RESEARCH GROUP IN MATHEMATICAL INEQUALITIES AND APPLICATIONS

PROBLEM CORNER

Problem 7, (2008)

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Open Problem. A Hadamard sequence, $(n_k)_{k\in\mathbb{N}}$, is a sequence of positive integers such that $\inf\limits_{k\geq 1}\frac{n_{k+1}}{n_k}>1$. Let $(n_k)_{k\in\mathbb{N}}$ be a Hadamard sequence, let $q>1$ and let $f(x)=\sum\limits_{k=0}^{\infty}a_k x^{n_k}$, where $a_k\geq 0$ for all $k\geq 0$. Prove or disprove that there is a constant C, independent of f, such that

$$\int_0^{\infty}\left(\sum_{k=0}^{\infty}a_k s^{n_k}\right)^q e^{-s}ds\leq C\sum_{k=0}^{\infty}a_k^q\Gamma\left(n_k q+1\right),$$

where Γ denotes the Gamma function.

Remark. It is worth mentioning that if f is not a function with gaps then the previous inequality does not hold, as the following counterexample proves it: $f(x)=\frac{e^{kx}+e^{-kx}}{2}$, where $k\in(0,1/2)$, and $q=2$.