# Patterns in Pythagorean Triples Using Double Variable Procedures

**Inder J. Taneja**[1]

**Abstract**

*The Pythagoras theorem is very famous in the literature of mathematics. The aim of this work is to extend in a symmetrical way the Pythagorean triples. The extensions are in such a way that they seem as patterns. In some cases, the final sum also give a good pattern. The patterns are obtained based on four procedures for double variable functions. Patterns based on single variable functions are done in [5].*

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[1]*Formerly, Professor of Mathematics, Universidade Federal de Santa Catarina, Florianópolis, SC, Brazil (1978-2012). Also worked at Delhi University, India (1976-1978).* ***E-mail:*** *ijtaneja@gmail.com;* ***Web-sites:*** *http://inderjtaneja.com; http://indertaneja.com;* ***Twitter:*** *@IJTANEJA.*

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# 1 Introduction

By Pythagoras theorem it is understood that

$$a^2 + b^2 = c^2, \ \forall a, \ b, \ c \in N_+$$

For simplicity, let's write it as $(a, b, c)$ . Let's consider the initial Pythagorean triple (3,4,5). This means that $3^2 + 4^2 = 5^2$.

The symmetric extensions of above triple we call **patterns in Pythagorean triples**. There two obvious ways of getting **patterns in Pythagorean triples**.

## 1.1 Simple Patterns

- **Multiplication by 10, 100, 1000, ... :**

See two examples

$$\begin{aligned} 3^2 + 4^2 &= 5^2 := 25 \\ 30^2 + 40^2 &= 50^2 := 2500 \\ 300^2 + 400^2 &= 500^2 := 250000 \end{aligned} \tag{1}$$

$$\begin{aligned} 9^2 + 40^2 &= 41^2 := 1681 \\ 90^2 + 400^2 &= 410^2 := 168100 \\ 900^2 + 4000^2 &= 4100^2 := 16810000 \end{aligned} \tag{2}$$

The difference in above two examples is that the first one is with single digit in each case, i.e., 3, 4 and 5, and in second example (2) not all the same digits are with same length, i.e., the first one is of length 1 and other two are with length 2. This process is generally true for all types of **Pythagorean triples**.

- **Repetition of Digits:**

$$\begin{aligned} 3^2 + 4^2 &= 5^2 := 25 \\ 33^2 + 44^2 &= 55^2 := 3025 \\ 333^2 + 444^2 &= 555^2 := 308035 \end{aligned} \tag{3}$$

$$\begin{aligned} 12^2 + 35^2 &= 37^2 := 1369 \\ 1212^2 + 3535^2 &= 3737^2 := 13965169 \\ 121212^2 + 353535^2 &= 373737^2 := 139679345169 \end{aligned} \tag{4}$$

$$\begin{aligned} 119^2 + 120^2 &= 169^2 := 28561 \\ 119119^2 + 120120^2 &= 169169^2 := 28618150561 \\ 119119119^2 + 120120120^2 &= 169169169^2 := 28618207740150561 \end{aligned} \tag{5}$$

In the examples (3), (4) and (5), we observe that the repetition of digits give us patterned results when we work with same length triples. Here we have (3,4,5), (12, 35, 27) and (119, 120, 169). In the example (3, 4, 5) each digit is of length 1. In the example (12, 35, 27) each digit is of length 2, and the triple (119, 120, 169) each digit is of length 3. This theory doesn't work when the triples are with different lengths, for example, (5, 12, 13). This gives

$$\begin{aligned} 5^2+12^2 &= 13^2 \quad \Rightarrow 169 = 169 \\ 55^2+1212^2 &\neq 1313^2 \Rightarrow 1471969 \neq 1723969 \end{aligned} \tag{6}$$

In this case the pattern (6) is not extendable as in case examples (3), (4) and (5).

**Remark 1.** *Analysing the final sums, we observe that the examples (1) and (2) are good for* ***patterns with final sums****. The example (3) is not so good but acceptable. In case of examples (4) and (5), the patterns with final sums are not good. In this work, we shall write patterns with final sums only if they are good.*

The aim of this work is to give different procedures to find Pythagorean triples, and then give examples giving patterns based on functional representations.

# 2 Two Variables Procedures for Pythagorean Triples

## 2.1 Procedure 1

For all $m, n \in N_+$, let's consider the following three functions:

$$\begin{aligned} F_1(m,n) &:= m(2n+m) \\ G_1(m,n) &:= 2n(n+m) \\ H_1(m,n) &:= m^2+2mn+2n^2 \end{aligned} \tag{7}$$

Then we can easily check that

$$\begin{aligned} F_1(m,n)^2+G_1(m,n)^2 &= (m(2n+m))^2+(2n(n+m))^2 \\ &= m^4+4m^3n+4m^2n^2+4m^2n^2+8mn^3+4n^4 \\ &= m^4+4m^3n+8m^2n^2+8mn^3+4n^4 \\ &= \left(m^2+2mn+2n^2\right)^2 = H_1(m,n)^2. \end{aligned}$$

This proves that the triple $(F_1, G_1, H_1)$ is a **Pythagorean triple** for all $m, n \in N_+$. In particular, we have

$$\begin{aligned} (F_1(1,n), G_1(1,n), H_1(1,n)) &= (f_1(n), g_1(n), h_1(n)) \\ (F_1(m,1), G_1(m,1), H_1(m,1)) &= (f_2(m), g_2(m), h_2(m)) \end{aligned}$$

where

$$\begin{aligned} f_1(n) &:= 2n+1 \\ g_1(n) &:= 2n(n+1) \\ h_1(n) &:= 2n^2+2n+1 \end{aligned} \tag{8}$$

and

$$\begin{aligned} f_2(m) &:= m(m+2) \\ g_2(m) &:= 2(m+1) \\ h_2(m) &:= m^2+2m+2 \end{aligned} \tag{9}$$

The Procedures (8) and (9) are two single variable procedures studied in [5].

Some particular cases Procedure (7 are as follows:

$$\begin{aligned} (F_1(1,2),G_1(1,2),H_1(1,2)) &\Rightarrow (5,12,13) \\ (F_1(3,1),G_1(3,1),H_1(3,1)) &\Rightarrow (5,8,17) \\ (F_1(3,2),G_1(3,2),H_1(3,2)) &\Rightarrow (21,20,29) \\ (F_1(2,5),G_1(2,5),H_1(2,5)) &\Rightarrow (24,70,74) \\ (F_1(3,4),G_1(3,4),H_1(3,4)) &\Rightarrow (33,56,65) \\ (F_1(2,6),G_1(2,6),H_1(2,6)) &\Rightarrow (28,96,100) \\ (F_1(5,5),G_1(5,5),H_1(5,5)) &\Rightarrow (75,100,125) \\ (F_1(3,7),G_1(3,7),H_1(3,7)) &\Rightarrow (51,140,149) \\ (F_1(6,7),G_1(6,7),H_1(6,7)) &\Rightarrow (120,182,218) \end{aligned}$$

Some patterned examples of the Procedure 1 are given in subsection 3.1. There the options of fixing $m$ or $n$ are considered.

## 2.2 Procedure 2

For all $m,n \in N_+, \ \ m > n \geq 1$, let's consider the following three functions:

$$\begin{aligned} F_2(m,n) &:= m^2-n^2 \\ G_2(m,n) &:= 2mn \\ H_2(m,n) &:= m^2+n^2 \end{aligned} \tag{10}$$

Then we can easily check that

$$\begin{aligned} F_2(m,n)^2+G_2(m,n)^2 &= \left(m^2-n^2\right)^2+(2mn)^2 \\ &= m^4-2m^2n^2+n^4+4m^2n^2 \\ &= m^4+2m^2n^2+n^4 \\ &= \left(m^2+n^2\right)^2 = H_2(m,n)^2. \end{aligned}$$

This proves that the triple $(F_2,G_2,H_2)$ is a **Pythagorean triple** for all $m,n \in N_+, \ \ m > n \geq 1$.

In particular, when $n=1$, we get $F_2(m,1)=f_3(m), \ G_2(m,1)=g_3(m)$ and $H_2(m,1)=h_3(m)$, i.e.,

$$(F_2(m,1),G_2(m,1),H_2(m,1)) = (f_3(m),g_3(m),h_3(m)).$$

where

$$\begin{aligned} f_3(m) &:= m^2 - 1 \\ g_3(m) &:= 2m \\ h_3(m) &:= m^2 + 1 \end{aligned} \tag{11}$$

The Procedure (11) is very famous in the literature [1]. Some patterned examples based on the Procedure (11) are studied by author [5]. Some particular cases of Procedure 2 are as follows:

$$\begin{aligned} &(F_2(3,2), G_2(3,2), H_2(3,2)) &&\Rightarrow (5,12,13) \\ &(F_2(4,2), G_2(4,2), H_2(4,2)) &&\Rightarrow (12,16,20) \\ &(F_2(5,2), G_2(5,2), H_2(5,2)) &&\Rightarrow (21,20,29) \\ &(F_2(6,2), G_2(6,2), H_2(6,2)) &&\Rightarrow (32,24,40) \\ &(F_2(9,3), G_2(9,3), H_2(9,3)) &&\Rightarrow (72,54,90) \\ &(F_2(9,7), G_2(9,7), H_2(9,7)) &&\Rightarrow (32,126,130) \\ &(F_2(10,9), G_2(10,9), H_2(10,9)) &&\Rightarrow (19,180,181) \end{aligned}$$

Some patterned examples of the Procedure 1 are given in subsection 3.2.

## 2.3 Procedure 3

For all $m, n \in N_+,\ \ 2m > n \geq 1$, let's consider the following three functions:

$$\begin{aligned} F_3(m,n) &:= 4m^2 - n^2 \\ G_3(m,n) &:= 4mn \\ H_3(m,n) &:= 4m^2 + n^2 \end{aligned} \tag{12}$$

Then we can easily check that

$$\begin{aligned} F_3(m,n)^2 + G_3(m,n)^2 &= \left(4m^2 - n^2\right)^2 + (4mn)^2 \\ &= 16m^4 - 8m^2n^2 + n^4 + 16m^2n^2 \\ &= 16m^4 + 8m^2n^2 + n^4 \\ &= \left(4m^2 + n^2\right)^2 = H_3(m,n)^2. \end{aligned}$$

This proves that the triple $(F_3, G_3, H_3)$ is a **Pythagorean triple** for all $m, n \in N_+,\ \ 2m > n \geq 1$. In particular, when $n = 1$, we get $F_3(m,1) = f_4(m),\ \ G_3(m,1) = g_4(m)$ and $H_3(m,1) = h_4(m)$, i.e.,

$$(F_3(m,1), G_3(m,1), H_3(m,1)) = (f_4(m), g_4(m), h_4(m)),$$

where

$$\begin{aligned} f_4(m) &:= 4m^2 - 1 \\ g_4(m) &:= 4m \\ h_4(m) &:= 4m^2 + 1 \end{aligned} \tag{13}$$

Some patterned examples based on the Procedure (13) are studied by author [5]. Some particular cases of Procedure 3 are as follows:

$$\begin{aligned}
&(F_3(2,1), G_3(2,1), H_3(2,1)) &&\Rightarrow (15, 8, 17)\\
&(F_3(3,1), G_3(3,1), H_3(3,1)) &&\Rightarrow (35, 12, 37)\\
&(F_3(3,2), G_3(3,2), H_3(3,2)) &&\Rightarrow (32, 24, 40)\\
&(F_3(5,2), G_3(5,2), H_3(5,2)) &&\Rightarrow (96, 40, 104)\\
&(F_3(6,2), G_3(6,2), H_3(6,2)) &&\Rightarrow (140, 48, 148)\\
&(F_3(9,3), G_3(9,3), H_3(9,3)) &&\Rightarrow (315, 108, 333)\\
&(F_3(9,7), G_3(9,7), H_3(9,7)) &&\Rightarrow (275, 252, 373)\\
&(F_3(10,9), G_3(10,9), H(10,9)) &&\Rightarrow (319, 360, 481)
\end{aligned}$$

For patterned examples of the Procedure 3 see subsection 3.3.

## 2.4 Procedure 4

For all $m, n \in N_+,\ \ 4\,m > n \geq 1$, let's consider the following three functions:

$$\begin{aligned}
F_4(m,n) &:= 16m^2 - n^2\\
G_4(m,n) &:= 8mn\\
H_4(m,n) &:= 16m^2 + n^2
\end{aligned} \tag{14}$$

Then we can easily check that

$$\begin{aligned}
F_4(m,n)^2 + G_4(m,n)^2 &= \left(16m^2 - n^2\right)^2 + (8mn)^2\\
&= 256m^4 - 32m^2n^2 + n^4 + 64m^2n^2\\
&= 256m^4 + 32m^2n^2 + n^4\\
&= \left(16m^2 + n^2\right)^2 = H_4(m,n)^2.
\end{aligned}$$

This proves that the triple $(F_4, G_4, H_4)$ is a **Pythagorean triple** for all $m, n \in N_+,\ \ 4\,m > n \geq 1$. When $n = 1$, we get $F_4(m,1) = f_5(m),\ G_4(m,1) = g_5(m)$ and $H_4(m,1) = h_5(m)$, i.e.,

$$(F_4(m,1), G_4(m,1), H_4(m,1)) = (f_5(m), g_5(m), h_5(m))\,,$$

where

$$\begin{aligned}
f_5(m) &:= 16m^2 - 1\\
g_5(m) &:= 8m\\
h_5(m) &:= 16m^2 + 1
\end{aligned} \tag{15}$$

Some patterned examples based on the Procedure (15) are studied by author [5]. Some particular cases of Procedure 4 are as follows:

$$\begin{aligned}
&(F_4(2,1), G_4(2,1), H_4(2,1)) &&\Rightarrow (63,16,65)\\
&(F_4(3,1), G_4(3,1), H_4(3,1)) &&\Rightarrow (143,24,145)\\
&(F_4(3,2), G_4(3,2), H_4(3,2)) &&\Rightarrow (140,48,148)\\
&(F_4(5,2), G_4(5,2), H_4(5,2)) &&\Rightarrow (396,80,404)\\
&(F_4(6,2), G_4(6,2), H_4(6,2)) &&\Rightarrow (572,96,580)\\
&(F_4(9,3), G_4(9,3), H_4(9,3)) &&\Rightarrow (1287,216,1305)\\
&(F_4(9,7), G_4(9,7), H_4(9,7)) &&\Rightarrow (1247,504,1345)\\
&(F_4(10,9), G_4(10,9), H_4(10,9)) &&\Rightarrow (1519,720,1681)
\end{aligned}$$

If we double the above values $m$ we get the same results for the triples $(F_3, G_3, H_3)$. See below:

$$\begin{aligned}
&(F_4(2,1), G_4(2,1), H_4(2,1)) &&= (F_3(4,1), G_3(4,1), H_3(4,1)) &&\Rightarrow (63,16,65)\\
&(F_4(3,1), G_4(3,1), H_4(3,1)) &&= (F_3(6,1), G_3(6,1), H_3(6,1)) &&\Rightarrow (143,24,145)\\
&(F_4(3,2), G_4(3,2), H_4(3,2)) &&= (F_3(6,2), G_3(6,2), H_3(6,2)) &&\Rightarrow (140,48,148)\\
&(F_4(5,2), G_4(5,2), H_4(5,2)) &&= (F_3(10,2), G_3(10,2), H_3(10,2)) &&\Rightarrow (396,80,404)\\
&(F_4(6,2), G_4(6,2), H_4(6,2)) &&= (F_3(12,2), G_3(12,2), H_3(12,2)) &&\Rightarrow (572,96,580)\\
&(F_4(9,3), G_4(9,3), H_4(9,3)) &&= (F_3(18,3), G_3(18,3), H_3(18,3)) &&\Rightarrow (1287,216,1305)\\
&(F_4(9,7), G_4(9,7), H_4(9,7)) &&= (F_3(18,7), G_3(18,7), H_3(18,7)) &&\Rightarrow (1247,504,1345)\\
&(F_4(10,9), G_4(10,9), H_4(10,9)) &&= (F_3(20,9), G_3(20,9), H_3(20,9)) &&\Rightarrow (1519,720,1681)
\end{aligned}$$

Equivalently, we have

$$(F_4(m,n), G_4(m,n), H_4(m,n)) = (F_3(2m,n), G_3(2m,n), H_3(2m,n))$$

For patterned examples of the Procedure 4 see subsection 3.4. The section below give **patterned Pythagorean triples** for all the four Procedures. In case of patterns with even numbers, division by 2, 4, 8, etc. are considered to get more patterns utill we reach patterns with primitive values.

# 3 Examples of Patterns

This section bring examples of **patterned Pythagorean triples** constructed according to above four Procedures.

## 3.1 Procedure 1

This subsection brings patterns based on Procedure (7) given in subsection 2.1. Below are examples based on particular values of $m$ and $n$. In this procedure we have facilities to fix one and vary other.

- **For $n = 1$; $m = 30, 300, 3000, 30000, ...$ in (7):**

$$\begin{aligned}
960^2+62^2 &= 962^2 \\
90600^2+602^2 &= 90602^2 &&:= 8208722404 \\
9006000^2+6002^2 &= 9006002^2 &&:= 81108072024004 \\
900060000^2+60002^2 &= 900060002^2 &&:= 810108007200240004 \\
90000600000^2+600002^2 &= 90000600002^2 &&:= 8100108000720002400004 \\
9000006000000^2+6000002^2 &= 9000006000002^2 &&:= 81000108000072000024000004
\end{aligned} \tag{16}$$

The first triple $(15, 8, 17)$ for $n = 1; m = 3$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
480^2+31^2 &= 481^2 \\
45300^2+301^2 &= 45301^2 \\
4503000^2+3001^2 &= 4503001^2 \\
450030000^2+30001^2 &= 450030001^2 &&:= 202527001800060001 \\
45000300000^2+300001^2 &= 45000300001^2 &&:= 2025027000180000600001 \\
4500003000000^2+3000001^2 &= 4500003000001^2 &&:= 20250027000018000006000001
\end{aligned} \tag{17}$$

- **For $n = 2;\ m = 500, 5000, 50000, ...$ in (7):**

$$\begin{aligned}
252000^2+2008^2 &= 252008^2 \\
25020000^2+20008^2 &= 25020008^2 \\
2500200000^2+200008^2 &= 2500200008^2 &&:= 6251000080003200064 \\
250002000000^2+2000008^2 &= 250002000008^2 &&:= 62501000008000032000064 \\
25000020000000^2+20000008^2 &= 25000020000008^2 &&:= 625001000000800000320000064
\end{aligned} \tag{18}$$

The first two triples $(45, 28, 53)$ and $(2700, 208, 2708)$ for $n = 2; m = 5$ and, 50 are not written above as they don't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
126000^2+1004^2 &= 126004^2 \\
12510000^2+10004^2 &= 12510004^2 \\
1250100000^2+100004^2 &= 1250100004^2 \\
125001000000^2+1000004^2 &= 125001000004^2 &&:= 15625250002000008000016 \\
12500010000000^2+10000004^2 &= 12500010000004^2 &&:= 156250250000200000080000016
\end{aligned} \tag{19}$$

▶ **Division by 4**

$$\begin{aligned}
63000^2 + 502^2 &= 63002^2 \\
6255000^2 + 5002^2 &= 6255002^2 \\
625050000^2 + 50002^2 &= 625050002^2 \\
62500500000^2 + 500002^2 &= 62500500002^2 \\
6250005000000^2 + 5000002^2 &= 6250005000002^2
\end{aligned} \tag{20}$$

▶ **Division by 8**

$$\begin{aligned}
31500^2 + 251^2 &= 31501^2 \\
3127500^2 + 2501^2 &= 3127501^2 \\
312525000^2 + 25001^2 &= 312525001^2 \\
31250250000^2 + 250001^2 &= 31250250001^2 \\
3125002500000^2 + 2500001^2 &= 3125002500001^2
\end{aligned} \tag{21}$$

- **For $n = 3;\ m = 500, 5000, 50000, ...$ in (7):**

$$\begin{aligned}
253000^2 + 3018^2 &= 253018^2 \\
25030000^2 + 30018^2 &= 25030018^2 &&:= 626501801080324 \\
2500300000^2 + 300018^2 &= 2500300018^2 &&:= 6251500180010800324 \\
250003000000^2 + 3000018^2 &= 250003000018^2 &&:= 62501500018000108000324 \\
25000030000000^2 + 30000018^2 &= 25000030000018^2 &&:= 625001500001800001080000324
\end{aligned} \tag{22}$$

The first two triples $(55, 48, 73)$ and $(2800, 318, 2818)$ for $n = 3; m = 5$ and, 50 are not written above as they don't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
126500^2 + 1509^2 &= 126509^2 \\
12515000^2 + 15009^2 &= 12515009^2 \\
1250150000^2 + 150009^2 &= 1250150009^2 \\
125001500000^2 + 1500009^2 &= 125001500009^2 &&:= 15625375004500027000081 \\
12500015000000^2 + 15000009^2 &= 12500015000009^2 &&:= 156250375000450000270000081
\end{aligned} \tag{23}$$

- **For $n = 3;\ m = 8, 80, 800, 8000, 80000, ...$ in (7):**

$$\begin{aligned}
644800^2 + 4818^2 &= 644818^2 \\
64048000^2 + 48018^2 &= 64048018^2 \\
6400480000^2 + 480018^2 &= 6400480018^2 &&:= 40966144460817280324 \\
640004800000^2 + 4800018^2 &= 640004800018^2 &&:= 409606144046080172800324 \\
64000048000000^2 + 48000018^2 &= 64000048000018^2 &&:= 4096006144004608001728000324
\end{aligned} \tag{24}$$

The first two triples $(112, 66, 130)$ and $(6880, 498, 6898)$ for $n = 3; m = 8$ and, 80 are not written above as they don't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
322400^2 + 2409^2 &= 322409^2 \\
32024000^2 + 24009^2 &= 32024009^2 \\
3200240000^2 + 240009^2 &= 3200240009^2 &&:= 10241536115204320081 \\
320002400000^2 + 2400009^2 &= 320002400009^2 &&:= 1024015360115200432000081 \\
32000024000000^2 + 24000009^2 &= 32000024000009^2 &&:= 10240015360011520004320000081
\end{aligned} \tag{25}$$

- **For $n = 7;\ m = 900, 9000, 90000, ...$ in (7):**

$$\begin{aligned}
822600^2 + 12698^2 &= 822698^2 \\
81126000^2 + 126098^2 &= 81126098^2 \\
8101260000^2 + 1260098^2 &= 8101260098^2 \\
810012600000^2 + 12600098^2 &= 810012600098^2 &&:= 656120412317522469609604 \\
81000126000000^2 + 126000098^2 &= 81000126000098^2 &&:= 6561020412031752024696009604
\end{aligned} \tag{26}$$

The first two triples $(207, 224, 305)$ and $(9360, 1358, 9458)$ for $n = 7; m = 9$ and, 90 are not written above as they don't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
40563000^2 + 63049^2 &= 40563049^2 \\
4050630000^2 + 630049^2 &= 4050630049^2 \\
405006300000^2 + 6300049^2 &= 405006300049^2 \\
40500063000000^2 + 63000049^2 &= 40500063000049^2
\end{aligned} \tag{27}$$

The first triple $(411300, 6349, 411349)$ is not written above as it doesn't give good pattern.

- **For $n = 7;\ m = 600, 6000, 60000, ...$ in (7):**

$$\begin{aligned}
368400^2 + 8498^2 &= 368498^2 \\
36084000^2 + 84098^2 &= 36084098^2 \\
3600840000^2 + 840098^2 &= 3600840098^2 \\
360008400000^2 + 8400098^2 &= 360008400098^2 &&:= 129606048141121646409604 \\
36000084000000^2 + 84000098^2 &= 36000084000098^2 &&:= 1296006048014112016464009604
\end{aligned} \tag{28}$$

The first two triples $(120, 182, 218)$ and $(4440, 938, 4538)$ for $n = 7; m = 6$ and, 60 are not written above as they don't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
184200^2 + 4249^2 &= 184249^2\\
18042000^2 + 42049^2 &= 18042049^2\\
1800420000^2 + 420049^2 &= 1800420049^2\\
180004200000^2 + 4200049^2 &= 180004200049^2 &&:= 32401512035280411602401\\
18000042000000^2 + 42000049^2 &= 18000042000049^2 &&:= 324001512003528004116002401
\end{aligned} \tag{29}$$

- **For $n = 7;\ m = 666, 6666, 66666, ...$ in (7):**

$$\begin{aligned}
452880^2 + 9422^2 &= 452978^2\\
44528880^2 + 93422^2 &= 44528978^2\\
4445288880^2 + 933422^2 &= 4445288978^2\\
444452888880^2 + 9333422^2 &= 444452888978^2\\
44444528888880^2 + 93333422^2 &= 44444528888978^2
\end{aligned} \tag{30}$$

The first two triples $(120, 182, 218)$ and $(5280, 1022, 5378)$ for $n = 7; m = 6$ and, $66$ are not written above as they don't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
226440^2 + 4711^2 &= 226489^2\\
22264440^2 + 46711^2 &= 22264489^2\\
2222644440^2 + 466711^2 &= 2222644489^2\\
222226444440^2 + 4666711^2 &= 222226444489^2\\
22222264444440^2 + 46666711^2 &= 22222264444489^2
\end{aligned} \tag{31}$$

- **For $n = 5;\ m = 303, 3003, 30003, ...$ in (7):**

$$\begin{aligned}
94839^2 + 3080^2 &= 94889^2\\
9048039^2 + 30080^2 &= 9048089^2\\
900480039^2 + 300080^2 &= 900480089^2 &&:= 810864390685447921\\
90004800039^2 + 3000080^2 &= 90004800089^2 &&:= 8100864039060854407921\\
9000048000039^2 + 30000080^2 &= 9000048000089^2 &&:= 81000864003906008544007921
\end{aligned} \tag{32}$$

The first two triples $(39, 80, 89)$ and $(1419, 380, 1469)$ for $n = 5; m = 3$ and, $33$ are not written above as they don't give good pattern.

- **For $n = 7;\ m = 6006, 60006, 600006, ...$ in (7):**

$$
\begin{aligned}
36156120^2 + 84182^2 &= 36156218^2 \\
3601560120^2 + 840182^2 &= 3601560218^2 \\
360015600120^2 + 8400182^2 &= 360015600218^2 &&:= 129611232400326801647524 \\
36000156000120^2 + 84000182^2 &= 36000156000218^2 &&:= 1296011232040032068016047524
\end{aligned} \tag{33}
$$

The first three triples $(120, 182, 218)$, $(5280, 1022, 5378)$ and $(375720, 8582, 375818)$ for $n = 7; m = 6, 66,$ and, $606$ are not written above as they don't give good pattern.

▶ **Division by 2**

$$
\begin{aligned}
18078060^2 + 42091^2 &= 18078109^2 \\
1800780060^2 + 420091^2 &= 1800780109^2 \\
180007800060^2 + 4200091^2 &= 180007800109^2 &&:= 32402808100081700411881 \\
18000078000060^2 + 42000091^2 &= 18000078000109^2 &&:= 3240028080100080170004011881
\end{aligned} \tag{34}
$$

- **For $n = 8;\ m = 9009, 90009, 900009, ...$ in (7):**

$$
\begin{aligned}
81306225^2 + 144272^2 &= 81306353^2 \\
8103060225^2 + 1440272^2 &= 8103060353^2 \\
810030600225^2 + 14400272^2 &= 810030600353^2 &&:= 656149573508241603724609 \\
81000306000225^2 + 144000272^2 &= 81000306000353^2 &&:= 6561049572150822216036124609
\end{aligned} \tag{35}
$$

The first three triples $(225, 272, 353)$, $(11385, 1712, 11513)$ and $(840825, 14672, 840953)$ for $n = 8; m = 9, 99,$ and, $909$ are not written above as they don't give good pattern.

- **For $n = 2;\ m = 33, 333, 3333, 33333, ...$ in (7):**

$$
\begin{aligned}
1221^2 + 140^2 &= 1229^2 \\
112221^2 + 1340^2 &= 112229^2 \\
11122221^2 + 13340^2 &= 11122229^2 \\
1111222221^2 + 133340^2 &= 1111222229^2 \\
111112222221^2 + 1333340^2 &= 111112222229^2 \\
11111122222221^2 + 13333340^2 &= 11111122222229^2
\end{aligned} \tag{36}
$$

The first triple $(21, 20, 29)$ for $n = 2; m = 3$ is not written above as it doesn't give good pattern.

- **For $n = 5$; $m = 33, 333, 3333, 33333, ...$ in (7):**

$$\begin{aligned}
1419^2 + 380^2 &= 1469^2\\
114219^2 + 3380^2 &= 114269^2\\
11142219^2 + 33380^2 &= 11142269^2\\
1111422219^2 + 333380^2 &= 1111422269^2\\
111114222219^2 + 3333380^2 &= 111114222269^2\\
11111142222219^2 + 33333380^2 &= 11111142222269^2
\end{aligned} \tag{37}$$

The first triple $(39, 80, 89)$ for $n = 5; m = 3$ is not written above as it doesn't give good pattern.

- **For $n = 7$; $m = 333, 3333, 33333, ...$ in (7):**

$$\begin{aligned}
115551^2 + 4760^2 &= 115649^2\\
11155551^2 + 46760^2 &= 11155649^2\\
1111555551^2 + 466760^2 &= 1111555649^2\\
111115555551^2 + 4666760^2 &= 111115555649^2\\
11111155555551^2 + 46666760^2 &= 11111155555649^2
\end{aligned} \tag{38}$$

The first two triples $(51, 140, 149)$ and $(1551, 560, 1649)$ for $n = 5; m = 3$ and $m = 33$ are not written above as they don't give good pattern.

- **For $n = 8$; $m = 333, 3333, 33333, ...$ in (7):**

$$\begin{aligned}
116217^2 + 5456^2 &= 116345^2\\
11162217^2 + 53456^2 &= 11162345^2\\
1111622217^2 + 533456^2 &= 1111622345^2\\
111116222217^2 + 5333456^2 &= 111116222345^2\\
11111162222217^2 + 53333456^2 &= 11111162222345^2
\end{aligned} \tag{39}$$

The first two triples $(57, 176, 185)$ and $(1617, 656, 1745)$ for $n = 8; m = 3$ and $m = 33$ are not written above as they don't give good pattern.

- **For $n = 4$; $m = 99, 999, 9999, 99999, ...$ in (7):**

$$\begin{aligned}
10593^2 + 824^2 &= 10625^2 &&\\
1005993^2 + 8024^2 &= 1006025^2 &&:= 1012086300625\\
100059993^2 + 80024^2 &= 100060025^2 &&:= 10012008603000625\\
10000599993^2 + 800024^2 &= 10000600025^2 &&:= 100012000860030000625\\
1000005999993^2 + 8000024^2 &= 1000006000025^2 &&:= 1000012000086000300000625\\
100000059999993^2 + 80000024^2 &= 100000060000025^2 &&:= 10000012000008600003000000625
\end{aligned} \tag{40}$$

The first triple $(153, 104, 185)$ for $n = 4; m = 9$ is not written above as it doesn't give good pattern.

- **For $n = 5;\ m = 9, 99, 999, 9999, 99999, ...$ in (7):**

$$\begin{aligned}
10791^2 + 1040^2 &= 10841^2 \\
1007991^2 + 10040^2 &= 1008041^2 &&:= 1016146657681 \\
100079991^2 + 100040^2 &= 100080041^2 &&:= 10016014606561681 \\
10000799991^2 + 1000040^2 &= 10000800041^2 &&:= 100016001460065601681 \\
1000007999991^2 + 10000040^2 &= 1000008000041^2 &&:= 1000016000146000656001681 \\
100000079999991^2 + 100000040^2 &= 100000080000041^2 &&:= 10000016000014600006560001681
\end{aligned} \tag{41}$$

The first triple $(171, 140, 221)$ for $n = 5; m = 9$ is not written above as it doesn't give good pattern.

- **For $n = 9;\ m = 99, 999, 9999, 99999, ...$ in (7):**

$$\begin{aligned}
1015983^2 + 18144^2 &= 1016145^2 &&:= 1032550661025 \\
100159983^2 + 180144^2 &= 100160145^2 &&:= 10032054646421025 \\
10001599983^2 + 1800144^2 &= 10001600145^2 &&:= 100032005460464021025 \\
1000015999983^2 + 18000144^2 &= 1000016000145^2 &&:= 1000032000546004640021025 \\
100000159999983^2 + 180000144^2 &= 100000160000145^2 &&:= 10000032000054600046400021025
\end{aligned} \tag{42}$$

The first two triples $(243, 324, 405)$ and $(11583, 1944, 11745)$ for $n = 9; m = 9$ and $m = 99$ are not written above as they don't give good pattern.

- **For $m = 2;\ n = 3, 33, 333, 3333, 33333, ...$ in (7):**

$$\begin{aligned}
16^2 + 30^2 &= 34^2 \\
136^2 + 2310^2 &= 2314^2 \\
1336^2 + 223110^2 &= 223114^2 \\
13336^2 + 22231110^2 &= 22231114^2 \\
133336^2 + 2222311110^2 &= 2222311114^2 \\
1333336^2 + 222223111110^2 &= 222223111114^2 \\
13333336^2 + 22222231111110^2 &= 22222231111114^2
\end{aligned} \tag{43}$$

► **Division by 2**

$$\begin{aligned}
8^2 + 15^2 &= 17^2 \\
68^2 + 1155^2 &= 1157^2 \\
668^2 + 111555,^2 &= 111557^2 \\
6668^2 + 11115555^2 &= 11115557^2 \\
66668^2 + 1111155555^2 &= 1111155557^2 \\
666668^2 + 111111555555^2 &= 111111555557^2 \\
6666668^2 + 11111115555555^2 &= 11111115555557^2
\end{aligned} \tag{44}$$

- **For $m = 3;\ n = 33, 333, 3333, 33333, ...$ in (7):**

$$\begin{aligned}
207^2 + 2376^2 &= 2385^2\\
2007^2 + 223776^2 &= 223785^2\\
20007^2 + 22237776^2 &= 22237785^2\\
200007^2 + 2222377776^2 &= 2222377785^2\\
2000007^2 + 222223777776^2 &= 222223777785^2\\
20000007^2 + 22222237777776^2 &= 22222237777785^2
\end{aligned} \tag{45}$$

The first triple $(27, 36, 45)$ for $m = 3; n = 3$ is not written above as it doesn't give good pattern.

- **For $m = 4;\ n = 33, 333, 3333, 33333, ...$ in (7):**

$$\begin{aligned}
280^2 + 2442^2 &= 2458^2\\
2680^2 + 224442^2 &= 224458^2\\
26680^2 + 22244442^2 &= 22244458^2\\
266680^2 + 2222444442^2 &= 2222444458^2\\
2666680^2 + 222224444442^2 &= 222224444458^2\\
26666680^2 + 22222244444442^2 &= 22222244444458^2
\end{aligned} \tag{46}$$

The first triple $(40, 42, 58)$ for $m = 4; n = 3$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
140^2 + 1221^2 &= 1229^2\\
1340^2 + 112221^2 &= 112229^2\\
13340^2 + 11122221^2 &= 11122229^2\\
133340^2 + 1111222221^2 &= 1111222229^2\\
1333340^2 + 111112222221^2 &= 111112222229^2\\
13333340^2 + 11111122222221^2 &= 11111122222229^2
\end{aligned} \tag{47}$$

- **For $m = 5;\ n = 33, 333, 3333, 33333, ...$ in (7):**

$$\begin{aligned}
355^2 + 2508^2 &= 2533^2\\
3355^2 + 225108^2 &= 225133^2\\
33355^2 + 22251108^2 &= 22251133^2\\
333355^2 + 2222511108^2 &= 2222511133^2\\
3333355^2 + 222225111108^2 &= 222225111133^2\\
33333355^2 + 22222251111108^2 &= 22222251111133^2
\end{aligned} \tag{48}$$

The first triple $(55, 48, 73)$ for $m = 5; n = 3$ is not written above as it doesn't give good pattern.

- **For $m = 2$; $n = 66, 666, 6666, 66666, \ldots$ in (7):**

$$\begin{aligned}
268^2 + 8976^2 &= 8980^2 \\
2668^2 + 889776^2 &= 889780^2 \\
26668^2 + 88897776^2 &= 88897780^2 \\
266668^2 + 8888977776^2 &= 8888977780^2 \\
2666668^2 + 888889777776^2 &= 888889777780^2 \\
26666668^2 + 88888897777776^2 &= 88888897777780^2
\end{aligned} \tag{49}$$

The first triple $(28, 96, 100)$ for $m = 2; n = 6$ is not written above as it doesn't give good pattern.

► **Division by 2**

$$\begin{aligned}
134^2 + 4488^2 &= 4490^2 \\
1334^2 + 444888^2 &= 444890^2 \\
13334^2 + 44448888^2 &= 44448890^2 \\
133334^2 + 4444488888^2 &= 4444488890^2 \\
1333334^2 + 444444888888^2 &= 444444888890^2 \\
13333334^2 + 44444448888888^2 &= 44444448888890^2
\end{aligned} \tag{50}$$

- **For $m = 5$; $n = 66, 666, 6666, 66666, \ldots$ in (7):**

$$\begin{aligned}
685^2 + 9372^2 &= 9397^2 \\
6685^2 + 893772^2 &= 893797^2 \\
66685^2 + 88937772^2 &= 88937797^2 \\
666685^2 + 8889377772^2 &= 8889377797^2 \\
6666685^2 + 888893777772^2 &= 888893777797^2 \\
66666685^2 + 88888937777772^2 &= 88888937777797^2
\end{aligned} \tag{51}$$

The first triple $(85, 132, 157)$ for $m = 5; n = 6$ is not written above as it doesn't give good pattern.

- **For $m = 6$; $n = 66, 666, 6666, 66666, \ldots$ in (7):**

$$\begin{aligned}
828^2 + 9504^2 &= 9540^2 \\
8028^2 + 895104^2 &= 895140^2 \\
80028^2 + 88951104^2 &= 88951140^2 \\
800028^2 + 8889511104^2 &= 8889511140^2 \\
8000028^2 + 888895111104^2 &= 888895111140^2 \\
80000028^2 + 88888951111104^2 &= 88888951111140^2
\end{aligned} \tag{52}$$

The first triple $(108, 144, 180)$ for $m = 6; n = 6$ is not written above as it doesn't give good pattern.

- **Division by 2**

$$\begin{aligned}
414^2 + 4752^2 &= 4770^2 \\
4014^2 + 447552^2 &= 447570^2 \\
40014^2 + 44475552^2 &= 44475570^2 \\
400014^2 + 4444755552^2 &= 4444755570^2 \\
4000014^2 + 444447555552^2 &= 444447555570^2 \\
40000014^2 + 44444475555552^2 &= 44444475555570^2
\end{aligned} \tag{53}$$

- **Division by 4**

$$\begin{aligned}
207^2 + 2376^2 &= 2385^2 \\
2007^2 + 223776^2 &= 223785^2 \\
20007^2 + 22237776^2 &= 22237785^2 \\
200007^2 + 2222377776^2 &= 2222377785^2 \\
2000007^2 + 222223777776^2 &= 222223777785^2 \\
20000007^2 + 22222237777776^2 &= 22222237777785^2
\end{aligned} \tag{54}$$

- **For $m = 5$; $n = 99, 999, 9999, 99999, ...$ in (7):**

$$\begin{aligned}
1015^2 + 20592^2 &= 20617^2 \\
10015^2 + 2005992^2 &= 2006017^2 &&:= 4024104204289 \\
100015^2 + 200059992^2 &= 200060017^2 &&:= 40024010402040289 \\
1000015^2 + 20000599992^2 &= 20000600017^2 &&:= 400024001040020400289 \\
10000015^2 + 2000005999992^2 &= 2000006000017^2 &&:= 4000024000104000204000289 \\
100000015^2 + 200000059999992^2 &= 200000060000017^2 &&:= 40000024000010400002040000289
\end{aligned} \tag{55}$$

The first triple $(115, 252, 277)$ for $m = 5; n = 9$ is not written above as it doesn't give good pattern.

- **For $m = 6$; $n = 99, 999, 9999, 99999, ...$ in (7):**

$$\begin{aligned}
1224^2 + 20790^2 &= 20826^2 \\
12024^2 + 2007990^2 &= 2008026^2 &&:= 4032168416676 \\
120024^2 + 200079990^2 &= 200080026^2 &&:= 40032016804160676 \\
1200024^2 + 20000799990^2 &= 20000800026^2 &&:= 400032001680041600676 \\
12000024^2 + 2000007999990^2 &= 2000008000026^2 &&:= 4000032000168000416000676 \\
120000024^2 + 200000079999990^2 &= 200000080000026^2 &&:= 40000032000016800004160000676
\end{aligned} \tag{56}$$

The first triple $(144, 270, 306)$ for $m = 6; n = 9$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
612^2+10395^2 &= 10413^2 \\
6012^2+1003995^2 &= 1004013^2 := 1008042104169 \\
60012^2+100039995^2 &= 100040013^2 := 10008004201040169 \\
600012^2+10000399995^2 &= 10000400013^2 := 100008000420010400169 \\
6000012^2+1000003999995^2 &= 1000004000013^2 := 1000008000042000104000169 \\
60000012^2+100000039999995^2 &= 100000040000013^2 := 10000008000004200001040000169
\end{aligned} \tag{57}$$

- **For $m=9;\ n=99,999,9999,99999,...$ in (7):**

$$\begin{aligned}
1863^2+21384^2 &= 21465^2 \\
18063^2+2013984^2 &= 2014065^2 \\
180063^2+200139984^2 &= 200140065^2 := 40056045618204225 \\
1800063^2+20001399984^2 &= 20001400065^2 := 400056004560182004225 \\
18000063^2+2000013999984^2 &= 2000014000065^2 := 4000056000456001820004225 \\
180000063^2+200000139999984^2 &= 200000140000065^2 := 40000056000045600018200004225
\end{aligned} \tag{58}$$

The first triple $(243, 324, 405)$ for $m=9; n=9$ is not written above as it doesn't give good pattern.

## 3.2 Procedure 2

This subsection brings patterns based on Procedure (10) given in subsection 2.2. See below some examples of patterns:

- **For $n=3;\ m=10,00,1000,10000,...$ in (10):**

$$\begin{aligned}
91^2+60^2 &= 109^2 := 11881 \\
9991^2+600^2 &= 10009^2 := 100180081 \\
999991^2+6000^2 &= 1000009^2 := 1000018000081 \\
99999991^2+60000^2 &= 100000009^2 := 10000001800000081
\end{aligned} \tag{59}$$

- **For $n=3;\ m=20,200,2000,20000,...$ in (10):**

$$\begin{aligned}
391^2+120^2 &= 409^2 := 167281 \\
39991^2+1200^2 &= 40009^2 := 1600720081 \\
3999991^2+12000^2 &= 4000009^2 := 16000072000081 \\
399999991^2+120000^2 &= 400000009^2 := 160000007200000081
\end{aligned} \tag{60}$$

- **For $n = 3;\ m = 50, 500, 5000, 50000, ...$ in (10):**

$$\begin{aligned}
2491^2 + 300^2 &= 2509^2 \\
249991^2 + 3000^2 &= 250009^2 := 62504500081 \\
24999991^2 + 30000^2 &= 25000009^2 := 625000450000081 \\
2499999991^2 + 300000^2 &= 2500000009^2 := 6250000045000000081
\end{aligned} \tag{61}$$

- **For $n = 4;\ m = 50, 500, 5000, 50000, ...$ in (10):**

$$\begin{aligned}
2484^2 + 400^2 &= 2516^2 \\
249984^2 + 4000^2 &= 250016^2 := 62508000256 \\
24999984^2 + 40000^2 &= 25000016^2 := 625000800000256 \\
2499999984^2 + 400000^2 &= 2500000016^2 := 6250000080000000256 \\
249999999984^2 + 4000000^2 &= 250000000016^2 := 62500000000800000000256 \\
24999999999984^2 + 40000000^2 &= 25000000000016^2 := 625000000000080000000000256
\end{aligned} \tag{62}$$

▶ **Division by 2**

$$\begin{aligned}
1242^2 + 200^2 &= 1258^2 \\
124992^2 + 2000^2 &= 125008^2 \\
12499992^2 + 20000^2 &= 12500008^2 := 156250200000064 \\
1249999992^2 + 200000^2 &= 1250000008^2 := 1562500020000000064 \\
124999999992^2 + 2000000^2 &= 125000000008^2 := 15625000002000000000064 \\
12499999999992^2 + 20000000^2 &= 12500000000008^2 := 156250000000200000000000064
\end{aligned} \tag{63}$$

▶ **Division by 4**

$$\begin{aligned}
62496^2 + 1000^2 &= 62504^2 \\
6249996^2 + 10000^2 &= 6250004^2 := 39062550000016 \\
624999996^2 + 100000^2 &= 625000004^2 := 390625005000000016 \\
62499999996^2 + 1000000^2 &= 62500000004^2 := 3906250000500000000016 \\
6249999999996^2 + 10000000^2 &= 6250000000004^2 := 39062500000050000000000016
\end{aligned} \tag{64}$$

Division of first term is excluded as it doesn't give good pattern.

▶ **Division by 8**

$$
\begin{aligned}
31248^2 + 500^2 &= 31252^2 \\
3124998^2 + 5000^2 &= 3125002^2 \\
312499998^2 + 50000^2 &= 312500002^2 &&:= 97656251250000004 \\
31249999998^2 + 500000^2 &= 31250000002^2 &&:= 976562500125000000004 \\
3124999999998^2 + 5000000^2 &= 3125000000002^2 &&:= 9765625000012500000000004
\end{aligned} \tag{65}
$$

▶ **Division by 16**

$$
\begin{aligned}
1562499^2 + 2500^2 &= 1562501^2 \\
156249999^2 + 25000^2 &= 156250001^2 \\
15624999999^2 + 250000^2 &= 15625000001^2 &&:= 244140625031250000001 \\
1562499999999^2 + 2500000^2 &= 1562500000001^2 &&:= 2441406250003125000000001
\end{aligned} \tag{66}
$$

We observe that that the first triple (15624, 250, 15626) doesn't give good pattern.

- **For $n = 3;\ m = 90, 900, 9000, 90000, ...$ in (10):**

$$
\begin{aligned}
8091^2 + 540^2 &= 8109^2 \\
809991^2 + 5400^2 &= 810009^2 &&:= 656114580081 \\
80999991^2 + 54000^2 &= 81000009^2 &&:= 6561001458000081 \\
8099999991^2 + 540000^2 &= 8100000009^2 &&:= 65610000145800000081
\end{aligned} \tag{67}
$$

- **For $n = 4;\ m = 5, 55, 555, 5555, ...$ in (10):**

$$
\begin{aligned}
9^2 + 40^2 &= 41^2 \\
3009^2 + 440^2 &= 3041^2 \\
308009^2 + 4440^2 &= 308041^2 \\
30858009^2 + 44440^2 &= 30858041^2
\end{aligned} \tag{68}
$$

### 3.2.1 Special Examples

This subsection brings special examples of patterns by fixing $n = 1, 2, ..., 9$ and varying m in terms 3, 6 and 9.

## (i) For $n = 1, 2, ..., 9;\ m = 3, 33, 333, 3333, 33333, ...$ :

- **For $n = 1;\ m = 3, 33, 333, 3333, 33333, ...$ in (10):**

$$\begin{aligned}
8^2+6^2 &= 10^2\\
1088^2+66^2 &= 1090^2\\
110888^2+666^2 &= 110890^2\\
11108888^2+6666^2 &= 11108890^2\\
1111088888^2+66666^2 &= 1111088890^2\\
111110888888^2+666666^2 &= 111110888890^2
\end{aligned} \tag{69}$$

In order to get primitive values, let's divide the above pattern by 2, we get a new pattern:

▶ **Division by 2**

$$\begin{aligned}
4^2+3^2 &= 5^2\\
544^2+33^2 &= 545^2\\
55444^2+333^2 &= 55445^2\\
5554444^2+3333^2 &= 5554445^2\\
555544444^2+33333^2 &= 555544445^2\\
55555444444^2+333333^2 &= 55555444445^2
\end{aligned} \tag{70}$$

- **For $n=2;\ m=3,33,333,3333,33333,...$ in (10):**

$$\begin{aligned}
5^2+12^2 &= 13^2\\
1085^2+132^2 &= 1093^2\\
110885^2+1332^2 &= 110893^2\\
11108885^2+13332^2 &= 11108893^2\\
1111088885^2+133332^2 &= 1111088893^2\\
111110888885^2+1333332^2 &= 111110888893^2
\end{aligned} \tag{71}$$

- **For $n=3;\ m=33,333,3333,33333,...$ in (10):**

$$\begin{aligned}
1080^2+198^2 &= 1098^2\\
110880^2+1998^2 &= 110898^2\\
11108880^2+19998^2 &= 11108898^2\\
1111088880^2+199998^2 &= 1111088898^2\\
111110888880^2+1999998^2 &= 111110888898^2
\end{aligned} \tag{72}$$

There is no value for $n=3; m=3$. In order to get primitive values, let's divide the above pattern by 2, we get a new pattern.

▶ **Division by 2**

$$\begin{aligned}
540^2 + 99^2 &= 549^2 \\
55440^2 + 999^2 &= 55449^2 \\
5554440^2 + 9999^2 &= 5554449^2 \\
555544440^2 + 99999^2 &= 555544449^2 \\
55555444440^2 + 999999^2 &= 55555444449^2
\end{aligned} \tag{73}$$

- **For $n = 4;\ m = 33, 333, 3333, 33333, ...$ in (10):**

$$\begin{aligned}
1073^2 + 264^2 &= 1105^2 \\
110873^2 + 2664^2 &= 110905^2 \\
11108873^2 + 26664^2 &= 11108905^2 \\
1111088873^2 + 266664^2 &= 1111088905^2 \\
111110888873^2 + 2666664^2 &= 111110888905^2
\end{aligned} \tag{74}$$

The first triple $(7, 24, 25)$ for $n = 4;\ m = 3$ is not written above as it doesn't give good pattern.

- **For $n = 5;\ m = 33, 333, 3333, 33333, ...$ in (10):**

$$\begin{aligned}
1064^2 + 330^2 &= 1114^2 \\
110864^2 + 3330^2 &= 110914^2 \\
11108864^2 + 33330^2 &= 11108914^2 \\
1111088864^2 + 333330^2 &= 1111088914^2 \\
111110888864^2 + 3333330^2 &= 111110888914^2
\end{aligned} \tag{75}$$

The first triple $(16, 30, 34)$ for $n = 5;\ m = 3$ is not written above as it doesn't give good pattern. In order to get primitive values, let's divide the above pattern by 2, we get a new pattern.

▶ **Division by 2**

$$\begin{aligned}
532^2 + 165^2 &= 557^2 \\
55432^2 + 1665^2 &= 55457^2 \\
5554432^2 + 16665^2 &= 5554457^2 \\
555544432^2 + 166665^2 &= 555544457^2 \\
55555444432^2 + 1666665^2 &= 55555444457^2
\end{aligned} \tag{76}$$

- **For $n = 6;\ m = 33, 333, 3333, 33333, ...$ in (10):**

$$\begin{aligned}
1053^2 + 396^2 &= 1125^2\\
110853^2 + 3996^2 &= 110925^2\\
11108853^2 + 39996^2 &= 11108925^2\\
1111088853^2 + 399996^2 &= 1111088925^2\\
111110888853^2 + 3999996^2 &= 111110888925^2
\end{aligned} \tag{77}$$

The first triple $(27, 36, 45)$ for $n = 6;\ m = 3$ is not written above as it doesn't give good pattern.

- **For $n = 7;\ m = 33, 333, 3333, 33333, ...$ in (10):**

$$\begin{aligned}
1040^2 + 462^2 &= 1138^2\\
110840^2 + 4662^2 &= 110938^2\\
11108840^2 + 46662^2 &= 11108938^2\\
1111088840^2 + 466662^2 &= 1111088938^2\\
111110888840^2 + 4666662^2 &= 111110888938^2
\end{aligned} \tag{78}$$

The first triple $(40, 42, 58)$ for $n = 7;\ m = 3$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
520^2 + 231^2 &= 569^2\\
55420^2 + 2331^2 &= 55469^2\\
5554420^2 + 23331^2 &= 5554469^2\\
555544420^2 + 233331^2 &= 555544469^2\\
55555444420^2 + 2333331^2 &= 55555444469^2
\end{aligned} \tag{79}$$

- **For $n = 8;\ m = 33, 333, 3333, 33333, ...$ in (10):**

$$\begin{aligned}
1025^2 + 528^2 &= 1153^2\\
110825^2 + 5328^2 &= 110953^2\\
11108825^2 + 53328^2 &= 11108953^2\\
1111088825^2 + 533328^2 &= 1111088953^2\\
111110888825^2 + 5333328^2 &= 111110888953^2
\end{aligned} \tag{80}$$

The first triple $(55, 48, 73)$ for $n = 8;\ m = 3$ is not written above as it doesn't give good pattern.

- **For $n = 9$; $m = 33, 333, 3333, 33333, ...$ in (10):**

$$\begin{aligned}
1008^2 + 594^2 &= 1170^2\\
110808^2 + 5994^2 &= 110970^2\\
11108808^2 + 59994^2 &= 11108970^2\\
1111088808^2 + 599994^2 &= 1111088970^2\\
111110888808^2 + 5999994^2 &= 111110888970^2
\end{aligned} \tag{81}$$

The first triple $(72, 54, 90)$ for $n = 9$; $m = 3$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
504^2 + 297^2 &= 585^2\\
55404^2 + 2997^2 &= 55485^2\\
5554404^2 + 29997^2 &= 5554485^2\\
555544404^2 + 299997^2 &= 555544485^2\\
55555444404^2 + 2999997^2 &= 55555444485^2
\end{aligned} \tag{82}$$

### (ii) For $n = 1, 2, ..., 9$; $m = 6, 66, 666, 6666, 66666, ...$ :

- **For $n = 1$; $m = 6, 66, 666, 6666, 66666, ...$ in (10):**

$$\begin{aligned}
35^2 + 12^2 &= 37^2\\
4355^2 + 132^2 &= 4357^2\\
443555^2 + 1332^2 &= 443557^2\\
44435555^2 + 13332^2 &= 44435557^2\\
4444355555^2 + 133332^2 &= 4444355557^2\\
444443555555^2 + 1333332^2 &= 444443555557^2
\end{aligned} \tag{83}$$

- **For $n = 2$; $m = 6, 66, 666, 6666, 66666, ...$ in (10):**

$$\begin{aligned}
32^2 + 24^2 &= 40^2\\
4352^2 + 264^2 &= 4360^2\\
443552^2 + 2664^2 &= 443560^2\\
44435552^2 + 26664^2 &= 44435560^2\\
4444355552^2 + 266664^2 &= 4444355560^2\\
444443555552^2 + 2666664^2 &= 444443555560^2
\end{aligned} \tag{84}$$

▶ **Division by 2**

$$\begin{aligned}
16^2+12^2 &= 20^2\\
2176^2+132^2 &= 2180^2\\
221776^2+1332^2 &= 221780^2\\
22217776^2+13332^2 &= 22217780^2\\
2222177776^2+133332^2 &= 2222177780^2\\
222221777776^2+1333332^2 &= 222221777780^2
\end{aligned} \tag{85}$$

▶ **Division by 4**

$$\begin{aligned}
8^2+6^2 &= 10^2\\
1088^2+66^2 &= 1090^2\\
110888^2+666^2 &= 110890^2\\
11108888^2+6666^2 &= 11108890^2\\
1111088888^2+66666^2 &= 1111088890^2\\
111110888888^2+666666^2 &= 111110888890^2
\end{aligned} \tag{86}$$

▶ **Division by 8**

$$\begin{aligned}
4^2+3^2 &= 5^2\\
544^2+33^2 &= 545^2\\
55444^2+333^2 &= 55445^2\\
5554444^2+3333^2 &= 5554445^2\\
555544444^2+33333^2 &= 555544445^2\\
55555444444^2+333333^2 &= 55555444445^2
\end{aligned} \tag{87}$$

- **For $n=3;\ m=66,666,6666,66666,...$ in (10):**

$$\begin{aligned}
4347^2+396^2 &= 4365^2\\
443547^2+3996^2 &= 443565^2\\
44435547^2+39996^2 &= 44435565^2\\
4444355547^2+399996^2 &= 4444355565^2\\
444443555547^2+3999996^2 &= 444443555565^2
\end{aligned} \tag{88}$$

The first triple $(27,36,45)$ for $n=3;\ m=6$ is not written above as it doesn't give good pattern.

- **For $n = 4;\ m = 66, 666, 6666, 66666, ...$ in (10):**

$$\begin{aligned}
4340^2 + 528^2 &= 4372^2 \\
443540^2 + 5328^2 &= 443572^2 \\
44435540^2 + 53328^2 &= 44435572^2 \\
4444355540^2 + 533328^2 &= 4444355572^2 \\
444443555540^2 + 5333328^2 &= 444443555572^2
\end{aligned} \tag{89}$$

The first triple $(20, 48, 52)$ for $n = 4;\ m = 6$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
10^2 + 24^2 &= 26^2 \\
2170^2 + 264^2 &= 2186^2 \\
221770^2 + 2664^2 &= 221786^2 \\
22217770^2 + 26664^2 &= 22217786^2 \\
2222177770^2 + 266664^2 &= 2222177786^2 \\
222221777770^2 + 2666664^2 &= 222221777786^2
\end{aligned} \tag{90}$$

▶ **Division by 4**

$$\begin{aligned}
5^2 + 12^2 &= 13^2 \\
1085^2 + 132^2 &= 1093^2 \\
110885^2 + 1332^2 &= 110893^2 \\
11108885^2 + 13332^2 &= 11108893^2 \\
1111088885^2 + 133332^2 &= 1111088893^2 \\
111110888885^2 + 1333332^2 &= 111110888893^2
\end{aligned} \tag{91}$$

- **For $n = 5;\ m = 6, 66, 666, 6666, 66666, ...$ in (10):**

$$\begin{aligned}
4331^2 + 660^2 &= 4381^2 \\
443531^2 + 6660^2 &= 443581^2 \\
44435531^2 + 66660^2 &= 44435581^2 \\
4444355531^2 + 666660^2 &= 4444355581^2 \\
444443555531^2 + 6666660^2 &= 444443555581^2
\end{aligned} \tag{92}$$

The first triple $(11, 60, 61)$ for $n = 5;\ m = 6$ is not written above as it doesn't give good pattern.

- **For $n = 6;\ m = 6, 66, 666, 6666, 66666, ...$ in (10):**

$$\begin{aligned} 4320^2 + 792^2 &= 4392^2 \\ 443520^2 + 7992^2 &= 443592^2 \\ 44435520^2 + 79992^2 &= 44435592^2 \\ 4444355520^2 + 799992^2 &= 4444355592^2 \\ 444443555520^2 + 7999992^2 &= 444443555592^2 \end{aligned} \tag{93}$$

In this case, there is no value for $n = 6;\ m = 6$.

▶ **Division by 2**

$$\begin{aligned} 2160^2 + 396^2 &= 2196^2 \\ 221760^2 + 3996^2 &= 221796^2 \\ 22217760^2 + 39996^2 &= 22217796^2 \\ 2222177760^2 + 399996^2 &= 2222177796^2 \\ 222221777760^2 + 3999996^2 &= 222221777796^2 \end{aligned} \tag{94}$$

▶ **Division by 4**

$$\begin{aligned} 1080^2 + 198^2 &= 1098^2 \\ 110880^2 + 1998^2 &= 110898^2 \\ 11108880^2 + 19998^2 &= 11108898^2 \\ 1111088880^2 + 199998^2 &= 1111088898^2 \\ 111110888880^2 + 1999998^2 &= 111110888898^2 \end{aligned} \tag{95}$$

▶ **Division by 8**

$$\begin{aligned} 540^2 + 99^2 &= 549^2 \\ 55440^2 + 999^2 &= 55449^2 \\ 5554440^2 + 9999^2 &= 5554449^2 \\ 555544440^2 + 99999^2 &= 555544449^2 \\ 55555444440^2 + 999999^2 &= 55555444449^2 \end{aligned} \tag{96}$$

- **For $n = 7;\ m = 6, 66, 666, 6666, 66666, ...$ in (10):**

$$\begin{aligned} 4307^2 + 924^2 &= 4405^2 \\ 443507^2 + 9324^2 &= 443605^2 \\ 44435507^2 + 93324^2 &= 44435605^2 \\ 4444355507^2 + 933324^2 &= 4444355605^2 \\ 444443555507^2 + 9333324^2 &= 444443555605^2 \end{aligned} \tag{97}$$

The first triple $(13, 84, 85)$ for $n = 7;\ m = 6$ is not written above as it doesn't give good pattern.

- **For $n = 8;\ m = 66, 666, 6666, 66666, ...$ in (10):**

$$\begin{aligned}
4292^2 + 1056^2 &= 4420^2\\
443492^2 + 10656^2 &= 443620^2\\
44435492^2 + 106656^2 &= 44435620^2\\
4444355492^2 + 1066656^2 &= 4444355620^2\\
444443555492^2 + 10666656^2 &= 444443555620^2
\end{aligned} \tag{98}$$

The first triple $(28, 96, 100)$ for $n = 8;\ m = 6$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
2146^2 + 528^2 &= 2210^2\\
221746^2 + 5328^2 &= 221810^2\\
22217746^2 + 53328^2 &= 22217810^2\\
2222177746^2 + 533328^2 &= 2222177810^2\\
222221777746^2 + 5333328^2 &= 222221777810^2
\end{aligned} \tag{99}$$

▶ **Division by 4**

$$\begin{aligned}
1073^2 + 264^2 &= 1105^2\\
110873^2 + 2664^2 &= 110905^2\\
11108873^2 + 26664^2 &= 11108905^2\\
1111088873^2 + 266664^2 &= 1111088905^2\\
111110888873^2 + 2666664^2 &= 111110888905^2
\end{aligned} \tag{100}$$

- **For $n = 9;\ m = 66, 666, 6666, 66666, ...$ in (10):**

$$\begin{aligned}
4275^2 + 1188^2 &= 4437^2\\
443475^2 + 11988^2 &= 443637^2\\
44435475^2 + 119988^2 &= 44435637^2\\
4444355475^2 + 1199988^2 &= 4444355637^2\\
444443555475^2 + 11999988^2 &= 444443555637^2
\end{aligned} \tag{101}$$

The first triple $(45, 108, 117)$ for $n = 9;\ m = 6$ is not written above as it doesn't give good pattern.

### (iii) For $n = 1, 2, ..., 9;\ m = 9, 99, 999, 9999, 99999, ...$ :

- **For $n = 1;\ m = 9, 99, 999, 9999, 99999, ....$ in (10):**

$$\begin{aligned}
80^2 + 18^2 &= 82^2 \\
9800^2 + 198^2 &= 9802^2 && := 96079204 \\
998000^2 + 1998^2 &= 998002^2 && := 996007992004 \\
99980000^2 + 19998^2 &= 99980002^2 && := 9996000799920004 \\
9999800000^2 + 199998^2 &= 9999800002^2 && := 99996000079999200004 \\
999998000000^2 + 1999998^2 &= 999998000002^2 && := 999996000007999992000004
\end{aligned} \tag{102}$$

▶ **Division by 2**

$$\begin{aligned}
40^2 + 9^2 &= 41^2 \\
4900^2 + 99^2 &= 4901^2 && := 24019801 \\
499000^2 + 999^2 &= 499001^2 && := 249001998001 \\
49990000^2 + 9999^2 &= 49990001^2 && := 2499000199980001 \\
4999900000^2 + 99999^2 &= 4999900001^2 && := 24999000019999800001 \\
499999000000^2 + 999999^2 &= 499999000001^2 && := 249999000001999998000001
\end{aligned} \tag{103}$$

- **For $n = 2;\ m = 9, 99, 999, 9999, 99999, ...$ in (10):**

$$\begin{aligned}
77^2 + 36^2 &= 85^2 \\
9797^2 + 396^2 &= 9805^2 && := 96138025 \\
997997^2 + 3996^2 &= 998005^2 && := 996013980025 \\
99979997^2 + 39996^2 &= 99980005^2 && := 9996001399800025 \\
9999799997^2 + 399996^2 &= 9999800005^2 && := 99996000139998000025 \\
999997999997^2 + 3999996^2 &= 999998000005^2 && := 999996000013999980000025
\end{aligned} \tag{104}$$

- **For $n = 3;\ m = 9, 99, 999, 9999, 99999, ...$ in (10):**

$$\begin{aligned}
72^2 + 54^2 &= 90^2 \\
9792^2 + 594^2 &= 9810^2 && := 96236100 \\
997992^2 + 5994^2 &= 998010^2 && := 996023960100 \\
99979992^2 + 59994^2 &= 99980010^2 && := 9996002399600100 \\
9999799992^2 + 599994^2 &= 9999800010^2 && := 99996000239996000100 \\
999997999992^2 + 5999994^2 &= 999998000010^2 && := 999996000023999960000100
\end{aligned} \tag{105}$$

► **Division by 2**

$$
\begin{aligned}
4896^2 + 297^2 &= 4905^2 &&:= 24059025 \\
498996^2 + 2997^2 &= 499005^2 &&:= 249005990025 \\
49989996^2 + 29997^2 &= 49990005^2 &&:= 2499000599900025 \\
4999899996^2 + 299997^2 &= 4999900005^2 &&:= 24999000059999000025 \\
499998999996^2 + 2999997^2 &= 499999000005^2 &&:= 249999000005999990000025
\end{aligned} \tag{106}
$$

The first triple $(36, 27, 45)$ is not written above as it doesn't give good pattern.

- **For $n = 4;\ m = 9, 99, 999, 9999, 99999, ...$ in (10):**

$$
\begin{aligned}
9785^2 + 792^2 &= 9817^2 &&:= 96373489 \\
997985^2 + 7992^2 &= 998017^2 &&:= 996037932289 \\
99979985^2 + 79992^2 &= 99980017^2 &&:= 9996003799320289 \\
9999799985^2 + 799992^2 &= 9999800017^2 &&:= 99996000379993200289 \\
999997999985^2 + 7999992^2 &= 999998000017^2 &&:= 999996000037999932000289
\end{aligned} \tag{107}
$$

The first triple $(65, 72, 97)$ for $n = 4;\ m = 9$ is not written above as it doesn't give good pattern.

- **For $n = 5;\ m = 9, 99, 999, 9999, 99999, ...$ in (10):**

$$
\begin{aligned}
9776^2 + 990^2 &= 9826^2 &&:= 96550276 \\
997976^2 + 9990^2 &= 998026^2 &&:= 996055896676 \\
99979976^2 + 99990^2 &= 99980026^2 &&:= 9996005598960676 \\
9999799976^2 + 999990^2 &= 9999800026^2 &&:= 99996000559989600676 \\
999997999976^2 + 9999990^2 &= 999998000026^2 &&:= 999996000055999896000676
\end{aligned} \tag{108}
$$

The first triple $(56, 90, 106)$ for $n = 5;\ m = 9$ is not written above as it doesn't give good pattern.

► **Division by 2**

$$
\begin{aligned}
4888^2 + 495^2 &= 4913^2 && \\
498988^2 + 4995^2 &= 499013^2 &&:= 249013974169 \\
49989988^2 + 49995^2 &= 49990013^2 &&:= 2499001399740169 \\
4999899988^2 + 499995^2 &= 4999900013^2 &&:= 24999000139997400169 \\
499998999988^2 + 4999995^2 &= 499999000013^2 &&:= 249999000013999974000169
\end{aligned} \tag{109}
$$

The first triple $(28^2, 45, 53)$ is not written above as it doesn't give good pattern.

- **For $n=6;\ m=99,999,9999,99999,...$ in (10):**

$$\begin{aligned}
9765^2+1188^2 &= 9837^2\\
997965^2+11988^2 &= 998037^2\\
99979965^2+119988^2 &= 99980037^2 := 9996007798521369\\
9999799965^2+1199988^2 &= 9999800037^2 := 99996000779985201369\\
999997999965^2+11999988^2 &= 999998000037^2 := 999996000077999852001369
\end{aligned} \tag{110}$$

The first triple $(45,108,117)$ for $n=6;\ m=9$ is not written above as it doesn't give good pattern.

- **For $n=7;\ m=99,999,9999,99999,...$ in (10):**

$$\begin{aligned}
9752^2+1386^2 &= 9850^2\\
997952^2+13986^2 &= 998050^2 := 996103802500\\
99979952^2+139986^2 &= 99980050^2 := 9996010398002500\\
9999799952^2+1399986^2 &= 9999800050^2 := 99996001039980002500\\
999997999952^2+13999986^2 &= 999998000050^2 := 999996000103999800002500
\end{aligned} \tag{111}$$

The first triple $(32,126,130)$ for $n=7;\ m=9$ is not written above as it doesn't give good pattern.

- ▶ **Division by 2**

$$\begin{aligned}
4876^2+693^2 &= 4925^2\\
498976^2+6993^2 &= 499025^2 := 249025950625\\
49989976^2+69993^2 &= 49990025^2 := 2499002599500625\\
4999899976^2+699993^2 &= 4999900025^2 := 24999002599995000625\\
499998999976^2+6999993^2 &= 499999000025^2 := 249999000025999950000625
\end{aligned} \tag{112}$$

- **For $n=8;\ m=99,999,9999,99999,...$ in (10):**

$$\begin{aligned}
9737^2+1584^2 &= 9865^2\\
997937^2+15984^2 &= 998065^2 := 996133744225\\
99979937^2+159984^2 &= 99980065^2 := 9996013397404225\\
9999799937^2+1599984^2 &= 9999800065^2 := 99996001339974004225\\
999997999937^2+15999984^2 &= 999998000065^2 := 999996000133999740004225
\end{aligned} \tag{113}$$

The first triple $(17,144,145)$ for $n=8;\ m=9$ is not written above as it doesn't give good pattern.

- **For $n = 9;\ m = 99, 999, 9999, 99999, ...$ in (10):**

$$\begin{aligned}
9720^2 + 1782^2 &= 9882^2 & \\
997920^2 + 17982^2 &= 998082^2 &:= 996167678724 \\
99979920^2 + 179982^2 &= 99980082^2 &:= 9996016796726724 \\
9999799920^2 + 1799982^2 &= 9999800082^2 &:= 99996001679967206724 \\
999997999920^2 + 17999982^2 &= 999998000082^2 &:= 999996000167999672006724
\end{aligned} \tag{114}$$

In this case, there is no value for $n = 9;\ m = 9$.

▶ **Division by 2**

$$\begin{aligned}
4860^2 + 891^2 &= 4941^2 & \\
498960^2 + 8991^2 &= 499041^2 &:= 249041919681 \\
49989960^2 + 89991^2 &= 49990041^2 &:= 2499004199181681 \\
4999899960^2 + 899991^2 &= 4999900041^2 &:= 24999000419991801681 \\
499998999960^2 + 8999991^2 &= 499999000041^2 &:= 249999000041999918001681
\end{aligned} \tag{115}$$

### 3.2.2 Fixed Number Multiple Patterns

In Procedure 2 given in (10), we observe that the middle value $G_2(m,n) = 2mn$. By fixing the value of $G_2(m,n)$, we get different possibilities for $m$ and $n$. This is the idea of this subsection. This means having the same middle term, how we can have more Pythagorean triples.

## (i) For $G_2(m,n) = 2mn = 110 = 2 \times 55 = 2 \times 5 \times 11:$

In this case, the possible combinations are

$$(m,n) := \{(55,1), (11,5)\}.$$

Based on these combinations, we have following Pythagorean triples:

$$\begin{aligned}
(F_2(55,1), G_2(55,1), H_2(55,1))/2 &\Rightarrow 1512^2 + 55^2 = 1513^2 \\
(F_2(11,5), G_2(11,5), H_2(11,5))/2 &\Rightarrow 48^2 + 55^2 = 73^2
\end{aligned}$$

Let's write patterns based on above two triples.

- **For $n = 1;\ m = 55, 555, 5555, 55555, ...$ in (10):**

$$\begin{aligned}
512^2 + 55^2 &= 1513^2 \\
154012^2 + 555^2 &= 154013^2 \\
15429012^2 + 5555^2 &= 15429013^2 \\
1543179012^2 + 55555^2 &= 1543179013^2
\end{aligned} \tag{116}$$

We observe that the above pattern is not so beautiful.

- **For $n = 5$; $m = 11, 111, 1111, 11111, \ldots$ in (10):**

$$\begin{aligned}
48^2 + 55^2 &= 73^2\\
6148^2 + 555^2 &= 6173^2\\
617148^2 + 5555^2 &= 617173^2\\
61727148^2 + 55555^2 &= 61727173^2
\end{aligned} \tag{117}$$

We observe that the above pattern is not so beautiful.

## (ii) For $G_2(m,n) = 2mn = 60 = 2 \times 30 = 2 \times 2 \times 3 \times 5$:

In this case the possible combinations are

$$(m,n) := \{(30,1),(15,2),(10,3),(6,5)\}$$

Based on these combinations, we have following Pythagorean triples:

$$\begin{aligned}
(F_2(30,1), G_2(30,1), H_2(30,1)) &\Rightarrow 899^2 + 60^2 = 901^2\\
(F_2(15,2), G_2(15,2), H_2(15,2)) &\Rightarrow 221^2 + 60^2 = 229^2\\
(F_2(10,3), G_2(10,3), H_2(10,3)) &\Rightarrow 91^2 + 60^2 = 109^2\\
(F_2(6,5), G_2(6,5), H_2(6,5)) &\Rightarrow 11^2 + 60^2 = 61^2
\end{aligned}$$

Let's write patterns based on above four triples.

- **For $n = 1$; $m = 30, 300, 3000, 30000, \ldots$ in (10):**

$$\begin{aligned}
899^2 + 60^2 &= 901^2 && := 811801\\
89999^2 + 600^2 &= 90001^2 && := 8100180001\\
8999999^2 + 6000^2 &= 9000001^2 && := 81000018000001\\
899999999^2 + 60000^2 &= 900000001^2 && := 810000001800000001
\end{aligned} \tag{118}$$

- **For $n = 2$; $m = 15, 155, 1555, 15555, \ldots$ in (10):**

$$\begin{aligned}
221^2 + 60^2 &= 229^2\\
24021^2 + 620^2 &= 24029^2\\
2418021^2 + 6220^2 &= 2418029^2\\
241958021^2 + 62220^2 &= 241958029^2
\end{aligned} \tag{119}$$

We observe that the above pattern is not so beautiful.

- **For $n = 3;\ m = 10, 100, 1000, 10000, ...$ in (10):**

$$
\begin{aligned}
91^2 + 60^2 &= 109^2 &&:= 11881 \\
9991^2 + 600^2 &= 10009^2 &&:= 100180081 \\
999991^2 + 6000^2 &= 1000009^2 &&:= 1000018000081 \\
99999991^2 + 60000^2 &= 100000009^2 &&:= 10000001800000081
\end{aligned} \tag{120}
$$

- **For $n = 5;\ m = 66, 666, 6666, 66666, ...$ in (10):**

$$
\begin{aligned}
4331^2 + 660^2 &= 4381^2 \\
443531^2 + 6660^2 &= 443581^2 \\
44435531^2 + 66660^2 &= 44435581^2 \\
4444355531^2 + 666660^2 &= 4444355581^2
\end{aligned} \tag{121}
$$

The first triple $(11, 60, 61)$ for $n = 5;\ m = 6$ is not written above as it doesn't give good pattern.

### (iii) For $G_2(m, n) = 2mn = 120 = 2 \times 60 = 2 \times 2 \times 2 \times 3 \times 5$ :

In this case the possible combinations are

$$(m, n) := \{(60, 1), (30, 2), (20, 3), (15, 4), (12, 5), (10, 6)\}$$

Based on these combinations, we have following Pythagorean triples:

$$
\begin{aligned}
(F_2(60,1), G_2(60,1), H_2(60,1)) &\Rightarrow 3599^2 + 120^2 = 3601^2 \\
(F_2(30,2), G_2(30,2), H_2(30,2)) &\Rightarrow 896^2 + 120^2 = 904^2 \\
(F_2(20,3), G_2(20,3), H_2(20,3)) &\Rightarrow 391^2 + 120^2 = 409^2 \\
(F_2(15,4), G_2(15,4), H_2(15,4)) &\Rightarrow 209^2 + 120^2 = 241^2 \\
(F_2(12,5), G_2(12,5), H_2(12,5)) &\Rightarrow 119^2 + 120^2 = 169^2 \\
(F_2(10,6), G_2(10,6), H_2(10,6)) &\Rightarrow 64^2 + 120^2 = 136^2
\end{aligned}
$$

Let's write patterns based on above six triples.

- **For $n = 1;\ m = 60, 600, 6000, 60000, ...$ in (10):**

$$
\begin{aligned}
3599^2 + 120^2 &= 3601^2 &&:= 12967201 \\
359999^2 + 1200^2 &= 360001^2 &&:= 129600720001 \\
35999999^2 + 12000^2 &= 36000001^2 &&:= 1296000072000001 \\
3599999999^2 + 120000^2 &= 3600000001^2 &&:= 12960000007200000001
\end{aligned} \tag{122}
$$

- **For $n = 2$; $m = 30, 300, 3000, 30000, ...$ in (10):**

$$\begin{aligned}
896^2 + 120^2 &= 904^2 &&:= 817216\\
89996^2 + 1200^2 &= 90004^2 &&:= 8100720016\\
8999996^2 + 12000^2 &= 9000004^2 &&:= 81000072000016\\
899999996^2 + 120000^2 &= 900000004^2 &&:= 810000007200000016\\
89999999996^2 + 1200000^2 &= 90000000004^2 &&:= 8100000000720000000016
\end{aligned} \tag{123}$$

- **Division by 2**

$$\begin{aligned}
448^2 + 60^2 &= 452^2 &&\\
44998^2 + 600^2 &= 45002^2 &&:= 2025180004\\
4499998^2 + 6000^2 &= 4500002^2 &&:= 20250018000004\\
449999998^2 + 60000^2 &= 450000002^2 &&:= 202500001800000004\\
44999999998^2 + 600000^2 &= 45000000002^2 &&:= 2025000000180000000004
\end{aligned} \tag{124}$$

- **Division by 4**

$$\begin{aligned}
22499^2 + 300^2 &= 22501^2 &&\\
2249999^2 + 3000^2 &= 2250001^2 &&:= 5062504500001\\
224999999^2 + 30000^2 &= 225000001^2 &&:= 50625000450000001\\
22499999999^2 + 300000^2 &= 22500000001^2 &&:= 506250000045000000001
\end{aligned} \tag{125}$$

The first triple $(224, 30, 226)$ is not written above as it doesn't give good pattern.

- **For $n = 3$; $m = 20, 200, 2000, 20000, ...$ in (10):**

$$\begin{aligned}
391^2 + 120^2 &= 409^2 &&:= 167281\\
39991^2 + 1200^2 &= 40009^2 &&:= 1600720081\\
3999991^2 + 12000^2 &= 4000009^2 &&:= 16000072000081\\
399999991^2 + 120000^2 &= 400000009^2 &&:= 160000007200000081
\end{aligned} \tag{126}$$

- **For $n = 4$; $m = 15, 155, 1555, 15555, 155555, ...$ in (10):**

$$\begin{aligned}
209^2 + 120^2 &= 241^2\\
24009^2 + 1240^2 &= 24041^2\\
2418009^2 + 12440^2 &= 2418041^2\\
241958009^2 + 124440^2 &= 241958041^2
\end{aligned} \tag{127}$$

We observe that the above pattern is not so beautiful.

- **For $n = 5;\ m = 122, 1222, 12222, 122222, ...$ in (10):**

$$\begin{aligned} 14859^2 + 1220^2 &= 14909^2 \\ 1493259^2 + 12220^2 &= 1493309^2 \\ 149377259^2 + 122220^2 &= 149377309^2 \\ 14938217259^2 + 1222220^2 &= 14938217309^2 \end{aligned} \tag{128}$$

The first triple $(119, 120, 169)$ for $n = 5;\ m = 12$ is not written above as it doesn't give good pattern.

- **For $n = 6;\ m = 10, 100, 1000, 10000, ...$ in (10):**

$$\begin{aligned} 64^2 + 120^2 &:= 136^2 \\ 9964^2 + 1200^2 &= 10036^2 := 100721296 \\ 999964^2 + 12000^2 &= 1000036^2 := 1000072001296 \\ 99999964^2 + 120000^2 &= 100000036^2 := 10000007200001296 \\ 9999999964^2 + 1200000^2 &= 10000000036^2 := 100000000720000001296 \end{aligned} \tag{129}$$

▶ **Division by 2**

$$\begin{aligned} 4982^2 + 600^2 &= 5018^2 \\ 499982^2 + 6000^2 &= 500018^2 := 250018000324 \\ 49999982^2 + 60000^2 &= 50000018^2 := 2500001800000324 \\ 4999999982^2 + 600000^2 &= 5000000018^2 := 25000000180000000324 \end{aligned} \tag{130}$$

The first triple $(32, 60, 68)$ for $n = 6;\ m = 10$ is not written above as it doesn't give good pattern.

▶ **Division by 4**

$$\begin{aligned} 2491^2 + 300^2 &= 2509^2 \\ 249991^2 + 3000^2 &= 250009^2 := 62504500081 \\ 24999991^2 + 30000^2 &= 25000009^2 := 625000450000081 \\ 2499999991^2 + 300000^2 &= 2500000009^2 := 6250000045000000081 \end{aligned} \tag{131}$$

The first triple $(16, 30, 34)$ for $n = 6;\ m = 10$ is not written above as it doesn't give good pattern.

### (iv) For $G_2(m,n) = 2mn = 330 = 2 \times 165 = 2 \times 3 \times 5 \times 11$ :

In this case the possible combinations are

$$(m,n) := \{(165,1),(55,3),(33,5),(15,11)\}$$

Based on these combinations, we have following Pythagorean triples:

$$\begin{aligned}
(F_2(165,1), G_2(165,1), H_2(165,1))/2 &\Rightarrow 13612^2 + 165^2 = 13613 \\
(F_2(55,3), G_2(55,3), H_2(55,3))/2 &\Rightarrow 1508^2 + 165^2 = 1517 \\
(F_2(33,5), G_2(33,5), H_2(33,5))/2 &\Rightarrow 532^2 + 165^2 = 557 \\
(F_2(15,11), G_2(15,11), H_2(15,11))/2 &\Rightarrow 52^2 + 165^2 = 173
\end{aligned}$$

Out of four triples, there are two triple don't give a good patterns. These are:

$$\begin{aligned}
&(i)\ n = 1;\ m = 165: \quad (13612, 165, 13613) \Rightarrow 13612^2 + 165^2 = 13613^2 \\
&(ii)\ n = 11;\ m = 15: \quad (52, 165, 173) \Rightarrow 52^2 + 165^2 = 173^2
\end{aligned}$$

The pattern for other two triples are as follows:

- **For $n = 3;\ m = 55, 555, 5555, 55555, ...$ in (10):**

$$\begin{aligned}
1508^2 + 165^2 &= 1517^2 \\
154008^2 + 1665^2 &= 154017^2 \\
15429008^2 + 16665^2 &= 15429017^2 \\
1543179008^2 + 166665^2 &= 1543179017^2
\end{aligned} \tag{132}$$

This pattern is not so beautiful as others.

- **For $n = 5;\ m = 33, 333, 3333, 33333, ...$ in (10):**

$$\begin{aligned}
532^2 + 165^2 &= 557^2 \\
5432^2 + 1665^2 &= 55457^2 \\
5554432^2 + 16665^2 &= 5554457^2 \\
555544432^2 + 166665^2 &= 555544457^2
\end{aligned} \tag{133}$$

- **For $n = 82000, 820000, 8200000, ...$ in (7):**

In order to get pattern for the triple $(13612, 165, 13613)$, we shall use Procedure 1 given in (7) instead of Procedure 2. See below:

$$\begin{aligned}
13448164000^2 + 164001^2 &= 13448164001^2 \\
1344801640000^2 + 1640001^2 &= 1344801640001^2 \\
134480016400000^2 + 16400001^2 &= 134480016400001^2 \\
13448000164000000^2 + 164000001^2 &= 13448000164000001^2
\end{aligned} \tag{134}$$

The first three triples for $n = 82, 820, 8200$ are not written above as they don't give good patterns. These are as follows:

$$\begin{aligned}
13612^2 + 165^2 &= 13613^2 \\
1346440^2 + 1641^2 &= 1346441^2 \\
134496400^2 + 16401^2 &= 134496401^2
\end{aligned}$$

## (v) For $G_2(m,n) = 2mn = 330 = 2 \times 261 = 2 \times 3 \times 3 \times 29$ :

In this case the possible combinations are

$$(m,n) := \{(261,1), (87,3), (29,9)\}$$

Based on these combinations, we have following Pythagorean triples:

$$\begin{aligned}
&(F_2(261,1), G_2(261,1), H_2(261,1))/2 \Rightarrow 34060^2 + 261^2 = 34061^2 \\
&(F_2(87,3), G_2(87,3), H_2(87,3))/2 \Rightarrow 3780^2 + 261^2 = 3789^2 \\
&(F_2(29,9), G_2(29,9), H_2(29,9))/2 \Rightarrow 380^2 + 261^2 = 461^2
\end{aligned}$$

Out of three triples, there are two triple don't give a good patterns. These are:

$$\begin{aligned}
&(i)\ n = 1;\ m = 261: \quad (34060, 261, 34061) \Rightarrow 34060^2 + 261^2 = 34061^2 \\
&(ii)\ n = 3;\ m = 87: \quad (3780, 261, 3789) \Rightarrow 3780^2 + 261^2 = 3789^2
\end{aligned}$$

- **For $n = 9;\ m = 299, 2999, 29999, 299999, ...$ in (10):**

$$\begin{aligned}
44660^2 + 2691^2 &= 44741 \\
4496960^2 + 26991^2 &= 4497041 \\
449969960^2 + 269991^2 &= 449970041 \\
44999699960^2 + 2699991^2 &= 44999700041
\end{aligned} \tag{135}$$

The first triple $(380, 261, 461)$ for $n = 9;\ m = 29$ is not written above as it doesn't give good pattern.

- **For $n = 1300, 1300, 13000, ...$ in (7):**

In order to get pattern for the triple $(134060, 261, 34061)$, we shall use Procedure 1 given in (7) instead of Procedure 2. See below:

$$\begin{aligned}
3382600^2 + 2601^2 &= 3382601^2 \\
338026000^2 + 26001^2 &= 338026001^2 \\
33800260000^2 + 260001^2 &= 33800260001^2 \\
3380002600000^2 + 2600001^2 &= 3380002600001^2
\end{aligned} \tag{136}$$

The first triple $(34060, 261, 34061)$ for $m = 1;\ n = 130$ is not written above as it doesn't give good pattern.

### (vi) For $G_2(m,n) = 2mn = 280 = 2 \times 140 = 2 \times 2 \times 5 \times 7:$

In this case the possible combinations are

$$(m,n) := \{(140,1),(70,2),(35,4),(28,5),(20,7)\}$$

Based on these combinations, we have following Pythagorean triples:

$$\begin{aligned}
(F_2(140,1), G_2(140,1), H_2(140,1)) &\Rightarrow 19599^2 + 280^2 = 19601^2 \\
(F_2(70,2), G_2(70,2), H_2(70,2)) &\Rightarrow 4896^2 + 280^2 = 4904^2 \\
(F_2(35,4), G_2(35,4), H_2(35,4)) &\Rightarrow 1209^2 + 280^2 = 1241^2 \\
(F_2(28,5), G_2(28,5), H_2(28,5)) &\Rightarrow 759^2 + 280^2 = 809^2 \\
(F_2(20,7), G_2(20,7), H_2(20,7)) &\Rightarrow 351^2 + 280^2 = 449^2
\end{aligned}$$

Below are patterns based on above triples:

- **For $n = 1$; $m = 140, 1400, 14000, 140000, ...$ in (10):**

$$\begin{aligned}
19599^2 + 280^2 &= 19601^2 \\
1959999^2 + 2800^2 &= 1960001^2 := 3841603920001 \\
195999999^2 + 28000^2 &= 196000001^2 := 38416000392000001 \\
19599999999^2 + 280000^2 &= 19600000001^2 := 384160000039200000001
\end{aligned} \tag{137}$$

- **For $n = 2$; $m = 70, 700, 7000, 70000, ...$ in (10):**

$$\begin{aligned}
4896^2 + 280^2 &= 4904^2 \\
489996^2 + 2800^2 &= 490004^2 := 240103920016 \\
48999996^2 + 28000^2 &= 49000004^2 := 2401000392000016 \\
4899999996^2 + 280000^2 &= 4900000004^2 := 24010000039200000016
\end{aligned} \tag{138}$$

▶ **Division by 2**

$$\begin{aligned}
2448^2 + 140^2 &= 2452^2 \\
244998^2 + 1400^2 &= 245002^2 := 60025980004 \\
24499998^2 + 14000^2 &= 24500002^2 := 600250098000004 \\
2449999998^2 + 140000^2 &= 2450000002^2 := 6002500009800000004 \\
244999999998^2 + 1400000^2 &= 245000000002^2 := 60025000000980000000004
\end{aligned} \tag{139}$$

▶ **Division by 4**

$$\begin{aligned}
122499^2 + 700^2 &= 122501^2 \\
12249999^2 + 7000^2 &= 12250001^2 &&:= 150062524500001 \\
1224999999^2 + 70000^2 &= 1225000001^2 &&:= 1500625002450000001 \\
122499999999^2 + 700000^2 &= 122500000001^2 &&:= 15006250000245000000001 \\
12249999999999^2 + 7000000^2 &= 12250000000001^2 &&:= 150062500000024500000000001
\end{aligned} \tag{140}$$

The first triple $(1224, 70, 1226)$ is not written above as it doesn't give good pattern.

- **For $n = 4$; $m = 350, 3500, 35000, ...$ in (10):**

$$\begin{aligned}
122484^2 + 2800^2 &= 122516^2 \\
12249984^2 + 28000^2 &= 12250016^2 &&:= 150062892000256 \\
1224999984^2 + 280000^2 &= 1225000016^2 &&:= 1500625039200000256 \\
122499999984^2 + 2800000^2 &= 122500000016^2 &&:= 15006250003920000000256 \\
12249999999984^2 + 28000000^2 &= 12250000000016^2 &&:= 150062500000392000000000256
\end{aligned} \tag{141}$$

The first triple $(1209, 280, 1241)$ for $n = 4$; $m = 35$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
61242^2 + 1400^2 &= 61258^2 \\
6124992^2 + 14000^2 &= 6125008^2 \\
612499992^2 + 140000^2 &= 612500008^2 &&:= 375156259800000064 \\
61249999992^2 + 1400000^2 &= 61250000008^2 &&:= 3751562500980000000064 \\
6124999999992^2 + 14000000^2 &= 6125000000008^2 &&:= 37515625000098000000000064
\end{aligned} \tag{142}$$

▶ **Division by 4**

$$\begin{aligned}
3062496^2 + 7000^2 &= 3062504^2 \\
306249996^2 + 70000^2 &= 306250004^2 \\
30624999996^2 + 700000^2 &= 30625000004^2 &&:= 937890625245000000016 \\
3062499999996^2 + 7000000^2 &= 3062500000004^2 &&:= 9378906250024500000000016 \\
306249999999996^2 + 70000000^2 &= 306250000000004^2 &&:= 93789062500002450000000000016
\end{aligned} \tag{143}$$

The first triple $(30621, 700, 30629)$ is not written above as it doesn't give good pattern.

▶ **Division by 8**

$$\begin{aligned}
1531248^2 + 3500^2 &= 1531252^2 \\
153124998^2 + 35000^2 &= 153125002^2 \\
15312499998^2 + 350000^2 &= 15312500002^2 \\
1531249999998^2 + 3500000^2 &= 1531250000002^2 &:= 2344726562506125000000004 \\
153124999999998^2 + 35000000^2 &= 153125000000002^2 &:= 23447265625000612500000000004
\end{aligned} \tag{144}$$

▶ **Division by 16**

$$\begin{aligned}
76562499^2 + 17500^2 &= 76562501^2 \\
7656249999^2 + 175000^2 &= 7656250001^2 \\
765624999999^2 + 1750000^2 &= 765625000001^2 \\
76562499999999^2 + 17500000^2 &= 76562500000001^2 &:= 5861816406250153125000000001 \\
7656249999999999^2 + 175000000^2 &= 7656250000000001^2 &:= 58618164062500015312500000000001
\end{aligned} \tag{145}$$

The first triple $(765624, 1750, 765626)$ is not written above as it doesn't give good pattern.

- **For $n = 5$; $m = 280, 2800, 28000, ...$ in (10):**

$$\begin{aligned}
78375^2 + 2800^2 &= 78425^2 \\
7839975^2 + 28000^2 &= 7840025^2 \\
783999975^2 + 280000^2 &= 784000025^2 &:= 614656039200000625 \\
78399999975^2 + 2800000^2 &= 78400000025^2 &:= 6146560003920000000625
\end{aligned} \tag{146}$$

The first triple $(759, 280, 809)$ for $n = 5$; $m = 28$ is not written above as it doesn't give good pattern.

- **For $Forn = 7$; $m = 20, 200, 2000, 20000, ...$ in (10):**

$$\begin{aligned}
351^2 + 280^2 &= 449^2 \\
39951^2 + 2800^2 &= 40049^2 &:= 1603922401 \\
3999951^2 + 28000^2 &= 4000049^2 &:= 16000392002401 \\
399999951^2 + 280000^2 &= 400000049^2 &:= 160000039200002401
\end{aligned} \tag{147}$$

## (vii) For $G_2(m,n) = 2mn = 360 = 2 \times 180 = 2 \times 2 \times 2 \times 3 \times 3 \times 5:$

In this case the possible combinations are

$$(m,n) := \{(180,1), (90,2), (60,3), (45,4), (38,5), (30,6), (20,9), (18,10), (15,12)\}$$

Based on these combinations, we have following Pythagorean triples:

$$
\begin{aligned}
&(F_2(180,1), G_2(180,1), H_2(180,1)) \Rightarrow 32399^2 + 360^2 = 32401^2\\
&(F_2(90,2), G_2(90,2), H_2(90,2)) \Rightarrow 8096^2 + 360^2 = 8104^2\\
&(F_2(60,3), G_2(60,3), H_2(60,3)) \Rightarrow 3591^2 + 360^2 = 3609^2\\
&(F_2(45,4), G_2(45,4), H_2(45,4)) \Rightarrow 2009^2 + 360^2 = 2041^2\\
&(F_2(36,5), G_2(36,5), H_2(36,5)) \Rightarrow 1271^2 + 360^2 = 1321^2\\
&(F_2(30,6), G_2(30,6), H_2(30,6)) \Rightarrow 864^2 + 360^2 = 936^2\\
&(F_2(20,9), G_2(20,9), H_2(20,9)) \Rightarrow 319^2 + 360^2 = 481^2\\
&(F_2(18,10), G_2(18,10), H_2(18,10)) \Rightarrow 224^2 + 360^2 = 424^2\\
&(F_2(15,12), G_2(15,12), H_2(15,12)) \Rightarrow 81^2 + 360^2 = 369^2
\end{aligned}
$$

Let's write patterns based on above triples:

- **For $n = 1$; $m = 180, 1800, 18000, 180000, ...$ in (10):**

$$
\begin{aligned}
32399^2 + 360^2 &= 32401^2\\
3239999^2 + 3600^2 &= 3240001^2 := 10497606480001\\
323999999^2 + 36000^2 &= 324000001^2 := 104976000648000001\\
32399999999^2 + 360000^2 &= 32400000001^2 := 1049760000064800000001
\end{aligned}
\tag{148}
$$

- **For $n = 2$; $m = 90, 900, 9000, 90000, ...$ in (10):**

$$
\begin{aligned}
8096^2 + 360^2 &= 8104^2\\
809996^2 + 3600^2 &= 810004^2 := 656106480016\\
80999996^2 + 36000^2 &= 81000004^2 := 6561000648000016\\
8099999996^2 + 360000^2 &= 8100000004^2 := 65610000064800000016\\
809999999996^2 + 3600000^2 &= 810000000004^2 := 656100000006480000000016\\
80999999999996^2 + 36000000^2 &= 81000000000004^2 := 6561000000000648000000000016
\end{aligned}
\tag{149}
$$

► **Division by 2**

$$
\begin{aligned}
4048^2 + 180^2 &= 4052^2\\
404998^2 + 1800^2 &= 405002^2\\
40499998^2 + 18000^2 &= 40500002^2 := 1640250162000004\\
4049999998^2 + 180000^2 &= 4050000002^2 := 16402500016200000004\\
404999999998^2 + 1800000^2 &= 405000000002^2 := 164025000001620000000004\\
40499999999998^2 + 18000000^2 &= 40500000000002^2 := 1640250000000162000000000004
\end{aligned}
\tag{150}
$$

▶ **Division by 4**

$$
\begin{aligned}
202499^2+900^2 &= 202501^2 \\
20249999^2+9000^2 &= 20250001^2 &&:= 410062540500001 \\
2024999999^2+90000^2 &= 2025000001^2 &&:= 4100625004050000001 \\
202499999999^2+900000^2 &= 202500000001^2 &&:= 41006250000405000000001 \\
20249999999999^2+9000000^2 &= 20250000000001^2 &&:= 410062500000040500000000001
\end{aligned} \tag{151}
$$

The first triple $(2024, 90, 2026)$ is not written above as it doesn't give good pattern.

- **For $n=3;\ m=60, 600, 6000, 60000, ...$ in (10):**

$$
\begin{aligned}
3591^2+360^2 &= 3609^2 \\
359991^2+3600^2 &= 360009^2 &&:= 129606480081 \\
35999991^2+36000^2 &= 36000009^2 &&:= 1296000648000081 \\
3599999991^2+360000^2 &= 3600000009^2 &&:= 12960000064800000081
\end{aligned} \tag{152}
$$

- **For $n=4;\ m=450, 4500, 45000, ...$ in (10):**

$$
\begin{aligned}
202484^2+3600^2 &= 202516^2 \\
20249984^2+36000^2 &= 20250016^2 \\
2024999984^2+360000^2 &= 2025000016^2 &&:= 4100625064800000256 \\
202499999984^2+3600000^2 &= 202500000016^2 &&:= 41006250006480000000256 \\
20249999999984^2+36000000^2 &= 20250000000016^2 &&:= 410062500000648000000000256
\end{aligned} \tag{153}
$$

The first triple $(2009, 360, 2041)$ for $n=4;\ m=45$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$
\begin{aligned}
101248^2+900^2 &= 101252^2 \\
10124998^2+9000^2 &= 10125002^2 \\
1012499998^2+90000^2 &= 1012500002^2 &&:= 1025156254050000004 \\
101249999998^2+900000^2 &= 101250000002^2 &&:= 10251562500405000000004 \\
10124999999998^2+9000000^2 &= 10125000000002^2 &&:= 102515625000040500000000004
\end{aligned} \tag{154}
$$

▶ **Division by 4**

$$
\begin{aligned}
5062499^2 + 4500^2 &= 5062501^2 \\
506249999^2 + 45000^2 &= 506250001^2 \\
50624999999^2 + 450000^2 &= 50625000001^2 &&:= 2562890625101250000001 \\
5062499999999^2 + 4500000^2 &= 5062500000001^2 &&:= 25628906250010125000000001 \\
506249999999999^2 + 45000000^2 &= 506250000000001^2 &&:= 256289062500001012500000000001
\end{aligned} \tag{155}
$$

The first triple $(50624, 450, 50626)$ is not written above as it doesn't give good pattern.

- **For $n = 5;\ m = 360, 3600, 36000, ...$ in (10):**

$$
\begin{aligned}
129575^2 + 3600^2 &= 129625^2 \\
12959975^2 + 36000^2 &= 12960025^2 \\
1295999975^2 + 360000^2 &= 1296000025^2 &&:= 1679616064800000625 \\
129599999975^2 + 3600000^2 &= 129600000025^2 &&:= 16796160006480000000625 \\
12959999999975^2 + 36000000^2 &= 12960000000025^2 &&:= 167961600000648000000000625
\end{aligned} \tag{156}
$$

The first triple $(1271, 360, 1321)$ for $n = 5;\ m = 36$ is not written above as it doesn't give good pattern.

- **For $n = 6;\ m = 30, 300, 3000, 30000, ...$ in (10):**

$$
\begin{aligned}
864^2 + 360^2 &= 936^2 \\
89964^2 + 3600^2 &= 90036^2 &&:= 8106481296 \\
8999964^2 + 36000^2 &= 9000036^2 &&:= 81000648001296 \\
899999964^2 + 360000^2 &= 900000036^2 &&:= 810000064800001296
\end{aligned} \tag{157}
$$

▶ **Division by 2**

$$
\begin{aligned}
44982^2 + 1800^2 &= 45018^2 \\
4499982^2 + 18000^2 &= 4500018^2 &&:= 20250162000324 \\
449999982^2 + 180000^2 &= 450000018^2 &&:= 202500016200000324 \\
44999999982^2 + 1800000^2 &= 45000000018^2 &&:= 2025000001620000000324 \\
4499999999982^2 + 18000000^2 &= 4500000000018^2 &&:= 20250000000162000000000324
\end{aligned} \tag{158}
$$

The first triple $(432, 180, 468)$ is not written above as it doesn't give good pattern.

▶ **Division by 4**

$$
\begin{aligned}
22491^2 + 900^2 &= 22509^2 \\
2249991^2 + 9000^2 &= 2250009^2 &&:= 5062540500081 \\
224999991^2 + 90000^2 &= 225000009^2 &&:= 50625004050000081 \\
22499999991^2 + 900000^2 &= 22500000009^2 &&:= 506250000405000000081 \\
2249999999991^2 + 9000000^2 &= 2250000000009^2 &&:= 5062500000040500000000081
\end{aligned} \tag{159}
$$

- **For $n = 9;\ m = 20, 200, 2000, 20000, ...$ in (10):**

$$\begin{aligned}
319^2 + 360^2 &= 481^2 \\
39919^2 + 3600^2 &= 40081^2 := 1606486561 \\
3999919^2 + 36000^2 &= 4000081^2 := 16000648006561 \\
399999919^2 + 360000^2 &= 400000081^2 := 160000064800006561
\end{aligned} \tag{160}$$

- **For $n = 10;\ m = 180, 1800, 18000, ...$ in (10):**

$$\begin{aligned}
32300^2 + 3600^2 &= 32500^2 \\
3239900^2 + 36000^2 &= 3240100^2 \\
323999900^2 + 360000^2 &= 324000100^2 := 104976064800010000 \\
32399999900^2 + 3600000^2 &= 32400000100^2 := 1049760006480000010000 \\
3239999999900^2 + 36000000^2 &= 3240000000100^2 := 10497600000648000000010000
\end{aligned} \tag{161}$$

The first triple $(224, 360, 424)$ for $n = 10;\ m = 18$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
16150^2 + 1800^2 &= 16250^2 \\
1619950^2 + 18000^2 &= 1620050^2 := 2624562002500 \\
161999950^2 + 180000^2 &= 162000050^2 := 26244016200002500 \\
16199999950^2 + 1800000^2 &= 16200000050^2 := 262440001620000002500 \\
1619999999950^2 + 18000000^2 &= 1620000000050^2 := 2624400000162000000002500
\end{aligned} \tag{162}$$

▶ **Division by 4**

$$\begin{aligned}
8075^2 + 900^2 &= 8125^2 \\
809975^2 + 9000^2 &= 810025^2 := 656140500625 \\
80999975^2 + 90000^2 &= 81000025^2 := 6561004050000625 \\
8099999975^2 + 900000^2 &= 8100000025^2 := 65610000405000000625 \\
809999999975^2 + 9000000^2 &= 810000000025^2 := 656100000040500000000625
\end{aligned} \tag{163}$$

- **For $n = 12;\ m = 1500, 15000, ...$ in (10):**

$$\begin{aligned}
2249856^2 + 36000^2 &= 2250144^2 \\
224999856^2 + 360000^2 &= 225000144^2 &&:= 50625064800020736 \\
22499999856^2 + 3600000^2 &= 22500000144^2 &&:= 506250006480000020736 \\
2249999999856^2 + 36000000^2 &= 2250000000144^2 &&:= 5062500000648000000020736 \\
224999999999856^2 + 360000000^2 &= 225000000000144^2 &&:= 50625000000064800000000020736
\end{aligned} \tag{164}$$

The first two triples $(81, 360, 369)$ and $(22356, 3600, 22644)$ for $n = 12;\ m = 15$ and 150 are not written above as they don't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
1124928^2 + 18000^2 &= 1125072^2 \\
112499928^2 + 180000^2 &= 112500072^2 \\
11249999928^2 + 1800000^2 &= 11250000072^2 &&:= 126562501620000005184 \\
1124999999928^2 + 18000000^2 &= 1125000000072^2 &&:= 1265625000162000000005184 \\
112499999999928^2 + 180000000^2 &= 112500000000072^2 &&:= 12656250000016200000000005184
\end{aligned} \tag{165}$$

▶ **Division by 4**

$$\begin{aligned}
562464^2 + 9000^2 &= 562536^2 \\
56249964^2 + 90000^2 &= 56250036^2 \\
5624999964^2 + 900000^2 &= 5625000036^2 &&:= 31640625405000001296 \\
562499999964^2 + 9000000^2 &= 562500000036^2 &&:= 316406250040500000001296 \\
56249999999964^2 + 90000000^2 &= 56250000000036^2 &&:= 3164062500004050000000001296
\end{aligned} \tag{166}$$

▶ **Division by 8**

$$\begin{aligned}
281232^2 + 4500^2 &= 281268^2 \\
28124982^2 + 45000^2 &= 28125018^2 \\
2812499982^2 + 450000^2 &= 2812500018^2 \\
281249999982^2 + 4500000^2 &= 281250000018^2 &&:= 79101562510125000000324 \\
28124999999982^2 + 45000000^2 &= 28125000000018^2 &&:= 791015625001012500000000324
\end{aligned} \tag{167}$$

▶ **Division by 16**

$$\begin{aligned}
14062491^2 + 22500^2 &= 14062509^2 \\
1406249991^2 + 225000^2 &= 1406250009^2 \\
140624999991^2 + 2250000^2 &= 140625000009^2 &&:= 19775390625253125000000081 \\
14062499999991^2 + 22500000^2 &= 14062500000009^2 &&:= 197753906250025312500000000081
\end{aligned} \tag{168}$$

The first triple $(140616, 2250, 140634)$ is not written as it doesn't give good pattern.

### (viii) For $G_2(m,n) = 2mn = 150 = 2 \times 75 = 2 \times 3 \times 5 \times 5:$

In this case, the possible combinations are

$$(m,n) := \{(15,5),(25,3),(75,1)\}.$$

Based on these combinations, we have following Pythagorean triples:

$$\begin{aligned}
(F_2(75,1), G_2(75,1), H_2(75,1)) &\Rightarrow 5624^2 + 150^2 = 5626^2 \\
(F_2(25,3), G_2(25,3), H_2(25,3)) &\Rightarrow 616^2 + 150^2 \quad = 634^2 \\
(F_2(15,5), G_2(15,5), H_2(15,5)) &\Rightarrow 200^2 + 150^2 \quad = 250^2
\end{aligned}$$

We observe that all the numbers appearing above are even numbers. Obviously, they can be divided by 2. Dividing by 2 the above value we get following triples:

$$\begin{aligned}
(F_2(75,1), G_2(75,1), H_2(75,1))\,/2 &\Rightarrow 2812^2 + 75^2 = 2813^2 \\
(F_2(25,3), G_2(25,3), H_2(25,3))\,/2 &\Rightarrow 308^2 + 75^2 \quad = 317^2 \\
(F_2(15,5), G_2(15,5), H_2(15,5))\,/2 &\Rightarrow 100^2 + 75^2 \quad = 125^2
\end{aligned}$$

Let's write patterns based on above triples:

- **For $n = 1$; $m = 750, 7500, 75000, \ldots$ in (10):**

$$\begin{aligned}
562499^2 + 1500^2 &= 562501^2 \\
56249999^2 + 15000^2 &= 56250001^2 &&:= 3164062612500001 \\
5624999999^2 + 150000^2 &= 5625000001^2 &&:= 31640625011250000001 \\
562499999999^2 + 1500000^2 &= 562500000001^2 &&:= 316406250001125000000001 \\
56249999999999^2 + 15000000^2 &= 56250000000001^2 &&:= 3164062500000112500000000001
\end{aligned} \tag{169}$$

The first triple $(5624, 150, 5626)$ for $n = 1$; $m = 75$ is not written above as it doesn't give good pattern.

The first term for $m = 75$ is not written above. It is [2].

- **For $n = 3$; $m = 250, 2500, 25000, \ldots$ in (10):**

$$\begin{aligned}
62491^2 + 1500^2 &= 62509^2 \\
6249991^2 + 15000^2 &= 6250009^2 \\
624999991^2 + 150000^2 &= 625000009^2 &&:= 390625011250000081 \\
62499999991^2 + 1500000^2 &= 62500000009^2 &&:= 3906250001125000000081 \\
6249999999991^2 + 15000000^2 &= 6250000000009^2 &&:= 39062500000112500000000081
\end{aligned} \tag{170}$$

The first triple $(616, 150, 634)$ for $n = 3$; $m = 25$ is not written above as it doesn't give good pattern.

- **For $n=5;\ m=15,150,1500,15000,...$ in (10):**

$$\begin{aligned}
22475^2+1500^2 &= 22525^2 \\
2249975^2+15000^2 &= 2250025^2 \\
224999975^2+150000^2 &= 225000025^2 &:= 50625011250000625 \\
22499999975^2+1500000^2 &= 22500000025^2 &:= 5062500011250000000625 \\
2249999999975^2+15000000^2 &= 2250000000025^2 &:= 506250000011250000000000625
\end{aligned} \tag{171}$$

The first triple $(200,150,250)$ for $n=5;\ m=25$ is not written above as it doesn't give good pattern.

## (ix) For $G_2(m,n)=2mn=444=2\times 222=2\times 2\times 3\times 37:$

In this case, the possible combinations are

$$(m,n):=\{(37,6),(74,3),(111,2),(222,1)\}.$$

Based on these combinations, we have following Pythagorean triples:

$$\begin{aligned}
(F_2(222,1),G_2(222,1),H_2(222,1)) &\Rightarrow 49283^2+444^2=49285^2 \\
(F_2(111,2),G_2(111,2),H_2(111,2)) &\Rightarrow 12317^2+444^2=12325^2 \\
(F_2(74,3),G_2(74,3),H_2(74,3)) &\Rightarrow 5467^2+444^2=5485^2 \\
(F_2(37,6),G_2(37,6),H_2(37,6)) &\Rightarrow 1333^2+444^2=1405^2
\end{aligned}$$

By no means can we say that this process is complete as there are more Pythagorean triples with 444, for example: $333^2+444^2=555^2$.

**Remark 2.** *The possibilities appearing in (12) don't include the one given in (1) and (2).*

Let's write patterns based on above triples:

- **For $n=1;\ m=2220,22200,222000,...$ in (10):**

$$\begin{aligned}
4928399^2+4440^2 &= 4928401^2 \\
492839999^2+44400^2 &= 492840001^2 \\
49283999999^2+444000^2 &= 49284000001^2 &:= 2428912656098568000001 \\
4928399999999^2+4440000^2 &= 4928400000001^2 &:= 24289126560009856800000001 \\
492839999999999^2+44400000^2 &= 492840000000001^2 &:= 242891265600000985680000000001
\end{aligned} \tag{172}$$

The first triple $(49283,444,49285)$ for $n=1;\ m=222$ is not written above as it doesn't give good pattern.

- **For $n=2;\ m=1110, 11100, 111000, ...$ in (10):**

$$\begin{aligned}
1232096^2 + 4440^2 &= 1232104^2 \\
123209996^2 + 44400^2 &= 123210004^2 \\
12320999996^2 + 444000^2 &= 12321000004^2 &&:= 151807041098568000016 \\
1232099999996^2 + 4440000^2 &= 1232100000004^2 &&:= 15180704100098568000000016 \\
123209999999996^2 + 44400000^2 &= 123210000000004^2 &&:= 1518070410000098568000000000016
\end{aligned} \tag{173}$$

The first triple $(12317, 444, 12325)$ for $n=2;\ m=111$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
616048^2 + 2220^2 &= 616052^2 \\
61604998^2 + 22200^2 &= 61605002^2 \\
6160499998^2 + 222000^2 &= 6160500002^2 \\
616049999998^2 + 2220000^2 &= 616050000002^2 &&:= 379517602502464200000004 \\
61604999999998^2 + 22200000^2 &= 61605000000002^2 &&:= 3795176025000246420000000004
\end{aligned} \tag{174}$$

▶ **Division by 4**

$$\begin{aligned}
308024^2 + 1110^2 &= 308026^2 \\
30802499^2 + 11100^2 &= 30802501^2 \\
3080249999^2 + 111000^2 &= 3080250001^2 \\
308024999999^2 + 1110000^2 &= 308025000001^2 &&:= 94879400625616050000001 \\
30802499999999^2 + 11100000^2 &= 30802500000001^2 &&:= 948794006250061605000000001
\end{aligned} \tag{175}$$

- **For $n=3;\ m=740, 7400, 74000, ...$ in (10):**

$$\begin{aligned}
547591^2 + 4440^2 &= 547609^2 \\
54759991^2 + 44400^2 &= 54760009^2 \\
5475999991^2 + 444000^2 &= 5476000009^2 &&:= 2998657609856800081 \\
547599999991^2 + 4440000^2 &= 547600000009^2 &&:= 299865760009856800000081 \\
54759999999991^2 + 44400000^2 &= 54760000000009^2 &&:= 2998657600000985680000000081
\end{aligned} \tag{176}$$

The first triple $(5467, 444, 5485)$ for $n=3;\ m=74$ is not written above as it doesn't give good pattern.

- **For $n = 6;\ m = 37, 370, 3700, 37000, ...$ in (10):**

$$\begin{aligned}
136864^2 + 4440^2 &= 136936^2 \\
13689964^2 + 44400^2 &= 13690036^2 \\
1368999964^2 + 444000^2 &= 1369000036^2 &&:= 18741610985680001296 \\
136899999964^2 + 4440000^2 &= 136900000036^2 &&:= 187416100098568000001296 \\
13689999999964^2 + 44400000^2 &= 13690000000036^2 &&:= 1874161000009856800000001296
\end{aligned} \tag{177}$$

The first triple $(1333, 444, 1405)$ for $n = 6;\ m = 37$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
68448^2 + 740^2 &= 68452^2 \\
6844998^2 + 7400^2 &= 6845002^2 \\
684499998^2 + 74000^2 &= 684500002^2 &&:= 468540252738000004 \\
68449999998^2 + 740000^2 &= 68450000002^2 &&:=, 4685402500273800000004 \\
6844999999998^2 + 7400000^2 &=, 6845000000002^2 &&:= 46854025000027380000000004
\end{aligned} \tag{178}$$

▶ **Division by 4**

$$\begin{aligned}
3422499^2 + 3700^2 &= 3422501^2 \\
342249999^2 + 37000^2 &= 342250001^2 \\
34224999999^2 + 370000^2 &= 34225000001^2 &&:= 1171350625068450000001 \\
3422499999999^2 + 3700000^2 &= 3422500000001^2 &&:= 11713506250006845000000001 \\
342249999999999^2 + 37000000^2 &= 342250000000001^2 &&:= 117135062500000684500000000001
\end{aligned} \tag{179}$$

The first triple $(34224, 370, 34226)$ is not written above as it doesn't give good pattern.

## 3.3 Procedure 3

This subsection brings patterns based on Procedure (12) given in subsection 2.3. Below are some examples of patterns:

### (i) For $n = 1, 2, 3, 4, 5;\ m = 3, 33, 333, 3333, 33333, ...$ :

- **For $n = 1;\ m = 3, 33, 333, 3333, 33333, ...$ in (12):**

$$\begin{aligned}
35^2 + 12^2 &= 37^2 \\
4355^2 + 132^2 &= 4357^2 \\
443555^2 + 1332^2 &= 443557^2 \\
44435555^2 + 13332^2 &= 44435557^2 \\
4444355555^2 + 133332^2 &= 4444355557^2
\end{aligned} \tag{180}$$

- **For $n = 2;\ m = 3, 33, 333, 3333, 33333, ...$ in (12):**

$$\begin{aligned}
32^2 + 24^2 &= 40^2 \\
4352^2 + 264^2 &= 4360^2 \\
443552^2 + 2664^2 &= 443560^2 \\
44435552^2 + 26664^2 &= 44435560^2 \\
4444355552^2 + 266664^2 &= 4444355560^2
\end{aligned} \tag{181}$$

▶ **Division by 2**

$$\begin{aligned}
16^2 + 12^2 &= 20^2 \\
2176^2 + 132^2 &= 2180^2 \\
221776^2 + 1332^2 &= 221780^2 \\
22217776^2 + 13332^2 &= 22217780^2 \\
2222177776,^2 + 133332^2 &= 2222177780^2 \\
222221777776^2 + 1333332^2 &= 222221777780^2
\end{aligned} \tag{182}$$

▶ **Division by 4**

$$\begin{aligned}
8^2 + 6^2 &= 10^2 \\
1088^2 + 66^2 &= 1090^2 \\
110888^2 + 666^2 &= 110890^2 \\
11108888^2 + 6666^2 &= 11108890^2 \\
1111088888^2 + 66666^2 &= 1111088890^2 \\
111110888888^2 + 666666^2 &= 111110888890^2
\end{aligned} \tag{183}$$

▶ **Division by 8**

$$\begin{aligned}
4^2 + 3^2 &= 5^2 \\
544^2 + 33^2 &= 545^2 \\
55444^2 + 333^2 &= 55445^2 \\
5554444^2 + 3333^2 &= 5554445^2 \\
555544444^2 + 33333^2 &= 555544445^2 \\
55555444444^2 + 333333^2 &= 55555444445^2
\end{aligned} \tag{184}$$

- **For $n = 3;\ m = 33, 333, 3333, 33333, ...$ in (12):**

$$\begin{aligned} 4347^2 + 396^2 &= 4365^2\\ 443547^2 + 3996^2 &= 443565^2\\ 44435547^2 + 39996^2 &= 44435565^2\\ 4444355547^2 + 399996^2 &= 4444355565^2 \end{aligned} \tag{185}$$

The first triple $(27, 36, 45)$ for $n = 3;\ m = 3$ is not written above as it doesn't give good pattern.

- **For $n = 4;\ m = 33, 333, 3333, 33333, ...$ in (12):**

$$\begin{aligned} 4340^2 + 528^2 &= 4372^2\\ 443540^2 + 5328^2 &= 443572^2\\ 44435540^2 + 53328^2 &= 44435572^2\\ 4444355540^2 + 533328^2 &= 4444355572^2 \end{aligned} \tag{186}$$

The first triple $(20, 48, 52)$ for $n = 4;\ m = 3$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned} 2170^2 + 264^2 &= 2186^2\\ 221770^2 + 2664^2 &= 221786^2\\ 22217770^2 + 26664^2 &= 22217786^2\\ 2222177770^2 + 266664^2 &= 2222177786^2\\ 222221777770^2 + 2666664^2 &= 222221777786^2 \end{aligned} \tag{187}$$

▶ **Division by 4**

$$\begin{aligned} 5^2 + 12^2 &= 13^2\\ 1085^2 + 132^2 &= 1093^2\\ 110885^2 + 1332^2 &= 110893^2\\ 11108885^2 + 13332^2 &= 11108893^2\\ 1111088885^2 + 133332^2 &= 1111088893^2\\ 111110888885^2 + 1333332^2 &= 111110888893^2 \end{aligned} \tag{188}$$

- **For $n = 5;\ m = 33, 333, 3333, 33333, ...$ in (12):**

$$\begin{aligned} 4331^2 + 660^2 &= 4381^2\\ 443531^2 + 6660^2 &= 443581^2\\ 44435531^2 + 66660^2 &= 44435581^2\\ 4444355531^2 + 666660^2 &= 4444355581^2. \end{aligned} \tag{189}$$

The first triple $(11, 60, 61)$ for $n = 5;\ m = 3$ is not written above as it doesn't give good pattern.

## (ii) For $n = 1, 2, 3, ..., 11;\ m = 6, 66, 666, 6666, 66666, ...$ :

- **For $n = 1;\ m = 6, 66, 666, 6666, 66666, ...$ in (12):**

$$\begin{aligned}
143^2 + 24^2 &= 145^2\\
17423^2 + 264^2 &= 17425^2\\
1774223^2 + 2664^2 &= 1774225^2\\
177742223^2 + 26664^2 &= 177742225^2\\
17777422223^2 + 266664^2 &= 17777422225^2
\end{aligned} \tag{190}$$

- **For $n = 2;\ m = 66, 666, 6666, 66666, ...$ in (12):**

$$\begin{aligned}
17420^2 + 528^2 &= 17428^2\\
1774220^2 + 5328^2 &= 1774228^2\\
177742220^2 + 53328^2 &= 177742228^2\\
17777422220^2 + 533328^2 &= 17777422228^2
\end{aligned} \tag{191}$$

The first triple $(140, 48, 148)$ for $n = 2;\ m = 6$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
70^2 + 24^2 &= 74^2\\
8710^2 + 264^2 &= 8714^2\\
887110^2 + 2664^2 &= 887114^2\\
88871110^2 + 26664^2 &= 88871114^2\\
8888711110^2 + 266664^2 &= 8888711114^2\\
888887111110^2 + 2666664^2 &= 888887111114^2
\end{aligned} \tag{192}$$

▶ **Division by 4**

$$\begin{aligned}
35^2 + 12^2 &= 37^2\\
4355^2 + 132^2 &= 4357^2\\
443555^2 + 1332^2 &= 443557^2\\
44435555^2 + 13332^2 &= 44435557^2\\
4444355555^2 + 133332^2 &= 4444355557^2\\
444443555555^2 + 1333332^2 &= 444443555557^2
\end{aligned} \tag{193}$$

- **For $n = 3$; $m = 66, 666, 6666, 66666, ...$ in (12):**

$$\begin{aligned}
17415^2 + 792^2 &= 17433^2 \\
1774215^2 + 7992^2 &= 1774233^2 \\
177742215^2 + 79992^2 &= 177742233^2 \\
17777422215^2 + 799992^2 &= 17777422233^2
\end{aligned} \tag{194}$$

The first triple $(135, 72, 153)$ for $n = 3$; $m = 6$ is not written above as it doesn't give good pattern.

- **For $n = 4$; $m = 66, 666, 6666, 66666, ...$ in (12):**

$$\begin{aligned}
17408^2 + 1056^2 &= 17440^2 \\
1774208^2 + 10656^2 &= 1774240^2 \\
177742208^2 + 106656^2 &= 177742240^2 \\
17777422208^2 + 1066656^2 &= 17777422240^2
\end{aligned} \tag{195}$$

The first triple $(128, 96, 160)$ for $n = 4$; $m = 6$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
8704^2 + 528^2 &= 8720^2 \\
887104^2 + 5328^2 &= 887120^2 \\
88871104^2 + 53328^2 &= 88871120^2 \\
8888711104^2 + 533328^2 &= 8888711120^2 \\
888887111104^2 + 5333328^2 &= 888887111120^2
\end{aligned} \tag{196}$$

▶ **Division by 4**

$$\begin{aligned}
32^2 + 24^2 &= 40^2 \\
4352^2 + 264^2 &= 4360^2 \\
443552^2 + 2664^2 &= 443560^2 \\
44435552^2 + 26664^2 &= 44435560^2 \\
4444355552^2 + 266664^2 &= 4444355560^2 \\
444443555552^2 + 2666664^2 &= 444443555560^2
\end{aligned} \tag{197}$$

▶ **Division by 8**

$$
\begin{aligned}
16^2+12^2 &= 20^2\\
2176^2+132^2 &= 2180^2\\
221776^2+1332^2 &= 221780^2\\
22217776^2+13332^2 &= 22217780^2\\
2222217777776^2+1333332^2 &= 2222217777780^2
\end{aligned} \tag{198}
$$

► **Division by 16**

$$
\begin{aligned}
8^2+6^2 &= 10^2\\
1088^2+66^2 &= 1090^2\\
110888^2+666^2 &= 110890^2\\
11108888^2+6666^2 &= 11108890^2\\
1111088888^2+66666^2 &= 1111088890^2\\
111110888888^2+666666^2 &= 111110888890^2
\end{aligned} \tag{199}
$$

► **Division by 32**

$$
\begin{aligned}
4^2+3^2 &= 5^2\\
544^2+33^2 &= 545^2\\
55444^2+333^2 &= 55445^2\\
5554444^2+3333^2 &= 5554445^2\\
555544444^2+33333^2 &= 555544445^2\\
55555444444^2+333333^2 &= 55555444445^2
\end{aligned} \tag{200}
$$

- **For $n=5;\ m=666, 6666, 66666, \ldots$ in (12):**

$$
\begin{aligned}
1774199^2+13320^2 &= 1774249^2\\
177742199^2+133320^2 &= 177742249^2\\
17777422199^2+1333320^2 &= 17777422249^2\\
1777774222199^2+13333320^2 &= 1777774222249^2
\end{aligned} \tag{201}
$$

The first two triple $(119, 120, 169)$ and $(17399, 1320, 17449)$ for $n=5;\ m=6$ and 66 are not written above as they don't give good pattern.

- **For $n = 6$; $m = 666, 6666, 66666, ...$ in (12):**

$$\begin{aligned}
1774188^2 + 15984^2 &= 1774260^2\\
177742188^2 + 159984^2 &= 177742260^2\\
17777422188^2 + 1599984^2 &= 17777422260^2\\
1777774222188^2 + 15999984^2 &= 1777774222260^2
\end{aligned} \tag{202}$$

The first two triple $(108, 144, 180)$ and $(17388, 1584, 17460)$ for $n = 6$; $m = 6$ and 66 are not written above as they don't give good pattern.

► **Division by 2**

$$\begin{aligned}
887094^2 + 7992^2 &= 887130^2\\
88871094^2 + 79992^2 &= 88871130^2\\
8888711094^2 + 799992^2 &= 8888711130^2\\
888887111094^2 + 7999992^2 &= 888887111130^2
\end{aligned} \tag{203}$$

► **Division by 4**

$$\begin{aligned}
4347^2 + 396^2 &= 4365^2\\
443547^2 + 3996^2 &= 443565^2\\
44435547^2 + 39996^2 &= 44435565^2\\
4444355547^2 + 399996^2 &= 4444355565^2\\
444443555547^2 + 3999996^2 &= 444443555565^2
\end{aligned} \tag{204}$$

- **For $n = 7$; $m = 666, 6666, 66666, ...$ in (12):**

$$\begin{aligned}
1774175^2 + 18648^2 &= 1774273^2\\
177742175^2 + 186648^2 &= 177742273^2\\
17777422175^2 + 1866648^2 &= 17777422273^2\\
1777774222175^2 + 18666648^2 &= 1777774222273^2
\end{aligned} \tag{205}$$

The first two triple $(95, 168, 193)$ and $(17375, 1848, 17473)$ for $n = 7$; $m = 6$ and 66 are not written above as they don't give good pattern.

- **For $n = 8$ $m = 666, 6666, 66666, ...$ in (12):**

$$\begin{aligned} 1774160^2 + 21312^2 &= 1774288^2 \\ 177742160^2 + 213312^2 &= 177742288^2 \\ 17777422160^2 + 2133312^2 &= 17777422288^2 \\ 1777774222160^2 + 21333312^2 &= 1777774222288^2 \end{aligned} \tag{206}$$

The first two triple $(80, 192, 208)$ and $(17360, 2112, 17488)$ for $n = 8$; $m = 6$ and 66 are not written above as they don't give good pattern.

▶ **Division by 2**

$$\begin{aligned} 8680^2 + 1056^2 &= 8744^2 \\ 887080^2 + 10656^2 &= 887144^2 \\ 88871080^2 + 106656^2 &= 88871144^2 \\ 8888711080^2 + 1066656^2 &= 8888711144^2 \\ 888887111080^2 + 10666656^2 &= 888887111144^2 \end{aligned} \tag{207}$$

▶ **Division by 4**

$$\begin{aligned} 4340^2 + 528^2 &= 4372^2 \\ 443540^2 + 5328^2 &= 443572^2 \\ 44435540^2 + 53328^2 &= 44435572^2 \\ 4444355540^2 + 533328^2 &= 4444355572^2 \\ 444443555540^2 + 5333328^2 &= 444443555572^2 \end{aligned} \tag{208}$$

▶ **Division by 8**

$$\begin{aligned} 10^2 + 24^2 &= 26^2 \\ 2170^2 + 264^2 &= 2186^2 \\ 221770^2 + 2664^2 &= 221786^2 \\ 22217770^2 + 26664^2 &= 22217786^2 \\ 2222177770^2 + 266664^2 &= 2222177786^2 \\ 222221777770^2 + 2666664^2 &= 222221777786^2 \end{aligned} \tag{209}$$

▶ **Division by 16**

$$\begin{aligned} 5^2 + 12^2 &= 13^2 \\ 1085^2 + 132^2 &= 1093^2 \\ 110885^2 + 1332^2 &= 110893^2 \\ 11108885^2 + 13332^2 &= 11108893^2 \\ 1111088885,^2 + 133332^2 &= 1111088893^2 \\ 111110888885^2 + 1333332^2 &= 111110888893^2 \end{aligned} \tag{210}$$

- **For $n = 9$; $m = 666, 6666, 66666, ...$ in (12):**

$$
\begin{aligned}
1774143^2 + 23976^2 &= 1774305^2 \\
177742143^2 + 239976^2 &= 177742305^2 \\
17777422143^2 + 2399976^2 &= 17777422305^2 \\
1777774222143^2 + 23999976^2 &= 1777774222305^2
\end{aligned} \tag{211}
$$

The first two triple $(63, 216, 225)$ and $(17343, 2376, 17505)$ for $n = 9$; $m = 6$ and 66 are not written above as they don't give good pattern.

- **For $n = 10$; $m = 666, 6666, 66666, ...$ in (12):**

$$
\begin{aligned}
1774124^2 + 26640^2 &= 1774324^2 \\
177742124^2 + 266640^2 &= 177742324^2 \\
17777422124^2 + 2666640^2 &= 17777422324^2 \\
1777774222124^2 + 26666640^2 &= 1777774222324^2
\end{aligned} \tag{212}
$$

The first two triple $(44, 240, 244)$ and $(17324, 2640, 17524)$ for $n = 10$; $m = 6$ and 66 are not written above as they don't give good pattern.

▶ **Division by 2**

$$
\begin{aligned}
8662^2 + 1320^2 &= 8762^2 \\
887062^2 + 13320^2 &= 887162^2 \\
88871062^2 + 133320^2 &= 88871162^2 \\
8888711062^2 + 1333320^2 &= 8888711162^2 \\
888887111062^2 + 13333320^2 &= 888887111162^2
\end{aligned} \tag{213}
$$

▶ **Division by 4**

$$
\begin{aligned}
4331^2 + 660^2 &= 4381^2 \\
443531^2 + 6660^2 &= 443581^2 \\
44435531^2 + 66660^2 &= 44435581^2 \\
4444355531^2 + 666660^2 &= 4444355581^2 \\
444443555531^2 + 6666660^2 &= 444443555581^2
\end{aligned} \tag{214}
$$

- **For $n = 11;\ m = 666, 6666, 66666, ...$ in (12):**

$$\begin{aligned}
1774103^2 + 29304^2 &= 1774345^2 \\
177742103^2 + 293304^2 &= 177742345^2 \\
17777422103^2 + 2933304^2 &= 17777422345^2 \\
1777774222103^2 + 29333304^2 &= 1777774222345^2
\end{aligned} \tag{215}$$

The first two triple $(23, 264, 265)$ and $(17303, 2904, 17545)$ for $n = 11;\ m = 6$ and $66$ are not written above as they don't give good pattern.

### (iii) For $n = 1, 2, 3, ..., 17;\ m = 9, 99, 999, 9999, 99999, ...$ :

- **For $n = 1;\ m = 9, 99, 999, 9999, 99999, ...$ in (12):**

$$\begin{aligned}
323^2 + 36^2 &= 325^2 \\
39203^2 + 396^2 &= 39205^2 \\
3992003^2 + 3996^2 &= 3992005^2 && := 15936103920025 \\
399920003^2 + 39996^2 &= 399920005^2 && := 159936010399200025 \\
39999200003^2 + 399996^2 &= +39999200005^2 && := 1599936001039992000025 \\
3999992000003^2 + 3999996^2 &= 3999992000005^2 && := 15999936000103999920000025
\end{aligned} \tag{216}$$

- **For $n = 2;\ m = 9, 99, 999, 9999, 99999, ...$ in (12):**

$$\begin{aligned}
320^2 + 72^2 &= 328^2 \\
39200^2 + 792^2 &= 39208^2 \\
3992000^2 + 7992^2 &= 3992008^2 && := 15936127872064 \\
399920000^2 + 79992^2 &= 399920008^2 && := 159936012798720064 \\
39999200000^2 + 799992^2 &= 39999200008^2 && := 1599936001279987200064 \\
3999992000000^2 + 7999992^2 &= 3999992000008^2 && := 15999936000127999872000064
\end{aligned} \tag{217}$$

▶ **Division by 2**

$$\begin{aligned}
160^2 + 36^2 &= 164^2 \\
19600^2 + 396^2 &= 19604^2 \\
1996000^2 + 3996^2 &= 1996004^2 && := 3984031968016 \\
199960000^2 + 39996^2 &= 199960004^2 && := 39984003199680016 \\
19999600000^2 + 399996,^2 &= 19999600004^2 && := 399984000319996800016 \\
1999996000000^2 + 3999996^2 &= 1999996000004,^2 && := 3999984000031999968000016
\end{aligned} \tag{218}$$

▶ **Division by 4**

$$\begin{aligned}
80^2 + 18^2 &= 82^2 \\
9800^2 + 198^2 &= 9802^2 &&:= 96079204 \\
998000^2 + 1998^2 &= 998002^2 &&:= 996007992004 \\
99980000^2 + 19998^2 &= 99980002^2 &&:= 9996000799920004 \\
9999800000^2 + 199998^2 &= 9999800002^2 &&:= 99996000079999200004 \\
999998000000^2 + 1999998^2 &= 999998000002^2 &&:= 999996000007999992000004
\end{aligned} \tag{219}$$

▶ **Division by 8**

$$\begin{aligned}
40^2 + 9^2 &= 41^2 \\
4900^2 + 99^2 &= 4901^2 &&:= 24019801 \\
499000^2 + 999^2 &= 499001^2 &&:= 249001998001 \\
49990000^2 + 9999^2 &= 49990001^2 &&:= 2499000199980001 \\
4999900000^2 + 99999^2 &= 4999900001^2 &&:= 24999000019999800001 \\
499999000000^2 + 999999^2 &= 499999000001^2 &&:= 249999000001999998000001
\end{aligned} \tag{220}$$

- **For $n = 3$; $m = 99, 999, 9999, 99999, ...$ in (12):**

$$\begin{aligned}
39195^2 + 1188^2 &= 39213^2 \\
3991995^2 + 11988^2 &= 3992013^2 &&:= 15936167792169 \\
399919995^2 + 119988^2 &= 399920013^2 &&:= 159936016797920169 \\
39999199995^2 + 1199988^2 &= 39999200013^2 &&:= 1599936001679979200169 \\
3999991999995^2 + 11999988^2 &= 3999992000013^2 &&:= 15999936000167999792000169
\end{aligned} \tag{221}$$

The first triple $(315, 108, 333)$ for $n = 3$; $m = 9$ is not written above as it doesn't give good pattern.

- **For $n = 4$; $m = 99, 999, 9999, 99999, ...$ in (12):**

$$\begin{aligned}
39188^2 + 1584^2 &= 39220^2 \\
3991988^2 + 15984^2 &= 3992020^2 &&:= 15936223680400 \\
399919988^2 + 159984^2 &= 399920020^2 &&:= 159936022396800400 \\
39999199988^2 + 1599984^2 &= 39999200020^2 &&:= 1599936002239968000400 \\
3999991999988^2 + 15999984^2 &= 3999992000020^2 &&:= 15999936000223999680000400
\end{aligned} \tag{222}$$

The first triple $(308, 144, 340)$ for $n = 4$; $m = 9$ is not written above as it doesn't give good pattern.

► **Division by 2**

$$\begin{aligned}
19594^2 + 792^2 &= 19610^2 \\
1995994^2 + 7992^2 &= 1996010^2 &:= 3984055920100 \\
199959994^2 + 79992^2 &= 199960010^2 &:= 39984005599200100 \\
19999599994^2 + 799992^2 &= 19999600010^2 &:= 399984000559992000100 \\
1999995999994^2 + 7999992^2 &= 1999996000010^2 &:= 3999984000055999920000100
\end{aligned} \tag{223}$$

► **Division by 4**

$$\begin{aligned}
77^2 + 36^2 &= 85^2 \\
9797^2 + 396^2 &= 9805^2 &:= 96138025 \\
997997^2 + 3996^2 &= 998005^2 &:= 996013980025 \\
99979997^2 + 39996^2 &= 99980005^2 &:= 9996001399800025 \\
9999799997^2 + 399996^2 &= 9999800005^2 &:= 99996000139998000025 \\
999997999997^2 + 3999996^2 &= 999998000005^2 &:= 999996000013999980000025
\end{aligned} \tag{224}$$

- **For $n = 5;\ m = 99, 999, 9999, 99999, ...$ in (12):**

$$\begin{aligned}
39179^2 + 1980^2 &= 39229^2 \\
3991979^2 + 19980^2 &= 3992029^2 &:= 15936295536841 \\
399919979^2 + 199980^2 &= 399920029^2 &:= 159936029595360841 \\
39999199979^2 + 1999980^2 &= 39999200029^2 &:= 1599936002959953600841 \\
3999991999979^2 + 19999980^2 &= 3999992000029^2 &:= 15999936000295999536000841
\end{aligned} \tag{225}$$

The first triple $(299, 180, 349)$ for $n = 5;\ m = 9$ is not written above as it doesn't give good pattern.

- **For $n = 6;\ m = 99, 999, 9999, 99999, ...$ in (12):**

$$\begin{aligned}
39168^2 + 2376^2 &= 39240^2 \\
3991968^2 + 23976^2 &= 3992040^2 &:= 15936383361600 \\
399919968^2 + 239976^2 &= 399920040^2 &:= 159936038393601600 \\
39999199968^2 + 2399976^2 &= 39999200040^2 &:= 1599936003839936001600 \\
3999991999968^2 + 23999976^2 &= 3999992000040^2 &:= 15999936000383999360001600
\end{aligned} \tag{226}$$

The first triple $(288, 216, 360)$ for $n = 6;\ m = 9$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$
\begin{aligned}
19584^2 + 1188^2 &= 19620^2 \\
1995984^2 + 11988^2 &= 1996020^2 &&:= 3984095840400 \\
199959984^2 + 119988^2 &= 199960020^2 &&:= 39984009598400400 \\
19999599984^2 + 1199988^2 &= 19999600020^2 &&:= 399984000959984000400 \\
1999995999984^2 + 11999988^2 &= 1999996000020^2 &&:= 3999984000095999840000400
\end{aligned} \tag{227}
$$

▶ **Division by 4**

$$
\begin{aligned}
72^2 + 54^2 &= 90^2 \\
9792^2 + 594^2 &= 9810^2 &&:= 96236100 \\
997992^2 + 5994^2 &= 998010^2 &&:= 996023960100 \\
99979992^2 + 59994^2 &= 99980010^2 &&:= 9996002399600100 \\
9999799992^2 + 599994^2 &= 9999800010^2 &&:= 99996000239996000100 \\
999997999992^2 + 5999994^2 &= 999998000010^2 &&:= 999996000023999960000100
\end{aligned} \tag{228}
$$

▶ **Division by 8**

$$
\begin{aligned}
4896^2 + 297^2 &= 4905^2 &&:= 24059025 \\
498996^2 + 2997^2 &= 499005^2 &&:= 249005990025 \\
49989996^2 + 29997^2 &= 49990005^2 &&:= 2499000599900025 \\
4999899996^2 + 299997^2 &= 4999900005^2 &&:= 24999000059999000025 \\
499998999996^2 + 2999997^2 &= 499999000005^2 &&:= 249999000005999990000025
\end{aligned} \tag{229}
$$

- **For $n = 7;\ m = 99, 999, 9999, 99999, ...$ in (12):**

$$
\begin{aligned}
39155^2 + 2772^2 &= 39253^2 \\
3991955^2 + 27972^2 &= 3992053^2 \\
399919955^2 + 279972^2 &= 399920053^2 &&:= 159936048791522809 \\
39999199955^2 + 2799972^2 &= 39999200053^2 &&:= 1599936004879915202809 \\
3999991999955^2 + 27999972^2 &= 3999992000053^2 &&:= 15999936000487999152002809
\end{aligned} \tag{230}
$$

The first triple $(275, 252, 373)$ for $n = 7;\ m = 9$ is not written above as it doesn't give good pattern.

- **For $n = 8;\ m = 99, 999, 9999, 99999, ...$ in (12):**

$$
\begin{aligned}
39140^2 + 3168^2 &= 39268^2 \\
3991940^2 + 31968^2 &= 3992068^2 \\
399919940^2 + 319968^2 &= 399920068^2 && := 1599360607891246 24 \\
39999199940^2 + 3199968^2 &= 39999200068^2 && := 1599936006079891204624 \\
3999991999940^2 + 31999968^2 &= 3999992000068^2 && := 15999936000607998912004624
\end{aligned} \tag{231}
$$

The first triple $(260, 288, 388)$ for $n = 8;\ m = 9$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$
\begin{aligned}
19570^2 + 1584^2 &= 19634^2 \\
1995970^2 + 15984^2 &= 1996034^2 \\
199959970^2 + 159984^2 &= 199960034^2 && := 39984015197281156 \\
19999599970^2 + 1599984^2 &= 19999600034^2 && :=, 3999840015199728 01156 \\
1999995999970^2 + 15999984^2 &= 1999996000034^2 && := 3999984000151999728001156
\end{aligned} \tag{232}
$$

▶ **Division by 4**

$$
\begin{aligned}
9785^2 + 792^2 &= 9817^2 \\
997985^2 + 7992^2 &= 998017^2 && := 996037932289 \\
99979985^2 + 79992^2 &= 99980017^2 && := 9996003799320289 \\
9999799985^2 + 799992^2 &= 9999800017^2 && := 99996000379993200289 \\
999997999985^2 + 7999992^2 &= 999998000017^2 && := 999996000037999932000289
\end{aligned} \tag{233}
$$

- **For $n = 9;\ m = 999, 9999, 99999, ...$ in (12):**

$$
\begin{aligned}
3991923^2 + 35964^2 &= 3992085^2 \\
399919923^2 + 359964^2 &= 399920085^2 && := 159936074386407225 \\
39999199923^2 + 3599964^2 &= 39999200085^2 && := 1599936007439864007225 \\
3999991999923^2 + 35999964^2 &= 3999992000085^2 && := 15999936000743998640007225
\end{aligned} \tag{234}
$$

The first two triple $(243, 324, 405)$ and $(39123, 3564, 39285)$ for $n = 9;\ m = 9$ and 99 are not written above as they don't give good pattern.

- **For $n = 10$; $m = 999, 9999, 99999, ...$ in (12):**

$$\begin{aligned}
3991904^2 + 39960^2 &= 3992104^2 \\
399919904^2 + 399960^2 &= 39920104^2 && := 159936089583370816 \\
39999199904^2 + 3999960^2 &= 39999200104^2 && := 1599936008959833610816 \\
3999991999904^2 + 39999960^2 &= 3999992000104^2 && := 159999360008959998336010816
\end{aligned} \tag{235}$$

The first two triple $(224, 360, 424)$ and $(39104, 3960, 39304)$ for $n = 10$; $m = 9$ and 99 are not written above as they don't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
19552^2 + 1980^2 &= 19652^2 \\
1995952^2 + 19980^2 &= 1996052^2 \\
199959952^2 + 199980^2 &= 199960052^2 && := 39984022395842704 \\
19999599952^2 + 1999980^2 &= 19999600052^2 && := 399984002239958402704 \\
1999995999952^2 + 19999980^2 &= 1999996000052^2 && := 3999984000223999584002704
\end{aligned} \tag{236}$$

▶ **Division by 4**

$$\begin{aligned}
9776^2 + 990^2 &= 9826^2 \\
997976^2 + 9990^2 &= 998026^2 && := 996055896676 \\
99979976^2 + 99990^2 &= 99980026^2 && := 9996005598960676 \\
9999799976^2 + 999990^2 &= 9999800026^2 && := 99996000559989600676 \\
999997999976^2 + 9999990^2 &= 999998000026^2 && := 999996000055999896000676
\end{aligned} \tag{237}$$

▶ **Division by 8**

$$\begin{aligned}
4888^2 + 495^2 &= 4913^2 \\
498988^2 + 4995^2 &= 499013^2 && := 249013974169 \\
49989988^2 + 49995^2 &= 49990013^2 && := 2499001399740169 \\
4999899988^2 + 499995^2 &= 4999900013^2 && := 24999000139997400169 \\
499998999988^2 + 4999995^2 &= 499999000013^2 && := 249999000013999974000169
\end{aligned} \tag{238}$$

- **For $n = 11$; $m = 999, 9999, 99999, ...$ in (12):**

$$\begin{aligned}
3991883^2 + 43956^2 &= 3992125^2 \\
399919883^2 + 439956^2 &= 399920125^2 && := 159936106380015625 \\
39999199883^2 + 4399956^2 &= 39999200125^2 && := 1599936010639800015625 \\
3999991999883^2 + 43999956^2 &= 3999992000125^2 && := 159999360010639998000015625
\end{aligned} \tag{239}$$

The first two triple $(203, 396, 445)$ and $(39083, 4356, 39325)$ for $n = 11$; $m = 9$ and 99 are not written above as they don't give good pattern.

- **For $n = 12;\ m = 999, 9999, 99999, ...$ in (12):**

$$
\begin{aligned}
3991860^2 + 47952^2 &= 3992148^2 \\
399919860^2 + 479952^2 &= 399920148^2 \\
39999199860^2 + 4799952^2 &= 39999200148^2 := 1599936012479763221904 \\
3999991999860^2 + 47999952^2 &= 3999992000148^2 := 15999936001247997632021904
\end{aligned}
\tag{240}
$$

The first two triple $(180, 432, 468)$ and $(39060, 4752, 39348)$ for $n = 12;\ m = 9$ and 99 are not written above as they don't give good pattern.

► **Division by 2**

$$
\begin{aligned}
19530^2 + 2376^2 &= 19674^2 \\
1995930^2 + 23976^2 &= 1996074^2 \\
199959930^2 + 239976^2 &= 199960074^2 := 39984031194085476 \\
19999599930^2 + 2399976^2 &= 19999600074^2 := 399984003119940805476 \\
1999995999930^2 + 23999976^2 &= 1999996000074^2 := 3999984000311999408005476
\end{aligned}
\tag{241}
$$

► **Division by 4**

$$
\begin{aligned}
9765^2 + 1188^2 &= 9837^2 \\
997965^2 + 11988^2 &= 998037^2 \\
99979965^2 + 119988^2 &= 99980037^2 := 9996007798521369 \\
9999799965^2 + 1199988^2 &= 9999800037^2 := 99996000779985201369 \\
999997999965^2 + 11999988^2 &= 999998000037^2 := 999996000077999852001369
\end{aligned}
\tag{242}
$$

- **For $n = 13;\ m = 999, 9999, 99999, ...$ in (12):**

$$
\begin{aligned}
3991835^2 + 51948^2 &= 3992173^2 \\
399919835^2 + 519948^2 &= 399920173^2 \\
39999199835^2 + 5199948^2 &= 39999200173^2 := 1599936014479723229929 \\
3999991999835^2 + 51999948^2 &= 3999992000173^2 := 15999936001447997232029929
\end{aligned}
\tag{243}
$$

The first two triple $(155, 468, 493)$ and $(39035, 5148, 39373)$ for $n = 13;\ m = 9$ and 99 are not written above as they don't give good pattern.

- **For $n = 14;\ m = 999, 9999, 99999, ...$ in (12):**

$$\begin{aligned}
3991808^2 + 55944^2 &= 3992200^2 \\
399919808^2 + 559944^2 &= 399920200^2 \\
39999199808^2 + 5599944^2 &= 39999200200^2 &:= 1599936016639680040000 \\
3999991999808^2 + 55999944^2 &= 3999992000200^2 &:= 15999936001663996800040000
\end{aligned} \tag{244}$$

The first two triple $(128, 504, 520)$ and $(39008, 5544, 39400)$ for $n = 14;\ m = 9$ and 99 are not written above as they don't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
19504^2 + 2772^2 &= 19700^2 \\
1995904^2 + 27972^2 &= 1996100^2 &:= 3984415210000 \\
199959904^2 + 279972,^2 &= 199960100^2 &:= 39984041592010000 \\
19999599904^2 + 2799972^2 &= 19999600100^2 &:= 399984004159920010000 \\
1999995999904^2 + 27999972^2 &= 1999996000100^2 &:= 99998400041599920001000
\end{aligned} \tag{245}$$

▶ **Division by 4**

$$\begin{aligned}
9752^2 + 1386^2 &= 9850^2 \\
997952^2 + 13986,^2 &= 998050^2 &:= 996103802500 \\
99979952^2 + 139986^2 &= 99980050^2 &:= 9996010398002500 \\
9999799952^2 + 1399986^2 &= 9999800050^2 &:= 99996001039980002500 \\
999997999952^2 + 13999986^2 &= 999998000050^2 &:= 999996000103999800002500
\end{aligned} \tag{246}$$

▶ **Division by 8**

$$\begin{aligned}
4876^2 + 693^2 &= 4925^2 \\
498976^2 + 6993^2 &= 499025^2 &:= 249025950625 \\
49989976^2 + 69993^2 &= 49990025^2 &:= 2499002599500625 \\
4999899976^2 + 699993^2 &= 4999900025^2 &:= 24999000259995000625 \\
499998999976^2 + 6999993^2 &= 499999000025^2 &:= 249999000025999950000625
\end{aligned} \tag{247}$$

- **For $n = 15;\ m = 999, 9999, 99999, ...$ in (12):**

$$\begin{aligned}
3991779^2 + 59940^2 &= 3992229^2 \\
399919779^2 + 599940^2 &= 399920229^2 \\
39999199779^2 + 5999940^2 &= 39999200229^2 &:= 1599936018959633652441 \\
3999991999779^2 + 59999940^2 &= 3999992000229^2 &:= 15999936001895996336052441
\end{aligned} \tag{248}$$

The first two triple $(99, 540, 549)$ and $(38979, 5940, 39429)$ for $n = 15;\ m = 9$ and 99 are not written above as they don't give good pattern.

- **For $n = 16$; $m = 999, 9999, 99999, ...$ in (12):**

$$
\begin{aligned}
3991748^2 + 63936^2 &= 3992260^2 \\
399919748^2 + 639936^2 &= 399920260^2 \\
39999199748^2 + 6399936^2 &= 39999200260^2 &&:= 1599936021439584067600 \\
3999991999748^2 + 63999936^2 &= 3999992000260^2 &&:= 15999936002143995840067600
\end{aligned} \tag{249}
$$

The first two triple $68^2, 576, 580)$ and $(38948, 6336, 39460)$ for $n = 16$; $m = 9$ and 99 are not written above as they don't give good pattern.

▶ **Division by 2**

$$
\begin{aligned}
19474^2 + 3168^2 &= 19730^2 \\
1995874^2 + 31968^2 &= 1996130^2 \\
199959874^2 + 319968^2 &= 199960130^2 &&:= 39984053589616900 \\
19999599874^2 + 3199968^2 &= 19999600130^2 &&:= 399984005359896016900 \\
1999995999874^2 + 31999968^2 &= 1999996000130^2 &&:= 3999984000535998960016900
\end{aligned} \tag{250}
$$

▶ **Division by 4**

$$
\begin{aligned}
9737^2 + 1584^2 &= 9865^2 \\
997937^2 + 15984^2 &= 998065^2 &&:= 996133744225 \\
99979937^2 + 159984^2 &= 99980065^2 &&:= 9996013397404225 \\
9999799937^2 + 1599984^2 &= 9999800065^2 &&:= 99996001339974004225 \\
999997999937^2 + 15999984^2 &= 999998000065^2 &&:=, 999996000133999740004225
\end{aligned} \tag{251}
$$

- **For $n = 17$; $m = 999, 9999, 99999, ...$ in (12):**

$$
\begin{aligned}
3991715^2 + 67932^2 &= 3992293^2 \\
399919715^2 + 679932^2 &= 399920293^2 \\
39999199715^2 + 6799932^2 &= 39999200293^2 &&:= 1599936024079531285849 \\
3999991999715^2 + 67999932^2 &= 3999992000293^2 &&:= 15999936002407995312085849
\end{aligned} \tag{252}
$$

The first two triple $35, 612, 613)$ and $(38915, 6732, 39493)$ for $n = 17$; $m = 9$ and 99 are not written above as they don't give good pattern.

### 3.4 Procedure 4

This subsection brings patterns based on Procedure (14) given in subsection 2.4. Below are some examples of patterns:

## (i) For $n = 1, 2, 3, 4, 5;\ m = 6, 60, 600, 6000, 60000, ...$ :

- **For $n = 1;\ m = 60, 600, 6000, 60000, ...$ in (14):**

$$\begin{aligned}
57599^2 + 480^2 &= 57601^2 \\
5759999^2 + 4800^2 &= 5760001^2 &&:= 33177611520001 \\
575999999^2 + 48000^2 &= 576000001^2 &&:= 331776001152000001 \\
57599999999^2 + 480000^2 &= 57600000001^2 &&:= 3317760000115200000001 \\
5759999999999^2 + 4800000^2 &= 5760000000001^2 &&:= 33177600000011520000000001
\end{aligned} \tag{253}$$

The first triple $(575, 48, 577)$ for $n = 1;\ m = 6$ is not written above as it doesn't give good pattern.

- **For $n = 2;\ m = 60, 600, 6000, 60000, ...$ in (14):**

$$\begin{aligned}
57596^2 + 960^2 &= 57604^2 \\
5759996^2 + 9600^2 &= 5760004^2 &&:= 33177646080016 \\
575999996^2 + 96000^2 &= 576000004^2 &&:= 331776004608000016 \\
57599999996^2 + 960000^2 &= 57600000004^2 &&:= 3317760000460800000016 \\
5759999999996^2 + 9600000^2 &= 5760000000004^2 &&:= 33177600000046080000000016
\end{aligned} \tag{254}$$

The first triple $(572, 96, 580)$ for $n = 2;\ m = 6$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
28798^2 + 480^2 &= 28802^2 \\
2879998^2 + 4800^2 &= 2880002^2 &&:= 16588823040008 \\
287999998^2 + 48000^2 &= 288000002^2 &&:= 165888002304000008 \\
28799999998^2 + 480000^2 &= 28800000002^2 &&:= 1658880000230400000008 \\
2879999999998^2 + 4800000^2 &= 2880000000002^2 &&:= 16588800000023040000000008 \\
287999999999998^2 + 48000000^2 &= 288000000000002^2 &&:= 165888000000002304000000000008
\end{aligned} \tag{255}$$

▶ **Division by 4**

$$\begin{aligned}
14399^2 + 240^2 &= 14401^2 \\
1439999^2 + 2400^2 &= 1440001^2 &&:= 4147205760002 \\
143999999^2 + 24000^2 &= 144000001^2 &&:= 41472000576000002 \\
14399999999^2 + 240000^2 &= 14400000001^2 &&:= 414720000057600000002 \\
1439999999999^2 + 2400000^2 &= 1440000000001^2 &&:= 4147200000005760000000002 \\
143999999999999^2 + 24000000^2 &= 144000000000001^2 &&:= 41472000000000576000000000002
\end{aligned} \tag{256}$$

- **For $n = 3;\ m = 60, 600, 6000, 60000, ...$ in (14):**

$$\begin{aligned}
57591^2 + 1440^2 &= 57609^2 \\
5759991^2 + 14400^2 &= 5760009^2 \qquad (257)\\
575999991^2 + 144000^2 &= 576000009^2 := 331776010368000081 \\
57599999991^2 + 1440000^2 &= 57600000009^2 := 3317760001036800000081 \\
5759999999991^2 + 14400000^2 &= 5760000000009^2 := 33177600000103680000000081 \qquad (258)
\end{aligned}$$

The first triple $(567, 144, 585)$ for $n = 3;\ m = 6$ is not written above as it doesn't give good pattern.

- **For $n = 4;\ m = 60, 600, 6000, 60000, ...$ in (14):**

$$\begin{aligned}
57584^2 + 1920^2 &= 57616^2 \\
5759984^2 + 19200^2 &= 5760016^2 \\
575999984^2 + 192000^2 &= 576000016^2 := 331776018432000256 \\
57599999984^2 + 1920000^2 &= 57600000016^2 := 3317760001843200000256 \\
5759999999984^2 + 19200000^2 &= 5760000000016^2 := 33177600000184320000000256 \qquad (259)
\end{aligned}$$

The first triple $(560, 192, 592)$ for $n = 4;\ m = 6$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
28792^2 + 960^2 &= 28808^2 \\
2879992^2 + 9600^2 &= 2880008^2 := 4147223040032 \\
287999992^2 + 96000^2 &= 288000008^2 := 41472002304000032 \\
28799999992^2 + 960000^2 &= 28800000008^2 := 414720000230400000032 \\
2879999999992^2 + 9600000^2 &= 2880000000008^2 := 4147200000023040000000032 \\
287999999999992^2 + 96000000^2 &= 288000000000008^2 := 41472000000002304000000000032 \qquad (260)
\end{aligned}$$

▶ **Division by 4**

$$\begin{aligned}
14396^2 + 480^2 &= 14404^2 \\
1439996^2 + 4800^2 &= 1440004^2 := 4147223040032 \\
143999996^2 + 48000^2 &= 144000004^2 := 41472002304000032 \\
14399999996^2 + 480000^2 &= 14400000004^2 := 414720000230400000032 \\
1439999999996^2 + 4800000^2 &= 1440000000004^2 := 4147200000023040000000032 \\
143999999999996^2 + 48000000^2 &= 144000000000004^2 := 41472000000002304000000000032 \qquad (261)
\end{aligned}$$

► **Division by 8**

$$
\begin{aligned}
7198^2 + 240^2 &= 7202^2 \\
719998^2 + 2400^2 &= 720002^2 &&:= 1036805760008 \\
71999998^2 + 24000^2 &= 72000002^2 &&:= 10368000576000008 \\
7199999998^2 + 240000^2 &= 7200000002^2 &&:= 103680000057600000008 \\
719999999998^2 + 2400000^2 &= 720000000002^2 &&:= 1036800000005760000000008 \\
71999999999998^2 + 24000000^2 &= 72000000000002^2 &&:= 10368000000000576000000000008
\end{aligned} \tag{262}
$$

► **Division by 16**

$$
\begin{aligned}
3599^2 + 120^2 &= 3601^2 \\
359999^2 + 1200^2 &= 360001^2 &&:= 259201440002 \\
35999999^2 + 12000^2 &= 36000001^2 &&:= 2592001440000002 \\
3599999999^2 + 120000^2 &= 3600000001^2 &&:= 25920000144000000002 \\
359999999999^2 + 1200000^2 &= 360000000001^2 &&:= 259200000014400000000002 \\
35999999999999^2 + 12000000^2 &= 36000000000001^2 &&:= 2592000000001440000000000002
\end{aligned} \tag{263}
$$

- **For $n = 5;\ m = 60, 600, 6000, 60000, \ldots$ in (14):**

$$
\begin{aligned}
57575^2 + 2400^2 &= 57625^2 \\
5759975^2 + 24000^2 &= 5760025^2 \\
575999975^2 + 240000^2 &= 576000025^2 &&:= 331776028800000625 \\
57599999975^2 + 2400000^2 &= 57600000025^2 &&:= 3317760002880000000625 \\
5759999999975^2 + 24000000^2 &= 5760000000025^2 &&:= 33177600000288000000000625
\end{aligned} \tag{264}
$$

The first triple $(551, 240, 601)$ for $n = 5;\ m = 6$ is not written above as it doesn't give good pattern.

## (ii) For $n = 1, 2, 3, 4, 5;\ m = 7, 70, 700, 7000, 70000, \ldots$ :

- **For $n = 1;\ m = 70, 700, 7000, 70000, \ldots$ in (14):**

$$
\begin{aligned}
78399^2 + 560^2 &= 78401^2 \\
7839999^2 + 5600^2 &= 7840001^2 &&:= 61465615680001 \\
783999999^2 + 56000^2 &= 784000001^2 &&:= 614656001568000001 \\
78399999999^2 + 560000^2 &= 78400000001^2 &&:= 6146560000156800000001 \\
7839999999999^2 + 5600000^2 &= 7840000000001^2 &&:= 61465600000015680000000001
\end{aligned} \tag{265}
$$

The first triple $(783, 56, 7852)$ for $n = 1;\ m = 7$ is not written above as it doesn't give good pattern.

- **For $n = 2$; $m = 70, 700, 7000, 70000, ...$ in (14):**

$$\begin{aligned}
78396^2 + 1120^2 &= 78404^2 \\
7839996^2 + 11200^2 &= 7840004^2 && := 61465662720016 \\
783999996^2 + 112000^2 &= 784000004^2 && := 614656006272000016 \\
78399999996^2 + 1120000^2 &= 78400000004^2 && := 6146560000627200000016 \\
7839999999996^2 + 11200000^2 &= 7840000000004^2 && := 61465600000062720000000016
\end{aligned} \tag{266}$$

The first triple $(780, 112, 788)$ for $n = 2$; $m = 7$ is not written above as it doesn't give good pattern.

- **Division by 2**

$$\begin{aligned}
39198^2 + 560^2 &= 39202^2 \\
3919998^2 + 5600^2 &= 3920002^2 && := 30732831360008 \\
391999998^2 + 56000^2 &= 392000002^2 && := 307328003136000008 \\
39199999998^2 + 560000^2 &= 39200000002^2 && := 3073280000313600000008 \\
3919999999998^2 + 5600000^2 &= 3920000000002^2 && := 30732800000031360000000008 \\
391999999999998^2 + 56000000^2 &= 392000000000002^2 && := 307328000000003136000000000008
\end{aligned} \tag{267}$$

- **Division by 4**

$$\begin{aligned}
19599^2 + 280^2 &= 19601^2 \\
1959999^2 + 2800^2 &= 1960001^2 && := 7683207840002 \\
195999999^2 + 28000^2 &= 196000001^2 && := 76832000784000002 \\
19599999999^2 + 280000^2 &= 19600000001^2 && := 768320000078400000002 \\
1959999999999^2 + 2800000^2 &= 1960000000001^2 && := 7683200000007840000000002 \\
195999999999999^2 + 28000000^2 &= 196000000000001^2 && := 76832000000000784000000000002
\end{aligned} \tag{268}$$

- **For $n = 3$; $m = 70, 700, 7000, 70000, ...$ in (14):**

$$\begin{aligned}
78391^2 + 1680^2 &= 78409^2 \\
7839991^2 + 16800^2 &= 7840009^2 \\
783999991^2 + 168000^2 &= 784000009^2 && := 614656014112000081 \\
78399999991^2 + 1680000^2 &= 78400000009^2 && := 6146560001411200000081 \\
7839999999991^2 + 16800000^2 &= 7840000000009^2 && := 61465600000141120000000081
\end{aligned} \tag{269}$$

The first triple $(775, 168, 793)$ for $n = 3$; $m = 6$ is not written above as it doesn't give good pattern.

- **For $n = 4$; $m = 70, 700, 7000, 70000, ...$ in (14):**

$$\begin{aligned}
78384^2 + 2240^2 &= 78416^2\\
7839984^2 + 22400^2 &= 7840016^2\\
783999984^2 + 224000^2 &= 784000016^2 &&:= 614656025088000256\\
78399999984^2 + 2240000^2 &= 78400000016^2 &&:= 6146560002508800000256\\
7839999999984^2 + 22400000^2 &= 7840000000016^2 &&:= 61465600000250880000000256
\end{aligned} \tag{270}$$

The first triple $(768, 224, 800)$ for $n = 4$; $m = 7$ is not written above as it doesn't give good pattern.

- **Division by 2**

$$\begin{aligned}
39192^2 + 1120^2 &= 39208^2\\
3919992^2 + 11200^2 &= 3920008^2 &&:= 15366462720064\\
391999992^2 + 112000^2 &= 392000008^2 &&:= 153664006272000064\\
39199999992^2 + 1120000^2 &= 39200000008^2 &&:= 1536640000627200000064\\
3919999999992^2 + 11200000^2 &= 3920000000008^2 &&:= 15366400000062720000000064
\end{aligned} \tag{271}$$

- **Division by 4**

$$\begin{aligned}
19596^2 + 560^2 &= 19604^2\\
1959996^2 + 5600^2 &= 1960004^2 &&:= 3841615680016\\
195999996^2 + 56000^2 &= 196000004^2 &&:= 38416001568000016\\
19599999996^2 + 560000^2 &= 19600000004^2 &&:= 384160000156800000016\\
1959999999996^2 + 5600000^2 &= 1960000000004^2 &&:= 3841600000015680000000016
\end{aligned} \tag{272}$$

- **Division by 8**

$$\begin{aligned}
9798^2 + 280^2 &= 9802^2\\
979998^2 + 2800^2 &= 980002^2 &&:= 960403920004\\
97999998^2 + 28000^2 &= 98000002^2 &&:= 9604000392000004\\
9799999998^2 + 280000^2 &= 9800000002^2 &&:= 96040000039200000004\\
979999999998^2 + 2800000^2 &= 980000000002^2 &&:= 960400000003920000000004
\end{aligned} \tag{273}$$

- **Division by 16**

$$\begin{aligned}
4899^2 + 140^2 &= 4901^2 &&:= 24019801\\
489999^2 + 1400^2 &= 490001^2 &&:= 240100980001\\
48999999^2 + 14000^2 &= 49000001^2 &&:= 2401000098000001\\
4899999999^2 + 140000^2 &= 4900000001^2 &&:= 24010000009800000001\\
489999999999^2 + 1400000^2 &= 490000000001^2 &&:= 240100000000980000000001
\end{aligned} \tag{274}$$

- **For $n = 5;\ m = 70, 700, 7000, 70000, ...$ in (14):**

$$\begin{aligned}
78375^2 + 2800^2 &= 78425^2\\
7839975^2 + 28000^2 &= 7840025^2\\
783999975^2 + 280000^2 &= 784000025^2 &&:= 614656039200000625\\
78399999975^2 + 2800000^2 &= 78400000025^2 &&:= 6146560003920000000625\\
7839999999975^2 + 28000000^2 &= 7840000000025^2 &&:= 61465600000392000000000625
\end{aligned} \tag{275}$$

The first triple $(759, 280, 809)$ for $n = 5;\ m = 7$ is not written above as it doesn't give good pattern.

## (iii) For $n = 1, 2, 3, 4, 5;\ m = 6, 66, 666, 6666, 66666, ...$ :

- **For $n = 1;\ m = 666, 6666, 66666, ...$ in (14):**

$$\begin{aligned}
7096895^2 + 5328^2 &= 7096897^2\\
710968895^2 + 53328^2 &= 710968897^2\\
71109688895^2 + 533328^2 &= 71109688897^2\\
7111096888895^2 + 5333328^2 &= 7111096888897^2
\end{aligned} \tag{276}$$

The first two triple $(575, 48, 577)$ and $(69695, 528, 69697)$ for $n = 1;\ m = 6$ and 66 are not written above as they don't give good pattern.

- **For $n = 2;\ m = 666, 6666, 66666, ...$ in (14):**

$$\begin{aligned}
7096892^2 + 10656^2 &= 7096900^2\\
710968892^2 + 106656^2 &= 710968900^2\\
71109688892^2 + 1066656^2 &= 71109688900^2\\
7111096888892^2 + 10666656^2 &= 7111096888900^2\\
711110968888892^2 + 106666656^2 &= 711110968888900^2
\end{aligned} \tag{277}$$

The first two triple $(572, 96.580)$ and $(69692, 1056, 69700)$ for $n = 2;\ m = 6$ and 66 are not written above as they don't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
34846^2 + 528^2 &= 34850^2\\
3548446^2 + 5328^2 &= 3548450^2\\
355484446^2 + 53328^2 &= 355484450^2\\
35554844446^2 + 533328^2 &= 35554844450^2\\
3555548444446^2 + 5333328^2 &= 3555548444450^2\\
355555484444446^2 + 53333328^2 &= 355555484444450^2
\end{aligned} \tag{278}$$

▶ **Division by 4**

$$\begin{aligned}
143^2+24^2 &= 145^2\\
17423^2+264^2 &= 17425^2\\
1774223^2+2664^2 &= 1774225^2\\
177742223^2+26664^2 &= 177742225^2\\
17777422223^2+266664^2 &= 17777422225^2\\
1777774222223^2+2666664^2 &= 1777774222225^2\\
177777742222223^2+26666664^2 &= 177777742222225^2
\end{aligned} \tag{279}$$

- **For $n=3;\ m=666,6666,66666,...$ in (14):**

$$\begin{aligned}
7096887^2+15984^2 &= 7096905^2\\
710968887^2+159984^2 &= 710968905^2\\
71109688887^2+1599984^2 &= 71109688905^2\\
7111096888887^2+15999984^2 &= 7111096888905^2
\end{aligned} \tag{280}$$

The first two triple $(567,144,585)$ and $(69687,1584,69705)$ for $n=3;\ m=6$ and 66 are not written above as they don't give good pattern.

- **For $n=4;\ m=666,6666,66666,...$ in (14):**

$$\begin{aligned}
70968802^2+21312^2 &= 7096912^2\\
7109688802^2+213312^2 &= 710968912^2\\
711096888802^2+2133312^2 &= 71109688912^2\\
71110968888802^2+21333312^2 &= 7111096888912^2\\
7111109688888880^2+213333312^2 &= 711110968888912^2
\end{aligned} \tag{281}$$

The first two triple $(5602,192,592)$ and $(696802,2112,69712)$ for $n=4;\ m=6$ and 66 are not written above as they don't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
34840^2+1056^2 &= 34856^2\\
3548440^2+10656^2 &= 3548456^2\\
355484440^2+106656^2 &= 355484456^2\\
35554844440^2+1066656^2 &= 35554844456^2\\
3555548444440^2+10666656^2 &= 3555548444456^2\\
355555484444440^2+106666656^2 &= 355555484444456^2
\end{aligned} \tag{282}$$

▶ **Division by 4**

$$\begin{aligned}
17420^2+528^2 &= 17428^2\\
1774220^2+5328^2 &= 1774228^2\\
177742220^2+53328^2 &= 177742228^2\\
17777422220^2+533328^2 &= 17777422228^2\\
1777774222220^2+5333328^2 &= 1777774222228^2\\
177777742222220^2+53333328^2 &= 177777742222228^2
\end{aligned} \tag{283}$$

▶ **Division by 8**

$$\begin{aligned}
70^2+24^2 &= 74^2\\
8710^2+264^2 &= 8714^2\\
887110^2+2664^2 &= 887114^2\\
88871110^2+26664^2 &= 88871114^2\\
8888711110^2+266664^2 &= 8888711114^2\\
888887111110^2+2666664^2 &= 888887111114^2\\
88888871111110^2+26666664^2 &= 88888871111114^2
\end{aligned} \tag{284}$$

▶ **Division by 16**

$$\begin{aligned}
35^2+12^2 &= 37^2\\
4355^2+132^2 &= 4357^2\\
443555^2+1332^2 &= 443557^2\\
44435555^2+13332^2 &= 44435557^2\\
4444355555^2+133332^2 &= 4444355557^2\\
444443555555^2+1333332^2 &= 444443555557^2\\
44444435555555^2+13333332^2 &= 44444435555557^2
\end{aligned} \tag{285}$$

- **For $n=5;\ m=666,6666,66666,\ldots$ in (14):**

$$\begin{aligned}
70968712^2+26640^2 &= 7096921^2\\
7109688712^2+266640^2 &= 710968921^2\\
711096888712^2+2666640^2 &= 71109688921^2\\
71110968888712^2+26666640^2 &= 7111096888921^2
\end{aligned} \tag{286}$$

The first two triple $(551,240,601)$ and $(69671,2640,69721)$ for $n=5;\ m=6$ and 66 are not written above as they don't give good pattern.

### (iv) For $n = 1,2,3,4,5;\ m = 9,99,999,9999,99999,...$ :

- **For $n = 1;\ m = 99,999,9999,99999,...$ in (14):**

$$
\begin{aligned}
156815^2 + 792^2 &= 156817^2 \\
15968015^2 + 7992^2 &= 15968017^2 \\
1599680015^2 + 79992^2 &= 1599680017^2 &&:= 2558976156789120289 \\
159996800015^2 + 799992^2 &= 159996800017^2 &&:= 25598976015679891200289 \\
15999968000015^2 + 7999992^2 &= 15999968000017^2 &&:= 255998976001567998912000289
\end{aligned} \tag{287}
$$

The first triple $(1295, 72, 1297)$ for $n = 1;\ m = 9$ is not written above as it doesn't give good pattern.

- **For $n = 2;\ m = 99,999,9999,99999,...$ in (14):**

$$
\begin{aligned}
156812^2 + 1584^2 &= 15682^2 \\
15968012^2 + 15984^2 &= 15968020^2 \\
1599680012^2 + 159984^2 &= 1599680020^2 &&:= 2558976166387200400 \\
159996800012^2 + 1599984^2 &= 159996800020^2 &&:= 25598976016639872000400 \\
15999968000012^2 + 15999984^2 &= 15999968000020^2 &&:= 255998976001663998720000400
\end{aligned} \tag{288}
$$

The first triple $(1292, 144, 1300)$ for $n = 2;\ m = 9$ is not written above as it doesn't give good pattern.

► **Division by 2**

$$
\begin{aligned}
78406^2 + 792^2 &= 78410^2 \\
7984006^2 + 7992^2 &= 7984010^2 \\
799840006^2 + 79992^2 &= 799840010^2 &&:= 639744041596800100 \\
79998400006^2 + 799992^2 &= 79998400010^2 &&:= 6399744004159968000100 \\
7999984000006^2 + 7999992^2 &= 7999984000010,^2 &&:= 63999744000415999680000100
\end{aligned} \tag{289}
$$

► **Division by 4**

$$
\begin{aligned}
323^2 + 36^2 &= 325^2 \\
39203^2 + 396^2 &= 39205^2 \\
3992003^2 + 3996^2 &= 3992005^2 &&:= 15936103920025 \\
399920003^2 + 39996^2 &= 399920005,^2 &&:= 159936010399200025 \\
39999200003^2 + 399996^2 &= 39999200005^2 &&:= 1599936001039992000025 \\
3999992000003^2 + 3999996^2 &= 3999992000005^2 &&:= 15999936000103999920000025
\end{aligned} \tag{290}
$$

▶ **Division by 8**

$$\begin{aligned}
156807^2 + 2376^2 &= 156825^2 \\
15968007^2 + 23976^2 &= 15968025^2 \\
1599680007^2 + 239976^2 &= 1599680025^2 &&:= 2558976182384000625 \\
159996800007^2 + 2399976^2 &= 159996800025^2 &&:= 25598976018239840000625 \\
15999968000007^2 + 23999976^2 &= 15999968000025^2 &&:= 255998976001823998400000625
\end{aligned} \tag{291}$$

• **For $n = 3;\ m = 99, 999, 9999, 99999, ...$ in (14):**

$$\begin{aligned}
156807^2 + 2376^2 &= 156825^2 \\
15968007^2 + 23976^2 &= 15968025^2 \\
1599680007^2 + 239976^2 &= 1599680025^2 &&:= 2558976182384000625 \\
159996800007^2 + 2399976^2 &= 159996800025^2 &&:= 25598976018239840000625 \\
15999968000007^2 + 23999976^2 &= 15999968000025^2 &&:= 255998976001823998400000625
\end{aligned} \tag{292}$$

The first triple $(1287, 216, 1305)$ for $n = 3;\ m = 9$ is not written above as it doesn't give good pattern.

• **For $n = 4;\ m = 99, 999, 9999, 99999, ...$ in (14):**

$$\begin{aligned}
156800^2 + 3168^2 &= 156832^2 \\
15968000^2 + 31968^2 &= 15968032^2 \\
1599680000^2 + 319968^2 &= 1599680032^2 &&:= 2558976204779521024 \\
159996800000^2 + 3199968^2 &= 159996800032^2 &&:= 25598976020479795201024 \\
15999968000000^2 + 31999968^2 &= 15999968000032^2 &&:= 255998976002047997952001024
\end{aligned} \tag{293}$$

The first triple $(1280, 288, 1312)$ for $n = 4;\ m = 9$ is not written above as it doesn't give good pattern.

▶ **Division by 2**

$$\begin{aligned}
78400^2 + 1584^2 &= 78416^2 \\
7984000^2 + 15984^2 &= 7984016^2 \\
799840000^2 + 159984^2 &= 799840016^2 &&:= 639744051194880256 \\
79998400000^2 + 1599984^2 &= 79998400016^2 &&:= 6399744005119948800256 \\
7999984000000^2 + 15999984^2 &= 7999984000016^2 &&:= 63999744000511999488000256
\end{aligned} \tag{294}$$

▶ **Division by 4**

$$
\begin{aligned}
320^2 + 72^2 &= 328^2 \\
39200^2 + 792^2 &= 39208^2 \\
3992000^2 + 7992^2 &= 3992008^2 &&:= 15936127872064 \\
399920000^2 + 79992^2 &= 399920008^2 &&:= 159936012798720064 \\
39999200000^2 + 799992^2 &= 39999200008^2 &&:= 1599936001279987200064 \\
3999992000000^2 + 7999992^2 &= 3999992000008^2 &&:= 15999936000127999872000064
\end{aligned} \tag{295}
$$

► **Division by 8**

$$
\begin{aligned}
160^2 + 36^2 &= 164^2 \\
19600^2 + 396^2 &= 19604^2 \\
1996000^2 + 3996^2 &= 1996004^2 &&:= 3984031968016 \\
199960000^2 + 39996^2 &= 199960004^2 &&:= 39984003199680016 \\
19999600000^2 + 399996^2 &= 19999600004^2 &&:= 399984000319996800016 \\
1999996000000^2 + 3999996^2 &= 1999996000004^2 &&:= 3999984000031999968000016
\end{aligned} \tag{296}
$$

► **Division by 16**

$$
\begin{aligned}
80^2 + 18^2 &= 82^2 \\
9800^2 + 198^2 &= 9802^2 &&:= 96079204 \\
998000^2 + 1998^2 &= 998002^2 &&:= 996007992004 \\
99980000^2 + 19998^2 &= 99980002^2 &&:= 9996000799920004 \\
9999800000^2 + 199998^2 &= 9999800002^2 &&:= 99996000079999200004 \\
999998000000^2 + 1999998^2 &= 999998000002^2 &&:= 999996000007999992000004
\end{aligned} \tag{297}
$$

► **Division by 32**

$$
\begin{aligned}
40^2 + 9^2 &= 41^2 \\
4900^2 + 99^2 &= 4901^2 &&:= 24019801 \\
499000^2 + 999^2 &= 499001^2 &&:= 249001998001 \\
49990000^2 + 9999^2 &= 49990001^2 &&:= 2499000199980001 \\
4999900000^2 + 99999^2 &= 4999900001^2 &&:= 24999000019999800001 \\
499999000000^2 + 999999^2 &= 499999000001^2 &&:= 249999000001999998000001
\end{aligned} \tag{298}
$$

- **For $n = 5;\ m = 999, 9999, 99999, ...$ in (14):**

$$\begin{aligned}
156791^2 + 3960^2 &= 156841^2 \\
15967991^2 + 39960^2 &= 15968041^2 \\
1599679991^2 + 399960^2 &= 1599680041^2 &&:= 2558976233573761681 \\
159996799991^2 + 3999960^2 &= 159996800041^2 &&:= 25598976023359737601681 \\
15999967999991^2 + 39999960^2 &= 15999968000041^2 &&:= 255998976002335997376001681
\end{aligned} \tag{299}$$

The first triple $(1271, 360, 1321)$ for $n = 5;\ m = 9$ is not written above as it doesn't give good pattern.

# 4 Pandigital Palindromic-Type Patterns

## 4.1 Part I

Below are some examples of pattern in Pythagorean triples with palindromic type numbers having all the digits from 1 to 9.

- **For $n = 1;\ m = 55, 555, 5555, 55555, ...$ in (12):**

$$\begin{aligned}
099^2 + 20^2 &= \mathbf{1}\,01^2 \\
\mathbf{12}\,099^2 + 220^2 &= \mathbf{121}\,01^2 \\
\mathbf{1232}\,099^2 + 2220^2 &= \mathbf{12321}\,01^2 \\
\mathbf{123432}\,099^2 + 22220^2 &= \mathbf{1234321}\,01^2 \\
\mathbf{12345432}\,099^2 + 222220^2 &= \mathbf{123454321}\,01^2 \\
\mathbf{1234565432}\,099^2 + 2222220^2 &= \mathbf{12345654321}\,01^2 \\
\mathbf{123456765432}\,099^2 + 22222220^2 &= \mathbf{1234567654321}\,01^2 \\
\mathbf{12345678765432}\,099^2 + 222222220^2 &= \mathbf{123456787654321}\,01^2 \\
\mathbf{1234567898765432}\,099^2 + 2222222220^2 &= \mathbf{12345678987654321}\,01^2
\end{aligned} \tag{300}$$

Here we have total 9 examples os similar kind. See below more 8 examples:

- **For $n = 2;\ m = 55, 555, 5555, 55555, ...$ in (12) :**

$$\begin{aligned}
096^2 + 40^2 &= 1\,04^2\\
12\,096^2 + 440^2 &= 121\,04^2\\
1232\,096^2 + 4440^2 &= 12321\,04^2\\
123432\,096^2 + 44440^2 &= 1234321\,04^2\\
12345432\,096^2 + 444440^2 &= 123454321\,04^2\\
1234565432\,096^2 + 4444440^2 &= 12345654321\,04^2\\
123456765432\,096^2 + 44444440^2 &= 1234567654321\,04^2\\
12345678765432\,096^2 + 444444440^2 &= 123456787654321\,04^2\\
1234567898765432\,096^2 + 4444444440^2 &= 12345678987654321\,04^2 \qquad (301)
\end{aligned}$$

- **For $n = 3;\ m = 55, 555, 5555, 55555, ...$ in (12) :**

$$\begin{aligned}
091^2 + 60^2 &= 1\,09^2\\
12\,091^2 + 660^2 &= 121\,09^2\\
1232\,091^2 + 6660^2 &= 12321\,09^2\\
123432\,091^2 + 66660^2 &= 1234321\,09^2\\
12345432\,091^2 + 666660^2 &= 123454321\,09^2\\
1234565432\,091^2 + 6666660^2 &= 12345654321\,09^2\\
123456765432\,091^2 + 66666660^2 &= 1234567654321\,09^2\\
12345678765432\,091^2 + 666666660^2 &= 123456787654321\,09^2\\
1234567898765432\,091^2 + 6666666660^2 &= 12345678987654321\,09^2 \qquad (302)
\end{aligned}$$

- **For $n = 4;\ m = 55, 555, 5555, 55555, ...$ in (12) :**

$$\begin{aligned}
084^2 + 80^2 &= 1\,16^2\\
12\,084^2 + 880^2 &= 121\,16^2\\
1232\,084^2 + 8880^2 &= 12321\,16^2\\
123432\,084^2 + 88880^2 &= 1234321\,16^2\\
12345432\,084^2 + 888880^2 &= 123454321\,16^2\\
1234565432\,084^2 + 8888880^2 &= 12345654321\,16^2\\
123456765432\,084^2 + 88888880^2 &= 1234567654321\,16^2\\
12345678765432\,084^2 + 888888880^2 &= 123456787654321\,16^2\\
1234567898765432\,084^2 + 8888888880^2 &= 12345678987654321\,16^2 \qquad (303)
\end{aligned}$$

- **For $n = 5;\ m = 55, 555, 5555, 55555, ...$ in (12) :**

$$
\begin{aligned}
075^2 + 100^2 &= \mathbf{1}\,25^2\\
\mathbf{12}\,075^2 + 1100^2 &= \mathbf{121}\,25^2\\
\mathbf{1232}\,075^2 + 11100^2 &= \mathbf{12321}\,25^2\\
\mathbf{123432}\,075^2 + 111100^2 &= \mathbf{1234321}\,25^2\\
\mathbf{12345432}\,075^2 + 1111100^2 &= \mathbf{123454321}\,25^2\\
\mathbf{1234565432}\,075^2 + 11111100^2 &= \mathbf{12345654321}\,25^2\\
\mathbf{123456765432}\,075^2 + 111111100^2 &= \mathbf{1234567654321}\,25^2\\
\mathbf{12345678765432}\,075^2 + 1111111100^2 &= \mathbf{123456787654321}\,25^2\\
\mathbf{1234567898765432}\,075^2 + 11111111100^2 &= \mathbf{12345678987654321}\,25^2
\end{aligned} \tag{304}
$$

- **For $n = 6$; $m = 55, 555, 5555, 55555, ...$ in (12) :**

$$
\begin{aligned}
064^2 + 120^2 &= \mathbf{1}\,36^2\\
\mathbf{12}\,064^2 + 1320^2 &= \mathbf{121}\,36^2\\
\mathbf{1232}\,064^2 + 13320^2 &= \mathbf{12321}\,36^2\\
\mathbf{123432}\,064^2 + 133320^2 &= \mathbf{1234321}\,36^2\\
\mathbf{12345432}\,064^2 + 1333320^2 &= \mathbf{123454321}\,36^2\\
\mathbf{1234565432}\,064^2 + 13333320^2 &= \mathbf{12345654321}\,36^2\\
\mathbf{123456765432}\,064^2 + 133333320^2 &= \mathbf{1234567654321}\,36^2\\
\mathbf{12345678765432}\,064^2 + 1333333320^2 &= \mathbf{123456787654321}\,36^2\\
\mathbf{1234567898765432}\,064^2 + 13333333320^2 &= \mathbf{12345678987654321}\,36^2
\end{aligned} \tag{305}
$$

- **For $n = 7$; $m = 55, 555, 5555, 55555, ...$ in (12) :**

$$
\begin{aligned}
051^2 + 140^2 &= \mathbf{1}\,49^2\\
\mathbf{12}\,051^2 + 1540^2 &= \mathbf{121}\,49^2\\
\mathbf{1232}\,051^2 + 15540^2 &= \mathbf{12321}\,49^2\\
\mathbf{123432}\,051^2 + 155540^2 &= \mathbf{1234321}\,49^2\\
\mathbf{12345432}\,051^2 + 1555540^2 &= \mathbf{123454321}\,49^2\\
\mathbf{1234565432}\,051^2 + 15555540^2 &= \mathbf{12345654321}\,49^2\\
\mathbf{123456765432}\,051^2 + 155555540^2 &= \mathbf{1234567654321}\,49^2\\
\mathbf{12345678765432}\,051^2 + 1555555540^2 &= \mathbf{123456787654321}\,49^2\\
\mathbf{1234567898765432}\,051^2 + 15555555540^2 &= \mathbf{12345678987654321}\,49^2
\end{aligned} \tag{306}
$$

- **For $n = 8$; $m = 55, 555, 5555, 55555, ...$ in (12) :**

$$
\begin{aligned}
036^2 + 160^2 &= \mathbf{1}\,64^2\\
\mathbf{12}\,036^2 + 1760^2 &= \mathbf{121}\,64^2\\
\mathbf{1232}\,036^2 + 17760^2 &= \mathbf{12321}\,64^2\\
\mathbf{123432}\,036^2 + 177760^2 &= \mathbf{1234321}\,64^2\\
\mathbf{12345432}\,036^2 + 1777760^2 &= \mathbf{123454321}\,64^2\\
\mathbf{1234565432}\,036^2 + 17777760^2 &= \mathbf{12345654321}\,64^2\\
\mathbf{123456765432}\,036^2 + 177777760^2 &= \mathbf{1234567654321}\,64^2\\
\mathbf{12345678765432}\,036^2 + 1777777760^2 &= \mathbf{123456787654321}\,64^2\\
\mathbf{1234567898765432}\,036^2 + 17777777760^2 &= \mathbf{12345678987654321}\,64^2
\end{aligned} \tag{307}
$$

- **For $n = 9$; $m = 55, 555, 5555, 55555, ...$ in (12) :**

$$
\begin{aligned}
019^2 + 180^2 &= \mathbf{1}\,81^2\\
\mathbf{12}\,019^2 + 1980^2 &= \mathbf{121}\,81^2\\
\mathbf{1232}\,019^2 + 19980^2 &= \mathbf{12321}\,81^2\\
\mathbf{123432}\,019^2 + 199980^2 &= \mathbf{1234321}\,81^2\\
\mathbf{12345432}\,019^2 + 1999980^2 &= \mathbf{123454321}\,81^2\\
\mathbf{1234565432}\,019^2 + 19999980^2 &= \mathbf{12345654321}\,81^2\\
\mathbf{123456765432}\,019^2 + 199999980^2 &= \mathbf{1234567654321}\,81^2\\
\mathbf{12345678765432}\,019^2 + 1999999980^2 &= \mathbf{123456787654321}\,81^2\\
\mathbf{1234567898765432}\,019^2 + 19999999980^2 &= \mathbf{12345678987654321}\,81^2
\end{aligned} \tag{308}
$$

For extended study of above type of patterns refer to author's wrok [6].

**Remark 3.** *We observe the last number in each row is a perfect square, for example, i.e.,*

$$01 = 1^2,\ 04 = 2^2,\ 09 = 3^2,\ \ldots,\ 81 = 9^2.$$

*Moreover, the sum of last two digits of first and third rows are always 100. See below:*

► ***Sum of Last Two Digits: First and Third Rows***

$$
\begin{aligned}
n = 1 &\quad\Rightarrow\quad 99 + 01 = 100\\
n = 2 &\quad\Rightarrow\quad 96 + 04 = 100\\
n = 3 &\quad\Rightarrow\quad 91 + 09 = 100\\
n = 4 &\quad\Rightarrow\quad 84 + 16 = 100\\
n = 5 &\quad\Rightarrow\quad 75 + 25 = 100\\
n = 6 &\quad\Rightarrow\quad 64 + 36 = 100\\
n = 7 &\quad\Rightarrow\quad 51 + 49 = 100\\
n = 8 &\quad\Rightarrow\quad 19 + 64 = 100\\
n = 9 &\quad\Rightarrow\quad 19 + 81 = 100
\end{aligned}
$$

## 4.2 Part II

Below are some examples of pattern in Pythagorean triples with palindromic type numbers having all the digits from 1 to 9 with 0 in between.

- **For $n = 1$; $m = 10, 1010, 101010, 10101010, 1010101010, \ldots$ in (10):**

$$
\begin{aligned}
0099^2 + 20^2 &= 1\,01^2\\
102\,0099^2 + 2020^2 &= 10201\,01^2\\
1020302\,0099^2 + 202020^2 &= 102030201\,01^2\\
10203040302\,0099^2 + 20202020^2 &= 1020304030201\,01^2\\
102030405040302\,0099^2 + 2020202020^2 &= 10203040504030201\,01^2\\
1020304050605040302\,0099^2 + 202020202020^2 &= 102030405060504030201\,01^2\\
10203040506070605040302\,0099^2 + 20202020202020^2 &= 1020304050607060504030201\,01^2\\
102030405060708070605040302\,0099^2 + 2020202020202020^2 &= 10203040506070807060504030201\,01^2\\
1020304050607080908070605040302\,0099^2 + 202020202020202020^2 &= 102030405060708090807060504030201\,01^2
\end{aligned}
\tag{309}
$$

- **For $n = 2$; $m = 10, 1010, 101010, 10101010, 1010101010, \ldots$ in (10):**

$$
\begin{aligned}
0096^2 + 40^2 &= 1\,04^2\\
102\,0096^2 + 4040^2 &= 10201\,04^2\\
1020302\,0096^2 + 404040^2 &= 102030201\,04^2\\
10203040302\,0096^2 + 40404040^2 &= 1020304030201\,04^2\\
102030405040302\,0096^2 + 4040404040^2 &= 10203040504030201\,04^2\\
1020304050605040302\,0096^2 + 404040404040^2 &= 102030405060504030201\,04^2\\
10203040506070605040302\,0096^2 + 40404040404040^2 &= 1020304050607060504030201\,04^2\\
102030405060708070605040302\,0096^2 + 4040404040404040^2 &= 10203040506070807060504030201\,04^2\\
1020304050607080908070605040302\,0096^2 + 404040404040404040^2 &= 102030405060708090807060504030201\,04^2
\end{aligned}
\tag{310}
$$

- **For $n = 3$; $m = 10, 1010, 101010, 10101010, 1010101010, \ldots$ in (10):**

$$
\begin{aligned}
0091^2 + 60^2 &= 1\,09^2\\
102\,0091^2 + 6060^2 &= 10201\,09^2\\
1020302\,0091^2 + 606060^2 &= 102030201\,09^2\\
10203040302\,0091^2 + 60606060^2 &= 1020304030201\,09^2\\
102030405040302\,0091^2 + 6060606060^2 &= 10203040504030201\,09^2\\
1020304050605040302\,0091^2 + 606060606060^2 &= 102030405060504030201\,09^2\\
10203040506070605040302\,0091^2 + 60606060606060^2 &= 1020304050607060504030201\,09^2\\
102030405060708070605040302\,0091^2 + 6060606060606060^2 &= 10203040506070807060504030201\,09^2\\
1020304050607080908070605040302\,0091^2 + 606060606060606060^2 &= 102030405060708090807060504030201\,09^2
\end{aligned}
\tag{311}
$$

- **For $n = 4$; $m = 10, 1010, 101010, 10101010, 1010101010, ...$ in (10):**

$$\begin{aligned}
0084^2 + 80^2 &= \mathbf{1}\,16^2\\
\mathbf{102}\,0084^2 + 8080^2 &= \mathbf{10201}\,16^2\\
\mathbf{1020302}\,0084^2 + 808080^2 &= \mathbf{102030201}\,16^2\\
\mathbf{10203040302}\,0084^2 + 80808080^2 &= \mathbf{1020304030201}\,16^2\\
\mathbf{102030405040302}\,0084^2 + 8080808080^2 &= \mathbf{10203040504030201}\,16^2\\
\mathbf{1020304050605040302}\,0084^2 + 808080808080^2 &= \mathbf{102030405060504030201}\,16^2\\
\mathbf{10203040506070605040302}\,0084^2 + 80808080808080^2 &= \mathbf{1020304050607060504030201}\,16^2\\
\mathbf{102030405060708070605040302}\,0084^2 + 8080808080808080^2 &= \mathbf{10203040506070807060504030201}\,16^2\\
\mathbf{1020304050607080908070605040302}\,0084^2 + 808080808080808080^2 &= \mathbf{102030405060708090807060504030201}\,16^2
\end{aligned} \tag{312}$$

- **For $n = 5$; $m = 10, 1010, 101010, 10101010, 1010101010, ...$ in (10):**

$$\begin{aligned}
0075^2 + 100^2 &= \mathbf{1}\,25^2\\
\mathbf{102}\,0075^2 + 10100^2 &= \mathbf{10201}\,25^2\\
\mathbf{1020302}\,0075^2 + 1010100^2 &= \mathbf{102030201}\,25^2\\
\mathbf{10203040302}\,0075^2 + 101010100^2 &= \mathbf{1020304030201}\,25^2\\
\mathbf{102030405040302}\,0075^2 + 10101010100^2 &= \mathbf{10203040504030201}\,25^2\\
\mathbf{1020304050605040302}\,0075^2 + 1010101010100^2 &= \mathbf{102030405060504030201}\,25^2\\
\mathbf{10203040506070605040302}\,0075^2 + 101010101010100^2 &= \mathbf{1020304050607060504030201}\,25^2\\
\mathbf{102030405060708070605040302}\,0075^2 + 10101010101010100^2 &= \mathbf{10203040506070807060504030201}\,25^2\\
\mathbf{1020304050607080908070605040302}\,0075^2 + 1010101010101010100^2 &= \mathbf{102030405060708090807060504030201}\,25^2
\end{aligned} \tag{313}$$

- **For $n = 6$; $m = 10, 1010, 101010, 10101010, 1010101010, ...$ in (10):**

$$\begin{aligned}
0064^2 + 120^2 &= \mathbf{1}\,36^2\\
\mathbf{102}\,0064^2 + 12120^2 &= \mathbf{10201}\,36^2\\
\mathbf{1020302}\,0064^2 + 1212120^2 &= \mathbf{102030201}\,36^2\\
\mathbf{10203040302}\,0064^2 + 121212120^2 &= \mathbf{1020304030201}\,36^2\\
\mathbf{102030405040302}\,0064^2 + 12121212120^2 &= \mathbf{10203040504030201}\,36^2\\
\mathbf{1020304050605040302}\,0064^2 + 1212121212120^2 &= \mathbf{102030405060504030201}\,36^2\\
\mathbf{10203040506070605040302}\,0064^2 + 121212121212120^2 &= \mathbf{1020304050607060504030201}\,36^2\\
\mathbf{102030405060708070605040302}\,0064^2 + 12121212121212120^2 &= \mathbf{10203040506070807060504030201}\,36^2\\
\mathbf{1020304050607080908070605040302}\,0064^2 + 1212121212121212120^2 &= \mathbf{102030405060708090807060504030201}\,36^2
\end{aligned} \tag{314}$$

- **For $n = 7$; $m = 10, 1010, 101010, 10101010, 1010101010, ...$ in (10):**

$$
\begin{aligned}
0051^2 + 140^2 &= \mathbf{1}\,49^2\\
\mathbf{102}\,0051^2 + 14140^2 &= \mathbf{10201}\,49^2\\
\mathbf{1020302}\,0051^2 + 1414140^2 &= \mathbf{102030201}\,49^2\\
\mathbf{10203040302}\,0051^2 + 141414140^2 &= \mathbf{1020304030201}\,49^2\\
\mathbf{102030405040302}\,0051^2 + 14141414140^2 &= \mathbf{10203040504030201}\,49^2\\
\mathbf{1020304050605040302}\,0051^2 + 1414141414140^2 &= \mathbf{102030405060504030201}\,49^2\\
\mathbf{10203040506070605040302}\,0051^2 + 141414141414140^2 &= \mathbf{1020304050607060504030201}\,49^2\\
\mathbf{102030405060708070605040302}\,0051^2 + 14141414141414140^2 &= \mathbf{10203040506070807060504030201}\,49^2\\
\mathbf{1020304050607080908070605040302}\,0051^2 + 1414141414141414140^2 &= \mathbf{102030405060708090807060504030201}\,49^2
\end{aligned}
\tag{315}
$$

- **For $n = 8$; $m = 10, 1010, 101010, 10101010, 1010101010, \ldots$ in (10):**

$$
\begin{aligned}
0036^2 + 160^2 &= \mathbf{1}\,64^2\\
\mathbf{102}\,0036^2 + 16160^2 &= \mathbf{10201}\,64^2\\
\mathbf{1020302}\,0036^2 + 1616160^2 &= \mathbf{102030201}\,64^2\\
\mathbf{10203040302}\,0036^2 + 161616160^2 &= \mathbf{1020304030201}\,64^2\\
\mathbf{102030405040302}\,0036^2 + 16161616160^2 &= \mathbf{10203040504030201}\,64^2\\
\mathbf{1020304050605040302}\,0036^2 + 1616161616160^2 &= \mathbf{102030405060504030201}\,64^2\\
\mathbf{10203040506070605040302}\,0036^2 + 161616161616160^2 &= \mathbf{1020304050607060504030201}\,64^2\\
\mathbf{102030405060708070605040302}\,0036^2 + 16161616161616160^2 &= \mathbf{10203040506070807060504030201}\,64^2\\
\mathbf{1020304050607080908070605040302}\,0036^2 + 1616161616161616160^2 &= \mathbf{102030405060708090807060504030201}\,64^2
\end{aligned}
\tag{316}
$$

- **For $n = 9$; $m = 10, 1010, 101010, 10101010, 1010101010, \ldots$ in (10):**

$$
\begin{aligned}
0019^2 + 180^2 &= \mathbf{1}\,81^2\\
\mathbf{102}\,0019^2 + 18180^2 &= \mathbf{10201}\,81^2\\
\mathbf{1020302}\,0019^2 + 1818180^2 &= \mathbf{102030201}\,81^2\\
\mathbf{10203040302}\,0019^2 + 181818180^2 &= \mathbf{1020304030201}\,81^2\\
\mathbf{102030405040302}\,0019^2 + 18181818180^2 &= \mathbf{10203040504030201}\,81^2\\
\mathbf{1020304050605040302}\,0019^2 + 1818181818180^2 &= \mathbf{102030405060504030201}\,81^2\\
\mathbf{10203040506070605040302}\,0019^2 + 181818181818180^2 &= \mathbf{1020304050607060504030201}\,81^2\\
\mathbf{102030405060708070605040302}\,0019^2 + 18181818181818180^2 &= \mathbf{10203040506070807060504030201}\,81^2\\
\mathbf{1020304050607080908070605040302}\,0019^2 + 1818181818181818180^2 &= \mathbf{102030405060708090807060504030201}\,81^2
\end{aligned}
\tag{317}
$$

For extended study of above type of pattern refer to author's wrok [7].

**Remark 4.** *We observe the last number in each row is a perfect square, for example, i.e.,*

$$01 = 1^2,\ 04 = 2^2,\ 09 = 3^2,\ \ldots,\ 81 = 9^2.$$

*Moreover, the sum of last two digits of first and third rows are always 100. See below:*

► ***Sum of Last Two Digits: First and Third Rows***

$$
\begin{aligned}
n=1 &\quad\Rightarrow\quad 99+01=100\\
n=2 &\quad\Rightarrow\quad 96+04=100\\
n=3 &\quad\Rightarrow\quad 91+09=100\\
n=4 &\quad\Rightarrow\quad 84+16=100\\
n=5 &\quad\Rightarrow\quad 75+25=100\\
n=6 &\quad\Rightarrow\quad 64+36=100\\
n=7 &\quad\Rightarrow\quad 51+49=100\\
n=8 &\quad\Rightarrow\quad 19+64=100\\
n=9 &\quad\Rightarrow\quad 19+81=100
\end{aligned}
$$

## 4.3 Part III

This subsection brings pandigial palindromic-type patterns similar to one given in subsection 4.2. Only difference is in fixed values. Moreover, in this case can go up to 99 patterns instead of 9.

- **For $n=1$; $m=25, 2525, 252525, 25252525, \ldots$ in (14):**

$$
\begin{aligned}
009999^2+200^2 &= \mathbf{1}\,0001^2\\
\mathbf{102}\,009999^2+20200^2 &= \mathbf{10201}\,0001^2\\
\mathbf{1020302}\,009999^2+2020200^2 &= \mathbf{102030201}\,0001^2\\
\mathbf{10203040302}\,009999^2+202020200^2 &= \mathbf{1020304030201}\,0001^2\\
\mathbf{102030405040302}\,009999^2+20202020200^2 &= \mathbf{10203040504030201}\,0001^2\\
\mathbf{1020304050605040302}\,009999^2+2020202020200^2 &= \mathbf{102030405060504030201}\,0001^2\\
\mathbf{10203040506070605040302}\,009999^2+202020202020200^2 &= \mathbf{1020304050607060504030201}\,0001^2\\
\mathbf{102030405060708070605040302}\,009999^2+20202020202020200^2 &= \mathbf{10203040506070807060504030201}\,0001^2\\
\mathbf{1020304050607080908070605040302}\,009999^2+2020202020202020200^2 &= \mathbf{102030405060708090807060504030201}\,0001^2
\end{aligned}
\tag{318}
$$

- **For $n=2$; $m=25, 2525, 252525, 25252525, \ldots$ in (14):**

$$
\begin{aligned}
009996^2+400^2 &= \mathbf{1}\,0004^2\\
\mathbf{102}\,009996^2+40400^2 &= \mathbf{10201}\,0004^2\\
\mathbf{1020302}\,009996^2+4040400^2 &= \mathbf{102030201}\,0004^2\\
\mathbf{10203040302}\,009996^2+404040400^2 &= \mathbf{1020304030201}\,0004^2\\
\mathbf{102030405040302}\,009996^2+40404040400^2 &= \mathbf{10203040504030201}\,0004^2\\
\mathbf{1020304050605040302}\,009996^2+4040404040400^2 &= \mathbf{102030405060504030201}\,0004^2\\
\mathbf{10203040506070605040302}\,009996^2+404040404040400^2 &= \mathbf{1020304050607060504030201}\,0004^2\\
\mathbf{102030405060708070605040302}\,009996^2+40404040404040400^2 &= \mathbf{10203040506070807060504030201}\,0004^2\\
\mathbf{1020304050607080908070605040302}\,009996^2+4040404040404040400^2 &= \mathbf{102030405060708090807060504030201}\,0004^2
\end{aligned}
\tag{319}
$$

- **For $n = 3;\ m = 25, 2525, 252525, 25252525, \ldots$ in (14):**

$$
\begin{aligned}
009991^2 + 600^2 &= \mathbf{1}\,0009^2\\
\mathbf{102}\,009991^2 + 60600^2 &= \mathbf{10201}\,0009^2\\
\mathbf{1020302}\,009991^2 + 6060600^2 &= \mathbf{102030201}\,0009^2\\
\mathbf{10203040302}\,009991^2 + 606060600^2 &= \mathbf{1020304030201}\,0009^2\\
\mathbf{102030405040302}\,009991^2 + 60606060600^2 &= \mathbf{10203040504030201}\,0009^2\\
\mathbf{1020304050605040302}\,009991^2 + 6060606060600^2 &= \mathbf{102030405060504030201}\,0009^2\\
\mathbf{10203040506070605040302}\,009991^2 + 606060606060600^2 &= \mathbf{1020304050607060504030201}\,0009^2\\
\mathbf{102030405060708070605040302}\,009991^2 + 60606060606060600^2 &= \mathbf{10203040506070807060504030201}\,0009^2\\
\mathbf{1020304050607080908070605040302}\,009991^2 + 6060606060606060600^2 &= \mathbf{102030405060708090807060504030201}\,0009^2
\end{aligned}
\tag{320}
$$

- **For $n = 4;\ m = 25, 2525, 252525, 25252525, \ldots$ in (14):**

$$
\begin{aligned}
009984^2 + 800^2 &= \mathbf{1}\,0016^2\\
\mathbf{102}\,009984^2 + 80800^2 &= \mathbf{10201}\,0016^2\\
\mathbf{1020302}\,009984^2 + 8080800^2 &= \mathbf{102030201}\,0016^2\\
\mathbf{10203040302}\,009984^2 + 808080800^2 &= \mathbf{1020304030201}\,0016^2\\
\mathbf{102030405040302}\,009984^2 + 80808080800^2 &= \mathbf{10203040504030201}\,0016^2\\
\mathbf{1020304050605040302}\,009984^2 + 8080808080800^2 &= \mathbf{102030405060504030201}\,0016^2\\
\mathbf{10203040506070605040302}\,009984^2 + 808080808080800^2 &= \mathbf{1020304050607060504030201}\,0016^2\\
\mathbf{102030405060708070605040302}\,009984^2 + 80808080808080800^2 &= \mathbf{10203040506070807060504030201}\,0016^2\\
\mathbf{1020304050607080908070605040302}\,009984^2 + 8080808080808080800^2 &= \mathbf{102030405060708090807060504030201}\,0016^2
\end{aligned}
\tag{321}
$$

- **For $n = 5;\ m = 25, 2525, 252525, 25252525, \ldots$ in (14):**

$$
\begin{aligned}
009975^2 + 1000^2 &= \mathbf{1}\,0025^2\\
\mathbf{102}\,009975^2 + 10100^2 &= \mathbf{10201}\,0025^2\\
\mathbf{1020302}\,009975^2 + 1010100^2 &= \mathbf{102030201}\,0025^2\\
\mathbf{10203040302}\,009975^2 + 101010100^2 &= \mathbf{1020304030201}\,0025^2\\
\mathbf{102030405040302}\,009975^2 + 10101010100^2 &= \mathbf{10203040504030201}\,0025^2\\
\mathbf{1020304050605040302}\,009975^2 + 1010101010100^2 &= \mathbf{102030405060504030201}\,0025^2\\
\mathbf{10203040506070605040302}\,009975^2 + 101010101010100^2 &= \mathbf{1020304050607060504030201}\,0025^2\\
\mathbf{102030405060708070605040302}\,009975^2 + 10101010101010100^2 &= \mathbf{10203040506070807060504030201}\,0025^2\\
\mathbf{1020304050607080908070605040302}\,009975^2 + 1010101010101010100^2 &= \mathbf{102030405060708090807060504030201}\,0025^2
\end{aligned}
\tag{322}
$$

- **For $n = 6;\ m = 25, 2525, 252525, 25252525, \ldots$ in (14):**

$$
\begin{aligned}
009964^2 + 1200^2 &= 1\,0036^2\\
102\,009964^2 + 121200^2 &= 10201\,0036^2\\
1020302\,009964^2 + 12121200^2 &= 102030201\,0036^2\\
10203040302\,009964^2 + 1212121200^2 &= 1020304030201\,0036^2\\
102030405040302\,009964^2 + 121212121200^2 &= 10203040504030201\,0036^2\\
1020304050605040302\,009964^2 + 12121212121200^2 &= 102030405060504030201\,0036^2\\
10203040506070605040302\,009964^2 + 1212121212121200^2 &= 1020304050607060504030201\,0036^2\\
102030405060708070605040302\,009964^2 + 121212121212121200^2 &= 10203040506070807060504030201\,0036^2\\
1020304050607080908070605040302\,009964^2 + 12121212121212121200^2 &= 102030405060708090807060504030201\,0036^2
\end{aligned} \tag{323}
$$

- **For $n = 7$; $m = 25, 2525, 252525, 25252525, ...$ in (14):**

$$
\begin{aligned}
009951^2 + 1400^2 &= 1\,0049^2\\
102\,009951^2 + 141400^2 &= 10201\,0049^2\\
1020302\,009951^2 + 14141400^2 &= 102030201\,0049^2\\
10203040302\,009951^2 + 1414141400^2 &= 1020304030201\,0049^2\\
102030405040302\,009951^2 + 141414141400^2 &= 10203040504030201\,0049^2\\
1020304050605040302\,009951^2 + 14141414141400^2 &= 102030405060504030201\,0049^2\\
10203040506070605040302\,009951^2 + 1414141414141400^2 &= 1020304050607060504030201\,0049^2\\
102030405060708070605040302\,009951^2 + 141414141414141400^2 &= 10203040506070807060504030201\,0049^2\\
1020304050607080908070605040302\,009951^2 + 14141414141414141400^2 &= 102030405060708090807060504030201\,0049^2
\end{aligned} \tag{324}
$$

- **For $n = 8$; $m = 25, 2525, 252525, 25252525, ...$ in (14):**

$$
\begin{aligned}
009936^2 + 1600^2 &= 1\,0064^2\\
102\,009936^2 + 161600^2 &= 10201\,0064^2\\
1020302\,009936^2 + 16161600^2 &= 102030201\,0064^2\\
10203040302\,009936^2 + 1616161600^2 &= 1020304030201\,0064^2\\
102030405040302\,009936^2 + 161616161600^2 &= 10203040504030201\,0064^2\\
1020304050605040302\,009936^2 + 16161616161600^2 &= 102030405060504030201\,0064^2\\
10203040506070605040302\,009936^2 + 1616161616161600^2 &= 1020304050607060504030201\,0064^2\\
102030405060708070605040302\,009936^2 + 161616161616161600^2 &= 10203040506070807060504030201\,0064^2\\
1020304050607080908070605040302\,009936^2 + 16161616161616161600^2 &= 102030405060708090807060504030201\,0064^2
\end{aligned} \tag{325}
$$

- **For $n = 9$; $m = 25, 2525, 252525, 25252525, ...$ in (14):**

$$
\begin{aligned}
009919^2 + 1800^2 &= \mathbf{1}\,0081^2\\
\mathbf{102}\,009919^2 + 181800^2 &= \mathbf{10201}\,0081^2\\
\mathbf{1020302}\,009919^2 + 18181800^2 &= \mathbf{102030201}\,0081^2\\
\mathbf{10203040302}\,009919^2 + 1818181800^2 &= \mathbf{1020304030201}\,0081^2\\
\mathbf{102030405040302}\,009919^2 + 181818181800^2 &= \mathbf{10203040504030201}\,0081^2\\
\mathbf{1020304050605040302}\,009919^2 + 18181818181800^2 &= \mathbf{102030405060504030201}\,0081^2\\
\mathbf{10203040506070605040302}\,009919^2 + 1818181818181800^2 &= \mathbf{1020304050607060504030201}\,0081^2\\
\mathbf{102030405060708070605040302}\,009919^2 + 181818181818181800^2 &= \mathbf{10203040506070807060504030201}\,0081^2\\
\mathbf{1020304050607080908070605040302}\,009919^2 + 18181818181818181800^2 &= \mathbf{102030405060708090807060504030201}\,0081^2
\end{aligned} \tag{326}
$$

- **For $n = 10$; $m = 25, 2525, 252525, 25252525, \ldots$ in (14):**

$$
\begin{aligned}
009900^2 + 2000^2 &= \mathbf{1}\,0100^2\\
\mathbf{102}\,009900^2 + 202000^2 &= \mathbf{10201}\,0100^2\\
\mathbf{1020302}\,009900^2 + 20202000^2 &= \mathbf{102030201}\,0100^2\\
\mathbf{10203040302}\,009900^2 + 2020202000^2 &= \mathbf{1020304030201}\,0100^2\\
\mathbf{102030405040302}\,009900^2 + 202020202000^2 &= \mathbf{10203040504030201}\,0100^2\\
\mathbf{1020304050605040302}\,009900^2 + 20202020202000^2 &= \mathbf{102030405060504030201}\,0100^2\\
\mathbf{10203040506070605040302}\,009900^2 + 2020202020202000^2 &= \mathbf{1020304050607060504030201}\,0100^2\\
\mathbf{102030405060708070605040302}\,009900^2 + 202020202020202000^2 &= \mathbf{10203040506070807060504030201}\,0100^2\\
\mathbf{1020304050607080908070605040302}\,009900^2 + 20202020202020202000^2 &= \mathbf{102030405060708090807060504030201}\,0100^2
\end{aligned} \tag{327}
$$

- **For $n = 11$; $m = 25, 2525, 252525, 25252525, \ldots$ in (14):**

$$
\begin{aligned}
009879^2 + 2200^2 &= \mathbf{1}\,0121^2\\
\mathbf{102}\,009879^2 + 222200^2 &= \mathbf{10201}\,0121^2\\
\mathbf{1020302}\,009879^2 + 22222200^2 &= \mathbf{102030201}\,0121^2\\
\mathbf{10203040302}\,009879^2 + 2222222200^2 &= \mathbf{1020304030201}\,0121^2\\
\mathbf{102030405040302}\,009879^2 + 222222222200^2 &= \mathbf{10203040504030201}\,0121^2\\
\mathbf{1020304050605040302}\,009879^2 + 22222222222200^2 &= \mathbf{102030405060504030201}\,0121^2\\
\mathbf{10203040506070605040302}\,009879^2 + 2222222222222200^2 &= \mathbf{1020304050607060504030201}\,0121^2\\
\mathbf{102030405060708070605040302}\,009879^2 + 222222222222222200^2 &= \mathbf{10203040506070807060504030201}\,0121^2\\
\mathbf{1020304050607080908070605040302}\,009879^2 + 22222222222222222200^2 &= \mathbf{102030405060708090807060504030201}\,0121^2
\end{aligned} \tag{328}
$$

- **For $n = 12$; $m = 25, 2525, 252525, 25252525, \ldots$ in (14):**

$$
\begin{aligned}
009856^2 + 2400^2 &= \mathbf{1}\,0144^2 \\
\mathbf{102}\,009856^2 + 242400^2 &= \mathbf{10201}\,0144^2 \\
\mathbf{1020302}\,009856^2 + 24242400^2 &= \mathbf{102030201}\,0144^2 \\
\mathbf{10203040302}\,009856^2 + 2424242400^2 &= \mathbf{1020304030201}\,0144^2 \\
\mathbf{102030405040302}\,009856^2 + 242424242400^2 &= \mathbf{10203040504030201}\,0144^2 \\
\mathbf{1020304050605040302}\,009856^2 + 24242424242400^2 &= \mathbf{102030405060504030201}\,0144^2 \\
\mathbf{10203040506070605040302}\,009856^2 + 2424242424242400^2 &= \mathbf{1020304050607060504030201}\,0144^2 \\
\mathbf{102030405060708070605040302}\,009856^2 + 242424242424242400^2 &= \mathbf{10203040506070807060504030201}\,0144^2 \\
\mathbf{1020304050607080908070605040302}\,009856^2 + 24242424242424242400^2 &= \mathbf{102030405060708090807060504030201}\,0144^2
\end{aligned}
\tag{329}
$$

- **For $n = 13$; $m = 25, 2525, 252525, 25252525, ...$ in (14):**

$$
\begin{aligned}
009831^2 + 2600^2 &= \mathbf{1}\,0169^2 \\
\mathbf{102}\,009831^2 + 262600^2 &= \mathbf{10201}\,0169^2 \\
\mathbf{1020302}\,009831^2 + 26262600^2 &= \mathbf{102030201}\,0169^2 \\
\mathbf{10203040302}\,009831^2 + 2626262600^2 &= \mathbf{1020304030201}\,0169^2 \\
\mathbf{102030405040302}\,009831^2 + 262626262600^2 &= \mathbf{10203040504030201}\,0169^2 \\
\mathbf{1020304050605040302}\,009831^2 + 26262626262600^2 &= \mathbf{102030405060504030201}\,0169^2 \\
\mathbf{10203040506070605040302}\,009831^2 + 2626262626262600^2 &= \mathbf{1020304050607060504030201}\,0169^2 \\
\mathbf{102030405060708070605040302}\,009831^2 + 262626262626262600^2 &= \mathbf{10203040506070807060504030201}\,0169^2 \\
\mathbf{1020304050607080908070605040302}\,009831^2 + 26262626262626262600^2 &= \mathbf{102030405060708090807060504030201}\,0169^2
\end{aligned}
\tag{330}
$$

- **For $n = 14$; $m = 25, 2525, 252525, 25252525, ...$ in (14):**

$$
\begin{aligned}
009804^2 + 2800^2 &= \mathbf{1}\,0196^2 \\
\mathbf{102}\,009804^2 + 282800^2 &= \mathbf{10201}\,0196^2 \\
\mathbf{1020302}\,009804^2 + 28282800^2 &= \mathbf{102030201}\,0196^2 \\
\mathbf{10203040302}\,009804^2 + 2828282800^2 &= \mathbf{1020304030201}\,0196^2 \\
\mathbf{102030405040302}\,009804^2 + 282828282800^2 &= \mathbf{10203040504030201}\,0196^2 \\
\mathbf{1020304050605040302}\,009804^2 + 28282828282800^2 &= \mathbf{102030405060504030201}\,0196^2 \\
\mathbf{10203040506070605040302}\,009804^2 + 2828282828282800^2 &= \mathbf{1020304050607060504030201}\,0196^2 \\
\mathbf{102030405060708070605040302}\,009804^2 + 282828282828282800^2 &= \mathbf{10203040506070807060504030201}\,0196^2 \\
\mathbf{1020304050607080908070605040302}\,009804^2 + 28282828282828282800^2 &= \mathbf{102030405060708090807060504030201}\,0196^2
\end{aligned}
\tag{331}
$$

- **For $n = 15$; $m = 25, 2525, 252525, 25252525, ...$ in (14):**

$$
\begin{aligned}
009775^2 + 3000^2 &= 1\,0225^2\\
102\,009775^2 + 303000^2 &= 10201\,0225^2\\
1020302\,009775^2 + 30303000^2 &= 102030201\,0225^2\\
10203040302\,009775^2 + 3030303000^2 &= 1020304030201\,0225^2\\
102030405040302\,009775^2 + 303030303000^2 &= 10203040504030201\,0225^2\\
1020304050605040302\,009775^2 + 30303030303000^2 &= 102030405060504030201\,0225^2\\
10203040506070605040302\,009775^2 + 3030303030303000^2 &= 1020304050607060504030201\,0225^2\\
102030405060708070605040302\,009775^2 + 303030303030303000^2 &= 10203040506070807060504030201\,0225^2\\
1020304050607080908070605040302\,009775^2 + 30303030303030303000^2 &= 102030405060708090807060504030201\,0225^2
\end{aligned} \tag{332}
$$

- **For $n = 16$; $m = 25, 2525, 252525, 25252525, ...$ in (14):**

$$
\begin{aligned}
009744^2 + 3200^2 &= 1\,0256^2\\
102\,009744^2 + 323200^2 &= 10201\,0256^2\\
1020302\,009744^2 + 32323200^2 &= 102030201\,0256^2\\
10203040302\,009744^2 + 3232323200^2 &= 1020304030201\,0256^2\\
102030405040302\,009744^2 + 323232323200^2 &= 10203040504030201\,0256^2\\
1020304050605040302\,009744^2 + 32323232323200^2 &= 102030405060504030201\,0256^2\\
10203040506070605040302\,009744^2 + 3232323232323200^2 &= 1020304050607060504030201\,0256^2\\
102030405060708070605040302\,009744^2 + 323232323232323200^2 &= 10203040506070807060504030201\,0256^2\\
1020304050607080908070605040302\,009744^2 + 32323232323232323200^2 &= 102030405060708090807060504030201\,0256^2
\end{aligned} \tag{333}
$$

- **For $n = 17$; $m = 25, 2525, 252525, 25252525, ...$ in (14):**

$$
\begin{aligned}
009711^2 + 3400^2 &= 1\,0289^2\\
102\,009711^2 + 343400^2 &= 10201\,0289^2\\
1020302\,009711^2 + 34343400^2 &= 102030201\,0289^2\\
10203040302\,009711^2 + 3434343400^2 &= 1020304030201\,0289^2\\
102030405040302\,009711^2 + 343434343400^2 &= 10203040504030201\,0289^2\\
1020304050605040302\,009711^2 + 34343434343400^2 &= 102030405060504030201\,0289^2\\
10203040506070605040302\,009711^2 + 3434343434343400^2 &= 1020304050607060504030201\,0289^2\\
102030405060708070605040302\,009711^2 + 343434343434343400^2 &= 10203040506070807060504030201\,0289^2\\
1020304050607080908070605040302\,009711^2 + 34343434343434343400^2 &= 102030405060708090807060504030201\,0289^2
\end{aligned} \tag{334}
$$

- **For $n = 18$; $m = 25, 2525, 252525, 25252525, ...$ in (14):**

$$\begin{aligned}
009676^2+3600^2 &= 1\,0324^2\\
102\,009676^2+363600^2 &= 10201\,0324^2\\
1020302\,009676^2+36363600^2 &= 102030201\,0324^2\\
10203040302\,009676^2+3636363600^2 &= 1020304030201\,0324^2\\
102030405040302\,009676^2+363636363600^2 &= 10203040504030201\,0324^2\\
1020304050605040302\,009676^2+36363636363600^2 &= 102030405060504030201\,0324^2\\
10203040506070605040302\,009676^2+3636363636363600^2 &= 1020304050607060504030201\,0324^2\\
102030405060708070605040302\,009676^2+363636363636363600^2 &= 10203040506070807060504030201\,0324^2\\
1020304050607080908070605040302\,009676^2+36363636363636363600^2 &= 102030405060708090807060504030201\,0324^2
\end{aligned} \tag{335}$$

- **For $n=19$; $m=25,2525,252525,25252525,...$ in (14):**

$$\begin{aligned}
009639^2+3800^2 &= 1\,0361^2\\
102\,009639^2+383800^2 &= 10201\,0361^2\\
1020302\,009639^2+38383800^2 &= 102030201\,0361^2\\
10203040302\,009639^2+3838383800^2 &= 1020304030201\,0361^2\\
102030405040302\,009639^2+383838383800^2 &= 10203040504030201\,0361^2\\
1020304050605040302\,009639^2+38383838383800^2 &= 102030405060504030201\,0361^2\\
10203040506070605040302\,009639^2+3838383838383800^2 &= 1020304050607060504030201\,0361^2\\
102030405060708070605040302\,009639^2+383838383838383800^2 &= 10203040506070807060504030201\,0361^2\\
1020304050607080908070605040302\,009639^2+38383838383838383800^2 &= 102030405060708090807060504030201\,0361^2
\end{aligned} \tag{336}$$

- **For $n=20$; $m=25,2525,252525,25252525,...$ in (14):**

$$\begin{aligned}
009600^2+4000^2 &= 1\,0400^2\\
102\,009600^2+404000^2 &= 10201\,0400^2\\
1020302\,009600^2+40404000^2 &= 102030201\,0400^2\\
10203040302\,009600^2+4040404000^2 &= 1020304030201\,0400^2\\
102030405040302\,009600^2+404040404000^2 &= 10203040504030201\,0400^2\\
1020304050605040302\,009600^2+40404040404000^2 &= 102030405060504030201\,0400^2\\
10203040506070605040302\,009600^2+4040404040404000^2 &= 1020304050607060504030201\,0400^2\\
102030405060708070605040302\,009600^2+404040404040404000^2 &= 10203040506070807060504030201\,0400^2\\
1020304050607080908070605040302\,009600^2+40404040404040404000^2 &= 102030405060708090807060504030201\,0400^2
\end{aligned} \tag{337}$$

$$\begin{array}{cccccccccccc}
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{array}$$

- **For $n=99$; $m=25,2525,252525,25252525,...$ in (14):**

$$
\begin{aligned}
000199^2 + 19800^2 &= \mathbf{1}\,9801^2\\
\mathbf{102}\,000199^2 + 1999800^2 &= \mathbf{10201}\,9801^2\\
\mathbf{1020302}\,000199^2 + 199999800^2 &= \mathbf{102030201}\,9801^2\\
\mathbf{10203040302}\,000199^2 + 19999999800^2 &= \mathbf{1020304030201}\,9801^2\\
\mathbf{102030405040302}\,000199^2 + 1999999999800^2 &= \mathbf{10203040504030201}\,9801^2\\
\mathbf{1020304050605040302}\,000199^2 + 199999999999800^2 &= \mathbf{102030405060504030201}\,9801^2\\
\mathbf{10203040506070605040302}\,000199^2 + 19999999999999800^2 &= \mathbf{1020304050607060504030201}\,9801^2\\
\mathbf{102030405060708070605040302}\,000199^2 + 1999999999999999800^2 &= \mathbf{10203040506070807060504030201}\,9801^2\\
\mathbf{1020304050607080908070605040302}\,000199^2 + 199999999999999999800^2 &= \mathbf{102030405060708090807060504030201}\,9801^2
\end{aligned}
\quad \ldots \quad (414)
$$

**Remark 5.** *We observe the last four number in last column and each row is a perfect square, for example, i.e.,*

$$0001 = 1^2,\ 0004 = 2^2,\ 0009 = 3^2,\ \ldots,\ 9409 = 97^2,\ 9604 = 98^2,\ 9801 = 99^2.$$

*We have written only 20 patterned triples in a sequence and last pattern in number 99. We can go further, but in many cases, middle term of the first row don't follow the same pattern. For more details refer author's work [7].*

*Moreover, the sum of last four digits of first and third rows are always 10000. See below:*

- ***Sum of Last Four Digits: First and Third Rows***

$$
\begin{array}{lcl}
n = 1 & \Rightarrow & 9999 + 0001 = 10000\\
n = 2 & \Rightarrow & 9996 + 0004 = 10000\\
n = 3 & \Rightarrow & 9991 + 0009 = 10000\\
n = 4 & \Rightarrow & 9984 + 0016 = 10000\\
\ldots & \ldots & \ldots\ \ldots\ \ldots\ \ldots\\
\ldots & \ldots & \ldots\ \ldots\ \ldots\ \ldots\\
n = 97 & \Rightarrow & 0591 + 9409 = 10000\\
n = 98 & \Rightarrow & 0396 + 9604 = 10000\\
n = 99 & \Rightarrow & 0199 + 9801 = 10000
\end{array}
$$

# 5 Final Comments

The whole work is concentrated to bring pattered Pythagorean triples. Four Procedures of two variables are used to bring these results. Even though, we can write the fifth procedure but it don't bring much different results. Most of the results can be obtained just from the Procedures 1 and 2. See below the fifth procedure:

$$
\begin{aligned}
F_5(m) &:= 64m^2 - n^2\\
G_5(m) &:= 16mn\\
H_5(m) &:= 64m^2 + n^2
\end{aligned}
$$

In case of patterns with even numbers, divisions by 2, 4, 8, etc. are considered to reach patterns with primitive values. In some cases, there are same examples in two or more procedures.

In subsections 4.1, 4.2 and 4.3 we brought the patterns connected with palindromic- type pandigital pythagorean triples. In subsections 4.2 and 4.3, the results are similar and are depending in number 0 in between. While in 4.1 the results are not depends on 0 in between. In subsections 4.1 and 4.2, we have only 9 examples in each case, while in case of 4.3 there are total 99 examples. The Procedure 2 is well known in the literature, while the Procedures 1, 3 and 4 are not very much known. Moreover, all the results given in subsections 4.1, 4.2 and 4.3 can be obtained from the Procedure 2. For more details refer author's work [6, 7]. It is interesting to observe the ending numbers in each case:

- **Ending Numbers in Pandigital Patterns:**

$$\begin{aligned}
\text{Subsection } 4.1 \quad &\Rightarrow \quad 01 = 1^2,\ 04 = 2^2,\ 09 = 3^2,\ \ldots,\ 81 = 9^2 \\
\text{Subsection } 4.2 \quad &\Rightarrow \quad 01 = 1^2,\ 04 = 2^2,\ 09 = 3^2,\ \ldots,\ 81 = 9^2 \\
\text{Subsection } 4.3 \quad &\Rightarrow \quad 01 = 1^2,\ 04 = 2^2,\ 09 = 3^2,\ \ldots,\ 9801 = 99^2.
\end{aligned}$$

In case of single variable, there are five procedures studied by author in [5]. The connections of Pythagorean triples with magic squares refer author's another work [3].

# References


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