	bbbbbbb	cgcgcgc	dadadad	ekekeke	fefefef
ckckckc	aeaeaea	bababab	fdfdfdf	dcdcdcd	ehehehe	kbkbkbk	ggggggg	hfhfhfh
fbfbfbf	dgdgdgd	efefefe	kkkkkkk	gegegeg	hahahah	cdcdcdc	acacaca	bhbhbhb
kdkdkdk	gcgcgcg	hhhhhhh	cbcbcbc	agagaga	bfbfbfb	fkfkfkf	dededed	eaeaeae
bebebeb	cacacac	akakaka	ececece	fhfhfhf	ddddddd	hghghgh	kfkfkfk	gbgbgbg
egegege	fffffff	dbdbdbd	heheheh	kakakak	gkgkgkg	bcbcbcb	chchchc	adadada
hchchch	khkhkhk	gdgdgdg	bgbgbgb	cfcfcfc	abababa	eeeeeee	fafafaf	dkdkdkd

aaaaaaaa	bkbkkbkb	ceceecec	dhdhhdhd	ededdede	fcfccfcf	gfgffgfg	hbhbbhbh	kgkggkgk
dfdffdfd	ebebbebe	fgfggfgf	gagaagag	hkhkkhkh	kekeekek	ahahhaha	bdbddbdb	cccccccc
ghghhghg	hdhddhdh	kckcckck	afaffafa	bbbbbbbb	cgcggcgc	dadaadad	ekekkeke	fefeefef
ckckkckc	aeaeeaea	babaabab	fdfddfdf	dcdccdcd	ehehhehe	kbkbbkbk	gggggggg	hfhffhfh
fbfbbfbf	dgdggdgd	efeffefe	kkkkkkkk	gegeegeg	hahaahah	cdcddcdc	acaccaca	bhbhhbhb
kdkddkdk	gcgccgcg	hhhhhhhh	cbcbbcbc	agaggaga	bfbffbfb	fkfkkfkf	dedeeded	eaeaaeae
bebeebeb	cacaacac	akakkaka	ececcece	fhfhhfhf	dddddddd	hghgghgh	kfkffkfk	gbgbbgbg
egeggege	ffffffff	dbdbbdbd	heheeheh	kakaakak	gkgkkgkg	bcbccbcb	chchhchc	adaddada
hchcchch	khkhhkhk	gdgddgdg	bgbggbgb	cfcffcfc	ababbaba	eeeeeeee	fafaafaf	dkdkkdkd

If magic squares of order 9 exists, then magic sums are given by

Digits	Magic Sums
3	$S_{8\times8}(a,b,c,d,e,f,g,h,k) := (a+b+c+d+e+f+g+h+k)\times 111$
4	$S_{8\times8}(a,b,c,d,e,f,g,h,k) := (a+b+c+d+e+f+g+h+k)\times 1111$
5	$S_{8\times8}(a,b,c,d,e,f,g,h,k) := (a+b+c+d+e+f+g+h+k)\times 11111$
6	$S_{8\times8}(a,b,c,d,e,f,g,h,k) := (a+b+c+d+e+f+g+h+k)\times 111111$
7	$S_{8\times8}(a,b,c,d,e,f,g,h,k) := (a+b+c+d+e+f+g+h+k)\times 1111111$
8	$S_{8\times8}(a,b,c,d,e,f,g,h,k) := (a+b+c+d+e+f+g+h+k)\times 11111111$

Making proper choices of $a,b,c,d,e,f,g,h,k \in \{1,2,3,4,5,6,7,8,9\}$, the grids given in 8.1 and 8.2 lead us to a **palindromic bimagic** squares of order 9 resulting in a **number pattern** in magic sums. See below the examples.

Example 8.7. *For $a=1$, $b=2$, $c=3$, $d=4$, $e=5$, $f=6$, $g=7$, $h=8$ and $h=9$ in 8.1 and 8.2 the 3 to 8 digits **palindromic bimagic** squares of order 9 are given by*

									4995
111	292	353	484	545	636	767	828	979	4995
464	525	676	717	898	959	181	242	333	4995
787	848	939	161	222	373	414	595	656	4995
393	151	212	646	434	585	929	777	868	4995
626	474	565	999	757	818	343	131	282	4995
949	737	888	323	171	262	696	454	515	4995
252	313	191	535	686	444	878	969	727	4995
575	666	424	858	919	797	232	383	141	4995
838	989	747	272	363	121	555	616	494	4995
4995	4995	4995	4995	4995	4995	4995	4995	4995	4995

*In this case, the magic and **bimagic** sums are $S_{9\times9} := 4995$ and $Sb_{9\times9} := 3390285$ respectively.*

									49995
1111	2992	3553	4884	5445	6336	7667	8228	9779	49995
4664	5225	6776	7117	8998	9559	1881	2442	3333	49995
7887	8448	9339	1661	2222	3773	4114	5995	6556	49995
3993	1551	2112	6446	4334	5885	9229	7777	8668	49995
6226	4774	5665	9999	7557	8118	3443	1331	2882	49995
9449	7337	8888	3223	1771	2662	6996	4554	5115	49995
2552	3113	1991	5335	6886	4444	8778	9669	7227	49995
5775	6666	4224	8558	9119	7997	2332	3883	1441	49995
8338	9889	7447	2772	3663	1221	5555	6116	4994	49995
49995	49995	49995	49995	49995	49995	49995	49995	49995	49995

*In this case, the magic and **bimagic** sums are $S_{9\times9} := 49995$ and $Sb_{9\times9} := 338568285$ respectively.*

									499995
11111	29292	35353	48484	54545	63636	76767	82828	97979	499995
46464	52525	67676	71717	89898	95959	18181	24242	33333	499995
78787	84848	93939	16161	22222	37373	41414	59595	65656	499995
39393	15151	21212	64646	43434	58585	92929	77777	86868	499995
62626	47474	56565	99999	75757	81818	34343	13131	28282	499995
94949	73737	88888	32323	17171	26262	69696	45454	51515	499995
25252	31313	19191	53535	68686	44444	87878	96969	72727	499995
57575	66666	42424	85858	91919	79797	23232	38383	14141	499995
83838	98989	74747	27272	36363	12121	55555	61616	49494	499995
499995	499995	499995	499995	499995	499995	499995	499995	499995	499995

*In this case, the magic and **bimagic** sums are* $S_{9\times 9} := 499995$ *and* $Sb_{9\times 9} := 33960240285$ *respectively.*

									4999995
111111	292292	353353	484484	545545	636636	767767	828828	979979	4999995
464464	525525	676676	717717	898898	959959	181181	242242	333333	4999995
787787	848848	939939	161161	222222	373373	414414	595595	656656	4999995
393393	151151	212212	646646	434434	585585	929929	777777	868868	4999995
626626	474474	565565	999999	757757	818818	343343	131131	282282	4999995
949949	737737	888888	323323	171171	262262	696696	454454	515515	4999995
252252	313313	191191	535535	686686	444444	878878	969969	727727	4999995
575575	666666	424424	858858	919919	797797	232232	383383	141141	4999995
838838	989989	747747	272272	363363	121121	555555	616616	494494	4999995
4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995

*In this case, the magic and **bimagic** sums are* $S_{9\times 9} := 4999995$ *and* $Sb_{9\times 9} := 3397068960285$ *respectively.*

									49999995
1111111	2929292	3535353	4848484	5454545	6363636	7676767	8282828	9797979	49999995
4646464	5252525	6767676	7171717	8989898	9595959	1818181	2424242	3333333	49999995
7878787	8484848	9393939	1616161	2222222	3737373	4141414	5959595	6565656	49999995
3939393	1515151	2121212	6464646	4343434	5858585	9292929	7777777	8686868	49999995
6262626	4747474	5656565	9999999	7575757	8181818	3434343	1313131	2828282	49999995
9494949	7373737	8888888	3232323	1717171	2626262	6969696	4545454	5151515	49999995
2525252	3131313	1919191	5353535	6868686	4444444	8787878	9696969	7272727	49999995
5757575	6666666	4242424	8585858	9191919	7979797	2323232	3838383	1414141	49999995
8383838	9898989	7474747	2727272	3636363	1212121	5555555	6161616	4949494	49999995
49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995

*In this case, the magic and **bimagic** sums are* $S_{9\times 9} := 49999995$ *and* $Sb_{9\times 9} := 339608145240285$ *respectively.*

									499999995
11111111	29299292	35355353	48488484	54544545	63633636	76766767	82822828	97977979	499999995
46466464	52522525	67677676	71711717	89899898	95955959	18188181	24244242	33333333	499999995
78788787	84844848	93933939	16166161	22222222	37377373	41411414	59599595	65655656	499999995
39399393	15155151	21211212	64644646	43433434	58588585	92922929	77777777	86866868	499999995
62622626	47477474	56566565	99999999	75755757	81811818	34344343	13133131	28288282	499999995
94944949	73733737	88888888	32322323	17177171	26266262	69699696	45455454	51511515	499999995
25255252	31311313	19199191	53533535	68688686	44444444	87877878	96966969	72722727	499999995
57577575	66666666	42422424	85855858	91911919	79799797	23233232	38388383	14144141	499999995
83833838	98988989	74744747	27277272	36366363	12122121	55555555	61611616	49499494	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

In this case, the magic and **bimagic** sums are $S_{9\times 9} := 499999995$ and $Sb_{9\times 9} := 33959828108040285$ respectively.

According to above six magic squares of order 9, we have the following **number pattern** with magic sums:

Digits	Magic Sums	Bimagic Sums
3	4995	**339 0285**
4	49995	338 56 8285
5	499995	**339** 6024 **0285**
6	4999995	**339** 706896 **0285**
7	49999995	**339** 60814524 **0285**
8	499999995	**339** 5982810804 **0285**

We observe that there is a good pattern in magic sums, but in case of **bimagic** sums, the pattern is no so good. Moreover 4-digits case, it is totally different.

8.1.4 Composite Magic Squares

Grid 8.4. *Eliminating the third value in Grid 8.1, and then splitting in two Latin squares, we get*

a	b	c	d	e	f	g	h	k
d	e	f	g	h	k	a	b	c
g	h	k	a	b	c	d	e	f
c	a	b	f	d	e	k	g	h
f	d	e	k	g	h	c	a	b
k	g	h	c	a	b	f	d	e
b	c	a	e	f	d	h	k	g
e	f	d	h	k	g	b	c	a
h	k	g	b	c	a	e	f	d
				A				

a	k	e	h	d	c	f	b	g
f	b	g	a	k	e	h	d	c
h	d	c	f	b	g	a	k	e
k	e	a	d	c	h	b	g	f
b	g	f	k	e	a	d	c	h
d	c	h	b	g	f	k	e	a
e	a	k	c	h	d	g	f	b
g	f	b	e	a	k	c	h	d
c	h	d	g	f	b	e	a	k
				B				

aa	bk	ce	dh	ed	fc	gf	hb	kg
df	eb	fg	ga	hk	ke	ah	bd	cc
gh	hd	kc	af	bb	cg	da	ek	fe
ck	ae	ba	fd	dc	eh	kb	gg	hf
fb	dg	ef	kk	ge	ha	cd	ac	bh
kd	gc	hh	cb	ag	bf	fk	de	ea
be	ca	ak	ec	fh	dd	hg	kf	gb
eg	ff	db	he	ka	gk	bc	ch	ad
hc	kh	gd	bg	cf	ab	ee	fa	dk
				AB				

The grid AB can be written as

$$AB := 10 \times A + B$$

See below a particular case of Grid 8.4:

Example 8.8. *In particular for* $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 5$, $f = 6$, $g = 7$, $h = 8$ *and* $k = 9$ *in Grid 8.4 , we get*

1	2	3	4	5	6	7	8	9
4	5	6	7	8	9	1	2	3
7	8	9	1	2	3	4	5	6
3	1	2	6	4	5	9	7	8
6	4	5	9	7	8	3	1	2
9	7	8	3	1	2	6	4	5
2	3	1	5	6	4	8	9	7
5	6	4	8	9	7	2	3	1
8	9	7	2	3	1	5	6	4
				A				

1	9	5	8	4	3	6	2	7
6	2	7	1	9	5	8	4	3
8	4	3	6	2	7	1	9	5
9	5	1	4	3	8	2	7	6
2	7	6	9	5	1	4	3	8
4	3	8	2	7	6	9	5	1
5	1	9	3	8	4	7	6	2
7	6	2	5	1	9	3	8	4
3	8	4	7	6	2	5	1	9
				B				

11	29	35	48	54	63	76	82	97
46	52	67	71	89	95	18	24	33
78	84	93	16	22	37	41	59	65
39	15	21	64	43	58	92	77	86
62	47	56	99	75	81	34	13	28
94	73	88	32	17	26	69	45	51
25	31	19	53	68	44	87	96	72
57	66	42	85	91	79	23	38	14
83	98	74	27	36	12	55	61	49
				AB				

Note 8.1. *Applying* $9 \times (A-1) + B$ *over the elements of A and B given above, we get a magic square of order 9 given in Example 8.1. Below are some examples of composite magic squares. In some case, these are* ***upside down*** *and* ***mirror looking*** *magic squares. Moreover, A and B are* ***pairwise mutually orthogonal diagonal Latin squares****. The above Grid 8.4 is written for single letters* $a = 1, b = 2, c = 3, d = 4, e = 5, f = 6, g = 7, h = 8$ *and* $k = 9$ *. We can choose double digits numbers to write composite examples.*

Example 8.9. *Let's consider* $a = 11$, $b = 16$, $c = 19$, $d = 61$, $e = 66$, $f = 69$, $g = 91$, $h = 96$ *and* $k = 99$ *in Grid 8.4 , we get following* ***upside down bimagic*** *square:*

1111	1699	1966	6196	6661	6919	9169	9616	9991
6169	6616	6991	9111	9699	9966	1196	1661	1919
9196	9661	9919	1169	1616	1991	6111	6699	6966
1999	1166	1611	6961	6119	6696	9916	9191	9669
6916	6191	6669	9999	9166	9611	1961	1119	1696
9961	9119	9696	1916	1191	1669	6999	6166	6611
1666	1911	1199	6619	6996	6161	9691	9969	9116
6691	6969	6116	9666	9911	9199	1619	1996	1161
9619	9996	9161	1691	1969	1116	6666	6911	6199

In this case, the magic and **bimagic** sums are $S_{9\times 9} := 53328$ and $Sb_{9\times 9} := 414976074$.

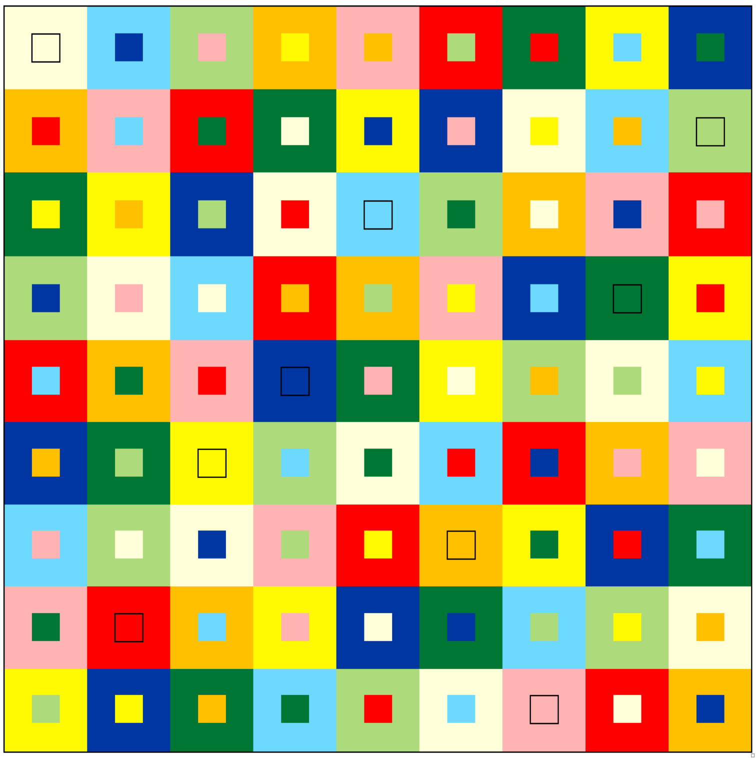
Example 8.10. *Let's consider* $a = 22$, $b = 25$, $c = 28$, $d = 52$, $e = 55$, $f = 58$, $g = 82$, $h = 85$ *and* $k = 88$ *in Grid 8.4 , we get following **upside down bimagic** and **mirror looking** square:*

2222	2588	2855	5285	5552	5828	8258	8525	8882
5258	5525	5882	8222	8588	8855	2285	2552	2828
8285	8552	8828	2258	2525	2882	5222	5588	5855
2888	2255	2522	5852	5228	5585	8825	8282	8558
5825	5282	5558	8888	8255	8522	2852	2228	2585
8852	8228	8585	2825	2282	2558	5888	5255	5522
2555	2822	2288	5528	5885	5252	8582	8858	8225
5582	5858	5225	8555	8822	8288	2528	2885	2252
8528	8885	8252	2582	2858	2225	5555	5822	5288

In this case, the magic and **bimagic** sums are $S_{9\times 9} := 49995$ and $Sb_{9\times 9} := 332267679$.

8.1.5 Superimposed Colored Pattern

The grid *AB* is a composite magic square of order 9. Based on it, here below is **double colored pattern**.



Looking above the **superimposed colored pattern** of order 9, we observe that it has **diagonal property**, as it is made from **pairwise mutually orthogonal diagonal Latin squares**.

8.2 Pan Diagonal Magic Squares of Order 9

In this subsection, we shall work with **pan diagonal** magic squares of order 9. This type of magic squares first constructed by Gakuho Abe in 1996 (see [4], [3]). For further studies, also refer [1, 5]. The idea is to do similar kind of work as done for **bimagic** squares in subsection 8.1.

Example 8.11. *Let's consider a **pan diagonal** magic square of order 9 given by*

		369	369	369	369	369	369	369	369	369
	1	13	25	30	42	54	56	68	80	369
369	29	41	53	55	67	79	3	15	27	369
369	57	69	81	2	14	26	28	40	52	369
369	22	7	10	51	36	39	77	62	65	369
369	50	35	38	76	61	64	24	9	12	369
369	78	63	66	23	8	11	49	34	37	369
369	16	19	4	45	48	33	71	74	59	369
369	44	47	32	70	73	58	18	21	6	369
369	72	75	60	17	20	5	43	46	31	369
	369	369	369	369	369	369	369	369	369	369

Additionally it has property that each 3×3 block is of same sum as of magic square, i.e., $S_9 = 495$. Also each 3×3 block is a semi-magic square of order 3 (only in rows and columns).

8.2.1 Palindromic Representations

Grid 8.5. *Using nine letters a, b, c, d, e, f, g and h, and 81 palindromes of 3-digits given in Table 8.1 , we get following palindromic grid of order 9:*

aaa	*bdb*	*cgc*	*dcd*	*efe*	*fkf*	*gbg*	*heh*	*khk*
dbd	*eee*	*fhf*	*gag*	*hdh*	*kgk*	*aca*	*bfb*	*ckc*
gcg	*hfh*	*kkk*	*aba*	*beb*	*chc*	*dad*	*ede*	*fgf*
cdc	*aga*	*bab*	*fff*	*dkd*	*ece*	*kek*	*ghg*	*hbh*
fef	*dhd*	*ebe*	*kdk*	*ggg*	*hah*	*cfc*	*aka*	*bcb*
kfk	*gkg*	*hch*	*cec*	*aha*	*bbb*	*fdf*	*dgd*	*eae*
bgb	*cac*	*ada*	*eke*	*fcf*	*dfd*	*hhh*	*kbk*	*geg*
ehe	*fbf*	*ded*	*hgh*	*kak*	*gdg*	*bkb*	*ccc*	*afa*
hkh	*kck*	*gfg*	*bhb*	*cbc*	*aea*	*ege*	*faf*	*ddd*

where $a, b, c, d, e, f, g, h, k \in \{1,2,3,4,5,6,7,8,9\}$

The grid given in 8.5 represents a **palindromic** magic square of order 9. If exists, then the magic sum is given by

$$S_{9\times 9}(a,b,c,d,e,f,g,h,k) := (a+b+c+d+e+f+g+h+k)\times 111.$$

Let's see some particular case of Grid 8.5.

Example 8.12. *For* $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 5$, $f = 6$, $g = 7$, $h = 8$ *and* $h = 9$ *in Grid 8.5, the 3-digits* ***palindromic pan diagonal*** *magic square of order 9 with magic sum* $S_{9\times 9}(1,2,3,4,5,6,7,8,9) := (1+2+3+4+5+6+7+8+9)\times 111 = 4995$ *is given by*

		4995	4995	4995	4995	4995	4995	4995	4995	4995
	111	242	373	434	565	696	727	858	989	4995
4995	424	555	686	717	848	979	131	262	393	4995
4995	737	868	999	121	252	383	414	545	676	4995
4995	343	171	212	666	494	535	959	787	828	4995
4995	656	484	525	949	777	818	363	191	232	4995
4995	969	797	838	353	181	222	646	474	515	4995
4995	272	313	141	595	636	464	888	929	757	4995
4995	585	626	454	878	919	747	292	333	161	4995
4995	898	939	767	282	323	151	575	616	444	4995
	4995	4995	4995	4995	4995	4995	4995	4995	4995	4995

Example 8.13. *For* $a = 0$, $b = 1$, $c = 2$, $d = 3$, $e = 4$, $f = 5$, $g = 7$, $h = 8$ *and* $h = 9$ *in Grid 8.1, the 3-digits* ***palindromic-type*** *magic square of order 9 with magic sum* $S_{9\times 9}(0,1,2,3,5,6,7,8,9) := (0+1+2+3+5+6+7+8+9)\times 111 = 4551$ *is given by.*

									4551
000	131	272	323	565	696	717	858	989	4551
313	555	686	707	838	979	020	161	292	4551
727	868	999	010	151	282	303	535	676	4551
232	070	101	666	393	525	959	787	818	4551
656	383	515	939	777	808	262	090	121	4551
969	797	828	252	080	111	636	373	505	4551
171	202	030	595	626	363	888	919	757	4551
585	616	353	878	909	737	191	222	060	4551
898	929	767	181	212	050	575	606	333	4551
4551	4551	4551	4551	4551	4551	4551	4551	4551	4551

We called this example as **palindromic-type** as it has terms like: 000, 010, 020, etc. Also, it is not **pan diagonal** due to the fact the numbers are not in consecutive way.

8.2.2 7-Digits Palindromic Magic Squares with 3 Letters

Grid 8.6. *Using three letters a, b and c, we have exactly 81 palindromes of 7-digits. This allows us to write as the following palindromic grid:*

aacbcaa	bcbabcb	cbacabc	cacacac	acbcbca	bbababb	bacccab	ccbbbcc	abaaaba
bbaaabb	cacccac	acbbbca	abacaba	bacbcab	ccbabcc	cbababc	aacacaa	bcbcbcb
ccbcbcc	abababa	bacacab	bcbbbcb	cbaaabc	aacccaa	acbabca	bbacabb	cacbcac
abbbbba	baaaaab	ccccccc	cbbabbc	aaacaaa	bccbccb	bbbcbbb	caabaac	accacca
bccaccb	cbbcbbc	aaabaaa	accccca	bbbbbbb	caaaaac	cccbccc	abbabba	baacaab
caacaac	accbcca	bbbabbb	baabaab	cccaccc	abbcbba	aaaaaaa	bcccccb	cbbbbbc
acabaca	bbcacbb	cabcbac	ccaaacc	abcccba	babbbab	bcacacb	cbcbcbc	aababaa
bababab	ccacacc	abcbcba	aabcbaa	bcabacb	cbcacbc	cabbbac	acaaaca	bbcccbb
cbcccbc	aabbbaa	bcaaacb	bbcbcbb	cababac	acacaca	abcacba	babcbab	ccabacc

where $a, b, c \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

The grid given in 8.6 represents a **palindromic** magic square of order 9. If it exists, then the magic sum is given by

$$S_{9\times 9}(a, b, c) := 3 \times (a + b + c) \times 1111111 = (a + b + c) \times 3333333.$$

Let's see some examples below:

Example 8.14. *Let's consider $a = 2$, $b = 3$ and $c = 7$ in Grid 8.6, we get following **pan diagonal palindromic** magic square just with three numbers 2, 3 and 7:*

		39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996
	2273722	3732373	7327237	7272727	2737372	3323233	3277723	7733377	2322232	39999996
39999996	3322233	7277727	2733372	2327232	3273723	7732377	7323237	2272722	3737373	39999996
39999996	7737377	2323232	3272723	3733373	7322237	2277722	2732372	3327233	7273727	39999996
39999996	2333332	3222223	7777777	7332337	2227222	3773773	3337333	7223227	2772772	39999996
39999996	3772773	7337337	2223222	2777772	3333333	7222227	7773777	2332332	3227223	39999996
39999996	7227227	2773772	3332333	3223223	7772777	2337332	2222222	3777773	7333337	39999996
39999996	2723272	3372733	7237327	7722277	2377732	3233323	3727273	7373737	2232322	39999996
39999996	3232323	7727277	2373732	2237322	3723273	7372737	7233327	2722272	3377733	39999996
39999996	7377737	2233322	3722273	3373733	7232327	2727272	2372732	3237323	7723277	39999996
	39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996

Example 8.15. *Let's consider $a = 1$, $b = 6$ and $c = 9$ in Grid 8.6, we get following **pan diagonal palindromic** magic square just with three numbers 1, 6 and 9:*

		76666659	76666659	76666659	76666659	76666659	76666659	76666659	76666659	76666659
	6698966	8986898	9869689	9696969	6989896	8868688	8699968	9988899	6866686	76666659
76666659	8866688	9699969	6988896	6869686	8698968	9986899	9868689	6696966	8989898	76666659
76666659	9989899	6868686	8696968	8988898	9866689	6699966	6986896	8869688	9698969	76666659
76666659	6888886	8666668	9999999	9886889	6669666	8998998	8889888	9668669	6996996	76666659
76666659	8996998	9889889	6668666	6999996	8888888	9666669	9998999	6886886	8669668	76666659
76666659	9669669	6998996	8886888	8668668	9996999	6889886	6666666	8999998	9888889	76666659
76666659	6968696	8896988	9689869	9966699	6899986	8688868	8969698	9898989	6686866	76666659
76666659	8686868	9969699	6898986	6689866	8968698	9896989	9688869	6966696	8899988	76666659
76666659	9899989	6688866	8966698	8898988	9686869	6969696	6896986	8689868	9968699	76666659
	76666659	76666659	76666659	76666659	76666659	76666659	76666659	76666659	76666659	76666659

Below is an **upside down** version of above magic square

6698966	8986898	9869689	9696969	6989896	8868688	8699968	9988899	6866686
8866688	9699969	6988896	6869686	8698968	9986899	9868689	6696966	8989898
9989899	6868686	8696968	8988898	9866689	6699966	6986896	8869688	9698969
6888886	8666668	9999999	9886889	6669666	8998998	8889888	9668669	6996996
8996998	9889889	6668666	6999996	8888888	9666669	9998999	6886886	8669668
9669669	6998996	8886888	8668668	9996999	6889886	6666666	8999998	9888889
6968696	8896988	9689869	9966699	6899986	8688868	8969698	9898989	6686866
8686868	9969699	6898986	6689866	8968698	9896989	9688869	6966696	8899988
9899989	6688866	8966698	8898988	9686869	6969696	6896986	8689868	9968699

Example 8.16. *Let's consider* $a = 2, b = 5$ *and* $c = 8$ *in Grid 8.6, we get following* ***pan diagonal palindromic*** *magic square just with three numbers 2, 5 and 8:*

		49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995
	2285822	5852585	8528258	8282828	2858582	5525255	5288825	8855588	2522252	49999995
49999995	5522255	8288828	2855582	2528252	5285825	8852588	8525258	2282822	5858585	49999995
49999995	8858588	2525252	5282825	5855585	8522258	2288822	2852582	5528255	8285828	49999995
49999995	2555552	5222225	8888888	8552558	2228222	5885885	5558555	8225228	2882882	49999995
49999995	5882885	8558558	2225222	2888882	5555555	8222228	8885888	2552552	5228225	49999995
49999995	8228228	2885882	5552555	5225225	8882888	2558552	2222222	5888885	8555558	49999995
49999995	2825282	5582855	8258528	8822288	2588852	5255525	5828285	8585858	2252522	49999995
49999995	5252525	8828288	2585852	2258522	5825285	8582858	8255528	2822282	5588855	49999995
49999995	8588858	2255522	5822285	5585855	8252528	2828282	2582852	5258525	8825288	49999995
	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995

Below is digital version of above magic square being **upside down** and **mirror looking**:

2285822	5852585	8528258	8282828	2858582	5525255	5288825	8855588	2522252
5522255	8288828	2855582	2528252	5285825	8852588	8525258	2282822	5858585
8858588	2525252	5282825	5855585	8522258	2288822	2852582	5528255	8285828
2555552	5222225	8888888	8552558	2228222	5885885	5558555	8225228	2882882
5882885	8558558	2225222	2888882	5555555	8222228	8885888	2552552	5228225
8228228	2885882	5552555	5225225	8882888	2558552	2222222	5888885	8555558
2825282	5582855	8258528	8822288	2588852	5255525	5828285	8585858	2252522
5252525	8828288	2585852	2258522	5825285	8582858	8255528	2822282	5588855
8588858	2255522	5822285	5585855	8252528	2828282	2582852	5258525	8825288

The Example 8.15 is**upside down**, while the Example 8.16 is **upside down** and **mirror looking** both.

8.2.3 Patterned Magic Squares

The examples for the patterned magic square sums can be constructed on similar lines as of subsection 8.1.3. It requires redistribution of entries to **pan diagonal palindromic** magic squares.

8.2.4 Composite Magic Squares

Grid 8.7. *Eliminating the third value in the Grid 8.5, and then splitting in two Latin squares, we get*

a	b	c	d	e	f	g	h	k
d	e	f	g	h	k	a	b	c
g	h	k	a	b	c	d	e	f
c	a	b	f	d	e	k	g	h
f	d	e	k	g	h	c	a	b
k	g	h	c	a	b	f	d	e
b	c	a	e	f	d	h	k	g
e	f	d	h	k	g	b	c	a
h	k	g	b	c	a	e	f	d
				A				

a	d	g	c	f	k	b	e	h
b	e	h	a	d	g	c	f	k
c	f	k	b	e	h	a	d	g
d	g	a	f	k	c	e	h	b
e	h	b	d	g	a	f	k	c
f	k	c	e	h	b	d	g	a
g	a	d	k	c	f	h	b	e
h	b	e	g	a	d	k	c	f
k	c	f	h	b	e	g	a	d
				B				

aa	bd	cg	dc	ef	fk	gb	he	kh
db	ee	fh	ga	hd	kg	ac	bf	ck
gc	hf	kk	ab	be	ch	da	ed	fg
cd	ag	ba	ff	dk	ec	ke	gh	hb
fe	dh	eb	kd	gg	ha	cf	ak	bc
kf	gk	hc	ce	ah	bb	fd	dg	ea
bg	ca	ad	ek	fc	df	hh	kb	ge
eh	fb	de	hg	ka	gd	bk	cc	af
hk	kc	gf	bh	cb	ae	eg	fa	dd
				AB				

The grid AB can be written as

$$AB := 10 \times A + B$$

See below a particular case of Grid 8.7:

Example 8.17. *In particular for* $a = 1$, $b = 2$, $c = 3$, $d = 4$, $e = 5$, $f = 6$, $g = 7$, $h = 8$ *and* $k = 9$ *in Grid 8.7 , we get*

1	2	3	4	5	6	7	8	9
4	5	6	7	8	9	1	2	3
7	8	9	1	2	3	4	5	6
3	1	2	6	4	5	9	7	8
6	4	5	9	7	8	3	1	2
9	7	8	3	1	2	6	4	5
2	3	1	5	6	4	8	9	7
5	6	4	8	9	7	2	3	1
8	9	7	2	3	1	5	6	4
				A				

1	4	7	3	6	9	2	5	8
2	5	8	1	4	7	3	6	9
3	6	9	2	5	8	1	4	7
4	7	1	6	9	3	5	8	2
5	8	2	4	7	1	6	9	3
6	9	3	5	8	2	4	7	1
7	1	4	9	3	6	8	2	5
8	2	5	7	1	4	9	3	6
9	3	6	8	2	5	7	1	4
				B				

11	24	37	43	56	69	72	85	98
42	55	68	71	84	97	13	26	39
73	86	99	12	25	38	41	54	67
34	17	21	66	49	53	95	78	82
65	48	52	94	77	81	36	19	23
96	79	83	35	18	22	64	47	51
27	31	14	59	63	46	88	92	75
58	62	45	87	91	74	29	33	16
89	93	76	28	32	15	57	61	44
				AB				

Note 8.2. *Applying* $9 \times (A-1) + B$ *over the elements of A and B given in Example 8.17, we get a magic square of order 9 given in Example 8.11. Below are some examples of composite magic squares. In some case, these are* ***upside down*** *and/or* ***mirror looking****. Moreover, A and B are* ***pairwise mutually orthogonal diagonal Latin squares****. The above Grid 8.7 is written for single letters. We can choose double digits numbers to write composite examples.*

Example 8.18. *Let's consider* $a = 66$, $b = 68$, $c = 69$, $d = 86$, $b = 88$, $c = 89$, $g = 96$, $h = 98$ *and* $k = 99$ *in Grid 8.7 , we get following* ***pan diagonal*** *magic square just with 3 digits, 6, 8 and 9:*

		76659	76659	76659	76659	76659	76659	76659	76659	76659
	6698	8986	9869	9696	6989	8868	8699	9988	6866	76659
76659	8866	9699	6988	6869	8698	9986	9868	6696	8989	76659
76659	9989	6868	8696	8988	9866	6699	6986	8869	9698	76659
76659	6888	8666	9999	9886	6669	8998	8889	9668	6996	76659
76659	8996	9889	6668	6999	8888	9666	9998	6886	8669	76659
76659	9669	6998	8886	8668	9996	6889	6666	8999	9888	76659
76659	6968	8896	9689	9966	6899	8688	8969	9898	6686	76659
76659	8686	9969	6898	6689	8968	9896	9688	6966	8899	76659
76659	9899	6688	8966	8898	9686	6969	6896	8689	9968	76659
	76659	76659	76659	76659	76659	76659	76659	76659	76659	76659

The above **pan diagonal** magic square is **upside down**. See below:

6698	8986	9869	9696	6989	8868	8699	9988	6866
8866	9699	6988	6869	8698	9986	9868	6696	8989
9989	6868	8696	8988	9866	6699	6986	8869	9698
6888	8666	9999	9886	6669	8998	8889	9668	6996
8996	9889	6668	6999	8888	9666	9998	6886	8669
9669	6998	8886	8668	9996	6889	6666	8999	9888
6968	8896	9689	9966	6899	8688	8969	9898	6686
8686	9969	6898	6689	8968	9896	9688	6966	8899
9899	6688	8966	8898	9686	6969	6896	8689	9968

Example 8.19. *Let's consider* $a = 00$, $b = 02$, $c = 05$, $d = 20$, $e = 22$, $f = 25$, $g = 50$, $h = 52$ *and* $k = 55$ *in Grid 8.7 , we get following* ***upside down*** *and* ***mirror looking pan diagonal*** *magic square:*

		23331	23331	23331	23331	23331	23331	23331	23331	23331
	0052	2520	5205	5050	0525	2202	2055	5522	0200	23331
23331	2200	5055	0522	0205	2052	5520	5202	0050	2525	23331
23331	5525	0202	2050	2522	5200	0055	0520	2205	5052	23331
23331	0222	2000	5555	5220	0005	2552	2225	5002	0550	23331
23331	2550	5225	0002	0555	2222	5000	5552	0220	2005	23331
23331	5005	0552	2220	2002	5550	0225	0000	2555	5222	23331
23331	0502	2250	5025	5500	0255	2022	2505	5252	0020	23331
23331	2020	5505	0252	0025	2502	5250	5022	0500	2255	23331
23331	5255	0022	2500	2252	5020	0505	0250	2025	5502	23331
	23331	23331	23331	23331	23331	23331	23331	23331	23331	23331

Writing in digital form, the above **pan diagonal** magic square is **upside down** and **mirror looking**. See below:

0052	2520	5205	5050	0525	2202	2055	5522	0200
2200	5055	0522	0205	2052	5520	5202	0050	2525
5525	0202	2050	2522	5200	0055	0520	2205	5052
0222	2000	5555	5220	0005	2552	2225	5002	0550
2550	5225	0002	0555	2222	5000	5552	0220	2005
5005	0552	2220	2002	5550	0225	0000	2555	5222
0502	2250	5025	5500	0255	2022	2505	5252	0020
2020	5505	0252	0025	2502	5250	5022	0500	2255
5255	0022	2500	2252	5020	0505	0250	2025	5502

8.2.5 Superimposed Colored Pattern

If follows similar lines of subsection 8.1.5.

9 Magic Squares of Order 10

Example 9.1. *Let's consider a magic square of order 10 is given by*

										505
1	80	65	97	39	22	48	86	53	14	505
98	12	9	66	90	74	55	33	41	27	505
47	81	23	79	16	35	94	60	62	8	505
70	57	88	34	2	91	29	15	76	43	505
84	99	52	11	45	68	73	7	30	36	505
13	38	44	10	77	56	82	21	95	69	505
75	46	40	83	28	19	67	92	4	51	505
59	24	96	42	61	3	20	78	37	85	505
26	5	17	58	93	50	31	64	89	72	505
32	63	71	25	54	87	6	49	18	100	505
505	505	505	505	505	505	505	505	505	505	505

9.1 Palindromic-Type Representations

Let's consider ten letters $a, b, c, d, e, f, g, h, k$ and n, where $a, b, c, d, e, f, g, h, k, n \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. We can make exactly 100 palindromes of 3-digits with these 10 letters. See the table below:

Table 9.1. *The palindromes are as follows:*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
aaa	aba	aca	ada	aea	afa	aga	aha	aka	ana	bab	bbb	bcb	bdb	beb	bfb	bgb	bhb	bkb	bnb
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	27	38	39	40
cac	cbc	ccc	cdc	cec	cfc	cgc	chc	ckc	cnc	dad	dbd	dcd	ddd	ded	dfd	dgd	dhd	dkd	dnd
41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
eae	ebe	ece	ede	eee	efe	ege	ehe	eke	ene	faf	fbf	fcf	fdf	fef	fff	fgf	fhf	fkf	fnf
61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
gag	gbg	gcg	gdg	geg	gfg	ggg	ghg	gkg	gng	hah	hbh	hch	hdh	heh	hfh	hgh	hhh	hkh	hnh
81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
kak	kbk	kck	kdk	kek	kfk	kgk	khk	kkk	knk	nan	nbn	ncn	ndn	nen	nfn	ngn	nhn	nkn	nnn

Replacing the above values with their respective palindromes in a magic square of order 10 given in Example 9.1 , we get a **palindromic-type grid** given below:

Grid 9.1. *Using nine letters $a, b, c, d, e, f, g, h, k$ and n, we have exactly 100 palindromes of 3-digits. This allows to write as the following palindromic grid:*

aaa	eke	bfb	nbn	khk	cdc	heh	dgd	fnf	gcg
dfd	bbb	ghg	kck	cnc	nkn	eae	fdf	aea	hgh
gbg	fhf	ccc	ene	nfn	aga	dkd	kak	hdh	beb
bnb	kgk	fef	ddd	hah	ece	nhn	aba	gfg	ckc
ndn	cac	aka	fgf	eee	hbh	kfk	gng	bcb	dhd
kek	gdg	dad	hkh	bgb	fff	ana	ncn	chc	ebe
ehe	aca	hnh	cfc	fbf	ded	ggg	bkb	nan	kdk
cgc	nen	ede	bab	gkg	knk	fcf	hhh	dbd	afa
hch	dnd	ngn	geg	ada	bhb	cbc	efe	kkk	faf
fkf	hfh	kbk	aha	dcd	gag	bdb	cec	ege	nnn

where $a,b,c,d,e,f,g,h,k,n \in \{1,2,3,4,5,6,7,8,9\}$

The grid given in Grid 9.1 represents a **palindromic** magic square of order 10. If exists, then the magic sum is given by

$$S_{10\times10}(a,b,c,d,e,f,g,h,k,n) := (a+b+c+d+e+f+g+h+k+n)\times 111.$$

Let's see some examples below:

Example 9.2. *For* $a=0$, $b=1$, $c=2$, $d=3$, $e=4$, $f=5$, $g=6$, $h=7$, $k=8$ *and* $n=9$ *in Grid 9.1, the 3-digits* ***palindromic-type*** *square of order 10 with magic sum* $S_{10\times10}(0,1,2,3,4,5,6,7,8,9) := (0+1+2+3+4+5+6+7+8+9)\times 111 = 4995$ *is given by.*

										4995
000	484	151	919	878	232	747	363	595	626	4995
353	111	676	828	292	989	404	535	040	767	4995
616	575	222	494	959	060	383	808	737	141	4995
191	868	545	333	707	424	979	010	656	282	4995
939	202	080	565	444	717	858	696	121	373	4995
848	636	303	787	161	555	090	929	272	414	4995
474	020	797	252	515	343	666	181	909	838	4995
262	949	434	101	686	898	525	777	313	050	4995
727	393	969	646	030	171	212	454	888	505	4995
585	757	818	070	323	606	131	242	464	999	4995
4995	4995	4995	4995	4995	4995	4995	4995	4995	4995	4995

We called this example as **palindromic-type** as it has terms like: 000, 010, 020, etc.

9.2 Patterned Magic Sums

The the palindromic grid 9.1 can be extended for the higher order palindromes in each cell. See below

Grid 9.2. *For all* $a,b,c,d,e,f,g,h,k \in \{0,1,2,3,4,5,6,7,8,9\}$*, we have following higher digits palindromic grids:*

aaaa	gddg	keek	bccb	nkkn	hbbh	dffd	enne	chhc	fggf
ecce	bbbb	dggd	akka	fddf	kaak	nhhn	cffc	hnnh	geeg
gnng	nffn	cccc	faaf	kggk	ehhe	heeh	bddb	abba	dkkd
neen	khhk	anna	dddd	caac	bggb	ebbe	fkkf	gffg	hcch
dhhd	Haah	bffb	knnk	eeee	gkkg	adda	nggn	fccf	cbbc
bkkb	cnnc	gbbg	hggh	ahha	ffff	kcck	deed	nddn	eaae
hddh	ekke	fhhf	ceec	bnnb	nccn	gggg	kbbk	daad	affa
fbbf	agga	naan	effe	gccg	dnnd	ckkc	hhhh	beeb	kddk
cggc	dccd	edde	nbbn	hffh	aeea	fnnf	gaag	kkkk	bhhb
kffk	feef	hkkh	ghhg	dbbd	cddc	baab	acca	egge	nnnn

aaaaa	gdgdg	kekek	bcbcb	nknkn	hbhbh	dfdfd	enene	chchc	fgfgf
ecece	bbbbb	dgdgd	akaka	fdfdf	kakak	nhnhn	cfcfc	hnhnh	gegeg
gngng	nfnfn	ccccc	fafaf	kgkgk	ehehe	heheh	bdbdb	ababa	dkdkd
nenen	khkhk	anana	ddddd	cacac	bgbgb	ebebe	fkfkf	gfgfg	hchch
dhdhd	hahah	bfbfb	knknk	eeeee	gkgkg	adada	ngngn	fcfcf	cbcbc
bkbkb	cncnc	gbgbg	hghgh	ahaha	fffff	kckck	deded	ndndn	eaeae
hdhdh	ekeke	fhfhf	cecec	bnbnb	ncncn	ggggg	kbkbk	dadad	afafa
fbfbf	agaga	nanan	efefe	gcgcg	dndnd	ckckc	hhhhh	bebeb	kdkdk
cgcgc	dcdcd	edede	nbnbn	hfhfh	aeaea	fnfnf	gagag	kkkkk	bhbhb
kfkfk	fefef	hkhkh	ghghg	dbdbd	cdcdc	babab	acaca	egege	nnnnn

aaaaaa	ekeeke	bfbbfb	nbnnbn	khkkhk	cdccdc	hehheh	dgddgd	fnffnf	gcggcg
dfddfd	bbbbbb	ghgghg	kckkck	cnccnc	nknnkn	eaeeae	fdffdf	aeaaea	hghhgh
gbggbg	fhffhf	cccccc	eneene	nfnnfn	agaaga	dkddkd	kakkak	hdhhdh	bebbeb
bnbbnb	kgkkgk	feffef	dddddd	hahhah	ececce	nhnnhn	abaaba	gfggfg	ckcckc
ndnndn	caccac	akaaka	fgffgf	eeeeee	hbhhbh	kfkkfk	gnggng	bcbbcb	dhddhd
kekkek	gdggdg	daddad	hkhhkh	bgbbgb	ffffff	anaana	ncnncn	chcchc	ebeebe
eheehe	acaaca	hnhhnh	cfccfc	fbffbf	dedded	gggggg	bkbbkb	nannan	kdkkdk
cgccgc	nennen	edeede	babbab	gkggkg	knkknk	fcffcf	hhhhhh	dbddbd	afaafa
hchhch	dnddnd	ngnngn	geggeg	adaada	bhbbhb	cbccbc	efeefe	kkkkkk	faffaf
fkffkf	hfhhfh	kbkkbk	ahaaha	dcddcd	gaggag	bdbbdb	ceccec	egeege	nnnnnn

aaaaaaa	ekekeke	bfbfbfb	nbnbnbn	khkhkhk	cdcdcdc	heheheh	dgdgdgd	fnfnfnf	gcgcgcg
dfdfdfd	bbbbbbb	ghghghg	kckckck	cncncnc	nknknkn	eaeaeae	fdfdfdf	aeaeaea	hghghgh
gbgbgbg	fhfhfhf	ccccccc	enenene	nfnfnfn	agagaga	dkdkdkd	kakakak	hdhdhdh	bebebeb
bnbnbnb	kgkgkgk	fefefef	ddddddd	hahahah	ececece	nhnhnhn	abababa	gfgfgfg	ckckckc
ndndndn	cacacac	akakaka	fgfgfgf	eeeeeee	hbhbhbh	kfkfkfk	gngngng	bcbcbcb	dhdhdhd
kekekek	gdgdgdg	dadadad	hkhkhkh	bgbgbgb	fffffff	ananana	ncncncn	chchchc	ebebebe
ehehehe	acacaca	hnhnhnh	cfcfcfc	fbfbfbf	dededed	ggggggg	bkbkbkb	nananan	kdkdkdk
cgcgcgc	nenenen	edededе	bababab	gkgkgkg	knknknk	fcfcfcf	hhhhhhh	dbdbdbd	afafafa
hchchch	dndndnd	ngngngn	gegegeg	adadada	bhbhbhb	cbcbcbc	efefefe	kkkkkkk	fafafaf
fkfkfkf	hfhfhfh	kbkbkbk	ahahaha	dcdcdcd	gagagag	bdbdbdb	cececec	egegege	nnnnnnn

aaaaaaaa	ekekkeke	bfbffbfb	nbnbbnbn	khkhhkhk	cdcddcdc	heheeheh	dgdggdgd	fnfnnfnf	gcgccgcg
dfdffdfd	bbbbbbbb	ghghhghg	kckcckck	cncnncnc	nknkknkn	eaeaaeae	fdfddfdf	aeaeeaea	hghgghgh
gbgbbgbg	fhfhhfhf	cccccccc	enennene	nfnffnfn	agaggaga	dkdkkdkd	kakaakak	hdhddhdh	bebeebeb
bnbnnbnb	kgkggkgk	fefeefef	dddddddd	hahaahah	ececcece	nhnhhnhn	ababbaba	gfgffgfg	ckckkckc
ndnddndn	cacaacac	akakhaka	fgfggfgf	eeeeeeee	hbhbbhbh	kfkffkfk	gngnngng	bcbccbcb	dhdhhdhd
kekeekek	gdgddgdg	dadaadad	hkhkkhkh	bgbggbgb	ffffffff	anannana	ncnccncn	chchhchc	ebebbebe
ehehhehe	acaccaca	hnhnnhnh	cfcffcfc	fbfbbfbf	dedeeded	gggggggg	bkbkkbkb	nanaanan	kdkddkdk
cgcggcgc	neneenen	ededdede	babaabab	gkgkkgkg	knknnknk	fcfccfcf	hhhhhhhh	dbdbbdbd	afaffafa
hchcchch	dndnndnd	ngnggngn	gegeegeg	adaddada	bhbhhbhb	cbcbbcbc	efeffefe	kkkkkkkk	fafaafaf
fkfkkfkf	hfhffhfh	kbkbbkbk	ahahhaha	dcdccdcd	gagaagag	bdbddbdb	ceceecec	egeggege	nnnnnnnn

Below is an example of magic squares of order 10:

Example 9.3. *For* $a = 0, b = 1, c = 2, d = 3, e = 4, f = 5, g = 6, h = 7, k = 8, n = 9$ *Grids 9.1 and 9.2, we have 3 to 8 digits magic squares of order 10:*

										4995
000	484	151	919	878	232	747	363	595	626	4995
353	111	676	828	292	989	404	535	040	767	4995
616	575	222	494	959	060	383	808	737	141	4995
191	868	545	333	707	424	979	010	656	282	4995
939	202	080	565	444	717	858	696	121	373	4995
848	636	303	787	161	555	090	929	272	414	4995
474	020	797	252	515	343	666	181	909	838	4995
262	949	434	101	686	898	525	777	313	050	4995
727	393	969	646	030	171	212	454	888	505	4995
585	757	818	070	323	606	131	242	464	999	4995
4995	4995	4995	4995	4995	4995	4995	4995	4995	4995	4995

										49995
0000	6336	8448	1221	9889	7117	3553	4994	2772	5665	49995
4224	1111	3663	0880	5335	8008	9779	2552	7997	6446	49995
6996	9559	2222	5005	8668	4774	7447	1331	0110	3883	49995
9449	8778	0990	3333	2002	1661	4114	5885	6556	7227	49995
3773	7007	1551	8998	4444	6886	0330	9669	5225	2112	49995
1881	2992	6116	7667	0770	5555	8228	3443	9339	4004	49995
7337	4884	5775	2442	1991	9229	6666	8118	3003	0550	49995
5115	0660	9009	4554	6226	3993	2882	7777	1441	8338	49995
2662	3223	4334	9119	7557	0440	5995	6006	8888	1771	49995
8558	5445	7887	6776	3113	2332	1001	0220	4664	9999	49995
49995	49995	49995	49995	49995	49995	49995	49995	49995	49995	49995

										499995
00000	63636	84848	12121	98989	71717	35353	49494	27272	56565	499995
42424	11111	36363	08080	53535	80808	97979	25252	79797	64646	499995
69696	95959	22222	50505	86868	47474	74747	13131	01010	38383	499995
94949	87878	09090	33333	20202	16161	41414	58585	65656	72727	499995
37373	70707	15151	89898	44444	68686	03030	96969	52525	21212	499995
18181	29292	61616	76767	07070	55555	82828	34343	93939	40404	499995
73737	48484	57575	24242	19191	92929	66666	81818	30303	05050	499995
51515	06060	90909	45454	62626	39393	28282	77777	14141	83838	499995
26262	32323	43434	91919	75757	04040	59595	60606	88888	17171	499995
85858	54545	78787	67676	31313	23232	10101	02020	46464	99999	499995
499995	499995	499995	499995	499995	499995	499995	499995	499995	499995	499995

										4999995
000000	484484	151151	919919	878878	232232	747747	363363	595595	626626	4999995
353353	111111	676676	828828	292292	989989	404404	535535	040040	767767	4999995
616616	575575	222222	494494	959959	060060	383383	808808	737737	141141	4999995
191191	868868	545545	333333	707707	424424	979979	010010	656656	282282	4999995
939939	202202	080080	565565	444444	717717	858858	696696	121121	373373	4999995
848848	636636	303303	787787	161161	555555	090090	929929	272272	414414	4999995
474474	020020	797797	252252	515515	343343	666666	181181	909909	838838	4999995
262262	949949	434434	101101	686686	898898	525525	777777	313313	050050	4999995
727727	393393	969969	646646	030030	171171	212212	454454	888888	505505	4999995
585585	757757	818818	070070	323323	606606	131131	242242	464464	999999	4999995
4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995	4999995

										49999995
0000000	4848484	1515151	9191919	8787878	2323232	7474747	3636363	5959595	6262626	49999995
3535353	1111111	6767676	8282828	2929292	9898989	4040404	5353535	0404040	7676767	49999995
6161616	5757575	2222222	4949494	9595959	0606060	3838383	8080808	7373737	1414141	49999995
1919191	8686868	5454545	3333333	7070707	4242424	9797979	0101010	6565656	2828282	49999995
9393939	2020202	0808080	5656565	4444444	7171717	8585858	6969696	1212121	3737373	49999995
8484848	6363636	3030303	7878787	1616161	5555555	0909090	9292929	2727272	4141414	49999995
4747474	0202020	7979797	2525252	5151515	3434343	6666666	1818181	9090909	8383838	49999995
2626262	9494949	4343434	1010101	6868686	8989898	5252525	7777777	3131313	0505050	49999995
7272727	3939393	9696969	6464646	0303030	1717171	2121212	4545454	8888888	5050505	49999995
5858585	7575757	8181818	0707070	3232323	6060606	1313131	2424242	4646464	9999999	49999995
49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995	49999995

										499999995
00000000	48488484	15155151	91911919	87877878	23233232	74744747	36366363	59599595	62622626	499999995
35355353	11111111	67677676	82822828	29299292	98988989	40400404	53533535	04044040	76766767	499999995
61611616	57577575	22222222	49499494	95955959	06066060	38388383	80800808	73733737	14144141	499999995
19199191	86866868	54544545	33333333	70700707	42422424	97977979	01011010	65655656	28288282	499999995
93933939	20200202	08087080	56566565	44444444	71711717	85855858	69699696	12122121	37377373	499999995
84844848	63633636	30300303	78788787	16166161	55555555	09099090	92922929	27277272	41411414	499999995
47477474	02022020	79799797	25255252	51511515	34344343	66666666	18188181	90900909	83833838	499999995
26266262	94944949	43433434	10100101	68688686	89899898	52522525	77777777	31311313	05055050	499999995
72722727	39399393	96966969	64644646	03033030	17177171	21211212	45455454	88888888	50500505	499999995
58588585	75755757	81811818	07077070	32322323	60600606	13133131	24244242	46466464	99999999	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

Since above examples contains 0 in the beginning of numbers, such as 010, 020, etc., we call these examples as **palindromic-type** magic squares. According to above six magic squares of order 10, we have the following **number pattern** with magic sums. It increases as the number of digits in each cell increases. See below:

Digits	Magic Sums
3	4995
4	49995
5	499995
6	4999995
7	49999995
8	499999995

9.3 Composite Magic Squares

Grid 9.3. *Eliminating the third value in Grid 9.1, and then splitting in two Latin squares, we get*

a	g	k	b	n	h	d	e	c	f
e	b	d	a	f	k	n	c	h	g
g	n	c	f	k	e	h	b	a	d
n	k	a	d	c	b	e	f	g	h
d	h	b	k	e	g	a	n	f	c
b	c	g	h	a	f	k	d	n	e
h	e	f	c	b	n	g	k	d	a
f	a	n	e	g	d	c	h	b	k
c	d	e	n	h	a	f	g	k	b
k	f	h	g	d	c	b	a	e	n
					A				

a	d	e	c	k	b	f	n	h	g
c	b	g	k	d	a	h	f	n	e
n	f	c	a	g	h	e	d	b	k
e	h	n	d	a	g	b	k	f	c
h	a	f	n	e	k	d	g	c	b
k	n	b	g	h	f	c	e	d	a
d	k	h	e	n	c	g	b	a	f
b	g	a	f	c	n	k	h	e	d
g	c	d	b	f	e	n	a	k	h
f	e	k	h	b	d	a	c	g	n
					B				

aa	gd	ke	bc	nk	hb	df	en	ch	fg
ec	bb	dg	ak	fd	ka	nh	cf	hn	ge
gn	nf	cc	fa	kg	eh	he	bd	ab	dk
ne	kh	an	dd	ca	bg	eb	fk	gf	hc
dh	ha	bf	kn	ee	gk	ad	ng	fc	cb
bk	cn	gb	hg	ah	ff	kc	de	nd	ea
hd	ek	fh	ce	bn	nc	gg	kb	da	af
fb	ag	na	ef	gc	dn	ck	hh	be	kd
cg	dc	ed	nb	hf	ae	fn	ga	kk	bh
kf	fe	hk	gh	db	cd	ba	ac	eg	nn
					AB				

The grid AB can be written as

$$AB := 10 \times A + B$$

Example 9.4. *In particular for $a = 0$, $b = 1$, $c = 2$, $d = 3$, $e = 4$, $f = 5$, $g = 6$, $h = 7$, $k = 8$ and $n = 9$ in 9.2, we get*

0	6	8	1	9	7	3	4	2	5
4	1	3	0	5	8	9	2	7	6
6	9	2	5	8	4	7	1	0	3
9	8	0	3	2	1	4	5	6	7
3	7	1	8	4	6	0	9	5	2
1	2	6	7	0	5	8	3	9	4
7	4	5	2	1	9	6	8	3	0
5	0	9	4	6	3	2	7	1	8
2	3	4	9	7	0	5	6	8	1
8	5	7	6	3	2	1	0	4	9
					A				

0	3	4	2	8	1	5	9	7	6
2	1	6	8	3	0	7	5	9	4
9	5	2	0	6	7	4	3	1	8
4	7	9	3	0	6	1	8	5	2
7	0	5	9	4	8	3	6	2	1
8	9	1	6	7	5	2	4	3	0
3	8	7	4	9	2	6	1	0	5
1	6	0	5	2	9	8	7	4	3
6	2	3	1	5	4	9	0	8	7
5	4	8	7	1	3	0	2	6	9
					B				

00	63	84	12	98	71	35	49	27	56
42	11	36	08	53	80	97	25	79	64
69	95	22	50	86	47	74	13	01	38
94	87	09	33	20	16	41	58	65	72
37	70	15	89	44	68	03	96	52	21
18	29	61	76	07	55	82	34	93	40
73	48	57	24	19	92	66	81	30	05
51	06	90	45	62	39	28	77	14	83
26	32	43	91	75	04	59	60	88	17
85	54	78	67	31	23	10	02	46	99
					AB				

Note 9.1. *Applying* $10 \times (A-1) + B$ *over the elements of* A *and* B *given above, we get a magic square of order 10 given in Example 9.1. Below are examples of composite* ***upside down*** *and* ***mirror looking*** *magic squares. Moreover,* A *and* B *are* ***pairwise mutually orthogonal diagonal Latin squares****. The above Grid 9.3 is written for single letters. We can choose double digits numbers to write composite examples. See below.*

Example 9.5. *Let's consider* $a = 11$, $b = 16$, $c = 19$, $d = 61$, $e = 66$, $f = 69$, $g = 88$, $h = 91$, $k = 96$ *and* $n = 99$ *in Grid 9.3 , we get following* ***upside down*** *magic square:*

1111	8891	6999	1916	6669	6119	9196	9966	1661	9688
9916	1919	9188	1169	9691	6911	6661	1696	6166	8899
8866	6696	1616	9611	6988	9961	6199	1991	1119	9169
6699	6961	1166	9191	1611	1988	9919	9669	8896	6116
9161	6111	1996	6966	9999	8869	1191	6688	9616	1619
1969	1666	8819	6188	1161	9696	6916	9199	6691	9911
6191	9969	9661	1699	1966	6616	8888	6919	9111	1196
9619	1188	6611	9996	8816	9166	1669	6161	1999	6991
1688	9116	9991	6619	6196	1199	9666	8811	6969	1961
6996	9699	6169	8861	9119	1691	1911	1116	9988	6666

In this case, the magic square sum is $S_{10\times 10} := 62216$.

Example 9.6. *Let's consider* $a = 00$, $b = 22$, $c = 25$, $d = 28$, $e = 52$, $f = 55$, $g = 58$, $h = 82$, $k = 85$ *and* $n = 88$ *in Grid 9.3 , we get following* ***upside down*** *and* ***mirror looking*** *magic square with magic sum* $S_{10\times 10} := 32219$*:*

0000	8820	5222	0205	5552	5002	2025	2255	0550	2588
2205	0202	2088	0052	2520	5200	5550	0525	5055	8822
8855	5525	0505	2500	5288	2250	5022	0220	0002	2052
5522	5250	0055	2020	0500	0288	2202	2552	8825	5005
2050	5000	0225	5255	2222	8852	0020	5588	2505	0502
0252	0555	8802	5088	0050	2525	5205	2022	5520	2200
5020	2252	2550	0522	0255	5505	8888	5202	2000	0025
2502	0088	5500	2225	8805	2055	0552	5050	0222	5220
0588	2005	2220	5502	5025	0022	2555	8800	5252	0250
5225	2522	5052	8850	2002	0520	0200	0005	2288	5555

Above two Examples 9.5 and ex10a5 are with four digits (1,6,8,9) and (0,2,5,8) respectively. With double digits choice with three numbers, we have maximum 9 possibilities. This is the reason we considered above two examples with 4 digits. With four digits, we can go maximum upto order 16 magic squares. If we make triple digits choice with 3 digits, i.e., (a,b,c) we can choose upto order 27 magic squares, i.e., 3^3. Let's select some possible digits to have symmetry with mirror looking magic square of order 10. See below the example.

Example 9.7. *Let's consider* $a = 222$, $b = 228$, $c = 258$, $d = 528$, $e = 555$, $f = 558$, $g = 822$, $h = 825$, $k = 852$ *and* $n = 855$ *in Grid 9.3 , we get following magic square:*

										5708703
222222	822528	852555	228258	855852	825228	528558	555855	258825	558822	5708703
555258	228228	528822	222852	558528	852222	855825	258558	825855	822555	5708703
822855	855558	258258	558222	852822	555825	825555	228528	222228	528852	5708703
855555	852825	222855	528528	258222	228822	555228	558852	822558	825258	5708703
528825	825222	228558	852855	555555	822852	222528	855822	558258	258228	5708703
228852	258855	822228	825822	222825	558558	852258	528555	855528	555222	5708703
825528	555852	558825	258555	228855	855258	822822	852228	528222	222558	5708703
558228	222822	855222	555558	822258	528855	258852	825825	228555	852528	5708703
258822	528258	555528	855228	825558	222555	558855	822222	852852	228825	5708703
852558	558555	825852	822825	528228	258528	228222	222258	555822	855855	5708703
5708703	5708703	5708703	5708703	5708703	5708703	5708703	5708703	5708703	5708703	5708703

The **upside down** and **mirror looking** version of above magic square is given by

222222	822528	852555	228258	855852	825228	528558	555855	258825	558822
555258	228228	528822	222852	558528	852222	855825	258558	825855	822555
822855	855558	258258	558222	852822	555825	825555	228528	222228	528852
855555	852825	222855	528528	258222	228822	555228	558852	822558	825258
528825	825222	228558	852855	555555	822852	222528	855822	558258	258228
228852	258855	822228	825822	222825	558558	852258	528555	855528	555222
825528	555852	558825	258555	228855	855258	822822	852228	528222	222558
558228	222822	855222	555558	822258	528855	258852	825825	228555	852528
258822	528258	555528	855228	825558	222555	558855	822222	852852	228825
852558	558555	825852	822825	528228	258528	228222	222258	555822	855855

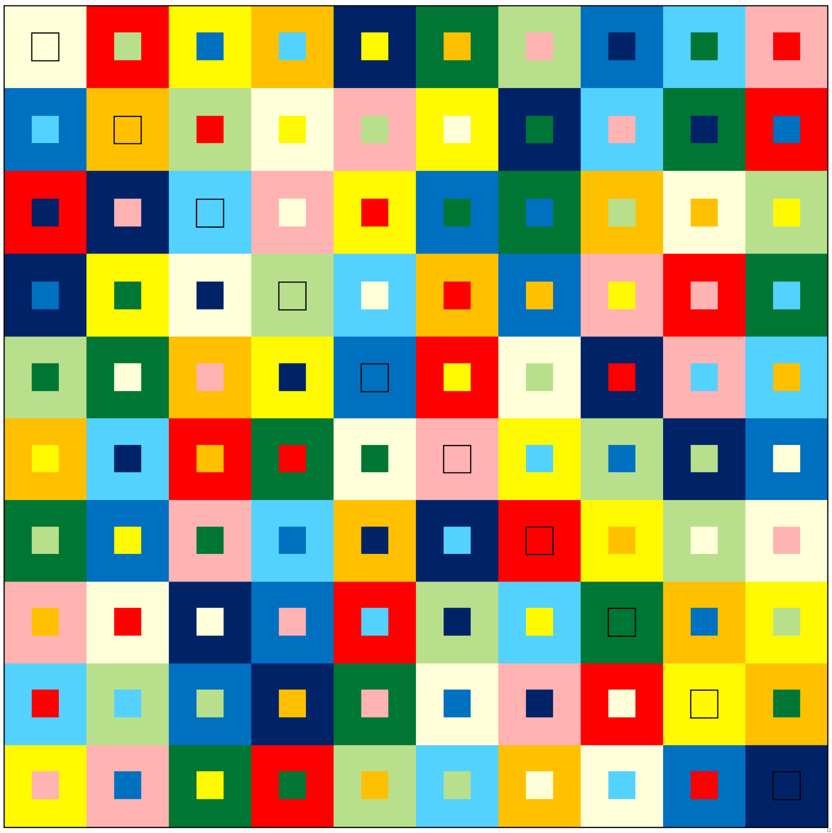
Since we know that in the mirror 2 becomes 5 and 5 as 2. The example below show that still it remains magic square even after changing 2 by 5 and 5 by 2.

Example 9.8. *Let's consider* $a = 555$, $b = 558$, $c = 528$, $d = 258$, $e = 222$, $f = 228$, $g = 855$, $h = 852$, $k = 825$ *and* $n = 822$ *in Grid* 9.3 *, we get following magic square:*

										5708703
555555	855258	825222	558528	822825	852558	258228	222822	528852	228855	5708703
222528	558558	258855	555825	228258	825555	822852	528228	852822	855222	5708703
855822	822228	528528	228555	825855	222852	852222	558258	555558	258825	5708703
822222	825852	555822	258258	528555	558855	222558	228825	855228	852528	5708703
258852	852555	558228	825822	222222	855825	555258	822855	228528	528558	5708703
558825	528822	855558	852855	555852	228228	825528	258222	822258	222555	5708703
852258	222825	228852	528222	558822	822528	855855	825558	258555	555228	5708703
228558	555855	822555	222228	855528	258822	528825	852852	558222	825258	5708703
528855	258528	222258	822558	852228	555222	228822	855555	825825	558852	5708703
825228	228222	852825	855852	258558	528258	558555	555528	222855	822822	5708703
5708703	5708703	5708703	5708703	5708703	5708703	5708703	5708703	5708703	5708703	5708703

9.4 Superimposed Colored Pattern

The grid *AB* is a composite magic square of order 10. Based on it, here below is **double colored pattern**



Looking from the above **superimposed colored pattern** of order 10, we observe that it has **diagonal property**, as it is made from **pairwise mutually orthogonal diagonal Latin squares**.

10 Final Comments

There are many ways of representing magic squares with palindromic type entries. This paper works with magic squares of orders 3 to 10. In each case, magic squares written for the entries from digits 3 to 8 with all palindromic numbers. These entries are arranged in such a way that their magic sums becomes **number patterns**. In some cases palindromic **upside down** and/or **mirror looking** magic squares are also given. The study is extended to **composite-type** magic squares resulting in **colored patterns**, **upside down**, and **mirror looking** magic squares. It is interesting to observe that, we have exactly perfect square numbers 3-digits palindromes depending on the number of letters (see the Table 1.2 given in the Section 1.1). This allows us to write palindromic magic squares of order 3 to 9. In case of order 10 it is **palindromic-type** magic square, as here we use numbers of type 010, 020, etc. Another interesting aspect of palindromes is that there are exactly 16 palindromes of 7-digits only with two letters. This gives magic squares of order 4 only with two digits, for example, (1, 8), (2, 5), etc. Also, there are exactly 64 palindromes of 11-digits only with two letters. This also give a palindromic magic and **bimagic** squares of order 8 only with two digits, for example, (1, 8), (2, 5), etc. Also, we have exactly, 64 palindromes of 5-digits just with 4 letters. The same happens with order 9 magic square. Here we have exactly 81 palindromes of 7-digits just with 3 letters. This gives magic squares using only 3 digits, for example, (1, 6, 9), (2, 5, 8), etc.

10.1 Author's Contributions to Magic Squares

The item-wise author's work on magic squares is as follows:

(i) **Digital Numbers** Magic Squares - [7, 8, 9, 10, 11, 12];

(ii) **Block-Wise Construction** of **Bimagic Squares** - [13];

(iii) Connections with **Genetic Tables** and **Shannon's entropy** - [14];

(iv) **Selfie** and **palindromic-type** Magic Squares - [15, 31];

(v) **Intervally Distributed** and **Block-Wise** Magic Squares - [16, 17, 18];

(vi) **Multi-digits** and **Number Patterns** Magic Squares - [19, 31];

(vii) **Perfect Square Sum** Magic Squares with **Uniformity**, **Minimum Sum** and **Pythagorean Triples** - [20, 21];

(viii) **Block-Wise** Constructions of Magic and Bimagic Squares - [22, 23, 24, 25, 27, 30];

(ix) **Magic Crosses:** Repeated and Non Repeated Entries - [26];

(x) Representations of **Letters** and **Numbers** With Equal Sums Magic Squares of Orders 4 and 6 - [28, 29].

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