A NOTE ON A GEOMETRIC INEQUALITY

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Abstract. In this note the author gives an alternative proof for a geometric inequalities obtained by M. Crasmareanu.

In a recent note "Weighted inequalities in triangle geometry", the *RGMIA Research Report*, **2**(7) (1999), pp. 1035–1037, Mircea Crasmareanu establishes the weighted triangle inequality

$$ma^2 + nb^2 + pc^2 \geq 4s\sqrt{mn + np + pm}, \tag{0.1}$$

where a, b, c, S are the sides and area of a triangle and $m + n > 0$, $n + 0 > 0$, $p + m > 0$, $mn + np + pm > 0$.

Firstly, the conditions on m, n, p can be simply stated as $m, n, p > 0$. Note that by letting $n + p = a_1$, $p + m = b_1$, and $m + n = c_1$, it follows that a_1 , b_1, c_1 are the sides of a triangle T_1 and then that

$$2m = b_1 + c_1 - a_1,\ 2n = c_1 + a_1 - b_1,\ 2p = a_1 + b_1 - c_1,$$
$$mn + np + pm = \frac{\left[2\sum b_1c_1 - \sum a_1^4\right]}{4} = 4\left(S_1'\right)^2,$$

where S_1' is the area of a triangle whose sides are the square roots of the sides of T_1. Inequality (0.1) now becomes

$$a^2\left(b_1 + c_1 - a_1\right) + b^2\left(c_1 + a_1 - b_1\right) + c^2\left(a_1 + b_1 - c_1\right) \geq 16SS_1'. \tag{0.2}$$

As known, the Neuberg-Pedoe inequality [1] is

$$a^2\left(b_1^2 + c_1^2 - a_1^2\right) + b^2\left(c_1^2 + a_1^2 - b_1^2\right) + c^2\left(a_1^2 + b_1^2 - c_1^2\right) \geq 16SS_1' \tag{0.3}$$

for two triangles of sides a, b, c and a_1, b_1, c_1. So that (0.2) follows from (0.3) by replacing the sides a_1, b_1, c_1 by their square roots. There is equality if and only if the triangles of sides a, b, c and $\sqrt{a_1}, \sqrt{b_1}, \sqrt{c_1}$ are similar.

References

[1] D.S. Mitrinović, J.E. Pečarić and V. Volenic, *Recent Advances in Geometric Inequalities,* Kluwer, Dordrecht, 1989, p. 355.

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