RESEARCH GROUP IN MATHEMATICAL INEQUALITIES AND APPLICATIONS

PROBLEM CORNER

Problem 5, (2008), Solution No. 1

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Solution of Problem 5, i. e. the following conjecture of F. M. Dannan:

Problem 5. *Prove or disprove the following inequality:*

$$(1+a_1)(1+a_2)\cdots(1+a_n) \le \frac{(a_1+a_n)^2}{4a_1a_n}(1+(a_1a_2\cdots a_n)^{1/n})^n,$$

where $0 < a_1 < a_2 < \cdots < a_n$.

We shall give a negative answer to the above problem.

Counterexample For a given n, define

$$a_k := \frac{k}{n},\ k = 1, 2, \cdots, n.$$

The above inequality reads

$$\left(\frac{(2n)!}{n!n^n}\right)^{1/n} \le \left(\frac{(n+1)^2}{4n}\right)^{1/n}\left(1+\left(\frac{n!}{n^n}\right)^{1/n}\right).$$

Letting $n \to \infty$, by Stirling formula we obtain

$$4/e \le 1 + 1/e.$$

Contradiction!

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