RGMIA Research Report Collection, Vol. 2, No. 1, 1999
http://sci.vu.edu.au/~rgmia

# A WEIGHTED VERSION OF OSTROWSKI INEQUALITY FOR MAPPINGS OF HÖLDER TYPE AND APPLICATIONS IN NUMERICAL ANALYSIS

S.S. Dragomir, P. Cerone, J. Roumeliotis and S. Wang

Abstract. In this paper we establish a weighted version of Ostrowski inequality for mappings of Hölder type and apply it in Numerical Integration. Some Examples for the most popular weights: Legendre, Logarithm, Jacobi, Chebyshev, Laguerre and Hermite are also given.

## 1 Introduction

In 1938, A. Ostrowski proved the following integral inequality [2, p. 468]

**Theorem 1.1.** *Let* $f:[a,b]\to\mathbf{R}$ *be continuous on* $[a,b]$ *and differentiable on* $(a,b)$ *and whose derivative* $f':(a,b)\to\mathbf{R}$ *is bounded on* $(a,b)$, *i.e.,* $\|f'\|_\infty := \sup_{t\in(a,b)}|f'(t)|<\infty$. *Then*

$$\left|f(x)-\frac{1}{b-a}\int_a^b f(t)\,dt\right| \le \left[\frac{1}{4}+\frac{\left(x-\frac{a+b}{2}\right)^2}{(b-a)^2}\right](b-a)\|f'\|_\infty \tag{1.1}$$

*for all* $x\in[a,b]$.

*The constant* $\frac{1}{4}$ *is the best possible.*

For some applications of Ostrowski's inequality to certain numerical quadrature rules, we refer to the recent paper [1] by S.S. Dragomir and S. Wang.

In this paper we establish a weighted version of Ostrowski inequality for mappings of $r-H$-Hölder type and apply it in Numerical Integration.

Some examples for the most popular weights: Legendre, Logarithm, Jacobi, Chebyshev, Laguerre and Hermite are also given.

For other results in connection to Ostrowski inequality, the reader is advised to consult [1-11].

## 2 The Results

The following theorem holds:

**Theorem 2.1.** *Let* $f,w:(a,b)\subseteq\mathbf{R}\to\mathbf{R}$ *be so that* $w(s)\ge 0$ *on* $(a,b)$, $w$ *is integrable on* $(a,b)$ *and* $\int_a^b w(s)\,ds>0$, $f$ *is of* $r-H$*-Hölder type, i.e.,*

$$|f(x)-f(y)|\le H|x-y|^r \qquad \text{for all } x,y\in(a,b) \tag{2.1}$$

*where* $H>0$ *and* $r\in(0,1]$ *are given. If* $wf\in L_1(a,b)$, *then we have the inequality:*

$$\left|f(x)-\frac{1}{\int_a^b w(s)\,ds}\int_a^b w(s)f(s)\,ds\right| \le H\cdot\frac{1}{\int_a^b w(s)\,ds}\int_a^b |x-s|^r w(s)\,ds \tag{2.2}$$

*for all* $x\in(a,b)$.

---

*1991 Mathematics Subject Classification.* Primary 26 D 15; Secondary 41 A 55.

*Key words and phrases.* Ostrowski's Inequality, Hölder Type Mappings, Numerical Quadrature.

*The constant factor $C = 1$ in the right hand side of the inequality can not be replaced by a smaller one.*

*Proof.* As $f$ is of $r-H-$Hölder type, we can state that

$$|f(x) - f(s)| \leq H|x-s|^r \qquad \text{for all } x, s \in (a,b). \tag{2.3}$$

Multiplying by $w(s) \geq 0$ and integrating over $s$ on $[a,b]$, we get

$$\int_a^b |f(x) - f(s)| w(s)\, ds \leq H \int_a^b |x-s|^s w(s)\, ds \tag{2.4}$$

for all $x \in (a,b)$.

On the other hand, by the integral's properties, we have

$$\int_a^b |f(x) - f(s)| w(s)\, ds \geq \left| \int_a^b (f(x) - f(s)) w(s)\, ds \right| \tag{2.5}$$

$$= \left| f(x) \int_a^b w(s)\, ds - \int_a^b f(s) w(s)\, ds \right|.$$

Now, using $(2.4)$ and $(2.5)$, we get the desired inequality $(2.2)$.

To prove that the constant factor $C = 1$ is sharp, let us assume that (2.2) holds with a constant $C > 0$, i.e.,

$$\left| f(x) - \frac{1}{\int_a^b w(s)\, ds} \int_a^b w(s) f(s)\, ds \right| \leq H \cdot \frac{C}{\int_a^b w(s)\, ds} \int_a^b |x-s|^r w(s)\, ds \tag{2.6}$$

for all $x \in (a,b)$.

Consider the mapping $f_0 : [0,1] \to \mathbf{R}, f_0 = x^r, r \in (0,1]$. Then

$$|f_0(x) - f_0(y)| = |x^r - y^r| \leq |x-y|^r,$$

for all $x, y \in [0,1]$, which shows that $f_0$ is of $r-$Hölder type with the constant $H = 1$.

Writing the inequality (2.6) for $f_0$, we get

$$\left| \int_0^1 (x^r - s^r) w(s)\, ds \right| \leq C \int_0^1 |x-s|^r w(s)\, ds \tag{2.7}$$

for all $x \in [0,1]$ and $w$ as above. Letting $x = 0$ in $(2.7)$, we deduce $C \geq 1$, which proves the sharpness of the constant. ∎

**Remark 2.1.** *If $r = 1$, i.e., the mapping $f$ is Lipschitzian with, let us say, the constant $L > 0$, then we get*

$$\left| f(x) - \frac{1}{\int_a^b w(s)\, ds} \int_a^b w(s) f(s)\, ds \right| \leq \frac{L}{\int_a^b w(s)\, ds} \int_a^b |x-s| w(s)\, ds. \tag{2.8}$$

Now, if in (2.8) we assume that the weight function $w(t) = 1$, then we get Ostrowski's inequality for Lipschitzian mappings (see also [12]):

$$\left| f(x) - \frac{1}{b-a} \int_a^b f(s)\, ds \right| \leq \left[ \frac{1}{4} + \frac{\left(x - \frac{a+b}{2}\right)^2}{(b-a)^2} \right] L(b-a), \quad x \in [a,b]. \tag{2.9}$$

The proof is obvious by (2.8) taking into account the fact that

$$\frac{1}{b-a}\int_a^b |x-s|\,ds = \frac{1}{b-a}\left[\int_a^x (x-s)\,ds + \int_x^b (s-x)\,ds\right]$$
$$= \frac{1}{b-a}\frac{(x-a)^2+(b-a)^2}{2}$$
$$= \left[\frac{1}{4}+\frac{\left(x-\frac{a+b}{2}\right)^2}{(b-a)^2}\right](b-a).$$

**Remark 2.2.** *If the mapping $f$ is differentiable on $(a,b)$ and whose derivative $f'$ is bounded on $(a,b)$, i.e., $\|f'\|_\infty := \sup_{t\in(a,b)} |f'(t)| < \infty$, then instead of $L$ in (2.8) we can put $\|f'\|_\infty$.*

The following corollary, which provides an Ostrowski type inequality for mappings of Hölder type holds.

**Corollary 2.2.** *Let $f:[a,b]\to\mathbf{R}$ be a mapping of $r-H-$Hölder type. Then we have the inequality*

$$\left|f(x)-\frac{1}{b-a}\int_a^b f(t)\,dt\right| \tag{2.9'}$$
$$\le H\frac{1}{r+1}\left[\left(\frac{x-a}{b-a}\right)^{r+1}+\left(\frac{b-x}{b-a}\right)^{r+1}\right](b-a)^r$$
$$\le \frac{H}{r+1}(b-a)^r$$

*The constant factor $C=1$ in the right hand side of the inequality can not be replaced by a smaller one.*

*Proof.* Put $w(s)=1$ in (2.2) to get, in the right hand side, that

$$\frac{1}{b-a}\int_a^b |x-s|^s\,ds = \frac{1}{b-a}\left[\int_a^x (x-s)^r\,ds + \int_x^b (s-x)^r\,ds\right]$$
$$= \frac{1}{(b-a)}\left[\frac{(x-a)^{r+1}+(b-x)^{r+1}}{r+1}\right]$$

and the inequality (2.9′) is proved. ∎

We give now some corollaries for the most popular weight functions.

## 2.1 Logarithm

**Corollary 2.3.** *Let $f:(0,1)\to\mathbf{R}$ be a differentiable mapping whose derivative is bounded and for which the integral $\int_0^1 \ln\left(\frac{1}{t}\right) f(t)\,dt$ is finite. Then*

$$\left|f(x)-\int_0^1 \ln\left(\frac{1}{t}\right) f(t)\,dt\right| \le \left[\frac{1}{4}-x+x^2\left(\frac{3}{2}-\ln x\right)\right]\|f'\|_\infty \tag{2.10}$$

*for all $x\in(0,1)$.*

*Proof.* We apply (2.8) for $L = \|f'\|_{\infty}, a = 0, b = 1, w(t) = \ln\left(\frac{1}{t}\right)$.
We have

$$\int_0^1 \ln\left(\frac{1}{t}\right) dt = 1,$$

$$\int_0^1 |x-s| \ln\left(\frac{1}{s}\right) ds = \int_0^x (s-x)\ln s ds + \int_x^1 (x-s)\ln s ds$$
$$= x^2\left(\frac{3}{2} - \ln x\right) - x + \frac{1}{4}$$

for all $x \in (0,1)$, and then (2.10) is obtained. ∎

**Remark 2.3.** *If $I(x) = x^2\left(\frac{3}{2} - \ln x\right) - x + \frac{1}{4}$, then $I'(x) = 2x(1-\ln x) - 1$, which shows that $I$ has its minimum on $(0,1)$ at the point $x_{\min} \approx 0.1866823$. At this point $I(x_{\min}) \approx 0.1740840$.*

Consequently, the best inequality we can get from (2.8) is

$$\left| f(0.1866823) - \int_0^1 \ln\left(\frac{1}{t}\right) f(t)\, dt \right| \le 0.1740840 \left\|f'\right\|_{\infty}. \tag{2.11}$$

## 2.2 Jacobi

**Corollary 2.4.** *Let $f : (0,1) \to \mathbf{R}$ be a differentiable mapping whose derivative is bounded and for which the integral $\int_0^1 \frac{f(t)}{\sqrt{t}} dt$ is finite.*
*Then*

$$\left| f(x) - \frac{1}{2}\int_0^1 \frac{f(t)}{\sqrt{t}} dt \right| \le \frac{1}{6}\left(8x^{3/2} - 6x + 2\right)\left\|f'\right\|_{\infty} \tag{2.12}$$

*for all $x \in (0,1)$.*

*Proof.* We apply (2.8) for $L = \|f'\|_{\infty}, a = 0, b = 1, w(t) = \frac{1}{\sqrt{t}}$. We have

$$\int_0^1 \frac{dt}{\sqrt{t}} = 2,$$

$$\int_0^1 \frac{|x-s|}{\sqrt{s}} ds = \frac{1}{3}\left(8x^{3/2} - 6x + 2\right)$$

for all $x \in (0,1)$, and then (2.12) is obtained. ∎

**Remark 2.4.** *If $J(x) := \frac{1}{6}\left(8x^{3/2} - 6x + 2\right)$, then $J'(x) = 2\sqrt{x} - 1$, which shows that $J$ has its minimum on $(0,1)$ at the point $x_{\min} = \frac{1}{4}$. $J(x_{\min}) = \frac{1}{4}$ and then, the best inequality we can get from (2.12) is*

$$\left| f\left(\frac{1}{4}\right) - \frac{1}{2}\int_0^1 \frac{f(t)}{\sqrt{t}} dt \right| \le \frac{1}{4}\left\|f'\right\|_{\infty}. \tag{2.13}$$

## 2.3 Chebyshev

**Corollary 2.5.** *Let $f : (-1,1) \to \mathbf{R}$ be a differentiable mapping whose derivative is bounded and for which the integral $\int_0^1 \frac{f(t)}{\sqrt{1-t^2}}dt$ is finite.*
*Then*

$$\left| f(x) - \frac{1}{\pi}\int_{-1}^{1} \frac{f(t)}{\sqrt{1-t^2}}dt \right| \le \frac{2}{\pi}\left( x\arcsin x + \sqrt{1-x^2} \right) \|f'\|_\infty \tag{2.14}$$

*for all $x \in (-1,1)$.*

*Proof.* We apply (2.8) for $L = \|f'\|_\infty, a = -1, b = 1, w(t) = \frac{1}{\sqrt{1-t^2}}$. We have

$$\int_{-1}^{1} \frac{dt}{\sqrt{1-t^2}} = \pi,$$

$$\int_{-1}^{1} \frac{|x-s|}{\sqrt{1-s^2}}ds = 2\left( x\arcsin x + \sqrt{1-x^2} \right)$$

for all $x \in (-1,1)$, and then (2.14) is obtained. ∎

**Remark 2.5.** *If $K(x) := \frac{2}{\pi}\left( x\arcsin x + \sqrt{1-x^2} \right)$, then $K'(x) = \frac{2}{\pi}\arcsin x$, which shows that $K$ has its minimum on $(-1,1)$ at the point $x_{\min} = 0$. $K(x_{\min}) = \frac{2}{\pi}$ and then, the best inequality we can get from (2.14) is*

$$\left| f(0) - \frac{1}{\pi}\int_{-1}^{1} \frac{f(t)}{\sqrt{1-t^2}}dt \right| \le \frac{2}{\pi}\|f'\|_\infty . \tag{2.15}$$

## 2.4 Laguerre

**Corollary 2.6.** *Let $f : [0,\infty) \to \mathbf{R}$ be a differentiable mapping whose derivative is bounded and for which the integral $\int_0^\infty e^{-t}f(t)\,dt$ is finite. Then*

$$\left| f(x) - \int_0^\infty e^{-t}f(t)\,dt \right| \le \left( 2e^{-x} + x - 1 \right)\|f'\|_\infty \tag{2.16}$$

*for all $x \in [0,\infty)$.*

*Proof.* We apply (2.8) for $L = \|f'\|_\infty, a = 0, b = +\infty, w(t) = e^{-t}$.
We have

$$\int_0^\infty e^{-t}dt = 1$$

$$\int_0^\infty |x-s|\,e^{-s}ds = 2e^{-x} + x - 1$$

for all $x \in [0,\infty)$, and then (2.16) is obtained. ∎

**Remark 2.6.** *If $L(x) := 2e^{-x} + x - 1$, then $L'(x) = -2e^{-x} + 1$ which shows that the mapping $L$ has its minimum on $(0,\infty)$ at the point $x_{\min} = \ln 2$. $L(x_{\min}) = \ln 2$ and then the best inequality we can get from (2.16) is*

$$\left| f(\ln 2) - \int_0^\infty e^{-t}f(t)\,dt \right| \le \|f'\|_\infty \ln 2. \tag{2.17}$$

## 2.5 Hermite

**Corollary 2.7.** *Let $f:\mathbf{R}\to\mathbf{R}$ be a differentiable mapping whose derivative is bounded and for which the integral $\int_{-\infty}^{\infty} e^{-t^2} f(t)\,dt$ is finite. Then*

$$\left| f(x) - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} f(t)\,dt \right| \le \frac{1}{\sqrt{\pi}} \left[ e^{-x^2} + \sqrt{\pi} x \operatorname{erf}(x) \right] \|f'\|_\infty \tag{2.18}$$

*for all $x\in\mathbf{R}$.*

*Proof.* We apply (2.8) for $L=\|f'\|_\infty, a=-\infty, b=+\infty, w(t)=e^{-t^2}$. We know

$$\int_{-\infty}^{\infty} e^{-t^2} dt = \sqrt{\pi},$$

$$\int_{-\infty}^{\infty} |x-s|\, e^{-s^2} ds = e^{-x^2} + \sqrt{\pi} x \operatorname{erf}(x)$$

for all $x\in\mathbf{R}$, and then (2.18) is obtained. ∎

**Remark 2.7.** *If $M(x) = \frac{1}{\sqrt{\pi}}\left(e^{-x^2} + \sqrt{\pi} x \operatorname{erf}(x)\right)$, then $M'(x) = \operatorname{erf}(x)$ which shows that $M$ has its minimum at $x_{\min}=0$ and $M(0)=\frac{1}{\sqrt{\pi}}$. Consequently, the best inequality we can get from (2.18) is*

$$\left| f(0) - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} f(t)\,dt \right| \le \frac{1}{\sqrt{\pi}} \|f'\|_\infty . \tag{2.19}$$

## 3 Applications in Numerical Integration

Let $I_n : a = x_0 < x_1 < ... < x_{n-1} < x_n = b$ be a division of $[a,b]$ and $\xi_i \in [x_i, x_{i+1}]$ $(i=0,...,n-1)$ intermediate points. Let $f,w:[a,b]\to\mathbf{R}$ and define the sum

$$A(f,w,I_n,\xi) := \sum_{i=0}^{n-1} f(\xi_i) \int_{x_i}^{x_{i+1}} w(s)\,ds.$$

The following result holds:

**Theorem 3.1.** *Let $f$ and $w$ be as in Theorem 2.1. Then we have the following quadrature rule:*

$$\int_a^b f(s)\,w(s)\,ds = A(f,w,I_n,\xi) + R(f,w,I_n,\xi) \tag{3.1}$$

*where $A(f,w,I_n,\xi)$ is given above and the remainder satisfies the estimate:*

$$\begin{aligned} |R(f,w,I_n,\xi)| &\le H \sum_{i=0}^{n-1} \int_{x_i}^{x_{i+1}} |\xi_i - s|^r\, w(s)\,ds \\ &\le \frac{H}{2^r} \sum_{i=0}^{n-1} h_i^r \int_{x_i}^{x_{i+1}} w(s)\,ds \le \frac{H}{2^r} \left[\nu(h)^r\right] \int_a^b w(s)\,ds, \end{aligned} \tag{3.2}$$

*where $h_i := x_{i+1} - x_i$ and $\nu(h) = \max_{i=\overline{0,n-1}} h_i$.*

*Proof.* We apply the inequality (2.2) on the interval $[x_i, x_{i+1}]$ $(i = 0, ..., n-1)$ to get

$$\left| f(\xi_i) \int_{x_i}^{x_{i+1}} w(s)\,ds - \int_{x_i}^{x_{i+1}} w(s) f(s)\,ds \right| \le H \int_{x_i}^{x_{i+1}} |\xi_i - s|^r w(s)\,ds.$$

Summing over $i$ from 0 to $n-1$ and using the generalized triangle inequality, we get the first part of (3.2).

The last part follows by the fact that

$$|\xi_i - s| \le \frac{h_i}{2} \le \frac{\nu(h)}{2}, \qquad i = 0, ..., n-1$$

and we omit the details. ∎

Suppose that the integral $\int_0^1 \frac{f(t)}{\sqrt{t}}dt$ is to be approximated. Let $\|f'\|_\infty := \sup_{t\in(0,1)} |f'(t)|$ and assume that $f' : (0,1) \to \mathbf{R}$ is bounded. If $I_n : 0 = x_0 < x_1 < ... < x_{n-1} < x_n = 1$ is a division of the interval $[0,1]$ and $\xi_i \in [x_i, x_{i+1}]$ are intermediate points, then

$$A\left(f, \frac{1}{\sqrt{\cdot}}, I_n, \xi\right) = 2\sum_{i=0}^{n-1} f(\xi_i)\left(\sqrt{x_{i+1}} - \sqrt{x_i}\right);$$

and

$$\begin{aligned}\int_{x_i}^{x_{i+1}} |\xi_i - s|\, s^{-\frac{1}{2}} ds &= \int_{x_i}^{\xi_i} (\xi_i - s)\, s^{-\frac{1}{2}} ds + \int_{\xi_i}^{x_{i+1}} (s - \xi_i)\, s^{-\frac{1}{2}} ds \\ &= 2\left(\sqrt{\xi_i} - \sqrt{x_i}\right)\left[\xi_i - \frac{1}{3}\left(\xi_i + \sqrt{\xi_i \cdot x_i} + x_i\right)\right] \\ &\quad + 2\left(\sqrt{x_{i+1}} - \sqrt{\xi_i}\right)\left[\frac{1}{3}\left(x_{i+1} + \sqrt{\xi_i \cdot x_{i+1}} + \xi_i\right) - \xi_i\right] \\ &=: \delta(h_i, \xi_i);\end{aligned}$$

and

$$\int_{x_i}^{x_{i+1}} w(s)\,ds = 2\left(\sqrt{x_{i+1}} - \sqrt{x_i}\right).$$

Consequently, we can approximate the integral $\int_0^1 \frac{f(t)}{\sqrt{t}}dt$ by

$$A\left(f, \frac{1}{\sqrt{\cdot}}, I_n, \xi\right) = 2\sum_{i=0}^{n-1} f(\xi_i)\left(\sqrt{x_{i+1}} - \sqrt{x_i}\right)$$

having an error $R\left(f, \frac{1}{\sqrt{\cdot}}, I_n, \xi\right)$ which satisfies the bound:

$$\left| R\left(f, \frac{1}{\sqrt{\cdot}}, I_n, \xi\right) \right| \le \|f'\|_\infty \sum_{i=0}^{n-1} \delta(h_i, \xi_i) \tag{3.3}$$

$$\le \|f'\|_\infty \sum_{i=0}^{n-1} h_i\left(\sqrt{x_{i+1}} - \sqrt{x_i}\right) \le \|f'\|_\infty\, \nu(h).$$

Consider the integral $\int_{-1}^1 \frac{f(t)}{\sqrt{1-t^2}}dt$ is to be approximated and assume that $f' : (-1,1) \to \mathbf{R}$ is bounded and $\|f'\|_\infty := \sup_{t\in(-1,1)} |f'(t)|$.

If $I_n : -1 = x_0 < x_1 < ... < x_{n-1} < x_n = 1$ is a division of the interval $[-1, 1]$ and $\xi_i \in [x_i, x_{i+1}]$ are intermediate points, then

$$A\left(f, \frac{1}{\sqrt{1-(\cdot)^2}}, I_n, \xi\right) = \sum_{i=0}^{n-1} f(\xi_i)(\arcsin x_{i+1} - \arcsin x_i),$$

and

$$\int_{x_i}^{x_{i+1}} |\xi_i - s| \frac{1}{\sqrt{1-s^2}} ds$$

$$= \int_{x_i}^{\xi_i} \frac{\xi_i - s}{\sqrt{1-s^2}} ds + \int_{\xi_i}^{x_{i+1}} \frac{s - \xi_i}{\sqrt{1-s^2}} ds$$

$$= \xi_i \left[\arcsin s|_{x_i}^{\xi_i}\right] + \frac{1}{2} \frac{\left(1-s^2\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Bigg|_{x_i}^{\xi_i}$$

$$- \frac{1}{2} \frac{\left(1-s^2\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\Bigg|_{\xi_i}^{x_{i+1}} - \xi_i \left[\arcsin s|_{\xi_i}^{x_i}\right]$$

$$= 2\xi_i \left(\arcsin \xi_i - \frac{\arcsin x_i + \arcsin x_{i+1}}{2}\right)$$

$$+2\left(\sqrt{1-\xi_i^2} - \frac{\sqrt{1-x_i^2} + \sqrt{1-x_{i+1}^2}}{2}\right) =: \beta(h_i, \xi_i)$$

and

$$\int_{x_i}^{x_{i+1}} w(s)\, ds = \arcsin x_{i+1} - \arcsin x_i.$$

Consequently, we can approximate the integral $\int_{-1}^{1} \frac{f(t)dt}{\sqrt{1-t^2}} dt$ by

$$A\left(f, \frac{1}{\sqrt{1-(\cdot)^2}}, I_n, \xi\right) = \sum_{i=0}^{n-1} f(\xi_i)(\arcsin x_{i+1} - \arcsin x_i)$$

having an error $R\left(f, \frac{1}{\sqrt{1-(\cdot)^2}}, I_n, \xi\right)$ which satisfies the bound

$$\left|R\left(f, \frac{1}{\sqrt{1-(\cdot)^2}}, I_n, \xi\right)\right| \le \|f'\|_\infty \sum_{i=0}^{n-1} \beta(h_i, \xi_i)$$

$$\le \frac{\|f'\|_\infty}{2} \sum_{i=0}^{n-1} h_i (\arcsin x_{i+1} - \arcsin x_i)$$

$$\le \frac{\|f'\|_\infty \pi}{2} \nu(h).$$

## References

[1] S.S. DRAGOMIR and S. WANG, Applications of Ostrowski's inequality to the estimation of error bounds for some special means and some numerical quadrature rules, *Appl. Math. Lett.*, **11**(1998), 105-109.

[2] S.S. DRAGOMIR and S. WANG, A new inequality of Ostrowski's type in $L_1$ norm and applications to some special means and to some numerical quadrature rules, *Tamkang J. of Math.*, **28**(1997), 239-244.

[3] S.S. DRAGOMIR and S. WANG, A new inequality of Ostrowski's type in $L_p$-norm, *submitted.*

[4] N.S. BARNETT and S.S. DRAGOMIR, An inequality of Ostrowski type for cumulative distribution function, *submitted.*

[5] N.S. BARNETT and S.S. DRAGOMIR, An Ostrowski type inequality for a random variable whose probability function belongs to $L_\infty [a, b]$, *submitted.*

[6] N.S. BARNETT and S.S. DRAGOMIR, An Ostrowski type inequality for double integrals and applications for cubature formulae, *submitted.*

[7] P. CERONE, S.S. DRAGOMIR and J. ROUMELIOTIS, An inequality of Ostrowski type for mappings whose second derivative are bounded and applications, *submitted.*

[8] P. CERONE, S.S. DRAGOMIR and J. ROUMELIOTIS, An Ostrowski type inequality for mappings whose second derivatives belong to $L_p(a, b)$ and applications, *submitted.*

[9] P. CERONE, S.S. DRAGOMIR and J. ROUMELIOTIS, Some Ostrowski type inequality for $n$-time differentiable mappings and applications, *submitted.*

[10] T. PEACHEY, A. Mc ANDREW and S.S. DRAGOMIR, The best constant in an inequality of Ostrowski type, *submitted.*

[11] J. ROUMELIOTIS, P. CERONE and S.S. DRAGOMIR, An Ostrowski type inequality for weighted mappings with bounded second derivatives, *submitted.*

[12] S.S. DRAGOMIR, Ostrowski's inequality for Lipschitzian mappings, *submitted.*

(Dragomir, Cerone and Roumeliotis) School of Communications and Informatics, Victoria University of Technology, PO Box 14428, Melbourne City MC, Victoria 8001, Australia.

(S. Wang) School of Mathematics and Statistics, Curtin University of Technology, GPO Box U 1987, Perth, Western Australia 6001, Australia.

*E-mail address:* S.S. Dragomir `sever@matilda.vu.edu.au`

P. Cerone `pc@matilda.vu.edu.au`

J. Roumeliotis `johnr@matilda.vu.edu.au`

S. Wang `swang@cs.curtin.edu.au`