A NOTE ON THE VOLUME OF SECTIONS OF B_p^n

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Abstract. Let B_p^n denote the unit ball in ℓ_p^n with $p \geq 1$. We prove that $\mathrm{Vol}_{n-1}(H \cap B_p^n) \geq \Big(\mathrm{Vol}_n(B_p^n)\Big)^{(n-1)/n}$ for any $(n-1)$-dimensional subspace H of $\mathbb{R}^n$. This is a consequence of bounding the isotropy constant of B_p^n above by $1/\sqrt{12}$ and we show that one can replace $1/\sqrt{12}$ by a possibly smaller number for $n \geq 2$.

1. Introduction

A symmetric convex body K in $\mathbb{R}^n$ is said to be in isotropic position if there is a constant (the isotropy constant) L_K such that

$$\int_K x_i x_j dx = L_K^2 \delta_{ij} \Big(\mathrm{Vol}_n(K)\Big)^{(n+2)/n}, \qquad (1 \leq i, j \leq n),$$

where δ_{ij} is the Kronecker symbol. A well-known conjecture is that there exists a universal constant $c > 0$ such that $L_K < c$ for all convex centrally symmetric bodies in all dimensions. The best estimate known to date is due to Bourgain [9] that

$$L_K < cn^{\frac{1}{4}} \ln n.$$

In addition, the conjecture was verified for large classes of bodies (see [16], [7], [13], [14]) and it is equivalent to the famous hyperplane conjecture, which states that there is a universal constant $c > 0$ such that, for any convex centrally symmetric body $K \subset \mathbb{R}^n$, there is an $(n-1)$-dimensional subspace H for which

$$\mathrm{Vol}_{n-1}(H \cap K) \geq c \cdot \Big(\mathrm{Vol}_n(K)\Big)^{(n-1)/n}. \tag{1.1}$$

Now let K be the unit ball B_p^n in ℓ_p^n with $p \geq 1$, that is,

$$\begin{aligned} B_p^n &= \Big\{(x_1, \ldots, x_n) \in \mathbb{R}^n : \sum_{i=1}^n |x_i|^p \leq 1\Big\}, \quad 1 \leq p < +\infty, \\ B_\infty^n &= \Big\{(x_1, \ldots, x_n) \in \mathbb{R}^n : \max_{1 \leq i \leq n} |x_i| \leq 1\Big\}. \end{aligned}$$

In this case Meyer and Pajor [15] proved (1.1) (in fact for any $(n-1)$-dimensional subspace H) with $c = 1$ for $p = 1$ and $p \geq 2$. Later Schmuckenschläger [18] gave a proof for the case $1 < p < 2$ with $c = 1$ but the proof of the inequality he proposed was not correct and this was fixed by Bastero, Galve, Peña and Romance in [8]. The approach of Schmuckenschläger and Bastero et al. is based on an estimation of $L_{B_p^n}$, for which there is an explicit expression involving the gamma function $\Gamma(x)$. It is the goal of this paper to extend their results to all $p \geq 1$ via this approach and also to do it in a way that involves less direct computations.

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2. Gamma and Polygamma Functions

The digamma (or psi) function $\psi(x)$ for $x > 0$ is defined as the logarithmic derivative of $\Gamma(x)$ and the derivatives of $\psi(x)$ are known as polygamma functions. We note here that $\psi'(x)$ is completely monotonic on $(0, +\infty)$. (A function $f(x)$ is said to be completely monotonic on (a, b) if it has derivatives of all orders and $(-1)^n f^{(n)}(x) \geq 0, x \in (a, b), n = 0, 1, 2, \ldots$).

We now collect here a few facts about the gamma and polygamma functions, these can be found, for example, in [1, (7.1)], [2, (1.1)-(1.5), (3.39)].

Lemma 2.1. *For $x > 0$ we have*

$$\psi(x) = -\gamma + \int_0^\infty \frac{e^{-t} - e^{-xt}}{1 - e^{-t}} dt, \tag{2.1}$$

$$(-1)^{n+1}\psi^{(n)}(x) = \int_0^\infty e^{-xt} \frac{t^n}{1 - e^{-t}} dt = n! \sum_{k=0}^{\infty} \frac{1}{(x+k)^{n+1}}, \quad n \geq 1, \tag{2.2}$$

$$\psi^{(n)}(x+1) = \psi^{(n)}(x) + (-1)^n \frac{n!}{x^{n+1}}, \quad n \geq 0, \tag{2.3}$$

$$\ln \Gamma(x) = (x - \frac{1}{2}) \ln x - x + \frac{1}{2} \ln(2\pi) + O\left(\frac{1}{x}\right), \quad x \to +\infty, \tag{2.4}$$

$$\psi(x) = \ln x - \frac{1}{2x} - \frac{1}{12x^2} + O\left(\frac{1}{x^3}\right), \quad x \to +\infty, \tag{2.5}$$

$$(-1)^{n+1}\psi^{(n)}(x) = \frac{(n-1)!}{x^n} + \frac{n!}{2x^{n+1}} + O\left(\frac{1}{x^{n+2}}\right), \quad n \geq 1, \quad x \to +\infty, \tag{2.6}$$

$$\psi'(x) < \frac{1}{x} + \frac{1}{2x^2} + \frac{1}{6x^3}, \tag{2.7}$$

where $\gamma = 0.57721\ldots$ denotes Euler's constant.

Many interesting inequalities arise from the study of the asymptotic behavior of the polygamma functions. For example, one sees from (2.6) that $x^n(-1)^{n+1}\psi^{(n)}(x)$ is asymptotically $(n-1)!$, hence it's natural to ask how it approaches this constant. For $n = 1$, a result of Ronning [17] asserts that $x\psi'(x)$ is strictly decreasing. We note here this is also equivalent to a result of Alzer [2, Lemma 2.4], which asserts that $\psi(e^x)$ is strictly concave on $(-\infty, +\infty)$. The cases $n > 1$ have been studied in [11] and [3]. One can certainly ask a more general question on the behavior of $f_{a,n}(x) = x^n(-1)^{n+1}\psi^{(n)}(x+a)$ for any non-negative number a. When $a = 1$ and $n = 1$, this was investigated by Anderson and Qiu [5] and later proved to be strictly increasing for $x > -1$ by Elbert and Laforgia [12]. Borwein et al. showed that [10, Lemma 2.1] $f_{1,1}(x)$ is even completely monotonic on $(0, +\infty)$. Alzer and Ruehr [4] showed that $f_{a,1}(x)$ is strictly increasing for $a \geq 1/2$. We now summarize these results in the following lemma.

Lemma 2.2. *For fixed $n \geq 1, a \geq 0$, the function $f_{a,n}(x) = x^n(-1)^{n+1}\psi^{(n)}(x+a)$ is increasing on $[0, +\infty)$ if and only if $a \geq 1/2$. Also, $f_{0,n}(x)$ is decreasing on $(0, +\infty)$.*

Proof. From (2.6) we see that

$$\begin{aligned} \frac{f'_{a,n}(x)}{x^{n-1}} &= n(-1)^{n+1}\psi^{(n)}(x+a) - x(-1)^{n+2}\psi^{(n+1)}(x+a) \\ &= \frac{n!(a-1/2)}{(x+a)^{n+1}} + O\left(\frac{1}{(x+a)^{n+2}}\right), \quad x \to +\infty. \end{aligned}$$

It then follows that it is necessary to have $a \geq 1/2$ for $f_{a,n}(x)$ to be increasing on $[0, +\infty)$.

Assume now $a \geq 1/2$, we use the integral representation in (2.2) for $(-1)^{n+1}\psi^{(n)}(x)$ to deduce that

$$f_{a,n}(x) = \int_0^\infty e^{-(x+a)t}\frac{(xt)^n}{1-e^{-t}}dt.$$

It follows from this that $f'_{a,n}(0) = 0$. For $x > 0$, we make a change of variable $xt = s$ in the above integral to get

$$f_{a,n}(x) = \int_0^\infty e^{-s}s^{n-1}\frac{re^{-ar}}{1-e^{-r}}ds,$$

where $r = s/x$. We then obtain for $x > 0$,

$$f'_{a,n}(x) = \int_0^\infty e^{-s}s^n\frac{e^{-(a+1)r}\Big((ar-1)(e^r-1)+r\Big)}{\Big(x(1-e^{-r})\Big)^2}ds,$$

One then checks easily that $(ar-1)(e^r-1)+r \geq 0$ for $r \geq 0, a \geq 1/2$ and this implies $f'_{a,n}(x) \geq 0$ for $x > 0, a \geq 1/2$. Similarly, one shows that $f'_{0,n}(x) \leq 0$ for $x > 0$ and this completes the proof. □

Before we proceed to prove our main result in the next section, we state more auxiliary results here.

Lemma 2.3. *Let $0 \leq x \leq 1/4$ and $y \geq 0$. The function*

$$u(x,y) = \psi(1+xy) - \psi\Big(1+(y+2)x\Big) + \frac{(y+2)x}{2}\psi'(1+xy) - \frac{xy}{2}\psi'\Big(1+(y+2)x\Big)$$

is non-positive. Moreover, $u(1/2,y) < 0$ for $y \geq 0$ and $u(1,y) \leq 0$ for $y \geq 1$.

Proof. We have

$$\begin{aligned} & \frac{(y+2)x}{2}\psi'(1+xy) - \frac{xy}{2}\psi'\Big(1+(y+2)x\Big) \\ = \; & \frac{xy+1/2}{2}\psi'(1+xy) - \frac{x(y+2)+1/2}{2}\psi'\Big(1+(y+2)x\Big) \\ & +(x-\frac{1}{4})\psi'(1+xy) + (x+\frac{1}{4})\psi'\Big(1+(y+2)x\Big) \\ \leq \; & (x-\frac{1}{4})\psi'(1+xy) + (x+\frac{1}{4})\psi'\Big(1+(y+2)x\Big), \end{aligned}$$

where the inequality above follows from the case $n = 1, a = 1/2$ of Lemma 2.2. Also by Cauchy's mean value theorem, we obtain

$$\psi(1+xy) - \psi\Big(1+(y+2)x\Big) < -2x\psi'\Big(1+(y+2)x\Big). \tag{2.8}$$

These estimations yield

$$u(x,y) \leq (x-\frac{1}{4})\Bigg(\psi'(1+xy) - \psi'\Big(1+(y+2)x\Big)\Bigg) \leq 0,$$

for $0 \leq x \leq 1/4$ and $y \geq 0$.

In the case $x = 1/2$, we obtain by setting $z = y/2$ that

$$\begin{aligned} u(1/2,y) &= \psi(1+z) - \psi(1+z+1) + \frac{z+1}{2}\psi'(1+z) - \frac{z}{2}\psi'(1+z+1) \\ &= -\frac{1}{1+z} + \frac{1}{2}\psi'(1+z) + \frac{z}{2(1+z)^2}, \end{aligned}$$

where we have used (2.3) for $n = 0, 1$ above. We now use the bound (2.7) for $\psi'(x)$ to get for $z \geq 0$:

$$2u(1/2, y) < -\frac{1}{2(1+z)^2} + \frac{1}{6(1+z)^3} < 0.$$

Lastly, we use (2.3) to express $u(1, y)$ as

$$u(1, y) = -\frac{1}{y+1} - \frac{1}{y+2} + \psi'(1+y) + \frac{y}{2(y+1)^2} + \frac{y}{2(y+2)^2}.$$

We further apply (2.7) to get

$$\begin{aligned} u(1, y) &\leq \frac{1}{2(y+1)} - \frac{1}{2(y+2)} - \frac{1}{(y+2)^2} + \frac{1}{6(y+1)^3} \\ &= \frac{(y+2)^2 - 3y(y+1)^2}{6(y+1)^3(y+2)^2} \leq 0, \end{aligned}$$

where the last inequality follows since $3y \geq y+2$ and $(y+1)^2 \geq y+2$ for $y \geq 1$ and this completes the proof. □

Lemma 2.4. *Let $1/2 \leq x \leq 1$ and $y \geq 0$. The function*

$$v(x, y) = \psi(1+xy) - \psi\Big(1+(y+2)x\Big) + (y+2)x\psi'(1+xy) - (y+2)x\psi'\Big(1+(y+2)x\Big)$$

is non-negative.

Proof. Let $R = [1/2, 1] \times [0, +\infty)$ and we need to show $v(x, y) \geq 0$ for $(x, y) \in R$. Let $(x_0, y_0) \in R$ be the point in which the absolute minimum of $v(x, y)$ is reached and assume first that (x_0, y_0) is an interior point of R, then we obtain

$$\frac{\partial v}{\partial x}(x_0, y_0) = \frac{\partial v}{\partial y}(x_0, y_0) = 0.$$

Calculations yield

$$\begin{aligned} \frac{1}{x}\frac{\partial v}{\partial y} &= 2\psi'(1+xy) - 2\psi'\Big(1+(y+2)x\Big) + x(y+2)\psi''(1+xy) - x(y+2)\psi''\Big(1+(y+2)x\Big), \\ \frac{\partial v}{\partial x} &= 2(y+1)\psi'(1+xy) - 2(y+2)\psi'\Big(1+(y+2)x\Big) + xy(y+2)\psi''(1+xy) \\ &\quad -x(y+2)^2\psi'\Big(1+(y+2)x\Big). \end{aligned}$$

We then deduce from the above that

$$\psi'(1+x_0y_0) + x_0(y_0+2)\psi''(1+x_0y_0) = 0.$$

Note from Lemma 2.2 for the case $n = 1, a = 0$ we also have

$$\psi'(1+x_0y_0) + (1+x_0y_0)\psi''(1+x_0y_0) \leq 0 = \psi'(1+x_0y_0) + x_0(y_0+2)\psi''(1+x_0y_0),$$

which implies

$$(1-2x_0)\psi''(1+x_0y_0) \leq 0,$$

a contradiction. Thus we conclude that (x_0, y_0) is a boundary point of R. Hence we need to check $v(x, y) \geq 0$ for the cases $x = 1/2, 1$ or $y = 0, y \to +\infty$. It follows from the asymptotic expressions (2.6) and (2.5) that

$$\lim_{y \to +\infty} v(x, y) = 0.$$

Now for $x = 1/2$, using the relation (2.3) for $n = 0, 1$ and by setting $z = y/2$, we obtain

$$\begin{aligned} v(1/2, 2z) &= v(1/2, y) = \psi(1+z) - \psi(1+z+1) + (1+z)\psi'(1+z) - (1+z)\psi'(1+z+1) \\ &= -\frac{1}{1+z} + (1+z)\frac{1}{(1+z)^2} = 0. \end{aligned}$$

Similarly, for $x = 1$, we have

$$v(1,y) = -\frac{1}{y+2} - \frac{1}{y+1} + (y+2)\Big(\frac{1}{(y+2)^2} + \frac{1}{(y+1)^2}\Big) > 0.$$

It remains to check the case $y = 0$ and we get

$$v(x,0) = \psi(1) - \psi(1+2x) + 2x\psi'(1) - 2x\psi'(1+2x),$$

and that

$$\begin{aligned}\frac{1}{2}\frac{\partial v}{\partial x}(x,0) &= \psi'(1) - 2\psi'(1+2x) - 2x\psi''(1+2x) \\ &= \psi'(1) - \psi'(1+2x) + \psi''(1+2x) - \Big(\psi'(1+2x) + (1+2x)\psi''(1+2x)\Big) \\ &\geq \psi'(1) - \psi'(1+2x) + \psi''(1+2x),\end{aligned}$$

where the last inequality follows from the case $n = 1, a = 0$ of Lemma 2.2. Now by Cauchy's mean value theorem, we have

$$\psi'(1) - \psi'(1+2x) \geq -2x\psi''(1+2x),$$

which implies that

$$\frac{1}{2}\frac{\partial v}{\partial x}(x,0) \geq (1-2x)\psi''(1+2x) \geq 0.$$

Thus

$$v(x,0) \geq v(1/2,0) = 0,$$

and this completes the proof. □

Lemma 2.5. *Let $0 \leq x \leq 1$ and $y > 0$. For fixed x, the function*

$$f(x,y) = (1 + \frac{2}{y})\ln\Gamma(1+xy) - \ln\Gamma\Big(1 + (y+2)x\Big)$$

is a decreasing function of y for $y > 0$ when $0 \leq x \leq 1/2$ and for $y \geq 2$ when $1/2 < x \leq 1$.

Proof. We define

$$g(x,y) := y^2\frac{\partial f}{\partial y} = -2\ln\Gamma(1+xy) + y(y+2)x\psi(1+xy) - xy^2\psi\Big(1+(y+2)x\Big).$$

It suffices to show $g(x,y) \leq 0$ for $0 \leq x \leq 1/2, y \geq 0$ and $1/2 < x \leq 1, y \geq 2$. We show first that $g(x,y) \leq 0$ for $0 \leq x \leq 1/4$ and $y \geq 0$. Since $g(0,y) = g(x,0) = 0$, we may assume $x, y > 0$ and note that

$$\frac{1}{2xy}\frac{\partial g}{\partial y} = u(x,y), \quad \frac{1}{y^2}\frac{\partial g}{\partial x} = v(x,y),$$

where $u(x,y), v(x,y)$ are as defined in Lemma 2.3 and Lemma 2.4, respectively. By Lemma 2.3, $u(x,y) \leq 0$ for $0 \leq x \leq 1/4,\ y \geq 0$ and it follows that $g(x,y) \leq g(x,0) = 0$ for $0 \leq x \leq 1/4,\ y \geq 0$.

Now let $D = [1/4, 1/2] \times [0, +\infty)$. To show $g(x,y) \leq 0$ for $(x,y) \in D$, we let $(x_0, y_0) \in D$ be the point in which the absolute maximum of $g(x,y)$ is reached and assume first that (x_0, y_0) is an interior point of D, then we obtain

$$\frac{\partial g}{\partial x}(x_0,y_0) = \frac{\partial g}{\partial y}(x_0,y_0) = 0.$$

From our expressions for $u(x,y)$ and $v(x,y)$, one deduces that

$$(y_0+2)x_0\psi'(1+x_0y_0) = (y_0+4)x_0\psi'\Big(1+(y_0+2)x_0\Big),$$

which further implies that

$$\frac{1}{2x_0y_0}\frac{\partial g}{\partial y}(x_0,y_0) = \psi(1+x_0y_0) - \psi\Big(1+(y_0+2)x_0\Big) + 2x_0\psi'\Big(1+(y_0+2)x_0\Big) = 0,$$

which is certainly impossible in view of (2.8). Thus we conclude that (x_0, y_0) is a boundary point of D. Hence we need to check $g(x,y) \leq 0$ for the cases $x_0 = 1/4, 1/2$ or $y = 0, y \to +\infty$. The cases $g(x,0) = 0$ and $g(1/4,y) \leq 0$ follow from our discussion on the situation $x \leq 1/4, y \geq 0$ above and for the case $y \to +\infty$, using the asymptotic expression (2.4) and (2.5), we deduce via simple calculations that as $y \to +\infty$,

$$g(x,y) = -\ln y + O(1) < 0.$$

It thus remains to check the case $x = 1/2$. In this case it follows from Lemma 2.3 that $u(1/2,y) < 0$ so that $g(1/2,y) \leq g(1/2,0) = 0$.

Lastly, we need to show that $g(x,y) \leq 0$ for $1/2 < x \leq 1$ and $y \geq 1$. We note by Lemma 2.4 that in this case $g(x,y) \leq g(1,y)$ and also by Lemma 2.3 that $g(1,y)$ is a decreasing function of y. Hence it suffices to check that $g(1,2) \leq 0$. In this case one checks easily by using the well-known fact $\Gamma(n+1) = n!$, relation (2.3) and the observation that $\psi(1) = -\gamma$ from (2.1) that

$$\frac{g(1,2)}{2} = 3 - \ln 2 - 2\gamma - 1/2 - 2/3 < 0,$$

and this completes the proof. □

3. Volume of sections of B_p^n

We now apply Lemma 2.5 to estimate the volume of sections of B_p^n.

Theorem 3.1. *Let $n \in \mathbb{N}$, $n \geq 2$, $p \geq 1$ and let H be any $(n-1)$-dimensional subspace in $\mathbb{R}^n$. Then*

$$\frac{\mathrm{Vol}_{n-1}(H \cap B_p^n)}{\Big(\mathrm{Vol}_n(B_p^n)\Big)^{(n-1)/n}} \geq \sqrt{\frac{\Gamma(1+\frac{4}{p})\Gamma(1+\frac{1}{p})^3}{\Gamma(1+\frac{2}{p})^2\Gamma(1+\frac{3}{p})}} \geq 1. \tag{3.1}$$

Proof. Let H be a hyperplane in $\mathbb{R}^n$. A well-known result (see [6, (11)]) ensures that

$$\mathrm{Vol}_{n-1}(H \cap B_p^n) L_{B_p^n} \geq \frac{1}{\sqrt{12}}\Big(\mathrm{Vol}_n(B_p^n)\Big)^{(n-1)/n}$$

where $L_{B_p^n}$ is (see [18], [8])

$$L_{B_p^n}^2 = \frac{\Gamma(1+\frac{3}{p})\Gamma(1+\frac{n}{p})^{1+2/n}}{12\Gamma(1+\frac{n+2}{p})\Gamma(1+\frac{1}{p})^3}.$$

Now it follows from Lemma 2.5 with $x = 1/p, y = n$ that for $n \geq 2$,

$$L_{B_p^n}^2 \leq L_{B_p^2}^2,$$

from which one deduces the first inequality of (3.1). The second inequality of (3.1) now follows from Lemma 2.5 for the case $p \geq 2$ and [8, Proposition 1.2] for the case $1 \leq p < 2$. □

We remark here Theorem 3.1 recovers [18, Proposition 3.1] for the case $1 < p < 2$.

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References

[1] H. Alzer, On some inequalities for the gamma and psi functions, *Math. Comp.*, **66** (1997), 373–389.

[2] H. Alzer, Sharp inequalities for the digamma and polygamma functions, *Forum Math.*, **16** (2004), 181–221.

[3] H. Alzer, C. Berg and S. Koumandos, On a conjecture of Clark and Ismail, *J. Approx. Theory*, **134** (2005), 102–113.

[4] H. Alzer and O. G. Ruehr, A submultiplicative property of the psi function, *J. Comput. Appl. Math.*, **101** (1999), 53–60.

[5] G. D. Anderson and S.-L. Qiu, A monotoneity property of the gamma function, *Proc. Amer. Math. Soc.*, **125** (1997), 3355–3362.

[6] K. Ball, Logarithmically concave functions and sections of convex sets in R^n, *Studia Math.*, **88** (1988), 69–84.

[7] K. Ball, Normed spaces with a weak Gordon-Lewis property, *Lecture notes in Mathematics* **1470**, Springer, Berlin, 1991, 36-47.

[8] J. Bastero, F. Galve, A. Peña and M. Romance, Inequalities for the gamma function and estimates for the volume of sections of B_p^n, *Proc. Amer. Math. Soc.*, **130** (2002), 183–192.

[9] J. Bourgain, On the distribution of polynomials on high-dimensional convex sets, *Lecture notes in Mathematics* **1469**, Springer, Berlin, 1991, 127-137.

[10] D. Borwein, J. Borwein, G. Fee and R. Girgensohn, Refined convexity and special cases of the Blaschke-Santalo inequality, *Math. Inequal. Appl.*, **4** (2001), 631–638.

[11] W.E. Clark and M.E.H. Ismail, Inequalities involving gamma and psi functions, *Anal. Appl. (Singap.)*, **1** (2003), 129–140.

[12] Á. Elbert and A. Laforgia, On some properties of the gamma function, *Proc. Amer. Math. Soc.*, **128** (2000), 2667–2673.

[13] M. Junge, Hyperplane conjecture for quotient spaces of L_p, *Forum Math.*, **6** (1994), 617–635.

[14] H. König, M. Meyer and A. Pajor, The isotropy constants of the Schatten classes are bounded, *Math. Ann.*, **312** (1998), 773–783.

[15] M. Meyer and A. Pajor, Sections of the unit ball of L_p^n, *J. Funct. Anal.*, **80** (1988), 109–123.

[16] V.D. Milman and A. Pajor, Isotropic position and inertia ellipsoids and zonoids of the unit ball of a normed n-dimensional space, *Lecture notes in Mathematics* **1376**, Springer, Berlin, 1989, 64–104.

[17] G. Ronning, On the curvature of the trigamma function, *J. Comput. Appl. Math.*, **15** (1986), 397-399.

[18] M. Schmuckenschläger, Volume of intersections and sections of the unit ball of l_p^n, *Proc. Amer. Math. Soc.*, **126** (1998), 1527-1530.

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