HERMITE-HADAMARD TYPE INEQUALITIES FOR EXPONENTIAL CONVEX FUNCTIONS

LOREDANA TIRTIRAU

Abstract. Several Hermite-Hadamard type inequalities will be given in this work for functions whose second derivative in absolute value at certain power is exponential type convex.

1. Introduction

The inequality of Hermite-Hadamard type has been considered very useful in mathematical analysis being extended and generalized in many directions by many authors, see [6, 5, 10, 1, 15, 19, 12] and the references therein.

Using a recent concept of exponential type convex functions given in [13] some Hermite-Hadamard type inequalities will be presented for this kind of functions.

We begin by recalling below the classical definition for the convex functions.

Definition 1. *A function $f : I \subset \mathbb{R} \to \mathbb{R}$ is said to be convex on an interval I if the inequality*

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y) \tag{1}$$

holds for all $x, y \in I$ and $t \in [0,1]$. The function f is said to be concave on I if the inequality (1) takes place in reversed direction.

Definition 2. *(see [13]) A nonnegative function $f : I \to \mathbb{R}$ iscalled exponential type convex function if, for every $m, n \in I$ and $k \in [0,1]$,*

$$f(km + (1-k)n) \leq (e^k - 1)f(m) + (e^{1-k} - 1)f(n). \tag{1}$$

The class of all exponentially type convex functions on interval I is indicated by EXPC(I).

Definition 3. *([24]) Let $h : J \to \mathbf{R}$ be a nonnegative function and $h \neq 0$. We say that $f : I \to \mathbf{R}$ is an h-convex function, or that f belongs to the class SX(h,I), if is nonnegative and for all $m, n \in I,\ k \in [0,1]$ we have*

$$f(km + (1-k)n) \leq h(k)f(m) + h(1-k)f(n).$$

Date: April 3, 2020.

2000 *Mathematics Subject Classification.* 26D15.

Key words and phrases. Hermite-Hadamard inequality, exponential type convex functions, Holder's inequality .

RGMIA Res. Rep. Coll. 23 (2020), Art. 44, 7 pp. Received 07/04/20

When previous inequality is reversed then f is said to be a h-concave function, i.e. $f \in SV(h, I)$. It is obvious that when $h(u) = u$ then the h-convexity becomes convexity.

We know from [13] that every nonnegative convex function is exponential type convex function and that every exponential type convex function is an h-convex function with $h(k) = e^k - 1$.

It is necessary to recall below the definition of fractionals integrals, see [8, 11, 10, 20, 21]. For other type of convexity see also [22, 18].

The classical Hermite-Hadamard's inequality for convex functions is

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}. \tag{2}$$

Moreover, if the function f is concave then the inequality (2) hold in reversed direction.

Definition 4. *Let $f \in L[a, b]$. The Riemann-Liouville integrals $J^{\alpha}_{a+}f$ and $J^{\alpha}_{b-}f$ of order $\alpha > 0$ with $\alpha \geq 0$ are defined by*

$$J^{\alpha}_{a+}f(x) = \frac{1}{\Gamma(\alpha)}\int_a^x (x-t)^{\alpha-1} f(t)dt,\ x > a$$

and

$$J^{\alpha}_{b-}f(x) = \frac{1}{\Gamma(\alpha)}\int_x^b (t-x)^{\alpha-1} f(t)dt,\ x < b,$$

respectively, where $\Gamma(\alpha)$ is the Gamma function defined by $\Gamma(\alpha) = \int_0^{\infty} e^{-t}t^{\alpha-1}dt$ and $J^0_{a+}f(x) = J^0_{b-}f(x) = f(x)$.

Lemma 1. *(see [3]) Let $f : I^o \to \mathbf{R},\ : I^o \subset [0, \infty)$ be a twice differentiable function on $: I^o$ where $a, b \in I,\ a < b$. If $f^{''} \in L[a, b]$, then the following equality holds:*

$$-f\left(\frac{a+b}{2}\right) + \frac{1}{b-a}\int_a^b f(x)dx =$$

$$= \frac{(b-a)^2}{16}[\int_0^1 t^2 f^{''}\left(t\frac{a+b}{2} + (1-t)a\right)dt + \int_0^1 (t-1)^2 f^{''}\left(tb + (1-t)\frac{a+b}{2}\right)].$$

The following result is a generalization of Lemma 1 from [4] when $\alpha > n-1$ and $n \in \mathbb{N}$.

Lemma 2. *Let $n \in \mathbb{N}^*$ and $f : I \subset \mathbb{R} \to \mathbb{R}$ be o function such that $f^{(n)}$ exists on the interior I^0 of an interval I and $f^{(n)} \in L[a, b]$ with $a, b \in I^0,\ 0 < a < b$. Then for any $x \in [a, b]$, we have:*

$$I(f, x, a, b, \alpha, n) = (x-a)\int_0^1 t^{\alpha} f^{(n)}(tx+(1-t)a)dt + (b-x)\int_0^1 (1-t)^{\alpha} f^{(n)}(tb+(1-t)x)dt =$$

$$= \sum_{k=2}^{n} \alpha(\alpha-1)...(\alpha-k+2) f^{(n-k)}(x)\left(\frac{(-1)^{k-1}}{(x-a)^{k-1}} - \frac{1}{(b-x)^{k-1}}\right) +$$

$$+\Gamma(\alpha+1)[\frac{(-1)^n}{(x-a)^\alpha}J_{x^-}^{\alpha-n+1}f(a)+\frac{1}{(b-x)^\alpha}J_{x^+}^{\alpha-n+1}f(b)],$$

where $\alpha > n-1$.

Some Hermite-Hadamard type inequalities will be presented in this work in Theorem 1, 2, 3 and 4 for functions whose second derivative in absolute value at certain power is exponential type convex.

2. Some Hermite-Hadamard type inequalities for exponential type convex functions

The aim of this section is to present new inequalities that refine Hermite-Hadamard inequality for functions whose second derivative in absolute value at certain power is exponential type convex.

Theorem 1. *Let* $f : I \to \mathbf{R}$, $I^o \subset \mathbf{R}$ *be a twice differentiable function on* I^o *such that* $f^{''} \in L^1[a,b]$, *where* $a, b \in I$, $a < b$. *If* $|f^{''}|$ *is an exponential type convex function on* $[a,b]$ *then the following inequality*

$$|-f\left(\frac{a+b}{2}\right)+\frac{1}{b-a}\int_a^b f(x)dx| \leq$$

$$\leq \frac{(b-a)^2}{8}[(e-\frac{7}{3})|f^{''}\left(\frac{a+b}{2}\right)|+(e-\frac{8}{3})|f^{''}(a)|+(e-\frac{8}{3})|f^{''}(b)|]$$

holds.

Proof. By using Lemma 1 and the properties of the modulus we will find,

$$|-f\left(\frac{a+b}{2}\right)+\frac{1}{b-a}\int_a^b f(x)dx| \leq$$

$$\leq \frac{(b-a)^2}{16}\int_0^1 t^2|f^{''}\left(t\frac{a+b}{2}+(1-t)a\right)|dt+\int_0^1 (t-1)^2|f^{''}\left(tb+(1-t)\frac{a+b}{2}\right)|dt \leq$$

$$\leq \frac{(b-a)^2}{16}\{\int_0^1 t^2[(e^t-1)|f^{''}\left(\frac{a+b}{2}\right)|+(e^{1-t}-1)|f^{''}(a)|]dt+$$

$$+\int_0^1 (t-1)^2[(e^t-1)|f^{''}(b)|+(e^{1-t}-1)|f^{''}\left(\frac{a+b}{2}\right)|]dt\},$$

where we also used the definition of the exponential type convex functions in last inequality.

By calculus, we get:

$$|-f\left(\frac{a+b}{2}\right)+\frac{1}{b-a}\int_a^b f(x)dx| \leq$$

$$\leq \frac{(b-a)^2}{16}\{|f^{''}\left(\frac{a+b}{2}\right)|\int_0^1[t^2(e^t-1)+(t-1)^2(e^{1-t}-1)]dt+$$

$$+|f^{''}(a)|\int_0^1 t^2(e^{1-t}-1)dt+|f^{''}(b)|\int_0^1(t-1)^2(e^t-1)dt\}.$$

We can see that $\int_0^1 t^2(e^t-1)dt = \int_0^1 (t-1)^2(e^{1-t}-1)dt = e-\frac{7}{3}$,

$\int_0^1 t^2(e^{1-t}-1)dt = \int_0^1 (t-1)^2(e^t-1)dt = 2e - \frac{16}{3}$.

Therefore we obtain the desired inequailty by replacing these expressions in previous inequality .

∎

Theorem 2. *Let $f : I \to \mathbf{R}$, $I^o \subset \mathbf{R}$ be a twice differentiable function on I^o such that $f^{''} \in L^1[a,b]$, where $a, b \in I^o$, $a < b$ and $q > 1$ with $\frac{1}{p} + \frac{1}{q} = 1$. If $|f^{''}|^q$ is an exponential type convex function on $[a,b]$ then the following inequality*

$$|-f\left(\frac{a+b}{2}\right) + \frac{1}{b-a}\int_a^b f(x)dx| \leq$$

$$\leq \frac{(b-a)^2}{16}\frac{(e-2)^{\frac{1}{q}}}{(2p+1)^{\frac{1}{p}}}[A^{\frac{1}{q}}\left(|f^{''}\left(\frac{a+b}{2}\right)|^q, |f^{''}(a)|^q\right) + A^{\frac{1}{q}}\left(|f^{''}(b)|^q, |f^{''}\left(\frac{a+b}{2}\right)|^q\right)],$$

holds, where $A(u,v)$ is the arithmetic mean of u and v.

Proof. This time we use Holder's inequality. We start like in previous demonstration, by using Lemma 1 and then the modulus properties and we get:

$$|-f\left(\frac{a+b}{2}\right) + \frac{1}{b-a}\int_a^b f(x)dx| \leq$$

$$\leq \frac{(b-a)^2}{16}\int_0^1 t^2|f^{''}\left(t\frac{a+b}{2} + (1-t)a\right)|dt + \int_0^1 (t-1)^2|f^{''}\left(tb + (1-t)\frac{a+b}{2}\right)|dt.$$

Here we use the Holder's inequality for each of the two integrals and we have,

$$|-f\left(\frac{a+b}{2}\right) + \frac{1}{b-a}\int_a^b f(x)dx| \leq$$

$$\leq \frac{(b-a)^2}{16}[\left(\int_0^1 t^{2p}dt\right)^{\frac{1}{p}}\left(\int_0^1 |f^{''}\left(t\frac{a+b}{2} + (1-t)a\right)|^q dt\right)^{\frac{1}{q}} +$$

$$+\left(\int_0^1 (1-t)^{2p}dt\right)^{\frac{1}{p}}\left(\int_0^1 |f^{''}\left(tb + (1-t)\frac{a+b}{2}\right)|^q dt\right)^{\frac{1}{q}}].$$

By using the definition of the exponential type convex functions for the function $|f^{''}|^q$ and calculus we obtain,

$$|-f\left(\frac{a+b}{2}\right) + \frac{1}{b-a}\int_a^b f(x)dx| \leq$$

$$\leq \frac{(b-a)^2}{16}\frac{1}{(2p+1)^{\frac{1}{p}}}[\left(\int_0^1 \left((e^t-1)|f^{''}\left(\frac{a+b}{2}\right)|^q + (e^{1-t}-1)|f^{''}(a)|^q\right)dt\right)^{\frac{1}{q}} +$$

$$+\left(\int_0^1 \left((e^t-1)|f^{''}(b)|^q + (e^{1-t}-1)|f^{''}\left(\frac{a+b}{2}\right)|^q\right)dt\right)^{\frac{1}{q}}] =$$

$$= \frac{(b-a)^2}{16}\frac{(e-2)^{\frac{1}{q}}}{(2p+1)^{\frac{1}{p}}}[\left(|f^{''}\left(\frac{a+b}{2}\right)|^q + |f^{''}(a)|^q\right)^{\frac{1}{q}} + \left(|f^{''}(b)|^q + |f^{''}\left(\frac{a+b}{2}\right)|^q\right)^{\frac{1}{q}}] =$$

$$= \frac{(b-a)^2}{16}\frac{(e-2)^{\frac{1}{q}}}{(2p+1)^{\frac{1}{p}}}[A^{\frac{1}{q}}\left(|f^{''}\left(\frac{a+b}{2}\right)|^q, |f^{''}(a)|^q\right) + A^{\frac{1}{q}}\left(|f^{''}(b)|^q, |f^{''}\left(\frac{a+b}{2}\right)|^q\right)].$$

∎

Theorem 3. *Let $f : I \to \mathbf{R}$, $I^o \subset \mathbf{R}$ be a twice differentiable function on I^o such that $f^{''} \in L^1[a,b]$, where $a, b \in I^o$, $a < b$ and $q > 1$ with $\frac{1}{p} + \frac{1}{q} = 1$. If $|f^{''}|^q$ is an exponential type convex function on $[a,b]$ then the following inequality*

$$|-f\left(\frac{a+b}{2}\right) + \frac{1}{b-a}\int_a^b f(x)dx| \leq$$
$$\leq \frac{(b-a)^2}{16}\frac{1}{3^{\frac{1}{p}}}[A^{\frac{1}{q}}\left((e-\frac{7}{3})|f^{''}\left(\frac{a+b}{2}\right)|^q, 2(e-\frac{8}{3})|f^{''}(a)|^q\right)+$$
$$+A^{\frac{1}{q}}\left(2(e-\frac{8}{3})|f^{''}(b)|^q, (e-\frac{7}{3})|f^{''}\left(\frac{a+b}{2}\right)|^q\right)],$$

holds, where $A(u,v)$ is the arithmetic mean of u and v.

Proof. We use the same tools like in Theorem 2, i.e. Lemma 1, the modulus properties and the definition of the exponential type convex functions for the function $|f^{''}|^q$, but here we use the Holder's inequality like below:

$$|-f\left(\frac{a+b}{2}\right) + \frac{1}{b-a}\int_a^b f(x)dx| \leq$$
$$\leq \frac{(b-a)^2}{16}\int_0^1 t^2|f^{''}\left(t\frac{a+b}{2}+(1-t)a\right)|dt + \int_0^1 (t-1)^2|f^{''}\left(tb+(1-t)\frac{a+b}{2}\right)|dt \leq$$
$$\leq \frac{(b-a)^2}{16}[\left(\int_0^1 t^2 dt\right)^{1-\frac{1}{q}}\left(\int_0^1 t^2|f^{''}\left(t\frac{a+b}{2}+(1-t)a\right)|^q dt\right)^{\frac{1}{q}}+$$
$$+\left(\int_0^1 (1-t)^2 dt\right)^{1-\frac{1}{q}}\left(\int_0^1 (t-1)^2|f^{''}\left(tb+(1-t)\frac{a+b}{2}\right)|^q dt\right)^{\frac{1}{q}}] \leq$$
$$\leq \frac{(b-a)^2}{16}\left(\frac{1}{3}\right)^{1-\frac{1}{q}}[\left(|f^{''}\left(\frac{a+b}{2}\right)|^q\int_0^1 t^2(e^t-1)dt + |f^{''}(a)|^q\int_0^1 t^2(e^{1-t}-1)dt\right)^{\frac{1}{q}}+$$
$$+\left(|f^{''}(b)|^q\int_0^1 (t-1)^2(e^t-1)dt + |f^{''}\left(\frac{a+b}{2}\right)|^q\int_0^1 (t-1)^2(e^{1-t}-1)dt\right)^{\frac{1}{q}}] =$$
$$= \frac{(b-a)^2}{16}\left(\frac{1}{3}\right)^{1-\frac{1}{q}}[\left(|f^{''}\left(\frac{a+b}{2}\right)|^q(e-\frac{7}{3}) + |f^{''}(a)|^q(2e-\frac{16}{3})\right)^{\frac{1}{q}}+$$
$$+\left(|f^{''}(b)|^q(2e-\frac{16}{3}) + |f^{''}\left(\frac{a+b}{2}\right)|^q(e-\frac{7}{3})\right)^{\frac{1}{q}}] =$$
$$= \frac{(b-a)^2}{16}\frac{1}{3^{\frac{1}{p}}}[A^{\frac{1}{q}}\left((e-\frac{7}{3})|f^{''}\left(\frac{a+b}{2}\right)|^q, 2(e-\frac{8}{3})|f^{''}(a)|^q\right)+$$
$$+A^{\frac{1}{q}}\left(2(e-\frac{8}{3})|f^{''}(b)|^q, (e-\frac{7}{3})|f^{''}\left(\frac{a+b}{2}\right)|^q\right)].$$

∎

Theorem 4. *Let $n \in \mathbb{N}^*$ and $f : I \subset \mathbb{R} \to \mathbb{R}$ be o function such that $f^{(n)}$ exists on the interior I^0 of an interval I and $f^{(n)} \in L[a,b]$ with $a, b \in I^0$, $0 < a < b$. If $|f^{(n)}|^q$ is an exponential type convex function on $[a,b]$ for some fixed $q > 1$, where $\frac{1}{p} + \frac{1}{q} = 1$ then the following inequality*

$$|I(f,x,a,b,\alpha,n)| = |\sum_{k=2}^{n} \alpha(\alpha-1)...(\alpha-k+2) f^{(n-k)}(x) \left(\frac{(-1)^{k-1}}{(x-a)^{k-1}} - \frac{1}{(b-x)^{k-1}} \right) +$$

$$+\Gamma(\alpha+1)[\frac{(-1)^n}{(x-a)^\alpha} J_{x^-}^{\alpha-n+1} f(a) + \frac{1}{(b-x)^\alpha} J_{x^+}^{\alpha-n+1} f(b)]| \leq$$

$$\leq \frac{(e-2)^{\frac{1}{q}}}{2^{\frac{1}{q}}(\alpha p+1)^{\frac{1}{p}}} [(x-a) A^{\frac{1}{q}}(|f^{(n)}(x)|^q, |f^{(n)}(a)|^q) + (b-x) A^{\frac{1}{q}}(|f^{(n)}(b)|^q, |f^{(n)}(x)|^q)],$$

takes place, where $A(u,v)$ is the arithmetic mean of u and v.

Proof. Using now Lemma 2, the properties of modulus and Holder's inequality, we have,

$$|I(f,x,a,b,\alpha,n)| = |\sum_{k=2}^{n} \alpha(\alpha-1)...(\alpha-k+2) f^{(n-k)}(x) \left(\frac{(-1)^{k-1}}{(x-a)^{k-1}} - \frac{1}{(b-x)^{k-1}} \right) +$$

$$+\Gamma(\alpha+1)[\frac{(-1)^n}{(x-a)^\alpha} J_{x^-}^{\alpha-n+1} f(a) + \frac{1}{(b-x)^\alpha} J_{x^+}^{\alpha-n+1} f(b)]| \leq$$

$$\leq (x-a) \int_0^1 t^\alpha |f^{(n)}(tx+(1-t)a)| dt + (b-x) \int_0^1 (1-t)^\alpha |f^{(n)}(tb+(1-t)x)| dt \leq$$

$$\leq (x-a) \left(\int_0^1 t^{\alpha p} dt \right)^{\frac{1}{p}} \left(\int_0^1 |f^{(n)}(tx+(1-t)a)|^q dt \right)^{\frac{1}{q}} +$$

$$+(b-x) \left(\int_0^1 (1-t)^{\alpha p} dt \right)^{\frac{1}{p}} \left(\int_0^1 |f^{(n)}(tb+(1-t)x)|^q dt \right)^{\frac{1}{q}}.$$

Now, taking into account that $|f^{(n)}|^q$ is an exponential type convex function on $[a,b]$, we get by calculus,

$$|I(f,x,a,b,\alpha,n)| \leq \frac{(e-2)^{\frac{1}{q}}}{2^{\frac{1}{q}}(\alpha p+1)^{\frac{1}{p}}} [(x-a)(|f^{(n)}(x)|^q + |f^{(n)}(a)|^q)^{\frac{1}{q}} +$$

$$+(b-x)(|f^{(n)}(b)|^q + |f^{(n)}(x)|^q)^{\frac{1}{q}}] =$$

$$= \frac{(e-2)^{\frac{1}{q}}}{2^{\frac{1}{q}}(\alpha p+1)^{\frac{1}{p}}} [(x-a) A^{\frac{1}{q}}(|f^{(n)}(x)|^q, |f^{(n)}(a)|^q) + (b-x) A^{\frac{1}{q}}(|f^{(n)}(b)|^q, |f^{(n)}(x)|^q)],$$

i.e. the desired inequality.

∎

References

[1] Alomari, M., Darus, M., Kirmaci, U. S., Some inequalities of Hermite-Hadamard ty6pe for s-convex functions, *Acta Mathematica Scientia*, (2011) 31 B(4), 1643-1652.

[2] Alomari, M., Darus, M., Kirmaci, U. S., Refinements ofHadamard-type inequalities for quasi-convex functions with applications to trapezoidal formula and to specail means, *Computers and Mathematica with Applications*, **59** (2010) 225-232.

[3] Ciurdariu, L., On some Hermite-Hadamard type inequalities for functions whose power of absolute value of derivatives are $(\alpha, m)-$ convex, *Int. J. of Math. Anal.*, **6(48)** (2012), 2361-2383.

[4] Ciurdariu, L., A note concerning several Hermite-Hadamard inequalities for different types of convex functions, *Int. J. of Math. Anal.*, **6(33)** (2012), 1623-1639.

[5] Dragomir, S. S., Pearce, C. E. M., Selected topic on Hermite-Hadamard inequalities and applications, *Melbourne and Adelaide* December, (2001).

[6] Dragomir, S. S., Fitzpatrick, S., The Hadamard's inequality for s-convex functionsin the second sense, *Demonstratio Math.* , **32 (4)** (1999), 687-696.

[7] Latif, M. A., Dragomir, S. S., New inequalities of Hermite-Hadamard type for n-times differentiable convex and concave functions with applications, Res. Rep.Coll., 2014, pp. 17.

[8] Dahmani, Z., On Minkowski and Hermite-Hadamard integral inequalities via fractional integration, *Ann. Funct. Anal.,* **1(1)** (2010) 51-58.

[9] Hadamard, J., Etude sur le proprietes des fonctions entieres en particulier d' une fonction consideree par Riemann, J. Math. Pures Appl., 58, 171-215 (1893).

[10] Iscan Imdat, Generalizations of different type integral inequalities for s-convex functions via fractional integrals, *Appl. Anal.,* (2013) 1-17.

[11] Iscan Imdat, Generalization of different type integral inequalities via fractional integrals for functions whose second derivatives absolute value are quasi-convex, *Konuralp Journal of Mathematics*, **1(2)** (2013) 67-79.

[12] Iscan, Imdat, Kunt, M., Yazici, N., Gozutok, Tuncay, K., New general integral inequalities for Lipschitzian functions via Riemann-Liouville fractional integrals and applications, *Joirnal of Inequalities and Special Functions*, **7** 4, (2016), 1-12.

[13] Kadakal Mahir and Imdat Iscan, Exponential type convexity and some related inequalities, *Journal of Inequalities and Applications*, (2020) 2020:82, pp. 2-9.

[14] Kasvurmaci, H., Avci, M., Ozdemir, M. E., New inequalities of Hermite-Hadamard type for convex functions with applications, arXiv:1006.1593v1[math.CA].

[15] Kirmaci, U. S., Inequalities for differentiable mappings and applications to special means of real numbers and to midpoint formula, *Appl. Math. Comput.*, **147 (1)** (2014),137-146.

[16] Kirmaci, U. S., Klaricic, K., Bakula, Ozdemir, M. E., Pecaric, J., Hadamard-type inequalities for s-convex functions, *Appl. Math. Comput.*, **193 (1)** 2007, 26-35.

[17] Latif, M. A., Dragomir, S. S., New inequalities of Hermite-Hadanard type for functions whose derivatives in absolute value are convex with applications, *Acta Univ. Matthiae Belii, Series Math.*, (2013), 24-39.

[18] Mihesan, V. G., A generalization of the convexity, Seminar of Functional Equations, *Approx. and Convex,* Cluj-Napoca, Romania (1993).

[19] Set, E., New inequalities of Ostrowski type for mappings whose derivatives are s-convex in the second via fractional integrals, *Comput. Math. Appl.* **(2010)** Art ID:531976, 7 pages.

[20] Sarikaya, M. Z., Set, E., Yildiz, H., Basak, N., Hermite-Hadamard's inequalities for fractional integrals and related fractional inequalities, *Math. and Comput. Model.*, **2011** (2011).

[21] Set, E., Sarikaya, M. Z., Ozdemir, M. E., Some Ostrowski's type inequalities for functions whose second derivatives are s-convex in the second sense, arXiv:10006:2488v1[mathCA]12 June 2010.

[22] Toader, Gh., On a generalization of the convexity, *Mathematica*, 30 (53) (1988), 83-87.

[23] Tunc, M., On some new inequalities for convex functions, *Turk. J. Math.*, **35** (2011) , 1-7.

[24] Varosanec, S., On h-convexity, *J. Math. Anal. Appl.*, 326,303-311 (2007).

Department of Mathematics, "Politehnica" University of Timisoara, P-ta. Victoriei, No.2, 300006-Timisoara