

RESEARCH GROUP IN MATHEMATICAL INEQUALITIES AND APPLICATIONS

PROBLEM CORNER

Problem 6, (2008)

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Problem. Let $\{p_i\}_1^n$ be a positive weight sequence with $\sum_1^n p_i = 1$. Prove that

$$\prod_1^n (1 + a_i)^{p_i} \leq 1 + \frac{(a_1 + a_n)^2}{4a_1a_n} \prod_1^n a_i^{p_i},$$

where $0 < a_1 \leq a_2 \leq \dots \leq a_n$.

The solution is left to the readers.

Remark For $p_i = 1/n, i = 1, 2, \dots, n$, we get

$$(1 + a_1)(1 + a_2) \cdots (1 + a_n) \leq \left(1 + \frac{(a_1 + a_n)^2}{4a_1a_n} (a_1a_2 \cdots a_n)^{1/n}\right)^n.$$

Hence, this should be a correct form of Dannan's conjecture in Problem 5.