RESEARCH GROUP IN MATHEMATICAL INEQUALITIES AND APPLICATIONS

PROBLEM CORNER

Problem 9, (2009), Solution No. 1

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Conjecture. Let $x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n$ be positive real numbers and let $p \geq 1$, $q \geq 1$. Then

$$\sum_{i=1}^{n} \frac{\left(x_i^{p+1} + x_{i+1}^{p+1}\right)\left(y_i^{q+1} + y_{i+1}^{q+1}\right)}{\left(x_i^p + x_{i+1}^p\right)\left(y_i^q + y_{i+1}^q\right)} \geq \frac{1}{n^2} \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i$$

where $x_{n+1} = x_1$, $y_{n+1} = y_1$. If $p \leq -1$, $q \leq -1$ then the above inequality is reversed.

Counterexample. Fix $p \geq 1$, $q \geq 1$ and take $n = 4$. Let $t > 0$, set $(x_1, x_2, x_3, x_4) = (t, 1, 1, 1)$, and set $(y_1, y_2, y_3, y_4) = (1, 1, t, 1)$. Then the conjecture reduces to

$$2\frac{t^{p+1}+1}{t^p+1} + 2\frac{t^{q+1}+1}{t^q+1} \geq \frac{1}{16}(t+3)^2,$$

which clearly fails for large t.

Also, taking $x_i = y_i = 1$ for all i shows that the reversed inquality fails for all p and q.
