

Research Group in Mathematical Inequalities and Applications

$$v(G) > \sum_{m \in G} v(m)$$

*The value of the Group is greater than
the sum of the values of its members.*

Problem Corner

Problem 5, (2010)

Chao-Ping Chen

School of Mathematics and Informatics
Henan Polytechnic University
Jiaozuo City 454003
Henan Province, People's Republic of China
Email: chenchaoping@sohu.com

Received: 14 March, 2010

It is well-known that Binet's first formula for the $\ln \Gamma(x)$:

$$\ln \Gamma(x) = \left(x - \frac{1}{2}\right) \ln x - x + \ln \sqrt{2\pi} + \theta(x) \quad (x > 0),$$

where the remainder $\theta(x)$ is given by

$$\theta(x) = \int_0^\infty \left(\frac{1}{e^t - 1} - \frac{1}{t} + \frac{1}{2} \right) \frac{e^{-xt}}{t} dt.$$

Conjecture 1. For $x > 0$, we have

$$12 \left(\int_0^\infty \left(\frac{1}{e^t - 1} - \frac{1}{t} + \frac{1}{2} \right) \frac{e^{-xt}}{t} dt \right)^2 > \int_0^\infty \left(\frac{1}{e^t - 1} - \frac{1}{t} + \frac{1}{2} \right) e^{-xt} dt.$$